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Better Approximation of Functions by Genuine Baskakov Durrmeyer Operators

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Abstract — In this paper, we define a new genuine Baskakov-Durrmeyer operators. We give uniform convergence using the weighted modulus of continuity. Then we study direct approximation of the operators in terms of the moduli of smoothness. After that a Voronovskaya type result is studied.

Keywords — *Genuine Baskakov Durrmeyer operators, weighted modulus of continuity, Voronovskaya theorem*

1. Introduction

In the paper [1], the authors studied the sequences of linear Bernstein type operators defined for $f \in C[0,1]$ by $B_n(f \circ \sigma^{-1}) \circ \sigma$, B_n being the classical Bernstein operators and σ being any function satisfying some certain conditions. By this way, the Korovkin set is $\{1, \sigma, \sigma^2\}$ instead of $\{1, e_1, e_2\}$. It was shown that the B_n^σ actual a better degree of approximation. For this aim, have studied by a number of authors. For more details in this direction we can refer the readers to [2–9].

In [10], the authors introduced a general sequences of linear Baskakov Durrmeyer type operators by

$$G_n^\sigma(g;x) = (n-1)\sum_{l=0}^{\infty} P_{n,k}^\sigma(x)\int_0^\infty (g\circ\sigma^{-1})(u)\binom{n+k-1}{k}\frac{u^k}{(1+u)^{n+k}}du, \tag{1}$$

where $P_{n,k}^\sigma(x) = \binom{n+k-1}{k}\frac{(\sigma(x))^k}{(1+\sigma(x))^{n+k}}$, σ is a continuous infinite times differentiable function satisfying the condition $\sigma(1) = 0, \sigma(0) = 0$ and $\sigma'(x) > 0$ for $x \in [0,\infty)$.

In the present paper, we construct a genuine type modification of the operators in (1) which preserve the function σ, defined as

$$\begin{aligned} K_n^\sigma(g;x) &= \sum_{k=1}^{\infty} P_{n,k}^\sigma(x)\frac{1}{\beta(k,n+1)}\int_0^\infty \left(g\circ\sigma^{-1}\right)(t)\,\frac{t^{k-1}}{(1+t)^{n+k+1}}dt \\ &+P_{n,0}^\sigma(x)\left(g\circ\sigma^{-1}\right)(0) \end{aligned} \tag{2}$$

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The operators defined in (2) are linear and positive. In case of $\sigma(x) = x$, the operators in (2) reduce to the following operators introduced in [11]:

$$T_n(g;x) = \sum_{k=1}^{\infty} P_{n,k}(x) \frac{1}{\beta(k,n+1)} \int_0^{\infty} g(t) \frac{t^{k-1}}{(1+t)^{n+k+1}} dt + P_{n,0}(x) g(0)$$

2. Auxiliary lemmas

Lemma 2.1. We have

$$K_n^{\sigma}(1;x) = 1, \;\; K_n^{\sigma}(\sigma;x) = \sigma(x), \tag{3}$$

$$K_n^{\sigma}(\sigma^2;x) = \frac{\sigma^2(x)(n+1) + 2\sigma(x)}{n-1}, \tag{4}$$

$$K_n^{\sigma}(\sigma^3;x) = \frac{\sigma^3(x)(n+1)(n+2) + 6\sigma^2(x)(n+1) + 6\sigma(x)}{(n-1)(n-2)} \tag{5}$$

Lemma 2.2. If we describe the central moment operator by

$$M_{n,m}^{\sigma}(x) = K_n^{\sigma}\left((\sigma(t) - \sigma(x))^m; x\right)$$

then we get

$$M_{n,0}^{\sigma}(x) = 1, \;\; M_{n,1}^{\sigma}(x) = 0 \tag{6}$$

$$M_{n,2}^{\sigma}(x) = \frac{2\sigma(x)(\sigma(x)+1)}{n-1} \tag{7}$$

for all $n, m \in \mathbb{N}$.

3. Weighted Convergence of $K_n^{\sigma}(f)$

We suppose that:

(p_1) σ is a continuously differentiable function on $[0,\infty)$

(p_2) $\sigma(0) = 0$, $\inf_{x\in[0,\infty)} \sigma'(x) \geq 1$.

