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A Decomposition of α-continuity and $\mu\alpha$-continuity

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Abstract — The main purpose of this paper is to introduce the concepts of *η-sets, **η-sets, *η-continuity and **η-continuity and to obtain decomposition of α-continuity and $\mu\alpha$-continuity in topological spaces.

Keywords — *$\mu\alpha$-closed set, μp-closed set, *η-set, **η-set, *η-continuity and **η-continuity*

1. Introduction and Preliminaries

Tong [1] introduced the notions of A-sets and A-continuity in topological spaces and established a decomposition of continuity. In [2], he also introduced the notions of B-sets and B-continuity and used them to obtain a new decomposition of continuity and Ganster and Reill [3] improved Tong's decomposition result. Moreover, Noiri and Sayed [4] introduced the notions of η-sets and obtained some decompositions of continuity. Quite recently, Veera kumar [5] introduced and studied the notions of μp-sets in topological spaces. Quite recently, Ganesan [6] introduced and studied the notions of $\mu\alpha$-closed sets in topological spaces. In this paper, we introduce the notions of *η-sets, **η-sets, *η-continuity and **η-continuity and obtain decomposition of α-continuity and $\mu\alpha$-continuity. Throughout this paper (X, τ) and (Y, σ) (or X and Y)represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and A^C denote the closure of A, the interior of A and complement of A respectively.
We recall the following definitions which are useful in the sequel.

Definition 1.1. A subset A of a space (X, τ) is called:

1. a regular open [7] if A = int(cl(A)).

2. an α-open set [8] if A$\subseteq$ int(cl(int(A))).

3. a semi-open set [9] if A$\subseteq$ cl(int(A)).

4. a pre-open set [10] if A$\subseteq$ int(cl(A)).

The complements of the above mentioned open sets are called their respective closed sets.
The α-closure [8](resp. semi-closure [11],pre-closure [12]) of a subset A of X, denoted by αcl(A)

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(resp.scl(A), pcl(A)) is defined to be the intersection of all α-closed (resp. semi-closed, pre-closed) sets of (X, τ) containing A. For any subset A of an arbitrarily chosen topological space, the α-interior [8] (resp. semi-interior [11],pre-interior [12]) of a subset A of X, denoted by αint(A) (resp.sint(A), pint(A)) is defined to be the union of all α-open (resp. semi-open, pre-open) sets of (X, τ) contained A.

Definition 1.2. A subset A of a space X is called:

1. a t-set [2] if int(cl(A))=int(A).
2. an α^*-set [13] if int(A) = int(cl(int(A))).
3. an A-set [1] if A = V ∩ T where V is open and T is a regular closed set.
4. a B-set [2, 14] if A = V ∩ T where V is open and T is a t-set.
5. an αB-set [15] if A = V ∩ T where V is α-open and T is a t-set.
6. an η-set [4] if A = V ∩ T where V is open and T is an α-closed set.
7. a locally closed set [16] if A = V ∩ T where V is open and T is closed.

The collection of A-sets (resp. B-sets, αB-sets, η-sets, locally closed sets) in X is denoted by A(X) (resp. B(X), αB(X), η(X), LC(X)).

Definition 1.3. A subset A of a space (X, τ) is called:

1. a gα*-closed set [17, 18] if αcl(A) ⊆ int(U) whenever A ⊆ U and U is α-open in (X, τ). The complement of gα*-closed set is called gα*-open set.
2. a μ-preclosed (briefly μp-closed) set [5] if pcl(A) ⊆ U whenever A ⊆ U and U is gα*-open in (X, τ). The complement of μp-closed set is called μp-open set.
3. a $\mu\alpha$-closed set [6] if αcl(A) ⊆ U whenever A⊆U and U is gα*-open in (X, τ). The complement of $\mu\alpha$-closed set is called $\mu\alpha$-open set.

The collection of all $\mu\alpha$-open (resp. μp-open) sets in X will be denoted by $\mu\alpha$O(X)(resp. μpO(X)).

Remark 1.4. In a space X, the followings hold:

1. Every open set is gα*-open but not conversely [6].
2. Every α-open set is $\mu\alpha$-open but not conversely [6].
3. Every $\mu\alpha$-closed set is μp-closed but not conversely [6].
4. Every $\mu\alpha$-continuous map is μp-continuous but not conversely [6].
5. The intersection of two t-sets is a t-set [2].

Remark 1.5. In a space X,the followings hold:

1. A is α-closed set if and only if A = αcl(A).
2. Every regular closed set is closed but not conversely.
3. Every regular closed set is semi-closed (= t-set) but not conversely.
4. Every closed set is α-closed but not conversely.
5. Every α-closed set is semi-closed (= t-set) but not conversely.

