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A NEW CRITERION FOR STABILITY OF NEUTRAL-TYPE HOPFIELD NEURAL NETWORKS WITH CONSTANT DELAYS

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Abstract: This paper makes some contributions to the stability problem of neutral-type Hopfield neural network model having a constant time delay in states of neurons and a constant neutral delay in the time derivatives of states of neurons. With the help of a suitable Lyapunov functional, a novel stability criterion is derived for neutral-type Hopfield neural network model. This stability criterion only requires to check the positive defineteness of the matrices involving the system elements of this type of neural networks. The presented stability condition proved to be independently of these time and neutral delays. Therefore, this condition can be easily justified by applying the properties of some certain matrices. A numerical example for this type of neutral systems is studied to show the applicability of the presented stability result.

Keywords : Neutral-Type Systems, Hopfield Neural Networks, Lyapunov Functionals, Stability Analysis.

Sabit Gecikmeler İçeren Nötral-Tip Hopfield Yapay Sinir Ağlarının Kararlılığı için Yeni Bir Kriter

Oz: Bu makale, hem nöron durumlarının hem de nöron durumlarının türevlerinde sabit gecikmeler içeren nötral-tip Hopfield yapay sinir ağı modelinin kararlılık problemine yeni katkılar yapmaktadır. Uygun bir Lyapunov fonksiyoneli yardımıyla, nötral-tip Hopfield yapay sinir ağlarının kararlılığını sağlayan yeni bir kriter sunulmaktadır. Bu kararlılık kriterinin en önemli avantajı sadece sistem elemanlarından oluşan özel bir matrisin pozitif tanımlı olmasını test edilmesine dayandırılmış olmasıdır. Ayrıca, elde edilen kararlılık koşulu zaman ve nötral gecikmelerden bağımsızdır. Bu nedenle, elde edilen kararlılık kriterinin geçerliliği bazı özel matris özellikleri yardımıyla kolayca test edilebilir. Diğer yandan, önerilen kararlılık koşulunun uygulanabilirliğini göstermek amacıyla sayısal bir örnek verilmiştir.

Anahtar Kelimeler: Nötral-Tip Sistemler, Hopfield Sinir Ağları, Lyapunov Fonksiyonelleri, Kararlılık Analizi

1. INTRODUCTION

Hopfield neural network model studied in (Hopfield, 1982) has been effectively used for solving some typical practical problems associated with optimization problems, signal processing, associative memories design and control problems. When designing Hopfield neural networks in order to solve the related engineering problems, our main issue will be ensuring the intended stability criteria for these neural networks. If these neural networks are intended to be electronically implemented using the VLSI technology, then the main focus will be on the effect of finite switching speeds of operational amplifiers and neuronal communications within network as these two parameters may cause some unavoidable time delays in states of neurons.

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A well known fact is that these time delays might change the desired dynamics to some undesired complex dynamics. Therefore, in determining the exact dynamical behavior of a neural network, it is crucial to present time delays to the mathematical representation of this neural system. In the past years, various sets of stability criteria of delayed-type neural network models have been reported (Zhu ve Cao, 2010), (Song ve diğ., 2018), (Manivannan ve diğ., 2018), (Wang ve diğ., 2018), (Zhang ve diğ., 2018), (Manivannan ve diğ., 2017), (Ge ve diğ., 2014), (Arik, 2014a). However, neural systems involving delays only in states of neurons may not exactly exhibit the required stability properties of neural systems. This case leads us to also include the delays in time derivatives of states of neurons. The class of neural networks involving time delays in states of neurons and the neutral delays in time derivatives of states of neurons is known as neutral-type neural networks. Such neural systems are capable of being employed in different practical applications regarding population ecology, distributed networks having lossless transmission lines, propagation and diffusion models (Niculescu, 2001), (Kolmanovskii ve Nosov, 1986), (Kuang, 1993), (Shi ve diğ., 2013). In recent years, some important results regarding the stability conditions of these neural networks have been given (Muralisankar ve diğ., 2015), (Shu ve diğ., 2019), (Tu ve diğ., 2016), (Jian ve Duan, 2020), (Chen ve diğ., 2010), (Lakshmanan ve diğ., 2013), (Dharani ve diğ., 2015), (Shi ve diğ., 2015), (Zhang ve diğ., 2018), (Liao ve diğ., 2015), (Arik, 2014b), (Lien ve diğ., 2008), (Yang ve diğ., 2015), (Samli ve Arik, 2009), (Orman, 2012), (Cheng ve diğ., 2008), (Akca ve diğ., 2015), (Ozcan, 2018), (Ozcan, 2019). This work will employ an enhanced Lyapunov functional and study stability issues of neutral-type Hopfield neural systems possessing both constant time and constant neutral delays.