Let $\psi(x) = 1 + \sigma^2(x)$ and $B_\psi(\mathbb{R}^+) = \{f : \; |f(x)| \leq n_f \psi(x)\}$, where n_f is constant which may depend only on f. $C_\psi(\mathbb{R}^+)$ denote the subspace of all continuous functions in $B_\psi(\mathbb{R}^+)$. By $C_\psi^*(\mathbb{R}^+)$, we denote the subspace off all functions $f \in C_\psi(\mathbb{R}^+)$ for which $\lim_{x\to\infty} f(x)/\psi(x)$ is finite. Also let $U_\psi(\mathbb{R}^+)$ be the space of functions $f \in C_\psi(\mathbb{R}^+)$ such that f/ψ is uniformly continuous. $B_\psi(\mathbb{R}^+)$ is the linear normed space with the norm $\|f\|_\psi = \sup_{x\in\mathbb{R}^+} |f(x)|/\psi(x)$.

The weighted modulus of continuity defined in [12] is as follows

$$\omega_\sigma(f;\delta) = \sup_{\substack{x,t\in\mathbb{R}^+ \\ |\sigma(t)-\sigma(x)|\leq\delta}} \frac{|f(t) - f(x)|}{\psi(t) + \psi(x)}$$

for each $f \in C_\psi(\mathbb{R}^+)$ and for every $\delta > 0$. We observe that $\omega_\sigma(f;0) = 0$ for every $f \in C_\psi(\mathbb{R}^+)$ and the function $\omega_\sigma(f;\delta)$ is nonnegative and nondecreasing with respect to δ for $f \in C_\psi(\mathbb{R}^+)$ and also $\lim_{\delta\to 0} \omega_\sigma(f;\delta) = 0$ for every $f \in U_\psi(\mathbb{R}^+)$.

Let $\delta > 0$ and $W_\infty^2 = \{g \in C_B[0,\infty); g', g'' \in C_B[0,\infty)\}$. The Peetre's K functional is defined by

$$K_2(f,\delta) = \inf\left\{\|f-g\| + \delta\|g\|_{W_\infty^2}; g \in W_\infty^2\right\},$$

where

$$\|f\|_{W_\infty^2} := \|f\| + \|f'\| + \|f''\|$$

It was shown in [13], there exists an absolute constant $C > 0$ such that

$$K_2(f,\delta) \leq C\left\{w_2\left(f;\sqrt{\delta}\right) + \min(1,\delta)\left\|f\right\|\right\},$$

where the second order modulus of smoothness is defined by

$$w_2(f,\sqrt{\delta}) = \sup_{0\leq h\leq\sqrt{\delta}} \sup_{x\in[0,\infty)} |f(x+2h) - 2f(x+h) + f(x)|$$

The usual modulus of continuity of $f \in C_B[0,\infty)$ is defined by

$$w(f,\delta) = \sup_{0\leq h\leq\sqrt{\delta}} \sup_{x\in[0,\infty)} |f(x+h) - f(x)|$$

Lemma 3.1. [14] The positive linear operators L_n, $n \geq 1$, act from $C_\psi(\mathbb{R}^+)$ to $B_\psi(\mathbb{R}^+)$ if and only if the inequality

$$|L_n(\psi;x)| \leq P_n\psi(x),$$

holds, where P_n is a positive constant depending on n.

Theorem 3.2. [14] Let the sequence of linear positive operators (L_n), $n \geq 1$, acting from $C_\psi(\mathbb{R}^+)$ to $B_\psi(\mathbb{R}^+)$ satisfy the three conditions

$$\lim_{n\to\infty} \|L_n\sigma^\nu - \sigma^\nu\|_\psi = 0,\ \nu = 0,1,2.$$

Then for any function $g \in C^*_\psi(\mathbb{R}^+)$,

$$\lim_{n\to\infty} \|L_n g - g\|_\psi = 0$$

Theorem 3.3. For each function $g \in C^*_\psi(\mathbb{R}^+)$

$$\lim_{n\to\infty} \|K^\sigma_n g - g\|_\psi = 0$$

Proof. Using Theorem 3.2 we see that it is sufficient to verify the following three conditions

$$\lim_{n\to\infty} \|K^\sigma_n(\sigma^\nu) - \sigma^\nu\|_\psi = 0,\ \nu = 0,1,2. \tag{8}$$

