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New Exact Solutions of Cubic Nonlinear Schrödinger Equation by Using Extended Trial Equation Method

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Abstract

In this study, the extended trial equation method, which allows to obtain exact solutions of the partial differential equations, is investigated. This proposed method is applied to the cubic nonlinear Schrödinger equation and different new exact solutions are obtained. We can state that these new exact solutions are new exact solutions that are not find in the literature. In addition, two and three dimensional graphics drawn to show the physical behavior of these new exact solutions.

Genişletilmiş Deneme Denklemi Yöntemi ile Kübik Lineer Olmayan Schrödinger Denkleminin Yeni Tam Çözümleri

Anahtar kelimeler
Genişletilmiş Deneme Denklem Yöntemi; Kısmi Türevli Diferansiyel Denklemler; Kübik Lineer Olmayan Schrödinger Denklemi; Tam Çözümler

Öz

Bu çalışmada, kısmi türevli diferansiyel denklemlerin tam çözümlerinin elde edilmesine olanak sağlayan genişletilmiş deneme denklem yöntemi incelenmiştir. Önerilen bu yöntem kübik lineer olmayan Schrödinger denklemine uygulanmış ve farklı yeni tam çözümleri elde edilmiştir. Elde edilen bu yeni tam çözümlerin literatürde bulunmayan yeni tam çözümleri olduğunu ifade edebiliriz. Ayrıca, bulunan bu yeni tam çözümlerin fiziksel davranışlarını göstermek için iki ve üç boyutlu grafikleri çizilmiştir.

1. Introduction

It is important to integrate and solve nonlinear partial derivative differential equations that contain derivative by time, which is encountered in many fields of science. Recently, many studies in order to obtain solutions of the partial differential equations is made. We can use the concept of wave to make the solutions of such equations understandable. These nonlinear events encountered in nature are very common in the fields of fluid mechanics, plasma physics, optical fibers, solid state physics, biology, chemical kinematics, chemical physics, geochemistry and engineering. A single (solitary) wave is a wave that spreads without changing over time when it moves along with the current speed of the wave in the environment in which it occurs. We know that the application areas of the waves are quite high. For this reason, many different powerful and effective methods have been developed by different scientists, allowing exact solutions of nonlinear partial differential equations. As a result of these new developed methods, many physical events will be easier to understand with the determination of the new exact solution functions. Therefore, many different approach methods have been proposed and developed. Sine-cosine method Wang (1996), Wazwaz (2008), Hirota's bilinear transformation method Hietarinta (1997), Pashaev and Tanoglu (2005), (G'/G)-expansion method Akbar et al (2013), Shakeel and Mohyud-Din (2015), trial equation method Liu (2006), Liu (2010), *Gurefe et al*. (2011), Gurefe *et al*. (2012), extended trial equation method Pandir *et al*. (2012), Pandir *et al*. (2013), Gurefe *et al*. (2013), modified Kudryashov method Pandir (2014), Tandogan *et al*. (2013) can be given as an example for the proposed and developed complete solution methods.

In 2005, Liu C.S., proposed a powerful method to find complete solutions of nonlinear partial differential differential equations. Here, its main purpose was to find the solutions of an ordinary differential equation whose solution is not known by using different integration methods and to obtain the solutions of the equations with the finite series solution function that includes the functions consisting of them. Later, with the development of this proposed powerful method, different versions were brought into the literature by many scientists (Gurefe *et al*. 2011, Gurefe *et al*. 2012). Recently, the proposed method by Liu has been further developed and introduced into the literature as an extended trial equation method by Gürefe et al (Pandir *et al*. 2012, Pandir *et al*. 2013, Gurefe *et al*. 2013). This extended trial equation method developed has enabled the acquisition of new and exact solutions of the nonlinear partial differential equations.

