INVESTIGATION OF THE COEFFICIENT OF RESTITUTION OF DIFFERENT BALLS DROPPED ON FLOATING RECTANGULAR PRISM

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ABSTRACT

In this study, the effect of draft on the coefficient of restitution was investigated. 3 different balls (foam, plastic, table tennis) were dropped from 3 individual heights (50, 80, 110 cm) to a floating prism with 2 different drafts, the drafts being adjusted by adding weight. To control the results and see the effect of draft, the same experiment was conducted by dropping the balls on the prism on the floor. It was seen that as the draft of the rectangular prism was increased, the energy loss during collisions due to damping by the water was decreased.

Key words: coefficient of restitution, drop test, energy loss, balls, floating prism.

1. Introduction

Investigating collision mechanics involving fluids is a complex task. A collision is an isolated event in which two or more bodies exert relatively strong forces on each other for a relatively short time[1] (Sun et al., 2016). To further understand collisions involving fluids and compare them to collisions done on hard floor we have to investigate the physics behind this great phenomenon. Acknowledging the fact that a ball colliding a body floating in water is a dynamic process, due to measurement constraints, we have investigated this phenomenon statically. The balls were dropped vertically on to the surface and bounced vertically without spin as the bounce of a spinning ball is a more complicated process[2] (Cross, 2011). To investigate the relation between the collisions in water and on a hard surface floor, we have to be familiar with the Archimedes' Principle and the Coefficient of Restitution (COR). It is hypothesized that as the draft (T) (thus the displacement) of the rectangular prism increases, the energy loss during collisions due to damping by the water is expected to decrease. If the draft is increased to infinity, as there will be no heaving motion of the rectangular prism, it is expected that the energy loss during collisions be the same as on the hard floor.

2. Materials and Method

2.1 Archimedes' Principle

A body immersed in a fluid should be "buoyed up" with a force equal to the weight of the displaced fluid[3] (Sears & Zemansky, 1960). As it was first thought of by the Greek philosopher Archimedes, it is called the Archimedes' Principle. We can use the principle to calculate the mass of a rectangular prism submerged in a fluid (as given in Figure 1):

$$M_0 = w \cdot d \cdot T \cdot \rho \tag{1}$$

[1] Sun, Fen, Patrick T.K. Leung and Roy Vellaisamy. *Kung Fu and Science*. Hong Kong: City University of Hong Kong Press, 2016, p. 47.
[2] Cross, Rod. Physics of Baseball & Softball. New York: Springer, 2011, p. 120.
[3] Sears, Francis Weston and Mark W. Zemansky. *College Physics*. London: Addison-Wesley Publishing, 1960, p. 244.

Where (M_0) is the mass, (w) is beam, (d) is length, and (T) is the draft[4] of the prism inside the fluid and (ρ) is the density of the fluid.

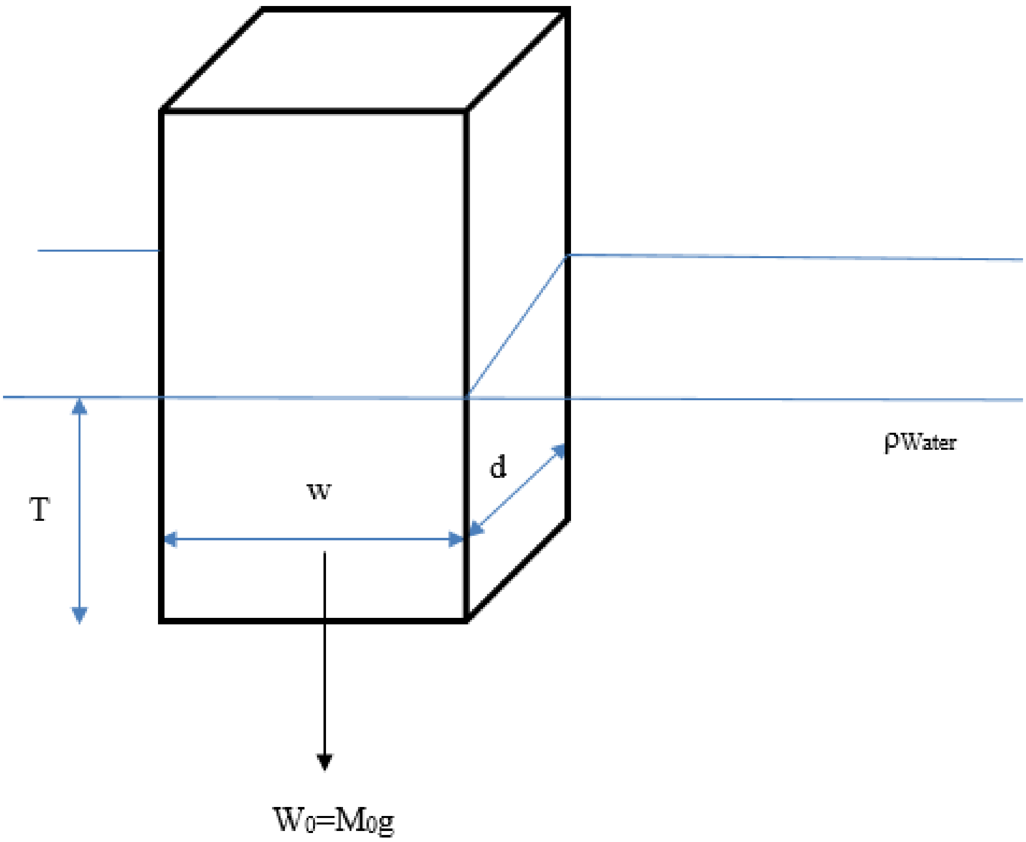

Figure 1: Rectangular Prism Floating in Water

2.2 The Coefficient of Restitution

For the collision of two bodies, the COR (*e*) is the ratio of the relative velocity of separation of the two bodies to their relative velocity of approach[5] (McLean & Nelson, 1952). The COR (*e*) is a constant with a value between 0 and 1. A value of 0 denotes a perfectly plastic collision while a value of 1 denotes a perfectly elastic collision. The COR can be used to observe energy loss and compare the energy efficiency of materials by comparing COR values.

$$0 \leq e \leq 1 \tag{2}$$

The formulation for calculating the COR (*e*) can be given as:

$$e = \frac{\text{velocity difference after collision}}{\text{velocity difference before collision}} = \left|\frac{v_2 - v_1}{u_1 - u_2}\right| \tag{3}$$

[4] Draft (T) of a rectangular prism is the vertical distance between the waterline and the bottom of the prism (Lewis, 1988).
[5] McLean, W.G. and E.W. Nelson. *Theory and Problems of Engineering Mechanics*. New York: Schaum Publishing, 1952, p.247.