Consider a Hopfield neural network model whose dynamics is determined by a set of nonlinear equations of delayed type:

$$\dot{x}_i(\mathrm{t}) + \sum_{j=1}^{n} e_{ij}\,\dot{x}_j(\mathrm{t}-\zeta) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j\left(x_j(t)\right) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t-\tau)\right) + u_i, i = 1,2,\ldots,n \quad (1)$$

where the state of the ith neuron is represented by $x_i(t)$, the c_i are positive constants determining the convergence rates of the states, the nondelayed interconnections parameters are denoted by a_{ij}, the delayed interconnections parameters are denoted by b_{ij}. τ is a constant time delay parameter and ζ is a constant neutral delay parameter. The constant values e_{ij} are the coefficients of time derivatives of states with neutral delay. The functions $f_j(x_j(t))$ represent the neuronal activations and u_i represents an external input to the ith neuron. In (1), if $\xi = max\{\tau,\zeta\}$, then neutral-type neural system defined by equation (1) will keep the initial conditions of $x_i(t) = \varphi_i(t)$ and $\dot{x}_i(\mathrm{t}) = \vartheta_i(t) \in C(\lfloor -\xi, 0\rfloor, R)$. We note here that $C(\lfloor -\xi, 0\rfloor, R)$ represents all the continuous real valued functions which are from $\lfloor -\xi, 0\rfloor$ to R.

A critical issue regarding the stability of system (1) is the properties of nonlinear functions $f_i(x_i(t))$ as stability conditions are mainly determined depending on these functions. Therefore, we first express the class of the functions to be used in system (1). In this paper, it will be supposed that there exist positive numbers ℓ_i that establish the following relations between the states and outputs of the neurons

$$\left|f_i\big(x_i(t)\big) - f_i\big(y_i(t)\big)\right| \le \ell_i |x_i(t) - y_i(t)|, \forall x_i(t), \forall y_i(t) \in R, x_i(t) \ne y_i(t), \forall i, \quad (2)$$

Neutral type system (1) has the mathematical nature of being represented in the vector and matrix form given below:

$$\dot{x}\,(t) + E\dot{x}\,(t-\zeta) = -Cx(t) + Af\big(x(t)\big) + Bf\big(x(t-\tau)\big) + u \quad (3)$$

where $C = diag(c_i > 0)$, $A = (a_{ij})_{nxn}$, $B = (b_{ij})_{nxn}$, $E = (e_{ij})_{nxn}$ and

$x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T,$

$\dot{x}(t) = (\dot{x}_1(t), \dot{x}_2(t), \dots, \dot{x}_n(t))^T,$

$f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t)))^T,$

$f(x(t-\tau)) = (f_1(x_1(t-\tau)), f_2(x_2(t-\tau)), \dots, f_n(x_n(t-\tau)))^T,$

$\dot{x}(t-\zeta) = (\dot{x}_1(t-\zeta), \dot{x}_2(t-\zeta), \dots, \dot{x}_n(t-\zeta))^T,$

$u = (u_1, u_2, \dots, u_n)^T.$

2. STABILITY ANALYSIS

Establishing stability criteria for equilibria of delayed neutral-type Hopfield neural system (1) is the essential purpose of this section. In order to derive the main result of the current work, the first step will be to transform the equilibrium points $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ of Hopfield neural system (1) to the origin. This will be done by exploiting the formula $z_i(t) = x_i(t) - x_i^*$, which transforms neutral-type neural network (1) into the following equivalent neutral-type neural network:

$$\dot{z}_i(\mathrm{t}) + \sum_{j=1}^{n} e_{ij}\,\dot{z}_j(\mathrm{t}-\zeta) = -c_i z_i(t) + \sum_{j=1}^{n} a_{ij} g_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij} g_j\left(z_j(t-\tau)\right), i = 1,2,\dots,n \qquad (4)$$

where $g_i\big(z_i(t)\big) = f_i(z_i(t) + x_i^*) - f_i(x_i^*), \forall i$. According to the conditions possessed by the activation functions of system (1) by (2), the new activation functions $g_i\big(z_i(t)\big)$ in (4) satisfy the following conditions:

$$\left|g_i\big(z_i(t)\big)\right| \le \ell_i |z_i(t)|, \forall z_i(t) \in R, \forall i \qquad (5)$$