In this study the developed method is applied to the cubic nonlinear Schrödinger equation. By creating the algorithms required for the extended trial equation method and writing the codes according to the created algorithm, new different exact solutions of the equation which is not found in the literature was obtained. In the next section, the extended trial equation method is described in detail.

2. Extended Trial Equation Method

In this section, it is aimed to find new exact solutions of the nonlinear partial differential equations with the extended trial equation method developed based on the trial equation method. The outlines of the extended trial equation method are expressed in detail. Let's suppose the general form of a nonlinear partial differential equation with independent variables $x, y, z, \ldots, t$ as

$$\mathrm{T}\left(u, u_x, u_y, u_z \ldots, u_t, \ldots, u_{xx}, u_{xy}, u_{xz}, \ldots, u_{xt}, \ldots\right) = 0. \quad (1)$$

The travelling wave transformation is used for the Eq.(1) as follows

$$u(x, y, z, \ldots, t) = u(\xi), \quad \xi = e_1 x + e_2 y + e_3 z + \ldots + e_m t \quad (2)$$

where $e_j \neq 0\left(j = 1, 2, 3, \ldots, m\right)$. Substituting Eq. (2) into Eq. (1) reduces a nonlinear ordinary differential equation

$$R\left(u, u', u'', u''', \ldots\right) = 0. \quad (3)$$

Let's consider the solution of the Eq. (3) with the finite series approach as follows

$$u(\xi) = \sum_{i=0}^{\delta} \tau_i \Gamma^i(\xi) \quad (4)$$

where

$$(\Gamma')^2 = \Lambda(\Gamma) = \frac{\Phi(\Gamma)}{\Psi(\Gamma)} = \frac{\varepsilon_0 + \varepsilon_1 \Gamma + \varepsilon_2 \Gamma^2 + \cdots + \varepsilon_\theta \Gamma^\theta}{\zeta_0 + \zeta_1 \Gamma + \zeta_2 \Gamma^2 + \cdots \zeta_\epsilon \Gamma^\epsilon}. \quad (5)$$

Here the $\Gamma(\xi)$ functions are the solution functions of the nonlinear ordinary elliptical differential equation. Using Eq (4) and Eq (5), we can write

$$(u')^2(\xi) = \frac{\Phi(\Gamma)}{\Psi(\Gamma)}\left(\sum_{i=0}^{\delta} i \tau_i \Gamma^{i-1}(\xi)\right)^2 \quad (6)$$

$$u''(\xi)=\frac{\Phi'(\Gamma)\Psi(\Gamma)-\Phi(\Gamma)\Psi'(\Gamma)}{2\Psi^2(\Gamma)}\sum_{i=0}^{\delta}i\tau_i\Gamma^{i-1}(\xi)+\frac{\Phi(\Gamma)}{\Psi(\Gamma)}\sum_{i=0}^{\delta}i(i-1)\tau_i\Gamma^{i-2}(\xi) \tag{7}$$

where $\Phi(\Gamma)$ and $\Psi(\Gamma)$ are polynomials. When the obtained derivatives in Eq. (6) and Eq. (7) are examined, as stated in the solution function (4) turns into a polynomial expression linked to a rational $\Gamma(\xi)$ function. The balance procedure according to Eq. (3) is applied in terms of the polynomial equivalent of the highest-order (non-linear) term with the highest-order derivative term. With the balancing procedure, δ in solution function (4), θ and $\in$ values in Eq. (5) will be calculated. Some balancing terms are calculated as follows

$$uu''\to\Gamma^{\theta+2\delta-\in-2},\ (u')^2\to\Gamma^{\theta+2\delta-\in-2},\ u^2\to\Gamma^{2\delta},\ u^3\to\Gamma^{3\delta}. \tag{8}$$