Where u_1 and u_2 are the initial velocities (before the collision) of bodies 1 (denotes balls) and 2 (denotes wooden prism) respectively, and v_1 and v_2 are the velocities of bodies 1 and 2 respectively after the collision. In this formulation, the velocities are normal to the striking surface. If the velocity is not normal to the striking surface, the normal component of the velocity is used in the Eq.3. During the collision, the force acting on both bodies are the same. The sum of the momentum before the collision must be equal to the sum of the momentum after the collision. This can be written as:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{4}$$

Where m_1 and m_2 are masses of bodies 1 and 2 respectively, u_1 and u_2 are the velocities before the collision of bodies 1 and 2 respectively and v_1 and v_2 are the velocities after the collision of bodies 1 and 2 respectively. In the first part of the experiment, the rectangular prism was put on a flat surface. Then different balls were dropped from different heights. In the experiments, because the rectangular prism didn't move, the COR (*e*), using Eq.4, was found to be:

$$e = \left|\frac{v_1}{u_1}\right| \tag{5}$$

In the second part of the experiment, the rectangular prism was put inside water. The same experimental procedure as the first part was applied. After the ball hit the surface of the rectangular prism, the box heaved. The COR (*e*) was found using Eq.3. In Figure 2, Point A is the drop point of the balls, Point B is just before collision and Point D is right after collision. Point C is the maximum bounce height of the balls and Point E is the minimum height of the prism after collision. To calculate the velocity u_1 of the ball, energy equation between two Points A & B (before collision) were written as follows:

$$E_{P.E.\ at\ A} + E_{K.E.\ at\ A} = E_{P.E.\ at\ B} + E_{K.E.\ at\ B} \tag{6}$$

$$m_1gh_A + \frac{m_1u_A^2}{2} = m_1gh_B + \frac{m_1u_B^2}{2} \tag{7}$$

Where m_1 is the mass of the ball, g the gravitational acceleration, h_A the height of ball, with reference to the rectangular prism, at Point A, u_A the velocity of the ball at Point A, h_B the height of ball, with reference to the rectangular prism, at Point B and u_B the velocity of the ball at Point B, all of which are before collision. As u_A equals to zero, the $E_{K.E.}$ value at A is zero. As the reference point is the top of the rectangular prism, and Point B just above the surface of the rectangular prism, h_B equals to zero, thus the $E_{P:E:}$value at B is zero.

$$m_1gh_A + 0 = 0 + \frac{m_1u_B^2}{2} \tag{8}$$

$$u_B = \sqrt{2gh_A} \tag{9}$$

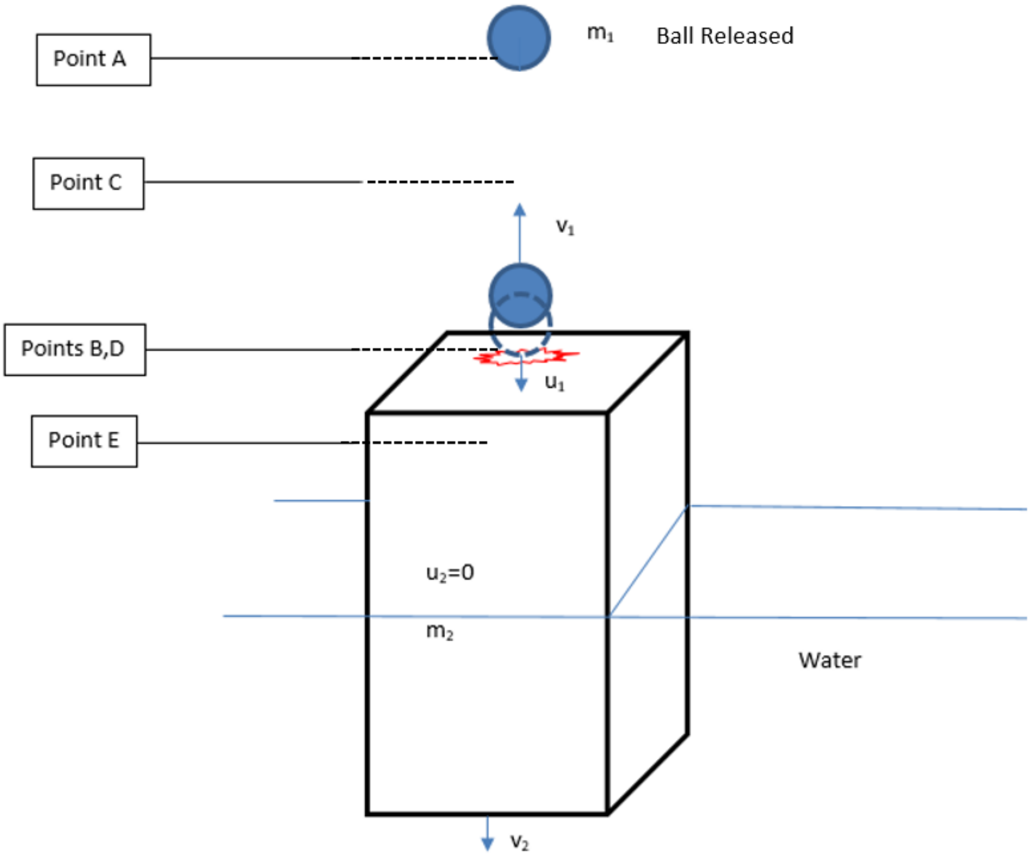

Figure 2: Collision of Ball and Floating Prism

As u_B is calculated, it is found that u_B equals to u_1. To calculate the velocity v_1 of the ball, energy equation between two Points B & C (after collision) were written as follows:

$$m_1 g h_B + \frac{m_1 v_B^2}{2} = m_1 g h_C + \frac{m_1 v_C^2}{2} \tag{10}$$

Where m_1 is the mass of the ball, g the gravitational acceleration, h_B the height of ball, with reference to the rectangular prism, at Point B, v_B the velocity of the ball at Point B, h_C the height of ball, with reference to the rectangular prism, at Point C and v_C the velocity of the ball at Point C, all of which are after collision. As the reference point is the top of the rectangular prism, and Point B just above the surface of the rectangular prism, h_B equals to zero, thus the $E_{P:E:}$value at B is zero. As v_C equals to zero, the $E_{K.E.}$ value at C is zero.

$$v_B = \sqrt{2 g h_C} \tag{11}$$

As v_B is calculated, it is found that v_B equals to v_1. For all experiments, as the wooden prism is at rest initially, u_2 equals to zero. To calculate the velocity v_2 of the ball, momentum equation, Eq.4, was written as follows:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \tag{12}$$

$$m_1 u_B + m_2 u_2 = m_1 v_B + m_2 v_D \tag{13}$$

Where m_1 and m_2 are the masses of the balls and the rectangular prism respectively, u_B the velocity of ball before the collision, at Point B, v_B the velocity of the ball after collision at Point B, and v_D the velocity of the ball at Point D, after the collision.

$$v_D = \frac{m_1}{m_2}\left(\sqrt{2gh_C} - \sqrt{2gh_A}\right) \tag{14}$$

As we calculate v_D, we find that v_D equals to v_2. So, the COR (e) equation can be given as:

$$e = \left| \frac{\frac{m_1}{m_2}\left(\sqrt{2gh_C} - \sqrt{2gh_A}\right) - \sqrt{2gh_C}}{\sqrt{2gh_A}} \right| \tag{15}$$

$$e = \left| \frac{\frac{m_1}{m_2}\left(\sqrt{h_C} - \sqrt{h_A}\right) - \sqrt{h_C}}{\sqrt{h_A}} \right| \tag{16}$$

For the collisions on hard floor, as the value of v_2 is zero, the Eq.16 can be simplified, in accordance to Eq.4, as follows:

$$e = \left| \frac{0 - \sqrt{2gh_C}}{\sqrt{2gh_A}} \right| = \left| \frac{\sqrt{2gh_C}}{\sqrt{2gh_A}} \right| = \left| \frac{\sqrt{h_C}}{\sqrt{h_A}} \right| \tag{17}$$

3. Experiment Setup and Procedure

The same wooden rectangular prism was used throughout the experiments. To control temperature, the experiments were done in the same room at room temperature (25°C). In the experiments, a water tank was used. The tank was filled with fresh water to a certain point and the water was not touched. The amount of water was always kept constant. As the prism was floating on the water, any breeze could easily damage the position of the prism and cause a deviation in the path of the projectile. To avoid errors, the experiments was carried out in a small room and all doors and windows were kept shut.

3 different balls were used throughout the experiments. A table tennis, a foam and a plastic ball was used as seen in Figure 3. As all these balls have different elastic properties, it was assumed that they all behaved differently upon collisions. The balls were dropped from 3 different drop heights, 50.0, 80.0 and 110.0 (±0.05) centimeters. The effect of increasing the drop height, thus the initial speed of the balls, on energy loss was observed. First a 5 kilogram weight was added and submerged in the water tank. Then to increase the draft, 5 more kilograms were added. Also, the balls were dropped on the rectangular prism standing on hard floor. All the apparatus used in the experiments are shown in Table 1.

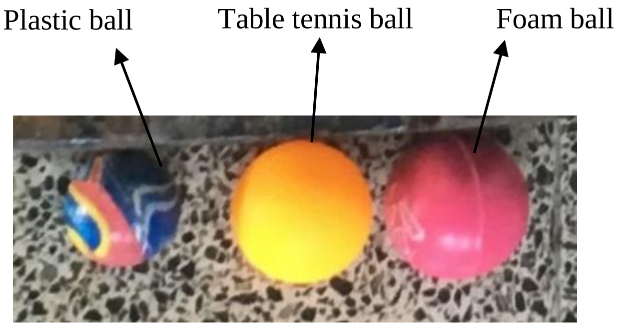

Figure 3: Different type of balls used in the experiments

Table 1: Apparatus used and their properties

Apparatus	Properties
1-Table tennis ball	2.62 ±0.01g
2-Plastic ball	(25.50 ±0.01g)
3-Foam ball	(45.77 ±0.01g)
Wooden rectangular prism	20.0cm x 20.5cm x 44.0cm
Wooden rectangular prism	2350.00 ±0.01g
Two 100cm rulers	(±0.05 cm) (to record bounce height)
Digital balance	(±0.01 g)
Water tank	(50cm x 110cm x 70cm)
Two iron weights	5000.00 ±0.01g
Slow motion camera	(shot at 240fps)

Experimental setup is made in two sections. First section of the experiment is done on hard floor. Two 100cm rulers are used and are taped against the wall. The wooden rectangular prism is put next to the rulers. The iPhone is setup on a tripod and ready to shoot. Using our DIY apparatus, the balls are dropped on the wooden rectangular prism from 3 different heights, 50cm, 80cm and 110cm respectively. The setup is shown in Figure 4. The second section of the experiment is done in a water tank (50cm x 110cm x 70cm) filled to about ¾ the way up with fresh water. The experimental setup of the second section is depicted in Figure 5. The wooden rectangular prism, coated with impermeable paint, is partially submerged in water by adding weights inside, 5kg and 10kg respectively. The weights are used so as to change the draft of the prism. The top part of the prism is nailed so that it acts as a lid. To add weight, this lid is utilized. The same rulers are used and are taped accordingly to the proper positions so that the "0" mark is at the edge of the top of the rectangular prism. The iPhone is setup on a tripod and ready to shoot. Using our DIY apparatus, the balls are dropped on the wooden rectangular prism from 3 different heights, 50cm, 80cm and 110cm respectively.