It is clear that from (3) and (4), $\|K^\sigma_n(1) - 1\|_\psi = 0$ and $\|K^\sigma_n(\sigma) - \sigma\|_\psi = 0$. Hence the conditions (8) are fullfilled for $\nu = 0,1$. Also using the property (4) we have

$$\begin{aligned}\left\|K^\sigma_n(\sigma^2) - \sigma^2\right\|_\psi &= \sup_{x\in\mathbb{R}^+} \frac{1}{(1+\sigma^2(x))}\left(\frac{\sigma^2(x)(n+1) + 2\sigma(x)}{(n-1)} - \sigma^2(x)\right)\\ &\leq \frac{4}{n-1}\end{aligned} \tag{9}$$

This means that the condition (8) holds also for $\nu = 2$ and by Theorem 3.2 the proof is completed. □

Theorem 3.4. [12] Let $L_n : C_\psi(\mathbb{R}^+) \to B_\psi(\mathbb{R}^+)$ be a sequence of positive linear operators with

$$\left\|L_n(\sigma^0) - \sigma^0\right\|_{\psi^0} = a_n, \tag{10}$$

$$\|L_n(\sigma) - \sigma\|_{\psi^{\frac{1}{2}}} = b_n,$$

$$\left\|L_n(\sigma^2) - \sigma^2\right\|_\psi = c_n,$$

$$\left\|L_n(\sigma^3) - \sigma^3\right\|_{\psi^{\frac{3}{2}}} = d_n, \tag{11}$$

where a_n, b_n, c_n and d_n tend to zero as $n \to \infty$. Then

$$\|L_n(g) - g\|_{\psi^{\frac{3}{2}}} \leq (7 + 4a_n + 2c_n)\,\omega_\sigma(g;\delta_n) + \|g\|_\psi a_n \tag{12}$$

for all $g \in C_\psi(\mathbb{R}^+)$, where

$$\delta_n = 2\sqrt{(a_n + 2b_n + c_n)(1+a_n)} + a_n + 3b_n + 3c_n + d_n$$

Theorem 3.5. For all $g \in C_\psi(\mathbb{R}^+)$ we get

$$\|K_n^\sigma(g) - g\|_{\psi^{\frac{3}{2}}} \leq \left(7 + \frac{2}{(n-1)}\right)\omega_\sigma\left(g; \frac{4}{\sqrt{(n-1)}} + \frac{24n^2+4n-8}{(n-1)(n-2)}\right)$$

PROOF. On account of apply Theorem 3.4, we must calculate the sequences a_n, b_n, c_n and d_n. Using (3) and (4) we find

$$\left\|K_n^\sigma(\sigma^0) - \sigma^0\right\|_{\psi^0} = a_n = 0$$

and

$$\|K_n^\sigma(\sigma) - \sigma\|_{\psi^{\frac{1}{2}}} = b_n = 0$$

Also from (9)

$$c_n = \left\|\widetilde{C}_n^\sigma(\sigma^2) - \sigma^2\right\|_\psi \leq \frac{4}{(n-1)}$$

Since

$$K_n^\sigma(\sigma^3; x) = \frac{\sigma^3(x)(n+1)(n+2) + 6\sigma^2(x)(n+1) + 6\sigma(x)}{(n-1)(n-2)} \tag{13}$$

we can write

$$\begin{aligned} d_n &= \left\|K_n^\sigma(\sigma^3) - \sigma^3\right\|_{\psi^{\frac{3}{2}}} \\ &= \sup_{x\in\mathbb{R}^+} \frac{1}{(1+\sigma^2(x))^{\frac{3}{2}}} \\ &\quad \times \frac{\sigma^3(x)(n+1)(n+2) + 6\sigma^2(x)(n+1) + 6\sigma(x) - \sigma^3(x)(n-1)(n-2)}{(n-1)(n-2)} \\ &\leq \frac{24n^2}{(n-1)(n-2)} \end{aligned}$$

Thus the conditions (10-11) are satisfied. From Theorem 3.4 we have

$$\|K_n^\sigma(g) - g\|_{\psi^{\frac{3}{2}}} \leq \left(7 + \frac{2}{(n-1)}\right)\omega_\sigma\left(g; \frac{4}{\sqrt{(n-1)}} + \frac{24n^2+4n-8}{(n-1)(n-2)}\right)$$

□

Remark 3.6. Using $\lim_{\delta\to 0}\omega_\sigma(f;\delta) = 0$ and Theorem 3.5, we have

$$\lim_{n\to\infty} \|K_n^\sigma(g) - g\|_{\psi^{\frac{3}{2}}} = 0$$

for $f \in U_\psi(\mathbb{R}^+)$.

