PROBABILISTIC DAMAGE STABILITY: Knowledge and Understanding

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ABSTRACT

The goal of this manuscript is to provide more knowledge for ship designers with respect to probabilistic damage stability (PDS). More precisely, the manuscript aims to give more insight to how certain changes in the arrangement and intact stability affect the PDS or A-index for a specific vessel. The following two objectives can be investigated: First, what is the effect on the A-index of changing the size of the particular tanks located along the ship, by changing the height of the horizontal deck?; secondly, what is the effect on the A-index of changing the intact stability of the ship, i.e. the ship's initial GM values for the three subdivision draughts d_S, d_P and d_L? The background for the abovementioned goal, is the introduction of the PDS regulations by IMO in 2009. Ship designers are forced to use the probabilistic approach instead of the deterministic approach (DDS), for certain vessel types when calculating damage stability. PDS offers more freedom than DDS in the design of the ship's internal watertight arrangement. However, since PDS calculations usually are conducted at late design stages, it may be challenging to utilise this flexibility due to time pressure. Thus, ship designers often rely on experience, since there is little time for research and optimisation. The author would therefore like to contribute with more knowledge and understanding to damage stability.

Keywords: Arrangement design, Damage Stability, Probabilistic Damage Stability

1. Introduction

Probabilistic damage stability (PDS) is a methodology based on accident statistics on ship-ship collisions. A probabilistic approach involves some degrees of uncertainty. Thus, 'random variables' are required to develop prediction models, means that accidents and the damage extent of accidents are unpredictable. The influence of these random factors is different for ships with different characteristics; for instance, differences in the range of permeability and service draught (IMO, 2008c). For each casualty, the size and location of the damage, and whether the vessel has sufficient buoyancy to remain afloat after the damage has occurred is noted. The philosophy behind the method is that two different ships with the same attained index of subdivision are equally safe. The initial ideas of regulations for damage stability that would be based on accidents statistics came from the German professor Kurt Wendel in 1960. He published an article with the title ''The Probability of Survival from Damages''. The International Maritime Organization (IMO) has later developed probabilistic regulations for damage stability based in this approach. The foundation for the probabilistic method is the probability that the vessel will suffer a certain damage multiplied with the survivability of the vessel after the damage has occurred. The method calculates the individual probabilities for all possible damage cases the ship can encounter multiplied with the survivability of each individual damage case. Survivability is defined as the vessels capability to stay afloat after being rammed by an arbitrary ship. The attained index, A, is the summation of the probability and survivability for all the possible damage cases. This attained index describes whether the vessel can sustain certain damages in a sufficient manner to ensure the safety of the crew and passengers aboard. The probabilistic damage stability regulations require that the value of the attained index is at least the value of the required index, R. The required index is easily calculated for each vessel based on the ship length and the number of passengers the vessel can carry (Olufsen and Hjort, 2013).

The PDS regulations that entered into force on the 1st of January 2009 as a part of SOLAS Chapter II-1, Part B-1 Stability, applies to dry cargo ships with a length of 80 m or above and all passenger ships with keel laying on or after this date. A passenger ship is per definition a ship carrying more than 12 passengers. In addition, because the 'Code of Safety for Special Purpose Ships, 2008' (SPS code) was adopted in 2008 by IMO Resolution MSC.266(84), Special Purpose Ships (SPSs) are also covered. Furthermore, all the ships applicable to the PDS regulations are required to have double bottom and automatic cross flooding arrangements that stabilize the ship within 10 minutes. On top of this, if the ship is carrying over 36 passengers, there are additional deterministic requirements (IMO, 2006, 2008b). In SOLAS Chapter II-1, Part B-1 Stability, Reg. 7, A is defined as the 'Attained Subdivision Index' and R is defined as the 'Required Subdivision Index'. Two different ships are considered equally safe if they have the same value of A. The calculation of A is based on the probability of damage, i.e. flooding of compartments, and the survivability of the ship after flooding.