Neutral type neural system (4) may be rewritten in the matrix and vector form as stated below:

$$\dot{z}(t) + E\dot{z}(t-\zeta) = -Cz(t) + Ag(z(t) + Bg(z(t-\tau)) \qquad (6)$$

where

$z(t) = (z_1(t), z_2(t), \dots, z_n(t))^T,$

$\dot{z}(t) = (\dot{z}_1(t), \dot{z}_2(t), \dots, \dot{z}_n(t))^T,$

$g(z(t)) = (g_1(x_1(t)), g_2(z_2(t)), \dots, g_n(z_n(t)))^T,$

$g(z(t-\tau)) = (g_1(z_1(t-\tau)), g_2(z_2(t-\tau)), \dots, g_n(z_n(t-\tau)))^T,$

$\dot{z}(t-\zeta) = (\dot{z}_1(t-\zeta), \dot{z}_2(t-\zeta), \dots, \dot{z}_n(t-\zeta))^T.$

It can now be proceeded with the main stability criterion of this work:

Theorem 1: For neutral-type Hopfield neural system (6), assume that the activation functions $g_i\big(z_i(t)\big)$ satisfy (5). Let $\mathcal{L} = diag(\ell_1, \ell_2, \dots, \ell_n)$ and Υ be a positive diagonal matrix defined as $0 < \Upsilon < C$. In this case, the origin of system (6) is globally asymptotically stable, if there exist real constants $\alpha > 0, \beta > 0, \gamma > 0$ and $\delta > 0$ such that

$$\Theta = \mathcal{L}^{-2}\Upsilon - \left(\frac{1}{\beta} + \delta + 1\right)A^T C^{-1}\mathrm{A} - (\frac{1}{\gamma} + \frac{1}{\delta} + 1)B^T C^{-1}\mathrm{B} \geq 0$$

$$\Phi = C - \Upsilon - \frac{1}{\alpha}C - (\alpha + \beta + \gamma)E^T C\mathrm{E} > 0$$

Proof: Let us employ the state transformation $y(t) = \mathrm{z(t) + Ez(t - \zeta)}$. In this case, it follows that $\dot{y}(t) = \dot{z}(\mathrm{t}) + \mathrm{E}\dot{z}(\mathrm{t} - \zeta)$ implying that

$$\dot{y}\,(t) = -Cz(t) + Ag\big(z(t)\big) + Bg\big(z(t-\tau)\big) \tag{7}$$

We now make use of the positive valued Lyapunov functional:

$$V(t) = y^T(t)y(t) + \int_{t-\zeta}^{t} \dot{y}^T\,(s)C^{-1}\dot{y}(s)ds + (\alpha + \beta + \gamma)\int_{t-\zeta}^{t} z^T\,(s)E^T C\mathrm{Ez(s)ds}$$

$$+(\frac{1}{\gamma} + \frac{1}{\delta} + 1)\int_{t-\tau}^{t} g^T\big(z(s)\big)\,B^T C^{-1}\mathrm{Bg}\big(\mathrm{z(s)}\big)\mathrm{ds} \tag{8}$$

The time derivative of $V(t)$ is calculated as

$$\dot{V}(t) = 2y^T(t)\dot{y}(t) + \dot{y}^T(t)C^{-1}\dot{y}(t) - \dot{y}^T(t-\zeta)C^{-1}\dot{y}(t-\zeta)$$

$$+(\alpha + \beta + \gamma)z^T(t)E^T C\mathrm{Ez(t)} - (\alpha + \beta + \gamma)z^T(t-\zeta)E^T C\mathrm{Ez(t-\zeta)}$$

$$+(\frac{1}{\gamma} + \frac{1}{\delta} + 1)g^T(z(t))\,B^T C^{-1}\mathrm{Bg}\big(\mathrm{z(t)}\big)$$