The procedure to find the COR values on hard floor are as follows:

1. Build experimental setup as explained.
2. Drop table tennis ball from the first height (50cm) and record the bounce height.
3. Repeat step 2 for a total number of 8 trials.
4. Repeat step 2-3 for a drop height of 80cm and 110cm.
5. Repeat steps 2-4 for plastic and foam balls and record all collected data.

The procedure to find the COR values when the prism is submerged in water are as follows:

1. Build experimental setup. First add 5kg weight in the prism.

2. Drop table tennis ball from the first height (50cm) and record the bounce height.
3. Repeat step 2 for a total number of 8 trials.
4. Repeat step 2-3 for a drop height of 80cm and 110cm.
5. Repeat steps 2-4 for plastic and foam balls and record all data.
6. After all the measurements are done, add another 5kg (for a total of 10kg added weight) and repeat steps 2-5.

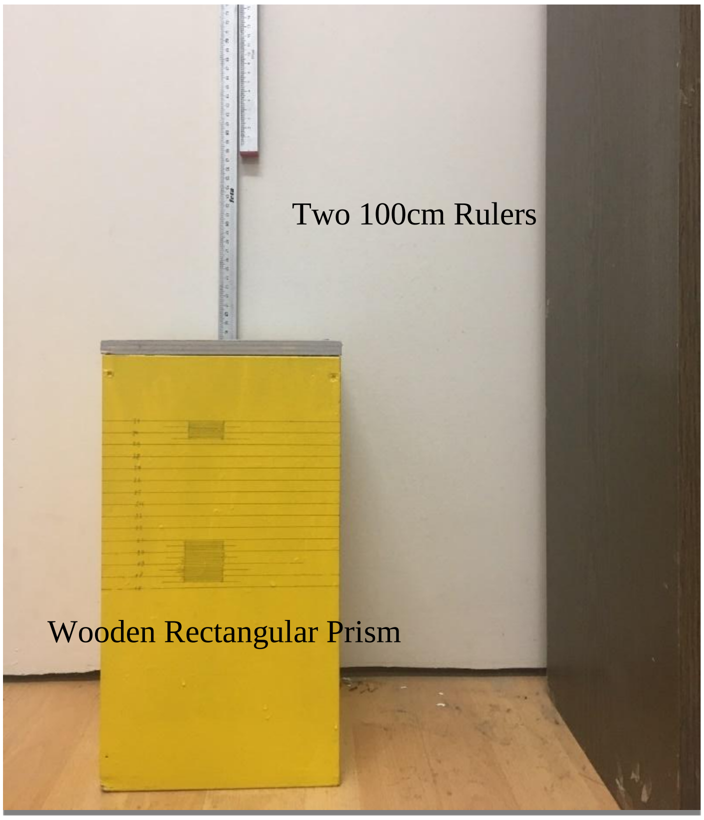

Figure 4: Snapshot of First Section of the Experiment Setup

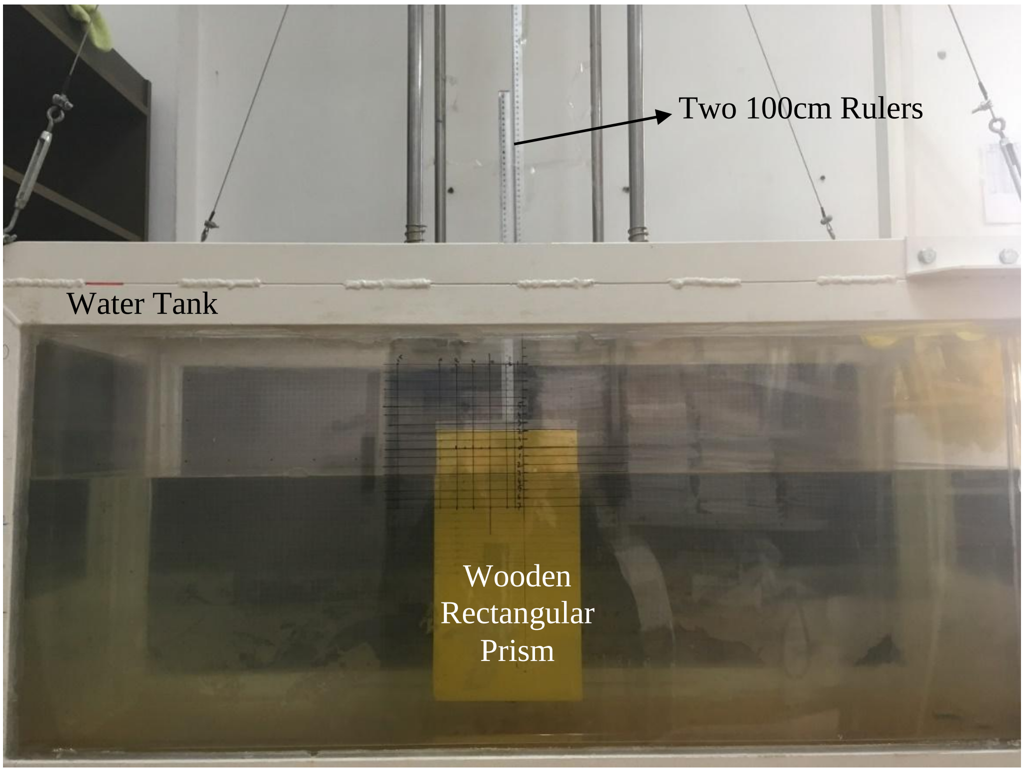

Figure 5: Snapshot of Second Section of the Experiment Setup

4. Results

The rectangular prism in the water tank with 2 different weights (thus two different drafts) is tested as given in Table 2. Table 3 was constructed for the average bounce height of each ball of 8 trials.