In order to fully understand probabilistic damage stability, it is vital to comprehend where the method came from. Deterministic damage stability (DDS) was the dominating method for damage stability calculations before PDS. Ship stability is defined as the ship's capability to return to its initial upright position after a force, external or internal, has been applied on the ship. There are two main elements to evaluate the stability of the vessel, moment acting on the vessel due to acting force and the righting moment. The righting moment is defined by the hull shape and geometry of superstructure, whereas the acting moment can be wind, sea conditions or water intrusion that causes the ship to heel. Intrusion of water is the dominating factor that influences the damage stability of the vessel. The main principles for calculation on stability are the acting gravity force and the change of buoyancy forces. All compartments under the waterline contribute to the buoyancy acting on the vessel. If a compartment is bilged, water will fill the volume and the vessel loose buoyancy that causes the vessel to sink down. The underwater volume increases so the buoyancy force increases accordingly until equilibrium with the gravity force. In case of damage to either side of the vessel, the ship will heel over because of unsymmetrical buoyancy. If the damage causes loss in buoyancy that is larger than the remaining buoyancy, the vessel will ultimately sink. Regulations for damage stability were formulated to limit the risk of sinking and ensure the safety of people aboard. The deterministic damage stability method controls if the ship is safe enough. The method calculates if the ship can withstand certain damage scenarios depending on the ship beam and length. Calculations are made for different damage conditions and the vessel should fulfil the criteria's given by SOLAS in order to be certified by the classification societies. The requirements for DDS are dependent on vessel type, number of passengers, cargo, etc. The parameters are the same for each ship type but will change in magnitude (Patterson and Ridley, 2014). The advantage for DDS is that the calculations do not require advanced damage stability calculations, and the method gives a rapid impression of the ship's capabilities to withstand damage. However, the method gives little flexibility in the design and the deterministic rules cannot be used as a quantification of risk (Olufsen and Hjort, 2013).

It has been demonstrated in several accidents that the concept of rule damages of a predefined size, such as in DDS, is not sufficient in real life accident scenarios. This has led to the development of probabilistic damage stability regulations. The first implementation of the regulations was done in the early 1970s in IMO Resolution A.265 as an alternative to the deterministic damage stability regulations for passenger vessels. The probabilistic approach was, however, seldom used as the method involved heavier demands and considered more damage cases than the deterministic approach (Lauridsen et al., 2001). As mentioned, Harmonization of Rules and Design Rationale (HARDER) and Goal Based Damage Stability (GOALDS) are research projects performed with the objective to improve the regulations regarding probabilistic damage stability and to increase the knowledge and understanding of ship casualties in order to develop regulations (Papanikolaou, 2009; Papanikolaou et al., 2012).

2. Calculation procedure

The attained index measures the residual stability of the vessel considering all possible sizes of the damage. Each of the damages are weighted by the probability that such a damage can be expected, measured in terms of the factor P. The survivability of the vessel after the damage has occurred, is measured in terms of the factor S, and calculated from the properties on the associated residual stability curve. The factors S and P does not take the vertical extent of damage into consideration, thus a factor V is implemented in the calculations. V represents the probability that a vertical deck above the waterline will remain intact after the damage has occurred. Thousands of damage cases must be considered for probabilistic damage stability calculations on a vessel, necessitating extensive use of dedicated computer programs for accurate calculations (Lauridsen et al., 2001).

Probabilistic damage stability gives more freedom in the design since the designer is not obliged to follow the damage extents known from deterministic damage stability. As mentioned, when following the probabilistic damage stability regulations, the attained index, A, needs to fulfil the requirement of the required index R in Equation 1.

$$A \geq R \qquad (1)$$

Figure 1 illustrates a seven-zone division of a ship with the corresponding possible single- and multi-zone damages. The bottom line triangles indicate single-zone damages, while the parallelograms indicate multi-zone damages (Djupvik et al., 2015; IMO, 2008c; Lützen, 2001).