$$-(\frac{1}{\gamma} + \frac{1}{\delta} + 1)g^T(z(t-\tau))\,B^T C^{-1}\mathrm{Bg}\big(\mathrm{z(t-\tau)}\big)$$

$$= (2C^{\frac{1}{2}}y(t) + C^{-\frac{1}{2}}\dot{y}(t))^T C^{-\frac{1}{2}}\dot{y}(t) \quad - \dot{y}^T(t-\zeta)C^{-1}\dot{y}(t-\zeta)$$

$$+(\alpha + \beta + \gamma)z^T(t)E^T C\mathrm{Ez(t)} - (\alpha + \beta + \gamma)z^T(t-\zeta)E^T C\mathrm{Ez(t-\zeta)}$$

$$+(\frac{1}{\gamma} + \frac{1}{\delta} + 1)g^T(z(t))\,B^T C^{-1}\mathrm{Bg}\big(\mathrm{z(t)}\big)$$

$$-(\frac{1}{\gamma} + \frac{1}{\delta} + 1)g^T(z(t-\tau))\,B^T C^{-1}\mathrm{Bg}\big(\mathrm{z(t-\tau)}\big)$$

$$\leq (2C^{\frac{1}{2}}y(t) + C^{-\frac{1}{2}}\dot{y}(t))^T C^{-\frac{1}{2}}\dot{y}(t)$$

$$+(\alpha + \beta + \gamma)z^T(t)E^T C\mathrm{Ez(t)} - (\alpha + \beta + \gamma)z^T(t-\zeta)E^T C\mathrm{Ez(t-\zeta)}$$

$$+(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t))\,B^T C^{-1}\mathrm{Bg}(z(t))$$

$$-(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t-\tau))\,B^T C^{-1}\mathrm{Bg}(z(t-\tau))$$

$$=(2C^{\frac{1}{2}}z(t)+2C^{\frac{1}{2}}Ez(t-\zeta)-C^{\frac{1}{2}}z(t)+C^{-\frac{1}{2}}A\mathrm{g}(z(t))+C^{-\frac{1}{2}}B\mathrm{g}(z(t-\tau)))^T$$

$$\times(-C^{\frac{1}{2}}z(t)+C^{-\frac{1}{2}}A\mathrm{g}(z(t))+C^{-\frac{1}{2}}B\mathrm{g}(z(t-\tau)))$$

$$+(\alpha+\beta+\gamma)z^T(t)E^T C\mathrm{Ez(t)}-(\alpha+\beta+\gamma)z^T(t-\zeta)E^T C\mathrm{Ez(t-\zeta)}$$

$$+(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t))\,B^T C^{-1}\mathrm{Bg}(z(t))$$

$$-(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t-\tau))\,B^T C^{-1}\mathrm{Bg}(z(t-\tau))$$

$$=(C^{\frac{1}{2}}z(t)+2C^{\frac{1}{2}}Ez(t-\zeta)+C^{-\frac{1}{2}}A\mathrm{g}(z(t))+C^{-\frac{1}{2}}B\mathrm{g}(z(t-\tau)))^T$$

$$\times(-C^{\frac{1}{2}}z(t)+C^{-\frac{1}{2}}A\mathrm{g}(z(t))+C^{-\frac{1}{2}}B\mathrm{g}(z(t-\tau)))$$

$$+(\alpha+\beta+\gamma)z^T(t)E^T C\mathrm{Ez(t)}-(\alpha+\beta+\gamma)z^T(t-\zeta)E^T C\mathrm{Ez(t-\zeta)}$$

$$+(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t))\,B^T C^{-1}\mathrm{Bg}(z(t))$$

$$-(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t-\tau))\,B^T C^{-1}\mathrm{Bg}(z(t-\tau))$$

$$=\quad -z^T(t)C\,\mathrm{z(t)}-2z^T(t-\zeta)E^T Cz(t)+2z^T(t-\zeta)E^T A\mathrm{g}(z(t))$$

$$+2z^T(t-\zeta)E^T B\mathrm{g}(z(t-\tau))+g^T(z(t))\,A^T C^{-1}\mathrm{Ag}(z(t))$$

$$+g^T(z(t-\tau))\,B^T C^{-1}\mathrm{Bg}(z(t-\tau))+2g^T(z(t))\,A^T C^{-1}\mathrm{Bg}(z(t-\tau))$$

$$+(\alpha+\beta+\gamma)z^T(t)E^T C\mathrm{Ez(t)}-(\alpha+\beta+\gamma)z^T(t-\zeta)E^T C\mathrm{Ez(t-\zeta)}$$

$$+(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t))\,B^T C^{-1}\mathrm{Bg}(z(t))$$

$$-(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t-\tau))\,B^T C^{-1}\mathrm{Bg}(z(t-\tau)) \tag{9}$$