Theorem 3.7. Let σ be a function satisfying the conditions p_1 and p_2 and $\|\sigma''\|$ is finite. If $f \in C_B[0,\infty)$, then we have

$$|K_n^\sigma(g;x) - g(x)| \leq C\left\{w_2\left(f; \sqrt{\frac{2\sigma(x)(\sigma(x)+1)}{n-1}}\right) + \min\left(1, \frac{2\sigma(x)(\sigma(x)+1)}{n-1}\right)\|g\|\right\}$$

PROOF. The classic Taylor's expansion of $g \in W_\infty^2$ yields for $t \in [0,\infty)$ that

$$\begin{aligned} g(t) &= (g\circ\sigma^{-1})(\sigma(t)) = (g\circ\sigma^{-1})(\sigma(x)) + D(g\circ\sigma^{-1})(\sigma(x))(\sigma(t)-\sigma(x)) \\ &\quad + \int_{\sigma(x)}^{\sigma(x)} (\sigma(t)-u)D^2(g\circ\sigma^{-1})(u)du \end{aligned}$$

Applying the operators K_n^σ to both sides of above equality and considering the fact (6) we obtain

$$K_n^\sigma(g;x) - g(x) = K_n^\sigma\left(\int_{\sigma(x)}^{\sigma(x)} (\sigma(t)-u)D^2(g\circ\sigma^{-1})(u)du; x\right)$$

On the other hand, with the change of variable $u=\sigma(y)$ we get

$$\int_{\sigma(x)}^{\sigma(x)}(\sigma(t)-u)D^2(g\circ\sigma^{-1})(u)du=\int_x^t(\sigma(t)-\sigma(y))D^2(g\circ\sigma^{-1})\sigma(y)\sigma'(y)dy$$

Using the equality

$$D^2(g\circ\sigma^{-1})(\sigma(y))=\frac{1}{\sigma'(y)}\frac{g''(y)\sigma'(y)-g'(y)\sigma''(y)}{(\sigma'(y))^2},$$

we can write

$$\begin{aligned}\int_{\sigma(x)}^{\sigma(x)}(\sigma(t)-u)D^2(g\circ\sigma^{-1})(u)du &= \int_x^t(\sigma(t)-\sigma(y))\left(\frac{1}{\sigma'(y)}\frac{g''(y)\sigma'(y)-g'(y)\sigma''(y)}{(\sigma'(y))^2}\right)dy\\ &= \int_{\sigma(x)}^{\sigma(x)}(\sigma(t)-u)\frac{g''(\sigma^{-1}(u))}{(\sigma^{-1}(\sigma^{-1}(u)))^3}du\\ &\quad -\int_{\sigma(x)}^{\sigma(x)}(\sigma(t)-u)\frac{g'(\sigma^{-1}(u))\sigma''(\sigma^{-1}(u))}{(\sigma'(\sigma^{-1}(u)))^3}du\end{aligned}$$

So we can write

$$\begin{aligned}K_n^\sigma(g;x)-g(x) &= K_n^\sigma\left(\int_{\sigma(x)}^{\sigma(x)}(\sigma(t)-u)\frac{g''(\sigma^{-1}(u))}{(\sigma^{-1}(\sigma^{-1}(u)))^3}du;x\right)\\ &\quad -K_n^\sigma\left(\int_{\sigma(x)}^{\sigma(x)}(\sigma(t)-u)\frac{g'(\sigma^{-1}(u))\sigma''(\sigma^{-1}(u))}{(\sigma'(\sigma^{-1}(u)))^3}du;x\right)\end{aligned}$$

Since σ is strictly increasing on $[0,\infty)$ and with the condition p_2, we get

$$\begin{aligned}|K_n^\sigma(g;x)-g(x)| &\le M_{n,2}^\sigma(x)\left(\|g''\|+\|g'\|\,\|\sigma''\|\right)\\ &\le \frac{2\sigma(x)(\sigma(x)+1)}{n-1}\left(\|g''\|+\|g'\|\,\|\sigma''\|\right)\end{aligned}$$