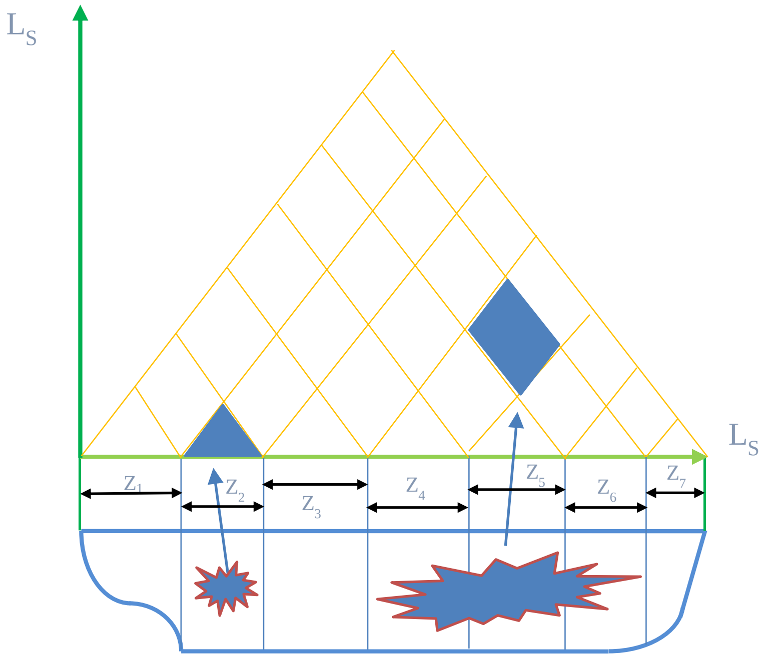


Figure 1. Possible single- and multi zone damages for a ship with 7 zones

It is not obvious how to use the regulations in a conceptual design process to obtain the most appropriate subdivision, as the method only results in a single measure in the attained index. The regulations for the calculation of the required and attained index are thoroughly described in the following sub-chapters. The regulations on probabilistic damage stability are taken from SOLAS chapter II-1 Part B: Stability. The explanatory notes on the probabilistic damage stability rules in SOLAS have been used to verify the interpretation of the regulations. As the regulations can be difficult to comprehend, it can support the understanding of the regulations by looking at a calculated damage case.

The current PDS regulations are based on damage statistics. More precisely, collision statistics. A ‘collision’ may be ship-to-ship or contact between a ship and an obstacle, e.g. an ice berg. For assessment of groundings there is no probabilistic approach available in the regulations, most likely due to lack of grounding statistics. Table 1 provides a complete overview of which ship types that follow the PDS approach and which ship types that follow the DDS approach.

Table 1. An overview of damage stability conventions and codes for different ship types (IMO, 2008b).

Code or Convention	**Ship Type**	**Method**
SOLAS - 2009	All Passenger Ships: • Pure passenger ships • Ro-Ro ships • Cruise ships	Probabilistic
SPS Code / SOLAS 2009	Special Purpose Ships	Probabilistic
SOLAS 2009	Dry Cargo Ships > 80 m in length: • Ro-Ro Cargo ships • Car carriers • General Cargo ships • Bulk carriers with reduced freeboard and deck cargo (IACS unified interpretation no.65) • Cable laying ships	Probabilistic
1966 Load Line Convention	Dry Cargo ships with reduced freeboard	Deterministic
1966 Load Line Convention / MARPOL 73/78 Annex I	Oil Tankers	Deterministic
International bulk chemical code	Chemical Tankers	Deterministic
International liquefied gas carrier code	liquefied gas carriers	Deterministic

Before explaining any further how the R- and A-indexes are calculated, it is useful to introduce a frequently used factor named subdivision length, which is denoted LS in the PDS regulations. It is important to distinguish between this length factor and the one used in the DDS regulations. Figure 2 illustrates how the subdivision length is determined for three different scenarios. As the figure shows, the subdivision length depends on the buoyant hull and the reserve buoyancy of the ship, and whether these ‘areas’ are harmed or not. The buoyant hull comprises the enclosed volume of the ship below the waterline, which is denoted ‘dS’ in the figure, while the reserve buoyancy is comprising the enclosed volume of the ship above the waterline. The maximum vertical damage extent is always equal to ds + 12.5 m measured from the baseline. (Hjort & Olufsen, 2014).