The following inequalities can be noted :

$$-2z^T(t-\zeta)E^T Cz(t)\leq\ \alpha z^T(t-\zeta)E^T CEz(t-\zeta)+\frac{1}{\alpha}z^T(t)C\,\mathrm{z(t)} \tag{10}$$

$$2z^T(t-\zeta)E^TAg(z(t)) \leq \beta z^T(t-\zeta)E^TCEz(t-\zeta)+\frac{1}{\beta}g^T(z(t))\,A^TC^{-1}\mathrm{Ag}(z(t)) \quad (11)$$

$$2z^T(t-\zeta)E^TBg\big(z(t-\tau)\big) \leq \gamma z^T(t-\zeta)E^TCEz(t-\zeta)$$
$$+\frac{1}{\gamma}g^T(z(t-\tau))\,B^TC^{-1}\mathrm{Bg}(z(t-\tau)) \quad (12)$$

$$2g^T(z(t))\,A^TC^{-1}\mathrm{Bg}(z(t-\tau)) \leq \delta g^T(z(t))A^TC^{-1}\mathrm{Ag}(z(t))$$
$$+\frac{1}{\delta}g^T(z(t-\tau))\,B^TC^{-1}\mathrm{Bg}(z(t-\tau)) \quad (13)$$

If we use (10)-(13) in (9), then we can obtain

$$\dot{V}(t) \leq -z^T(t)C\,\mathrm{z(t)}+\alpha z^T(t-\zeta)E^TCEz(t-\zeta)+\frac{1}{\alpha}z^T(t)C\,\mathrm{z(t)}$$
$$+\beta z^T(t-\zeta)E^TCEz(t-\zeta)+\frac{1}{\beta}g^T(z(t))\,A^TC^{-1}\mathrm{Ag}(z(t))$$
$$+\gamma z^T(t-\zeta)E^TCEz(t-\zeta)+\frac{1}{\gamma}g^T(z(t-\tau))\,B^TC^{-1}\mathrm{Bg}(z(t-\tau))$$
$$+\delta g^T(z(t))A^TC^{-1}\mathrm{Ag}(z(t))+\frac{1}{\delta}g^T(z(t-\tau))\,B^TC^{-1}\mathrm{Bg}(z(t-\tau))$$
$$+(\alpha+\beta+\gamma)z^T(t)E^TC\mathrm{Ez(t)}-(\alpha+\beta+\gamma)z^T(t-\zeta)E^TC\mathrm{Ez}(\mathrm{t}-\zeta)$$
$$+(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t))\,B^TC^{-1}\mathrm{Bg}(z(t))$$
$$-(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t-\tau))\,B^TC^{-1}\mathrm{Bg}(z(t-\tau))$$
$$=-z^T(t)C\,\mathrm{z(t)}+\frac{1}{\alpha}z^T(t)C\,\mathrm{z(t)}+\frac{1}{\beta}g^T(z(t))\,A^TC^{-1}\mathrm{Ag}(z(t))$$
$$+g^T(z(t))\,A^TC^{-1}\mathrm{Ag}(z(t))+\delta g^T(z(t))A^TC^{-1}\mathrm{Ag}(z(t))$$
$$+(\alpha+\beta+\gamma)z^T(t)E^TC\mathrm{Ez(t)}+(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t))\,B^TC^{-1}\mathrm{Bg}(z(t)) \quad (14)$$