Also, it is clear that

$$\|K_n^\sigma\|\le\|f\|$$

Hence we have

$$\begin{aligned}|K_n^\sigma(g;x)-g(x)| &\le |K_n^\sigma(g-f;x)|+|K_n^\sigma(f;x)-f(x)|+|-(g-f)(x)|\\ &\le 2\|f-g\|+\frac{2\sigma(x)(\sigma(x)+1)}{n-1}\left(\|g''\|+\|g'\|\,\|\sigma''\|\right)\end{aligned}$$

and choosing $C:=\max\{1,\|\sigma''\|\}$ we have

$$\begin{aligned}|K_n^\sigma(g;x)-g(x)| &\le C\left\{\|f-g\|+\frac{2\sigma(x)(\sigma(x)+1)}{n-1}\left(\|g''\|+\|g'\|+\|g\|\right)\right\}\\ &= C\left\{\|f-g\|+\frac{2\sigma(x)(\sigma(x)+1)}{n-1}\|g\|_{W^2_\infty}\right\}\end{aligned}$$

Taking the infimum on the right hand side over all $g\in W^2_\infty$ we obtain

$$\begin{aligned}|K_n^\sigma(g;x)-g(x)| &\le CK_2\left(f;\frac{2\sigma(x)(\sigma(x)+1)}{n-1}\right)\\ &\le C\left\{w_2\left(f;\sqrt{\frac{2\sigma(x)(\sigma(x)+1)}{n-1}}\right)+\min\left(1,\frac{2\sigma(x)(\sigma(x)+1)}{n-1}\right)\|g\|\right\}\end{aligned}$$

□

Lemma 3.8. [12] For every $g\in C_\psi(\mathbb{R}^+)$, for $\delta>0$ and for all $u,x\ge 0$,

$$|g(u)-g(x)|\le(\psi(u)+\psi(x))\left(2+\frac{|\sigma(u)-\sigma(x)|}{\delta}\right)\omega_\sigma(g,\delta) \tag{14}$$

holds.

Theorem 3.9. Let $g \in C_\psi(\mathbb{R}^+)$, $x \in I$ and suppose that the first and second derivatives of $g \circ \sigma^{-1}$ exist at $\sigma(x)$. If the second derivative of $g \circ \sigma^{-1}$ is bounded on $\mathbb{R}^+$, then we have

$$\lim_{n\to\infty} n\left[K_n^\sigma(g;x) - g(x)\right] = \sigma(x)(\sigma(x)+1)\left(g \circ \sigma^{-1}\right)''(\sigma(x))$$

PROOF. By the Taylor expansion of $g \circ \sigma^{-1}$ at the point $\sigma(x) \in \mathbb{R}^+$, there exists ξ lying between x and t such that

$$\begin{aligned} g(t) &= \left(g \circ \sigma^{-1}\right)(\sigma(t)) = \left(g \circ \sigma^{-1}\right)(\sigma(x)) \\ &\quad + \left(g \circ \sigma^{-1}\right)'(\sigma(x))(\sigma(t) - \sigma(x)) \\ &\quad + \frac{\left(g \circ \sigma^{-1}\right)''(\sigma(x))(\sigma(t) - \sigma(x))^2}{2} + \gamma_x(t)(\sigma(t) - \sigma(x))^2, \end{aligned}$$

where

$$\gamma_x(t) := \frac{\left(g \circ \sigma^{-1}\right)''(\sigma(\xi)) - \left(g \circ \sigma^{-1}\right)''(\sigma(x))}{2} \tag{15}$$

We get

$$\begin{aligned} K_n^\sigma(g;x) - g(x) &= \left(g \circ \sigma^{-1}\right)'(\sigma(x))\, K_n^\sigma(\sigma(t) - \sigma(x);x) \\ &\quad + \frac{\left(g \circ \sigma^{-1}\right)''(\sigma(x))\, K_n^\sigma((\sigma(t) - \sigma(x))^2;x)}{2} + K_n^\sigma\left(\gamma_x(t)(\sigma(t) - \sigma(x))^2;x\right) \end{aligned}$$

Using (6) and (7), we have

$$\lim_{n\to\infty} nK_n^\sigma(\sigma(t) - \sigma(x);x) = 0$$

$$\lim_{n\to\infty} nK_n^\sigma(((\sigma(t) - \sigma(x))^2;x) = 2\sigma(x)(\sigma(x)+1)$$

and thus

$$\begin{aligned} \lim_{n\to\infty} n\left[K_n^\sigma(g;x) - g(x)\right] &= \sigma(x)(\sigma(x)+1)\left(g \circ \sigma^{-1}\right)''(\sigma(x)) \\ &\quad + \lim_{n\to\infty} nK_n^\sigma\left(\gamma_x(t)(\sigma(t) - \sigma(x))^2;x\right) \end{aligned}$$