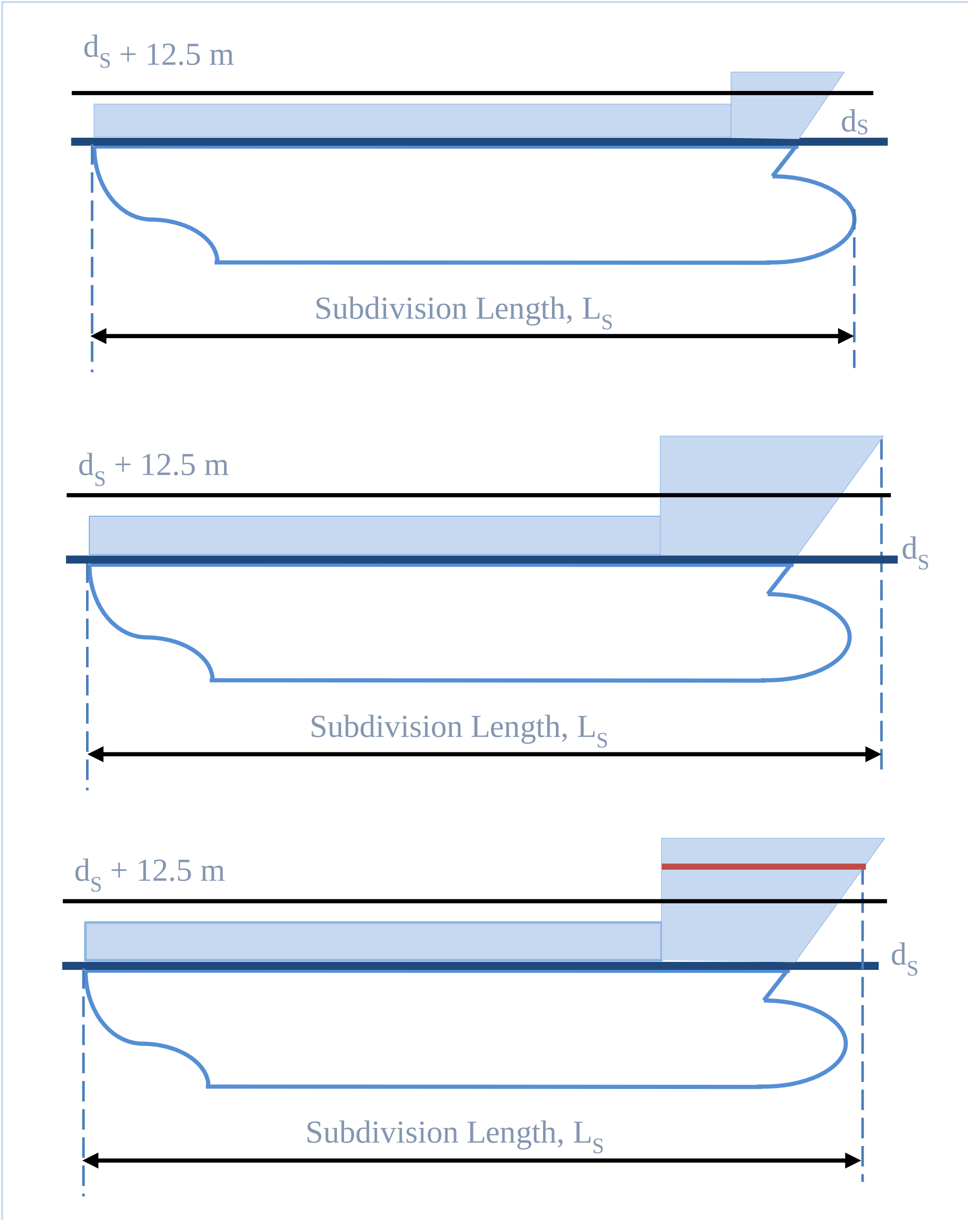


Figure 2. Examples of Subdivision Length.

2.1 Required subdivision index R (Reg. 6)

The required index for passenger vessels was formed through the HARDER project. Based on calculations on a sample of 40 passenger ships and 92 dry cargo ships, the degree of subdivision to be provided was proposed by formulas for the required subdivision index R. The subdivision length, Ls, is based on the buoyant hull and the reserve buoyancy of the hull. Explanation of how the subdivision length is found is presented in appendix B (Olufsen and Hjort, 2013). The formula is divided into three categories; passenger ships, cargo ships between 80 and 100 m and cargo ships larger than 100 m. The formula for passenger ships is calculated using equation 2 and depends on ship length and number of passengers the ship is certified for (IMO, 2014).

$$R = 1 - \frac{5000}{L_S + 2.5N + 15225} \qquad (2)$$

$N - N_1 + 2N_2$
$N_1 - Number\ of\ persons\ for\ whom\ lifeboats\ are\ provided$
$N_2 - Number\ of\ persons\ the\ ship\ is\ permitted\ in\ excess\ of\ N1$
$L_S -$ Subdivision $length$