(14) yields :

$$\dot{V}(t) \leq -z^T(t)(C-\Upsilon)\mathrm{z(t)}-z^T(t)\Upsilon\mathrm{z(t)}+\frac{1}{\alpha}z^T(t)C\,\mathrm{z(t)}$$
$$+(\frac{1}{\beta}+\delta+1)g^T(z(t))A^TC^{-1}\mathrm{Ag}(z(t))+(\alpha+\beta+\gamma)z^T(t)E^TC\mathrm{Ez(t)}$$

$$+(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t))\,B^T C^{-1}\mathrm{B}g(z(t))$$

$$\leq -z^T(t)(C-\Upsilon)\mathrm{z(t)} - g^T(z(t))\,\mathcal{L}^{-2}\Upsilon\mathrm{g}(z(t)) + \frac{1}{\alpha}z^T(t)C\,\mathrm{z(t)}$$

$$+(\frac{1}{\beta}+\delta+1)g^T(z(t))A^T C^{-1}\mathrm{A}g(z(t)) + (\alpha+\beta+\gamma)z^T(t)E^T C\mathrm{Ez(t)}$$

$$+(\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t))\,B^T C^{-1}\mathrm{B}g(z(t))$$

$$= -z^T(t)(C-\Upsilon-\frac{1}{\alpha}C-(\alpha+\beta+\gamma)E^T C\mathrm{E})\mathrm{z(t)}$$

$$-g^T(z(t))(\,\mathcal{L}^{-2}\Upsilon-(\frac{1}{\beta}+\delta+1)A^T C^{-1}\mathrm{A}-(\frac{1}{\gamma}+\frac{1}{\delta}+1)B^T C^{-1}\mathrm{B})\mathrm{g}(z(t))$$

$$= -z^T(t)\phi\mathrm{z(t)} - g^T(z(t)\Theta\mathrm{g}(z(t)) \tag{15}$$

In (15), the fact that $\Theta \geq 0, \forall\, \mathrm{z(t)} \neq 0$ will directly yield

$$\dot{V}(t) \leq -z^T(t)\phi\mathrm{z(t)} \tag{16}$$

In (16), the condition that $\phi > 0, \forall\, \mathrm{z(t)} \neq 0$ will directly yield that $\dot{V}(t) < 0$. It will now be assumed that $\mathrm{z(t)} = 0$. Thus, (9) yields

$$\dot{V}(t) = 2z^T(t-\zeta)E^T Bg(z(t-\tau) + g^T(z(t-\tau))\,B^T C^{-1}\mathrm{B}g(z(t-\tau))$$

$$-(\alpha+\beta+\gamma)z^T(t-\zeta)E^T C\mathrm{Ez}(\mathrm{t}-\zeta) - (\frac{1}{\gamma}+\frac{1}{\delta}+1)g^T(z(t-\tau))\,B^T C^{-1}\mathrm{B}g(z(t-\tau))$$

$$= 2z^T(t-\zeta)E^T B\mathrm{g}(z(t-\tau) - (\alpha+\beta+\gamma)z^T(t-\zeta)E^T C\mathrm{Ez}(\mathrm{t}-\zeta)$$

$$-(\frac{1}{\gamma}+\frac{1}{\delta})g^T(z(t-\tau))\,B^T C^{-1}\mathrm{B}g(z(t-\tau)) \tag{17}$$

Using (10) in (17) leads to

$$\dot{V}(t) = -(\alpha+\beta)z^T(t-\zeta)E^T CEz(t-\zeta) - \frac{1}{\delta}g^T(z(t-\tau))\,B^T C^{-1}\mathrm{B}g(z(t-\tau)) \tag{18}$$

In the light of (18), we get that

$$\dot{V}(t) \leq -(\alpha+\beta)z^T(t-\zeta)E^T CEz(t-\zeta) \tag{19}$$

Thus, (19) enables one to draw the conclusion of $\dot{V}(t) < 0$ if $z(t-\zeta) \neq 0$. Let $z(t-\zeta) = 0$. We now observe that (18) also yields the equality

$$\dot{V}(t) = -\frac{1}{\delta} g^T(z(t-\tau))\, B^T C^{-1} \mathrm{B} g(z(t-\tau)) \tag{20}$$