Thus, when the calculated values are written in place of the expressions, zero polynomial related to the Γ function is obtained. A system of algebraic equations is obtained by synchronizing the related terms in this zero polynomial to zero. When the created algebraic equation system is solved with the help of Mathematica 10 package program, $\varepsilon_0,\cdots,\varepsilon_\theta$, $\zeta_0,\cdots,\zeta_\in$ and $\tau_0,\cdots,\tau_\delta$ coefficients are obtained. When the obtained coefficients are replaced in the Eq. (5), $\Gamma(\xi)$ functions are obtained by calculating the integral

$$\pm(\xi-\xi_0)=\int\frac{d\Gamma}{\sqrt{\Lambda(\Gamma)}}=\int\sqrt{\frac{\Psi(\Gamma)}{\Phi(\Gamma)}}d\Gamma. \tag{9}$$

Then, $\Gamma(\xi)$ functions are replaced in solution function (4), respectively. Thus, by applying the transformation in the expression of (2) to the obtained $u(\xi)$ functions, new exact solutions of the Eq. (1) are obtained.

3. Application of the Extended Trial Equation Method

In this section, the extended trial equation method is applied to the cubic nonlinear Schrödinger equation. The cubic nonlinear Schrödinger equation is one of the most important universal nonlinear models that naturally occur in many physical systems. It is a general equation that occurs when a semi-monochromatic wave propagates in a diffuse and weakly nonlinear environment. It has also been used in hydrodynamics to describe various physical phenomena, especially in nonlinear optical fibers, heat transfer in a solid, nonlinear waves in a fluid-filled elastic tube, nonlinear imbalance problems, and solitary wave propagation in piezoelectric semiconductors (Imanli 2006, Sulem and Sulem 1999, Ablowitz *et al*. 2004). The most general form of the cubic nonlinear Schrödinger equation is

$$i\frac{\partial w}{\partial t}+\frac{\partial^2 w}{\partial x^2}+k|w|^2w=0, \tag{10}$$

where w is a complex valued function and k is a real arbitrary parameter (Chand and Malik 2012, Kaplan *et al*. 2016). First let's assume travelling wave transformation to apply the extended trial equation method to Eq. (10)

$$w(x,t)=e^{i\mu}w(\eta),\quad \mu=mx+nt,\quad \eta=lx+ct$$

where m,n,l,c are arbitrary constants. Derivatives and the unknown function in the Eq. (10) are calculated under transformation

$$i\frac{\partial w}{\partial t}=-ce^{i\mu}w-ine^{i\mu}w',$$

$$\frac{\partial^2 w}{\partial x^2}=-m^2e^{i\mu}w+2ilme^{i\mu}w'+l^2e^{i\mu}w'',\ |w|^2=w^2 \tag{11}$$

When these are substituted in the Eq. (10), a non-linear 3rd order ordinary differential equation is obtained

$$(-c-m^2)w+k^3w^3+l^2w''=0, \tag{12}$$

where $n = 2lm.$ Using the solution function (4) and differential equation (5), the related derivatives are calculated and replaced in Eq. (12). The balancing procedure is applied to determine the δ , θ and $\in$ values in solution function (4) and differential equation (5). According to the extended trial equation method, the balance process is determined between the term w'' containing the highest order derivative and the highest order nonlinear term w^3 for Eq. (12) as follows

$$w^3 \to \Gamma^{3\delta}, \; w'' \to \Gamma^{\theta+\delta-\in-2}. \qquad (13)$$

Accordingly, the balance term is obtained as $\theta = \varepsilon + 2\delta + 2$ from the equivalence of the obtained $w'' \approx w^3$ terms.