The required index for cargo ships is only dependent on ship length. For cargo ships greater than 100 m in length, the required index R is calculated using equation 3 (IMO, 2014).

$$R = 1 - \frac{128}{L_S + 152} \qquad (3)$$

In the case of cargo ships less than 100 m in length and not greater than 80 m in length, R is calculated using equation 4 (IMO, 2014).

$$R = 1 - \frac{1}{1 + \frac{L_S}{100} \times \frac{R_0}{1 - R_0}} \qquad (4)$$

R_0 – The value of R calculated in Equation 3

2.2 Attained subdivision index A (Reg. 7)

The attained subdivision index A is calculated for multiple damage scenarios depending on the geometric complexity of the watertight arrangement on the vessel. The calculation of A requires understanding of the ships parameters and divisions, and which formulas to use for different vessel types. The attained index is acquired by the summation of the partial indices for three predefined service draughts according to equation 5(IMO, 2014).

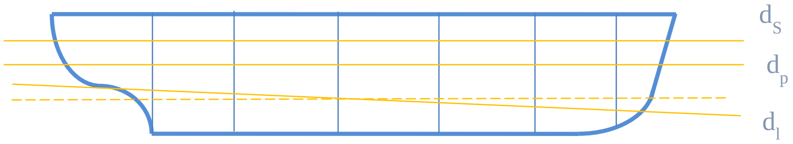


Figure 3. *Loading conditions (IMO, 2008c).*

$$A = 0.4A_S + 0.4A_P + 0.2A_L \qquad (5)$$

A_S – Attained index for deepest subdivision draught(on an even keel)
A_P – Attained index for partial subdivision draught(on an even keel)
A_L – Attained index for lightest service draught(not more than 1% of the length)

The attained indices are multiplied with factors representing the operation time in each loading condition. The factors are based on the assumption that the vessel operates 40% of its operation time in the deepest load line condition, 40% in the partial condition and 20% in the lightest service draught condition. For each partial index, the summation of all the possible damage cases must be calculated on the basis of the probability and survivability of damage, multiplied with the probability that the space above the horizontal subdivision will stay intact. The final attained index is calculated using equation 6 for the three draughts, and implementing the three attained indices in equation 5.

$$A_C = \sum_{1}^{i=t} P_i S_i V_i \tag{6}$$

where, $A_C > \begin{cases} 0.9R, & for\ passenger\ ships \\ 0.5R, & for\ cargo\ ships \end{cases}$

A_C - Attained index for particular loading condition
P_i - Accounts for the probability of flooding of a compartment or a group of compartments, disregarding any horizontal subdivision
S_i - Accounts for the probability of survival after the flooding of a compartment or a group of compartments, including the effect of horizontal subdivision (the V_i-factor);
V_i - The probability that the space above a horizontal subdivision is not flooded
i – Damage or damage zone under consideration
t – Number of damages that has to be investigated

To summarize, the P_i component depends on the geometry of the watertight arrangement of the ship and is a factor for the probability of suffering a specific damage. The S_i component depends on the survivability of the vessel after the damage has occurred for a specific damage case. The component V_i is implemented to include the vertical extent of the damage since P_i and S_i only includes the longitudinal and transverse extent. The V_i factor represents the probability that a deck above the damage will remain intact. The following paragraphs will explain the calculations behind these components. The definition of the different factors is repeated to ensure the reader's understanding of the central factors in the probabilistic damage stability regulations.

2.3 Calculation of the factor P_i (Reg. 7–1)

P_i is the probability of a specific damage on the vessel, i.e. that a compartment or group of compartments are flooded. The factor is solely dependent on the geometry of the watertight arrangement. The formula for calculation of P_i is shown in equation 7 (IMO, 2014).

$$P_i = p(x1_j, x2_j) \cdot \left[r(x1_j, x2_j, b_k) - r(x1_j, x2_j, b_{k-1}) \right] \tag{7}$$

j – The aftmost damage zone number involved in the damage starting with no.1 at stern
k – Number of particular longitudinal bulkhead as barrier for transverse penetration
x1 – Distance from aft end of the ship to the aft end of the zone in question
x2 – Distance from aft end of the ship to the forward end of the zone in question
b – Mean transverse distance from Shell to longitudinal barrier

r – Factor to account fort he transversal extent of damage

The formula in equation 7 applies for damages of single zones only. According to SOLAS Chapter II-1, Part B-1, Regulation 7-1, the below equations should be used to calculate the P_i-factor in case of multi-zone damages (IMO, 2006).