Table 2: Draft of Rectangular Prism

Rectangular Prism	Mass [g] (±0.02g)	Draft (T) [cm] (±0.15)
With 5kg added weight	7350.00	17.90
With 10kg added weight	12350.00	30.10

While calculating the uncertainty of the mass, the uncertainties of the weight and the rectangular prism itself were added. Because a digital balance of uncertainty ±0.01g was used to weigh both, the absolute uncertainty of both is ±0.01g. Thus the total absolute uncertainty is ±0.02g. To calculate the uncertainty of draft, the percentage uncertainties of mass, beam and length were added. The uncertainty of density of water was accepted as zero as it is a universally accepted value. The percentage uncertainty was then turned into absolute uncertainty and the highest value was accepted as the total absolute uncertainty of all the draft values.

Table 3: Trial Averages For All Data

Ball	Average Bounce Height (cm) (±2.6cm)								
	Hard Floor			17.90cm Draft			30.10cm Draft		
	50 cm	80 cm	110 cm	50 cm	80 cm	110 cm	50 cm	80 cm	110 cm
Foam Ball	36.3	54.3	70.6	31.3	49.2	66.6	33.4	52.7	69.5
Table Tennis Ball	34.3	52.5	64.7	30.3	48.3	62.7	31.0	49.3	63.8
Plastic Ball	23.1	36.0	47.8	18.0	27.9	36.0	20.0	31.5	42.4

While calculating COR values, Eq. 16 was used. The COR calculations were carried out for every individual ball and drop height. The COR values are essential to this investigation as it lets us examine the energy loss in different situations and provides ease of comparison.

As depicted in Figures 6-8, as the drop height increases, (thus the initial velocity), the COR values decreases. Typically, as the drop height (thus initial velocity) increases, the COR values decrease which means that more energy is lost in the collisions. The COR value of every ball decreased significantly when the targets' medium was changed from hard floor to water. As can be seen in Figures 6-8, if the draft of the target prism increases the COR value gets nearer to the COR value of target on hard floor. If the draft of the prism is increased to its maximum point, the COR value is expected to be really close to the COR values on hard floor, though never more. The COR and draft are directly proportional. The slopes of Figures 6-8 gives us the change in COR values per centimeter. All the slopes have negative values, denoting that as drop height increases, the COR values decrease. The COR and drop height are inversely proportional. The slope of foam ball and tennis ball are highest at -0.0008 and -0.0010 respectively when dropped on hard floor. In the water tank, the slopes decrease compared to hard floor. However, the slope of plastic ball is highest when dropped on prism submerged in the water tank with draft 17.90cm at -0.0005.

Table 6: COR (e) Values for All Data

Ball	Drop Height (cm) (±0.05cm)								
	50.00	80.00	110.00	50.00	80.00	110.00	50.00	80.00	110.00
	17.90cm Draft			30.10cm Draft			Hard Floor		
	COR Values (*e*) (±0.002)								
Foam Ball	0.790	0.783	0.776	0.817	0.811	0.794	0.852	0.824	0.801
Table Tennis Ball	0.778	0.777	0.755	0.787	0.785	0.762	0.828	0.810	0.767
Plastic Ball	0.599	0.589	0.571	0.632	0.627	0.620	0.680	0.671	0.659