To determine the new solution of the Eq. (10), if the balance terms are selected as $\varepsilon = 0$ and $\delta = 1$, then $\theta = 4$ is obtained. When these balancing terms are written in solution function (4) and differential equation (5) respectively, we acquire

$$w(\eta) = \tau_0 + \tau_1 \Gamma(\eta), \qquad (14)$$

$$(\Gamma')^2 = \frac{\Phi(\Gamma)}{\Psi(\Gamma)} = \frac{\varepsilon_0 + \varepsilon_1\Gamma + \varepsilon_2\Gamma^2 + \varepsilon_3\Gamma^3 + \varepsilon_4\Gamma^4}{\zeta_0}. \qquad (15)$$

The term w'' in the Eq. (12) is calculated as

$$w'' = \frac{\tau_1\left(\varepsilon_1 + 2\varepsilon_2\Gamma + 3\varepsilon_3\Gamma^2 + 4\varepsilon_4\Gamma^3\right)}{2\zeta_0} \qquad (16)$$

where $\varepsilon_4 \neq 0$ and $\zeta_0 \neq 0$.

When the calculated values are substituted in the Eq. (12), a polynomial expression based on the $\Gamma(\eta)$ function occurs. If we consider this polynomial as a zero polynomial, the coefficients of this polynomial are equalized to zero, resulting in a system of algebraic equations.

When this algebraic system is solved with the help of Mathematica 10 packet program, coefficients are found as follows

$$\varepsilon_0 = \varepsilon_0, \; \varepsilon_1 = -\frac{\varepsilon_3^3 - 4\varepsilon_2\varepsilon_3\varepsilon_4}{8\varepsilon_4^2}, \; \varepsilon_2 = \varepsilon_2, \; \varepsilon_3 = \varepsilon_3, \; \varepsilon_4 = \varepsilon_4,$$

$$\zeta_0 = -\frac{l^2\varepsilon_3^2}{8k\varepsilon_4\tau_0^2}, \; \tau_0 = \tau_0, \; \tau_1 = \frac{4\varepsilon_4\tau_0}{\varepsilon_3},$$

$$c = -m^2 + k\left(3 - \frac{8\varepsilon_2\varepsilon_4}{\varepsilon_3^2}\right)\tau_0^2. \qquad (17)$$

When these obtained coefficients are replaced in Eq. (5) and Eq. (9), an integral is obtained

$$\pm(\eta - \eta_0) = A\int \frac{d\Gamma}{\sqrt{\frac{\varepsilon_0}{\varepsilon_4} + \frac{\varepsilon_1}{\varepsilon_4}\Gamma(\eta) + \frac{\varepsilon_2}{\varepsilon_4}\Gamma^2(\eta) + \frac{\varepsilon_3}{\varepsilon_4}\Gamma^3(\eta) + \Gamma^4(\eta)}} \qquad (18)$$

where $A = \sqrt{-\frac{l^2\varepsilon_3^2}{8k\varepsilon_4\tau_0^2}}$. It is quite difficult to calculate the integral in Eq. (18). Integrating Eq. (18), we obtain the solutions of the Eq. (10) as follows:

$$\pm(\eta - \eta_0) = -\frac{A}{\Gamma - \alpha_1}, \qquad (19)$$

$$\pm(\eta - \eta_0) = \frac{2A}{\alpha_1 - \alpha_2}\sqrt{\frac{\Gamma - \alpha_2}{\Gamma - \alpha_1}}, \quad \alpha_2 > \alpha_1, \qquad (20)$$

$$\pm(\eta - \eta_0) = \frac{A}{\alpha_1 - \alpha_2}\ln\left|\frac{\Gamma - \alpha_1}{\Gamma - \alpha_2}\right|, \quad \alpha_1 > \alpha_2, \qquad (21)$$

$$\pm(\eta - \eta_0) = \frac{2A}{\sqrt{(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)}}\ln\left|\frac{\sqrt{(\Gamma - \alpha_2)(\alpha_1 - \alpha_3)} - \sqrt{(\Gamma - \alpha_3)(\alpha_1 - \alpha_2)}}{\sqrt{(\Gamma - \alpha_2)(\alpha_1 - \alpha_3)} + \sqrt{(\Gamma - \alpha_3)(\alpha_1 - \alpha_2)}}\right|$$
$$\alpha_1 > \alpha_2 > \alpha_3, \qquad (22)$$