2. $*^{\eta}$-sets and $**^{\eta}$-sets

In this section we introduce and study the notions of $*^{\eta}$-sets and$**^{\eta}$-sets in topological spaces.

Definition 2.1. A subset A of a space X is called:

1. an $*^{\eta}$-set if A = U ∩ T where U is gα*-open and T is α-closed in X.
2. an $**^{\eta}$-set if A = U ∩ T where U is $\mu\alpha$-open and T is a t-set in X.

The collection of all $*^{e}ta$-sets (resp.$**^{\eta}$-sets) in X will be denoted by$*^{\eta}(X)$ (resp. $**^{\eta}(X)$)

Theorem 2.2. For a subset A of a space X, the following are equivalent.

1. A is an $*^{\eta}$-set.
2. A = U ∩ αcl(A) for some gα*-open set U.

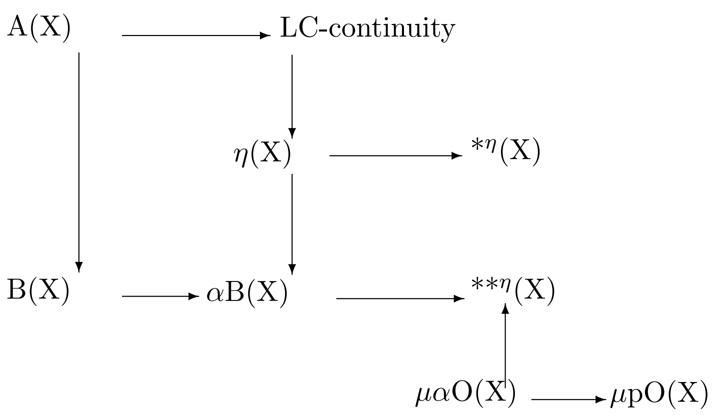
PROOF. (1)→ (2) Since A is an$*^{\eta}$-set, then A = U ∩ T, where U is gα*-open and T is α-closed. So, A ⊂ U and A ⊂ T. Hence αcl(A) ⊂ αcl(T). Therefore A ⊂ U ∩ αcl(A) ⊂ U ∩ αcl(T) = U ∩ T = A. Thus, A = U ∩ αcl(A).
(2)→ (1) It is obvious because αcl(A) is α-closed by Remark 1.5(1).

Remark 2.3. In a space X, the intersection of two$**^{\eta}$-sets is an$**^{\eta}$-set.

Remark 2.4. Union of two$**^{\eta}$-sets need not be an$**^{\eta}$-set as seen from the following example.

Example 2.5. Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$. The sets $\{a\}$, $\{c\}$ are $**^{\eta}$-sets in (X, τ) but their union $\{a, c\}$ is not an $**^{\eta}$-set in (X, τ).

Remark 2.6. We have the following implications.



where none of these implications is reversible as shown by [4] and the following examples.

Example 2.7.

1. In Example 2.5, the set {b} is an *η-set but not an η-set in (X, τ).
2. Let X = {a, b, c} with τ = {ϕ, {a, c}, X}. Clearly the set {b} is an **η-set but not an $\mu\alpha$-open set in (X, τ).
3. In Example 2.5, the set {a} is an **η-set but not an αB-set in (X, τ).

Remark 2.8.

1. The notions of *η-sets and $\mu\alpha$-closed sets are independent.
2. The notions of **η-sets and μp-closed sets are independent.

Example 2.9. In Example 2.5, the set {a, c} is $\mu\alpha$-closed but not an *η-set and also the set {a, b} is an *η-set but not a $\mu\alpha$-closed in (X, τ).

Example 2.10. In Example 2.7(2), the set {b} is an **η-set but not a μp-open set and also the set {a, b} is an μp-open set but not a **η-set in (X, τ).

Theorem 2.11. For a subset A of a space X, the following are equivalent:

1. A is α-closed.

2. A is an $*^{\eta}$-set and μ_{α}-closed.

PROOF. (1)$\rightarrow$ (2) It follows from Remark 1.4(1) and Definition 2.1(1).
(2) $\rightarrow$ (1) Since A is an $*^{\eta}$-set, then by Theorem 2.2, A = U $\cap$ αcl(A) where U is gα*-open in X. So, A $\subset$ U and since A is $\mu\alpha$-closed, then αcl(A) $\subset$ U. Therefore, αcl(A) $\subset$ U $\cap$ αcl(A) = A. But A $\subset$ αcl(A) always. Hence by Remark 1.5(1), A is α-closed.