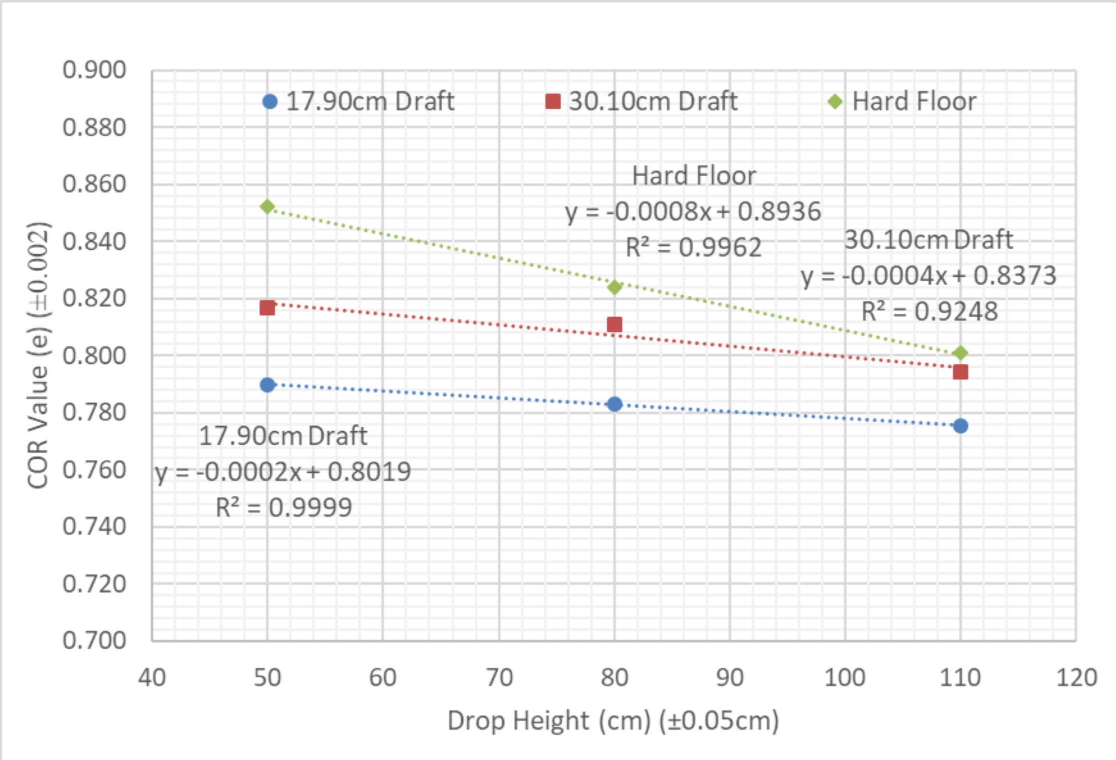

Figure 6: COR Value Versus Drop Height of Foam Ball

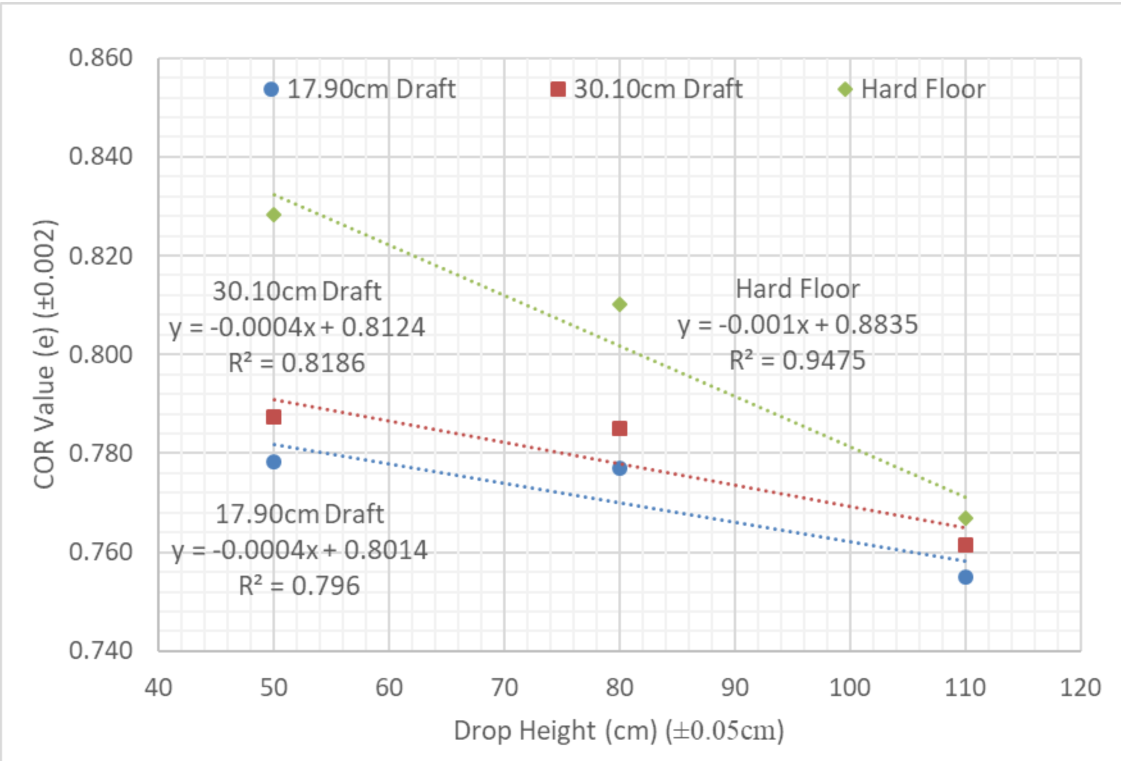

Figure 7: COR Value Versus Drop Height of Table Tennis Ball

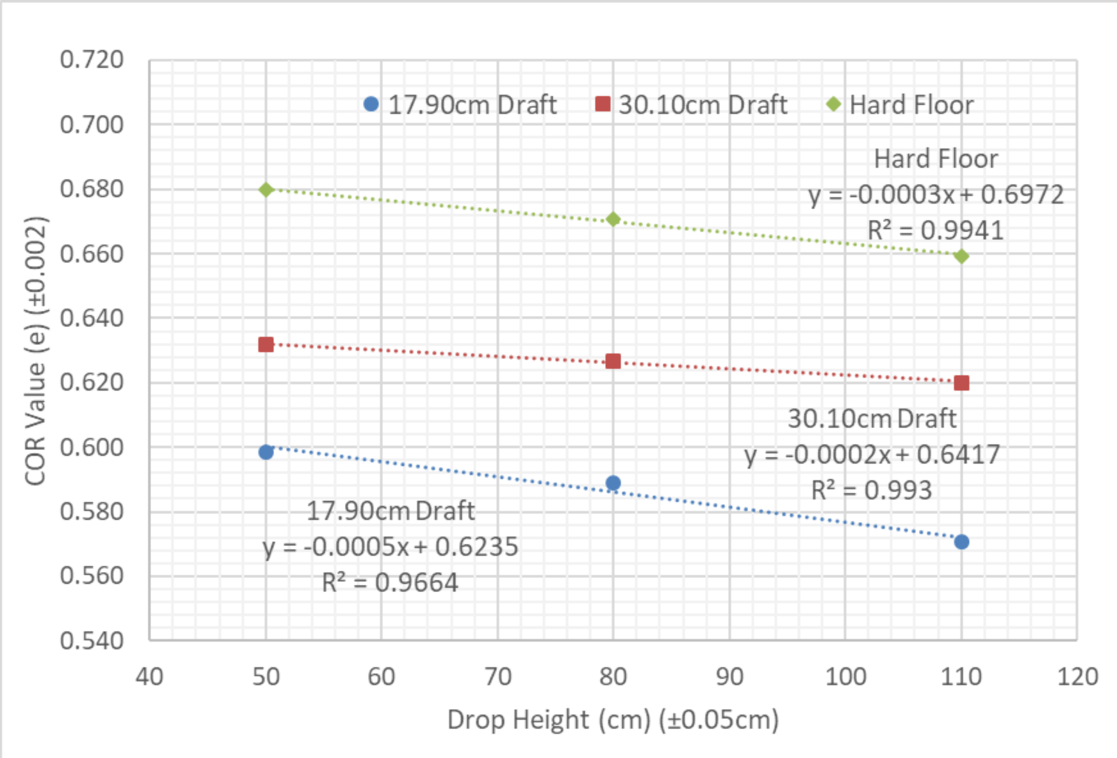

Figure 8: COR Value Versus Drop Height of Plastic Ball

As can be seen in the graphs, according to the individual elastic properties of the balls and their respective mass, the COR value showed variations. The typical results are given in Figures 9-11. Foam ball had the highest COR value at around 0.850-0.800 followed by table tennis ball and plastic ball. When the drop height increases, it is seen that the COR values decrease. Typical results are also shown in Cross (2011)[6]. The slopes of Figures 9-11 gives us the change in COR values per centimeter. All the slopes have negative values, denoting that as drop height increases, the COR values decrease. The COR and drop height are inversely proportional.

[6] Cross, Rod. Physics of Baseball & Softball. New York: Springer, 2011, p. 123, Fig. 8.5.