$$\pm(\eta - \eta_0) = \frac{2AF(\varphi, l)}{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}, \quad \alpha_1 > \alpha_2 > \alpha_3 > \alpha_4, \qquad (23)$$

where $F(\varphi,l)=\int_0^{\varphi}\frac{d\psi}{\sqrt{1-l^2\sin^2\psi}}$,

$$\varphi=\arcsin\sqrt{\frac{(\Gamma-\alpha_1)(\alpha_2-\alpha_4)}{(\Gamma-\alpha_2)(\alpha_1-\alpha_4)}},\quad l^2=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}.$$

Also $\alpha_1,\alpha_2,\alpha_3$ and α_4 are the roots of the polynomial equation

$$\Gamma^4+\frac{\varepsilon_3}{\varepsilon_4}\Gamma^3+\frac{\varepsilon_2}{\varepsilon_4}\Gamma^2+\frac{\varepsilon_1}{\varepsilon_4}\Gamma+\frac{\varepsilon_0}{\varepsilon_4}=0. \tag{24}$$

Substituting the solutions (19-23) into (4) and using travelling wave transformation, we have

$$w_1(x,t)=e^{i\mu_1}\left[\tau_0+\tau_1\left(\alpha_1\pm\frac{A}{\eta_1-\eta_0}\right)\right] \tag{25}$$

$$w_2(x,t)=e^{i\mu_1}\left[\tau_0+\tau_1\left(\alpha_1\pm\frac{4A(\alpha_2-\alpha_1)}{4-(\alpha_1-\alpha_2)^2(\eta_1-\eta_0)^2}\right)\right] \tag{26}$$

$$w_3(x,t)=e^{i\mu_1}\left[\tau_0+\tau_1\left(\frac{\alpha_2\,e^{\pm(\alpha_1-\alpha_2)(\eta_1-\eta_0)}-\alpha_1}{e^{\pm(\alpha_1-\alpha_2)(\eta_1-\eta_0)}-1}\right)\right] \tag{27}$$

$$w_4(x,t)=e^{i\mu_1}\left[\tau_0+\tau_1\left(\alpha_1-\frac{2A(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)\cosh\left[\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}(\eta_1-\eta_0)\right]}\right)\right] \tag{28}$$

$$w_5(x,t)=e^{i\mu_1}\left[\tau_0+\tau_1\left(\alpha_2+\frac{A(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)}{\alpha_4-\alpha_2+(\alpha_1-\alpha_4)sn^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\eta_1-\eta_0),\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}\right]}\right)\right] \tag{29}$$

where $\mu_1=lx+\left(k\left(3-\frac{8\varepsilon_2\varepsilon_4}{\varepsilon_3^2}\right)\tau_0^2-m^2\right)t$, $\eta_1=m(x+2lt)$. If taken as $\tau_0=-\tau_1\alpha_1$ and $\eta_0=0$, the equations (25)-(28) are obtained respectively rational function solutions:

$$w_1(x,t)=e^{i\left(lx+\left(k\left(3-\frac{8\varepsilon_2\varepsilon_4}{\varepsilon_3^2}\right)\tau_1^2\alpha_1^2-m^2\right)t\right)}\left(\pm\frac{A_1}{m(x+2lt)}\right), \tag{30}$$

$$w_2(x,t)=e^{i\left(lx+\left(k\left(3-\frac{8\varepsilon_2\varepsilon_4}{\varepsilon_3^2}\right)\tau_1^2\alpha_1^2-m^2\right)t\right)}\left(\pm\frac{4A_1(\alpha_2-\alpha_1)}{4-(\alpha_1-\alpha_2)^2m^2(x+2lt)^2}\right), \tag{31}$$

where $A_1=\tau_1A$, travelling wave solution:

$$w_3(x,t)=e^{i\left(lx+\left(k\left(3-\frac{8\varepsilon_2\varepsilon_4}{\varepsilon_3^2}\right)\tau_1^2\alpha_1^2-m^2\right)t\right)}C_1\coth\left[D_1(x+2lt)\right] \tag{32}$$

where $C_1=\pm\tau_1\alpha_2$ and $D_1=\frac{m(\alpha_1-\alpha_2)}{2A}$, soliton solution:

$$w_4(x,t)=e^{i\left(lx+\left(k\left(3-\frac{8\varepsilon_2\varepsilon_4}{\varepsilon_3^2}\right)\tau_1^2\alpha_1^2-m^2\right)t\right)}\frac{C_2}{E_1+\cosh\left[D_2(x+2lt)\right]} \tag{33}$$

where $C_2=\frac{-2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{\alpha_3-\alpha_2}$,

$E_1=\frac{2\alpha_1-\alpha_2-\alpha_3}{\alpha_3-\alpha_2}$ and $D_2=\frac{m\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A}$.

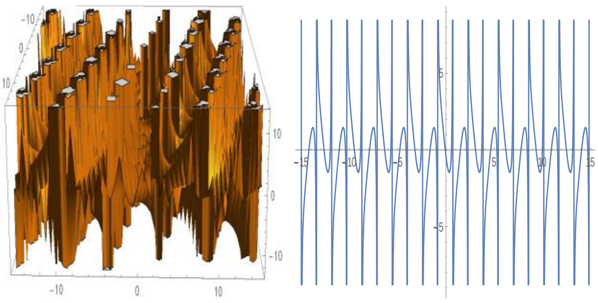

Fig. 1. Three and two dimensional graphical for $\xi_2=\tau_0=\tau_1=l=m=2,\ k=\frac{1}{2},\ \alpha_1=\xi_4=1,\ \alpha_2=\frac{3}{2}$, $\xi_3=4$ of the solution (32).

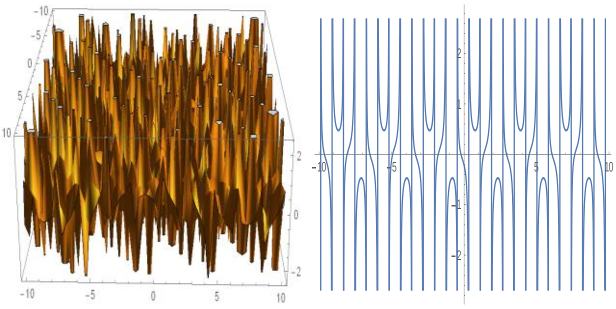

Fig. 2. Three and two dimensional graphical for $\xi_2=\tau_0=\tau_1=l=m=2,\ \alpha_3=k=\frac{1}{2},\ \alpha_1=\xi_4=1,\ \alpha_2=\frac{3}{2},\xi_3=4$ of the solution (33).

If $\tau_0=-\tau_1\alpha_2$ is taken in Eq. (29), an Jacobi elliptic function solution is found as

$$w_5(x,t)=e^{i\left(lx+\left(k\left(3-\frac{8\varepsilon_2\varepsilon_4}{\varepsilon_3^2}\right)\tau_1^2\alpha_1^2-m^2\right)t\right)}\frac{C_3}{E_2+sn^2(\varphi_1,l_1)} \tag{34}$$

where $C_3=\dfrac{\tau_1(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)}{\alpha_1-\alpha_4},\ E_2=\dfrac{\alpha_4-\alpha_2}{\alpha_1-\alpha_4},$

$\varphi_1=\dfrac{m\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(x+2lt),$