Proposition 2.12. [19] Let A and B be subsets of a space X. If B is an α^{*}-set, then αint(A $\cap$ B) = αint(A) $\cap$ int(B)

Theorem 2.13. For a subset S of a space X, the following are equivalent.

1. S is $\mu\alpha$-open.

2. S is an $**^{\eta}$-set and μp-open.

PROOF. Necessity: It follows from Remark 1.4(3) and Definition 2.1(2).
Sufficiency: Assume that S is μp-open and an $**^{\eta}$-set in X. Then S = A $\cap$ B where A is μ_{α}-open and B is a t-set in X. Let F $\subset$ S, where F is gα*-closed in X. Since S is μp-open in X, F $\subset$ pint(S) = S $\cap$ int(cl(S)) = (A $\cap$ B) $\cap$ int[cl(A $\cap$ B)] $\subset$ A $\cap$ B $\cap$ int(cl(A)) $\cap$ int(cl(B)) = A $\cap$ B $\cap$ int(cl(A)) $\cap$ int(B), since B is a t-set. This implies, F $\subset$ int(B). Note that A is $\mu\alpha$-open and that F$\subset$ A. So, F $\subset$ αint(A). Therefore, F $\subset$ αint(A) $\cap$ int(B) = αint(S) by Proposition 2.12. Hence S is $\mu\alpha$-open.

3. $*^{\eta}$-continuity and $**^{\eta}$-continuity

Definition 3.1. A function f : X $\rightarrow$ Y is said to be $*^{\eta}$-continuous (resp. $**^{\eta}$-continuous) if f^{-1}(V) is an $*^{\eta}$-set (resp. an $**^{\eta}$-set) in X for every open subset V of Y.

Definition 3.2. A function f : X $\rightarrow$ Y is said to be C$*^{\eta}$-continuous if f^{-1}(V) is an $*^{\eta}$-set in X for every closed subset V of Y.

We shall recall the definitions of some functions used in the sequel.

Definition 3.3. A function f : X $\rightarrow$ Y is said to be

1. A-continuous [1] if f^{-1}(V) is an A-set in X for every open set V of Y.

2. B-continuous [2, 14] if f^{-1}(V) is an B-set in X for every opens set V of Y.

3. α-continuous [20] if f^{-1}(V) is an α-open set in X for every open set V of Y.

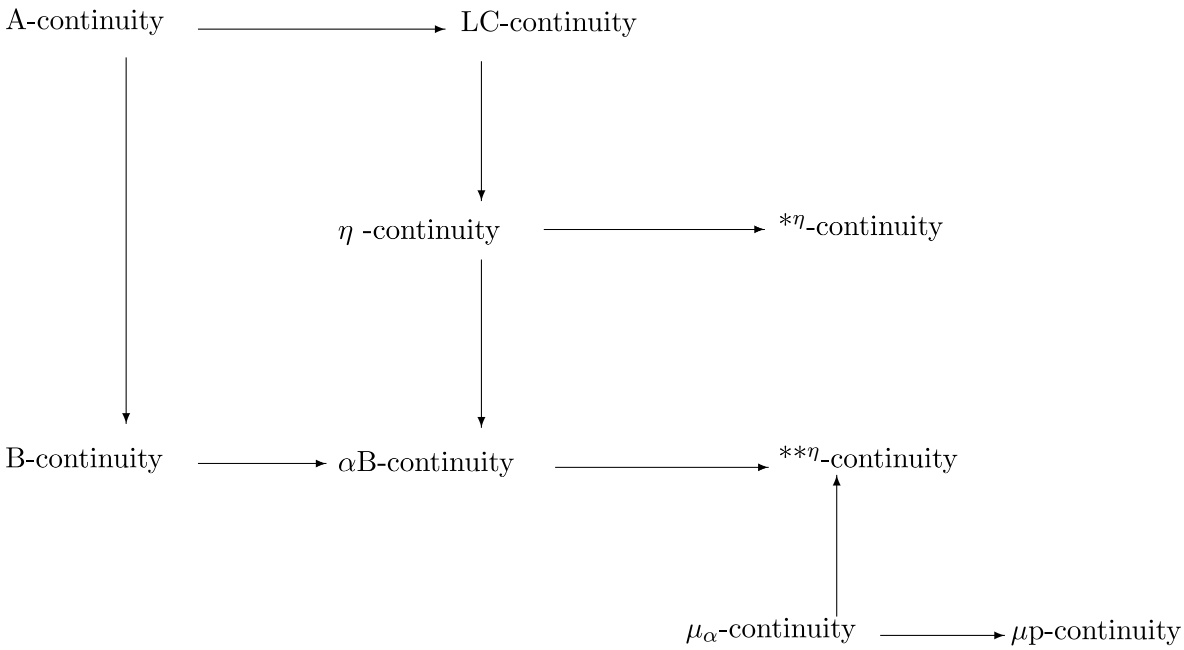
4. LC-continuous [16] (resp. αB-continuous [15] if f^{-1}(V) is an locally closed set (resp. αB-set) in X for every open set V of Y,

5. η-continuous [4] if f^{-1}(V) is an η-set in X for every open set V of Y.