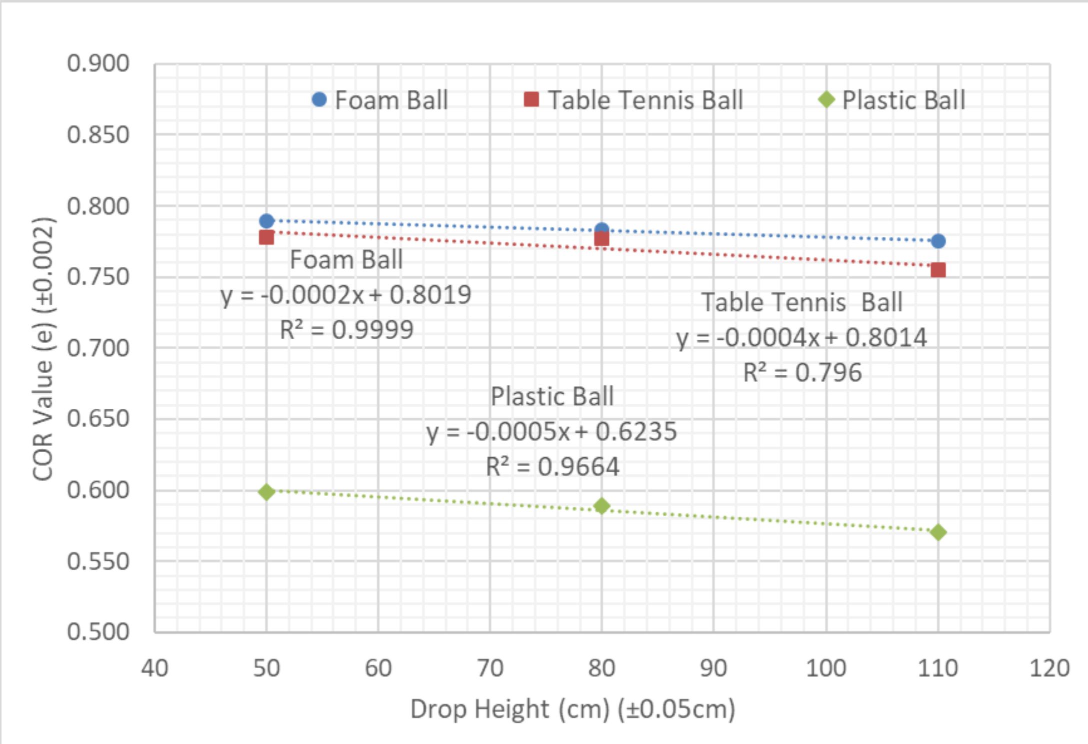

Figure 9: COR Values of All Balls, Submerged in Water, 17.90cm Draft

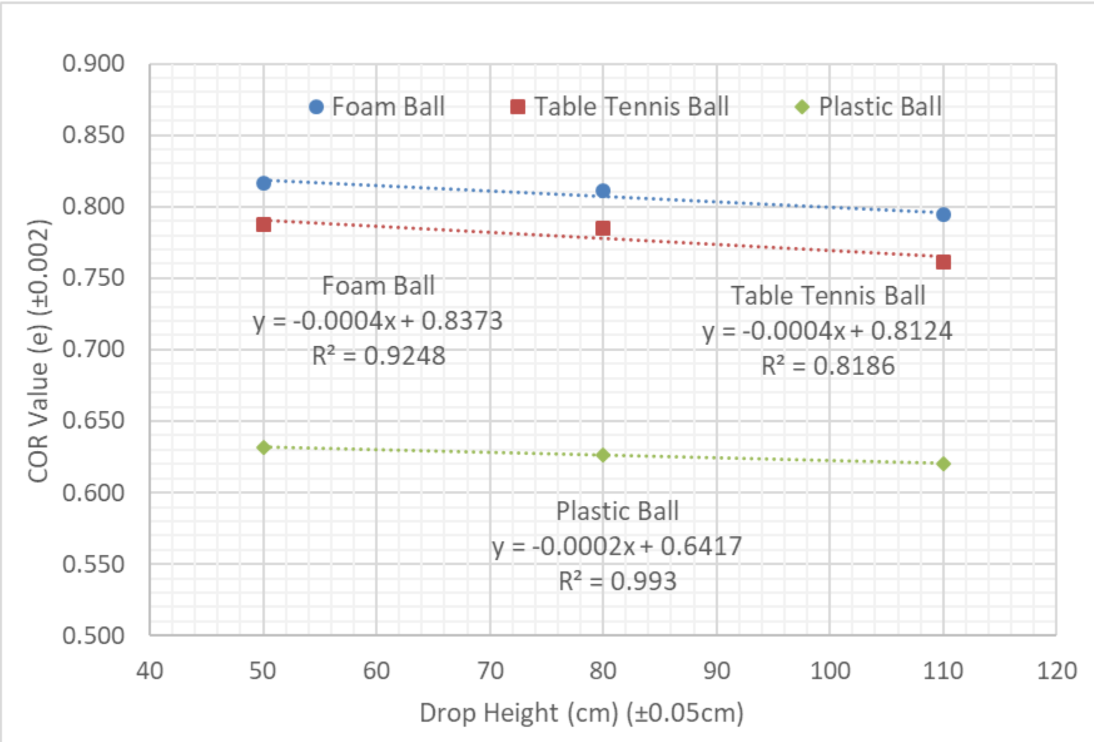

Figure 10: COR Values of All Balls, Submerged in Water, 30.10cm Draft

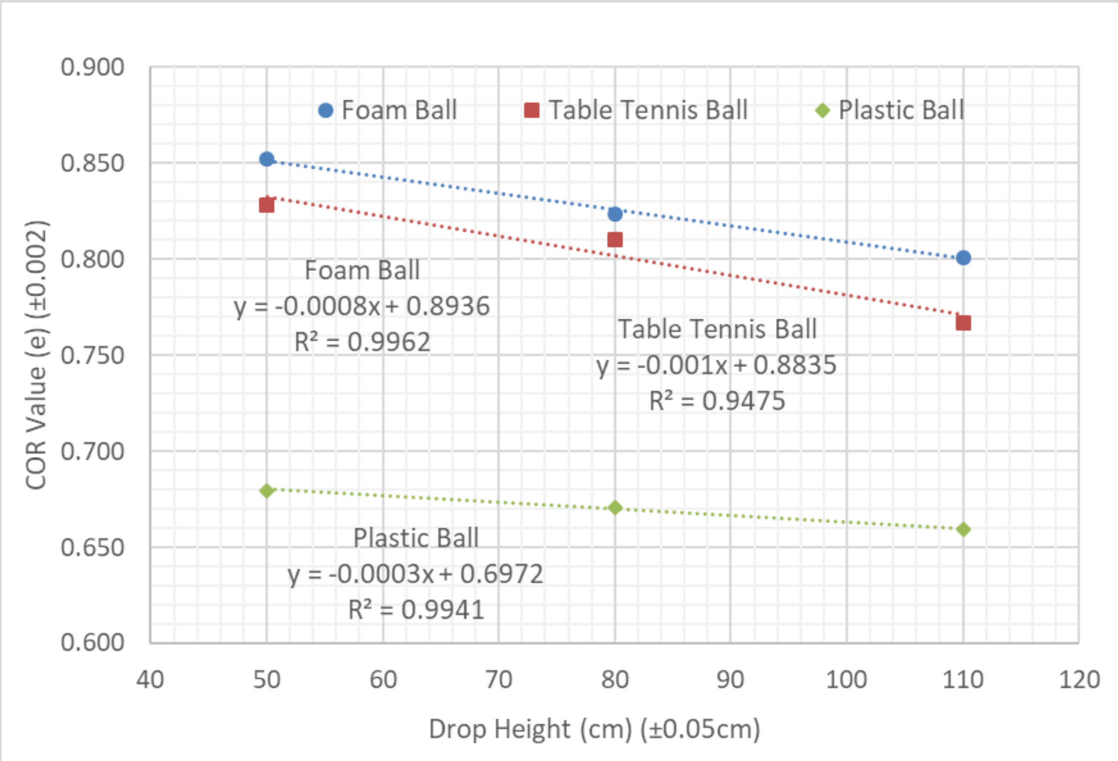

Figure 11: COR Values of All Balls, Hard Floor

5. Conclusion

Drop test experiments were conducted to investigate the variations of COR values between different types of balls and drop heights. Comparisons between target's mediums (i.e. hard floor and water tank) were done, thus enabling the observation of the effect of draft (T) on COR values and energy loss during collisions. It was seen that as the draft (T) of the rectangular prism was increased, the energy loss during collisions due to damping by the water was decreased. The collisions done on hard floor show the highest possible COR values for the balls and drop heights. As COR values give us a sense of energy loss, it can be safely assumed that the COR value difference between experiments done on hard floor and submerged in water is due to the energy dissipated in water.

The calculated COR values are related to the energy loss during the collisions. As the target's draft is increased, less energy is lost during the collisions. Compared to hard floor, collisions in the water tank get more and more elastic and COR values nearer that of those on hard floor as the draft is increased by means of weight additions. The COR values differed between the individual balls due to their materials. The total percentage uncertainty of the COR values were found to be ±0.002. The uncertainty is low as expected. As we used digital measurement devices whenever possible and analog devices with low uncertainties, the uncertainty of the COR values were also low. The R square values of most data was above 0.9 which shows that our data are strongly correlated. All the slopes have negative values as COR and drop height are inversely proportional. As drop height increases COR decreases. The slopes range from -0.0001 to -0.0010, which depicts the change in COR values per centimeter change in drop heights. The DIY dropping system proved to be a limitation as it took a long time to drop the balls. A more professional dropping system and experimental setup involving computer controlled suction cups can be used in future experiments.

For future experiments, it is proposed that digital measurement devices be used instead of rulers for more precise values. The energy loss can also be calculated and a relation between COR and energy loss can be more explicitly shown. Possible further investigations relevant to the topic may include the use of a cylindrical prism or a rectangular prism with a greater surface area for more in-depth research. Additionally, an investigation on the damping coefficient of water using data from this research can be studied.

References:

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