$l_1^2=\dfrac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}.$

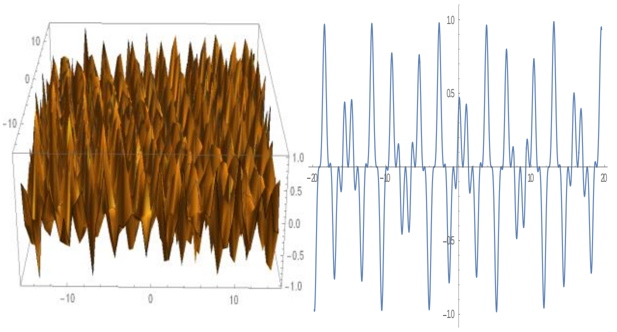

Fig. 3. Three and two dimensional graphical for $\xi_2=\tau_0=\tau_1=l=\alpha_4=m=2,\ \alpha_3=k=\frac{1}{2},\ \xi_3=4,\ \alpha_1=\xi_4=1,\ \alpha_2=\frac{3}{2}$ of the solution (34).

Here, C_2 indicates the amplitude of the soliton and D_2 indicates the inverse width of the solitons. In addition, if we take the module as $l\to 1$ in the elliptic solution Eq.(34), then the solution of the Eq. (10) turns into the following hyperbolic function solution

$$w_6(x,t)=e^{i\left(lx+\left(k\left(3-\frac{8\varepsilon_2\varepsilon_4}{\varepsilon_3^2}\right)\tau_1^2\alpha_1^2-m^2\right)t\right)}\frac{C_3}{E_2+\tanh^2\left[\frac{m\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(x+2lt)\right]} \tag{35}$$

where $\alpha_3=\alpha_4$.

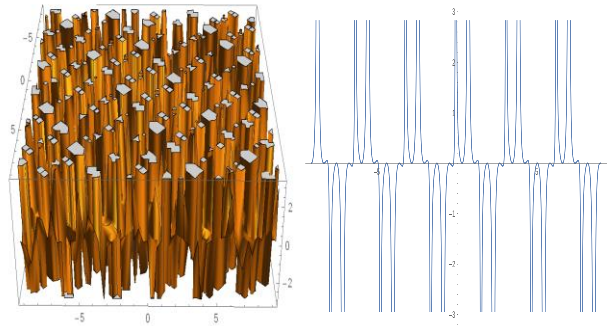

Fig. 4. Three and two dimensional graphical for $\xi_2=\tau_0=\tau_1=m=2,\ \alpha_3=k=\frac{1}{2},\ \alpha_1=\xi_4=1,\ \alpha_2=\frac{3}{2},\xi_3=4$ of the solution (35).

On the other hand; when the module is selected as $l\to 0$ in the jaccobi elliptic solution Eq. (34), then the solution of the Eq. (10) turns into the following periodic wave solution

$$w_7(x,t)=e^{i\left(lx+\left(k\left(3-\frac{8\varepsilon_2\varepsilon_4}{\varepsilon_3^2}\right)\tau_1^2\alpha_1^2-m^2\right)t\right)}\frac{C_3}{E_2+\sin^2\left[\frac{m\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(x+2lt)\right]} \tag{36}$$

where $\alpha_2=\alpha_3$.

When all the solutions obtained from the cubic nonlinear Schrödinger equation are examined; Eq. (33) exact solution is similar to the solution in the literature. Other solutions are the exact solutions that are not included in the literature. With the proposed method, new exact solutions of this equation are obtained. The graphs of the obtained solution functions are shown in Fig. 1-4.

4. Conclusion

In this paper, the extended trial equation method has been used to obtain a new exact solutions of the cubic nonlinear Schrödinger equation. This method makes it possible to achieve the rational function solutions, travelling wave solution, soliton solution and an Jacobi elliptic function solution. We suppose that this method can also be implemented to other nonlinear differential equations.

5. References

Ablowitz, M.J., Prinari, B. and Trubatch, A.D., 2004. Discrete and Continuous Nonlinear Schrödinger Systems, Cambridge Univ. Press, 18-45.