In (20), if $\mathrm{g}(z(t-\tau)) \neq 0$, (beacuse of the properties of the nonlinear activities of neurons, $\mathrm{g}(z(t-\tau)) \neq 0$ implies that $z(t-\tau) \neq 0$), then $\dot{V}(t) < 0$. Letting $z(t) = 0$, $z(t-\tau) = 0$ and $z(t-\zeta) = 0$ in (9) yields

$$\dot{V}(t) = -\dot{y}^T(t-\zeta) C^{-1} \dot{y}(t-\zeta) \tag{21}$$

In (21), if $\dot{y}(t-\zeta) \neq 0$, then $\dot{V}(t) < 0$. Thus, we observe from (9) that $\dot{V}(t) = 0$ wherever $\dot{y}(t-\zeta) = z(t) = z(t-\tau) = z(t-\zeta) = 0$, otherwise $\dot{V}(t) < 0$. Therefore, the origin of (6) is asymptotically stable. The Lyapunov functional stated in (8) satisfies :

$$V(t) \geq y^T(t)\mathrm{y}(t) = \|y(t)\|_2^2 \tag{22}$$

By the virtue of (22), $V(t) \to \infty$ as $\|y(t)\| \to \infty$. On the other hand, it has been adressed in (Lien ve diğ., 2008) that, under the condition of $E^T E < I$, $\|y(t)\| \to \infty$ if $\|z(t)\| \to \infty$. Therefore, $\|E\| < 1$ and $\|z(t)\| \to \infty$ will guarantee that $V(t) \to \infty$. This indicates the fact that $V(t) \to \infty$ is radially unbounded, concluding that the origin of (6) is globally asymptotically stable, Q.E.D.

3. AN EXAMPLE

This section studies an example to reveal the applicability of the stability criteria proposed in Theorem 1.

Example : Let neutral-type system (1) possess the system elements :

$$A = \begin{bmatrix} a & a & a & a \\ a & -a & a & -a \\ a & a & -a & -a \\ -a & a & a & -a \end{bmatrix}, \quad B = \begin{bmatrix} b & b & b & b \\ b & -b & b & -b \\ b & b & -b & -b \\ -b & b & b & -b \end{bmatrix}, \quad E = \begin{bmatrix} e & e & e & e \\ e & -e & e & -e \\ e & e & -e & -e \\ -e & e & e & -e \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \Upsilon = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathcal{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where a, b and e are some positive constants. We have

$$A^T C^{-1} \mathrm{A} = \begin{bmatrix} 4a^2 & 0 & 0 & 0 \\ 0 & 4a^2 & 0 & 0 \\ 0 & 0 & 4a^2 & 0 \\ 0 & 0 & 0 & 4a^2 \end{bmatrix}, B^T C^{-1} \mathrm{B} = \begin{bmatrix} 4b^2 & 0 & 0 & 0 \\ 0 & 4b^2 & 0 & 0 \\ 0 & 0 & 4b^2 & 0 \\ 0 & 0 & 0 & 4b^2 \end{bmatrix},$$

$$E^T C E = \begin{bmatrix} 4e^2 & 0 & 0 & 0 \\ 0 & 4e^2 & 0 & 0 \\ 0 & 0 & 4e^2 & 0 \\ 0 & 0 & 0 & 4e^2 \end{bmatrix}$$

For $\alpha = 4$, $\beta = 3$ and $\gamma = 2$, we have

$$\phi = \left(\frac{1}{4} - 36e^2\right) I$$

where I is the unity matrix of 4x4. Thus, according the second condition of Theorem 1, $e < \frac{1}{12}$ implies that $\phi > 0$. For $\delta = \frac{2}{3}$, Θ in Theorem 1 is determined as follows :

$$\Theta = \left(\frac{1}{2} - 8a^2 - 12a^2\right) I$$

where I is the unity matrix of 4x4. Thus, according the first condition of Theorem 1, the choices $a \leq \frac{1}{4\sqrt{2}}$ and $b \leq \frac{1}{4\sqrt{3}}$ imply that $\Theta \geq 0$. Hence, we have derived the constraints to be imposed on the elements of the example to establish the stability criteria in Theorem 1.

4. CONCLUSIONS

This paper has made some contributions to the stability problem of neutral-type Hopfield neural network model having a constant time delay in states of neurons and a constant neutral delay in the time derivatives of states of neurons. With the help of a suitable Lyapunov functional, a novel stability criterion has been derived for neutral-type Hopfield neural network model. This stability criterion only requires to check the positive definiteness of the matrices involving the system elements of this type of neural networks. The presented stability condition proved to be independently of these time and neutral delays. Therefore, this condition can be easily justified by applying the properties of some certain matrices. A constructive numerical example of this type of neural systems has been studied to show the applicability of the presented stability result.

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