6. $\mu\alpha$-continuous [6] (resp. μp-continuous [5]) if f^{-1}(V) is an $\mu\alpha$-open set (resp. μp-open set) in X for every open set V of Y.

Remark 3.4. It is clear that, a function f : X $\rightarrow$ Y is α-continuous if and only if f^{-1}(V) is an α-closed set in X for every closed set V of Y.

From the definitions stated above, we obtain the following diagram



Remark 3.5. None of the implications is reversible as shown by the following examples.

Example 3.6. Let X = Y = {a, b, c} with τ = {ϕ, {b, c}, X} and σ = {ϕ, {b}, {b, c}, Y}. Let f : X $\rightarrow$ Y be the identity function on X. Then f is *$^{\eta}$-continuous but not η-continuous.

Example 3.7. Let X, τ and f be as in Example 3.6. Let Y = {a, b, c} with σ = {ϕ, {c}, {b, c}, Y}. Then f is **$^{\eta}$-continuous but not αB-continuous.

Example 3.8. Let X, τ and f be as in Example 3.6. Let Y = {a, b, c} with σ = {ϕ, {a}, {b, c}, Y}. Then f is **$^{\eta}$-continuous but not $\mu\alpha$-continuous.

Remark 3.9. The following examples show that the concepts of

1. $\mu\alpha$-continuity and *$^{\eta}$-continuity are independent.

2. $\mu\alpha$-continuity and C*$^{\eta}$-continuity are independent.

3. *$^{\eta}$-continuity and C*$^{\eta}$-continuity are independent.

Example 3.10. Let X = Y = {a, b, c} with τ = {ϕ, {a}, X} and σ = {ϕ, {a}, {a, b}, {a, c}, Y}. Let f : X $\rightarrow$ Y be the identity function on X. Then f is $\mu\alpha$-continuous but not *$^{\eta}$-continuous.

Example 3.11. Let X, τ and f be as in Example 3.10. Let Y = {a, b, c} with σ = {ϕ, {b}, {c}, {b, c}, Y}. Then f is *$^{\eta}$-continuous but not $\mu\alpha$-continuous.

Example 3.12. Let X = Y = {a, b, c} with τ = {ϕ, {b}, X} and σ = {ϕ, {a}, {a, b}, Y}. Let f : X $\rightarrow$ Y be the identity function on X. Then f is $\mu\alpha$-continuous but not C*η-continuous.

Example 3.13. Let X, τ and f be as in Example 3.12. Let Y = {a, b, c} with σ = {ϕ, {b}, {a, c}, Y}. Then f is C*η-continuous but not $\mu\alpha$-continuous.

Example 3.14. Let X, τ and f be as in Example 2.5. Let Y = {a, b, c}with σ = {ϕ, {c}, {a, c}, Y}. Then f is C*η-continuous but not *$^{\eta}$-continuous.

Example 3.15. Let X, τ and f be as in Example 2.5. Let Y = {a, b, c} with σ = {ϕ, {b}, Y}. Then f is *$^{\eta}$-continuous but not C*η-continuous.

Remark 3.16. The following examples show that the concept of μp-continuity and **$^{\eta}$-continuity are independent.

Example 3.17. Let X and τ be as in Example 2.7(2). Let Y = {a, b, c} with σ = {ϕ, {a}, {a, b}, {a, c}, Y}. Let f : X → Y be the identity function on X. Then f is μp-continuous but not **η-continuous.

Example 3.18. Let X, τ and f be as in Example 3.6. Let Y = {a, b, c} with σ = {ϕ, {a}, Y}. Then f is **η-continuous but not μp-continuous.

Theorem 3.19. For a function f : X → Y, the following are equivalent.

1. f is α-continuous.

2. f is C*η-continuous and $\mu\alpha$-continuous.

Proof. The proof follows from Definitions 3.2 and 3.3(6), Remark 3.4 and Theorem 2.11.

Theorem 3.20. For a function f : X → Y, the following are equivalent.

1. f is $\mu\alpha$-continuous.

2. f is **$^{\eta}$-continuous and μp-continuous.

Proof. The proof follows from Theorem 2.13.

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