Akbar, M.A., Ali, N.H.M. and Mohyud-Din, S.T., 2013. The modified alternative (G'/G) -expansion method to nonlinear evolution equation: application to the (1+1)-dimensional Drinfel'd-Sokolov-Wilson equation. *SpringerPlus*, **327**, 2-16.

Chand, F. and Malik, A.K., 2012. Exact traveling wave solutions of some nonlinear equations using (G'/G)-expansion method. *International Journal of Nonlinear Science*, **14(4)**, 416-424.

Gurefe, Y., Sonmezoglu A. and Misirli, E., 2011. Application of trial equation method to the nonlinear partial differential equations arising in mathematical physics. *Pramana-Journal of Physics*, **77(6)**, 1023-1029.

Gurefe, Y., Sonmezoglu A. and Misirli, E., 2012. Application of an irrational trial equation method to high dimensional nonlinear evolution equations. *Journal of Advanced Mathematical Studies*, **5(1)**, 41-47.

Gurefe, Y., Misirli, E., Sonmezoglu, A., Ekici, M., 2013. Extended trial equation method to generalized nonlinear partial differential equations. *Applied Mathematics and Computation*, **219(10)**, 5253-5260.

Hietarinta, J., 1997. Hirota's bilinear method and its generalization. *International Journal of Modern Physics A*, **12(1)**, 43-51.

Imanli, M.I., 2006. Nonlinear Schrödinger equations in homogenous space. MSc thesis, Marmara University, Graduate School of Science and Engineering, Istanbul, 43.

Kaplan, M., Ünsal, Ö. and Bekir, A., 2016. Exact solutions of nonlinear Schrödinger equation by using symbolic computation. *Mathematical Methods in the Applied Science*, **39**, 2093-2099.

Liu, C.S., 2006. Trial equation method for nonlinear evolution equations with rank inhomogeneous: mathematical discussions and applications. *Communications in Theoretical Physics*, **45(2)**, 219-223.

Liu, C.S., 2010. Applications of complete discrimination system for polynomial for classifications of traveling wave solutions to nonlinear differential equations. *Computer Physics Communications*, **181(2)**, 317-324.

Pandir, Y., Gurefe, Y., Kadak, U. and Misirli, E., 2012. Classifications of exact solutions for some nonlinear partial differential equations with generalized evolution. *Abstract and Applied Analysis*, **2012**, Article ID 478531, 16 pp.

Pandir, Y., Gurefe, Y. and Misirli, E. 2013. Classification of exact solutions to the generalized Kadomtsev-Petviashvili equation. *Physica Scripta*, **87(2)**, 025003, 12 pp.

Pandir Y., 2014. Symmetric Fibonacci function solutions of some nonlinear partial differential equations. *Applied Mathematics & Information Science*, **8**, 2237-2241.

Pashaev, O. and Tanoglu, G., 2005. Vector shock soliton and the Hirota bilinear method. *Chaos, Solitons & Fractals*, **26**, 95-105.

Shakeel, M. and Mo hyud-Din, S. T., 2015. New (G'/G)-expansion method and its application to the Zakharov-Kuznetsov-Benjamin-Bona-Mahony (ZK–BBM) equation. *Journal of the Association of Arab Universities for Basic & Applied Science*, **18(1)**, 66-81.

Sulem C. and Sulem P.L. 1999. The nonlinear Schrödinger equation self-focusing and wave collapse. Springer, New-York, 161-186.

Tandogan, Y.A., Pandir, Y. and Gurefe, Y., 2013. Solutions of the nonlinear differential equations by use of modified Kudryashov method. *Turkish Journal of Mathematics and Computer Science*, **1**, 54-60.

Wang M. L., 1996. Exact solutions for compound KdV-Burgers equations. *Physics Letters A*, **213**, 279-287, 1996.

Wazwaz A. M., 2008. A sine-cosine method for handling nonlinear wave equations. *Mathematical and Computer Modellling*, **40(5-6)**, 499-508.