Let calculate the last term $\left|nK_n^\sigma\left(|\gamma_x(t)|(\sigma(t) - \sigma(x))^2;x\right)\right|$. Since $\lim_{t\to x}\gamma_x(t) = 0$ for every $\varepsilon > 0$, let $\delta > 0$ such that $|\gamma_x(t)| < \varepsilon$ for every $t \geq 0$. Cauchy-Schwarz inequality applied we have

$$\begin{aligned} \lim_{n\to\infty} nK_n^\sigma\left(|\gamma_x(t)|(\sigma(t) - \sigma(x))^2;x\right) &\leq \varepsilon \lim_{n\to\infty} nK_n^\sigma\left((\sigma(t) - \sigma(x))^2;x\right) \\ &\quad + \frac{C}{\delta^2}\lim_{n\to\infty} nK_n^\sigma\left((\sigma(t) - \sigma(x))^4;x\right) \end{aligned}$$

Since

$$\lim_{n\to\infty} nK_n^\sigma\left((\sigma(t) - \sigma(x))^4;x\right) = 0,$$

we get

$$\lim_{n\to\infty} nK_n^\sigma\left(|\gamma_x(t)|(\sigma(t) - \sigma(x))^2;x\right) = 0$$

□

Corollary 3.10. We have following particular case:

1. If we choose $\sigma(x) = x$, the operators (2) reduce to T_n operators defined in [11]. As a consequence of Theorem 3.9, we refined the following result.

$$\lim_{n\to\infty} n\left[T_n(g;x) - g(x)\right] = x(x+1)g''(x)$$

References

[1] D. Cárdenas-Morales, P. Garrancho, I. Raşa, *Bernstein-type Operators which Preserve Polynomials*, Computers and Mathematics with Applications 62 (2011) 158–163.

[2] T. Acar, A. Aral, I. Raşa, *Modified Bernstein-Durrmeyer Operators*, General Mathematics 22(1) (2014) 27–41.

[3] T. Acar, A. Aral, I. Raşa, *Positive Linear Operators Preserving* τ *and* τ^2, Constructive Mathematical Analysis 2(3) (2019) 98–102.

[4] T. Acar, G. Ulusoy, *Approximation by Modified Szasz Durrmeyer Operators*, Periodica Mathematica Hungarica 72 (2016) 64–75.

[5] T. Acar, V. Gupta, A. Aral, *Rate of Convergence for Generalized Szász Operators*, Bulletin of Mathematical Science 1(1) (2011) 99–113.

[6] A. Aral, D. Inoan, I. Raşa, *On the Generalized Szász-Mirakyan Operators*, Results in Mathematics 65 (2014) 441–452.

[7] M. Bodur, O. G. Yılmaz, A. Aral, *Approximation by Baskakov-Szász-Stancu Operators Preserving Exponential Functions*, Constructive Mathematical Analysis 1(1) (2018) 1–8.

[8] Z. Finta, *A Quantitative Variant of Voronovskaja's Theorem for King-Type Operators*, Constructive Mathematical Analysis 2(3) (2019) 124–129.

[9] R. Maurya, H. Sharma, C. Gupta, *Approximation Properties of Kantorovich Type Modifications of (p, q)-Meyer-König-Zeller Operators*, Constructive Mathematical Analysis 1(1) (2018) 58–72.

[10] G. Ulusoy Ada, *On the Generalized Baskakov Durrmeyer Operators*, Sakarya University Journal of Science 23(4) (2019) 549–553.

[11] Z. Finta, *On Converse Approximation Theorems*, Journal of Mathematical Analysis and Applications 312(1) (2005) 159–180.

[12] A. Holhos, *Quantitative Estimates for Positive Linear Operators in Weighted Space*, General Mathematics 16(4) (2008) 99–110.

[13] A. Ciupa, *A Class of Integral Favard Szasz Type Operators*, Studia Universitatis Babeş-Bolyai Mathematica 40(1) (1995) 39–47.

[14] A. D. Gadžiev, *The Convergence Problem for a Sequence of Positive Linear Operators on Unbounded Sets and Theorems Analogues to that of P. P. Korovkin*, Doklady Akademii Nauk SSSR 218 (1974) 1001-1004, Also in Soviet Mathematics Doklady 15 (1974) 1433–1436.