- If the damage involves two adjacent zones:

$$\begin{aligned} P_i = {} & P(x1_j, x2_{j+1}) \cdot \left[r(x1_j, x2_{j+1}, b_k) - r(x1_j, x2_{j+1}, b_{k-1}) \right] \\ & - P(x1_j, x2_j) \cdot \left[r(x1_j, x2_j, b_k) - r(x1_j, x2_j, b_{k-1}) \right] \\ & - P(x1_{j+1}, x2_{j+1}) \cdot \left[r(x1_{j+1}, x2_{j+1}, b_k) - r(x1_{j+1}, x2_{j+1}, b_{k-1}) \right] \end{aligned} \tag{8}$$

- If the damage involves three or more adjacent zones:

$$\begin{aligned} P_i = {} & P(x1_j, x2_{j+n-1}) \cdot \left[r(x1_j, x2_{j+n-1}, b_k) - r(x1_j, x2_{j+n-1}, b_{k-1}) \right] \\ & - P(x1_j, x2_{j+n-2}) \cdot \left[r(x1_j, x2_{j+n-2}, b_k) - r(x1_j, x2_{j+n-2}, b_{k-1}) \right] \\ & - P(x1_{j+1}, x2_{j+n-1}) \cdot \left[r(x1_{j+1}, x2_{j+n-1}, b_k) - r(x1_{j+1}, x2_{j+n-1}, b_{k-1}) \right] \\ & + P(x1_{j+1}, x2_{j+n-2}) \cdot \left[r(x1_{j+1}, x2_{j+n-2}, b_k) - r(x1_{j+1}, x2_{j+n-2}, b_{k-1}) \right] \end{aligned} \tag{9}$$

Where, $r(x1, x2, b_0) = 0$

p(x1, x2) is an expression for the probability of damage length in the longitudinal direction. The data on damage lengths collected from the HARDER project concluded in that the deterministic damage length used in the present SOLAS passenger ships regulations (0.03L + 3m, max 11m) did not give satisfactory results when compared to the actual damage length collected from collision accidents (Olufsen and Hjort, 2013). The work also concluded in that the damage location distribution was not significant. To simplify the calculations, the nondimensional damage location was set equal to 1, signifying an equal probability for damage along the whole ship length (Lützen, 2001; IMO, 2014). A bi-linear function has been used to describe the non-dimensional damage length. The parameters are described as fractions as it was considered easier to implement in the regulations. These bi-linear functions in equation 10, proposed by Lützen, were implemented in the SOLAS revision.

$$b(x) = \begin{cases} b_{11} \cdot x + b_{12} & for \quad x \le J_k \\ b_{21} \cdot x + b_{22} & for \quad x > J_k \end{cases} \tag{10}$$

J_k – Knuckle point on the red curve
x – nondimensional damage length

The subsequent coefficients are derived using the non-dimensional damage lengths and the bi-linear function in equation 8.

$$b_{11} = 4\frac{1-p_k}{(J_m - J_k)J_k} - 2\frac{p_k}{J_k^2}$$ b_{12} = independent on ship length

$$b_{21} = -2\frac{1-p_k}{(J_m - J_k)^2}$$ $b_{22} = -b_{21}J_m$

The J_m factor in the expressions for the coefficients is the maximum non-dimensional damage length for the ship under consideration, and J_{kn} is the knuckle point in the distribution. As the damage statistics varies with ship length, the factors J_m, J_k and b_{12} varies consequently (IMO, 2014).
In cases where $L_s \leq L^*$:

$$J_m = \min\left(J_{\max}, \frac{l_{\max}}{L_s}\right)$$

$$b_{12} = b_0 = 2\left(\frac{p_k}{J_{kn}} - \frac{1-p_k}{J_{\max} - J_{kn}}\right)$$

$$J_k = \frac{J_m}{2} + \frac{\sqrt{1 + (1-2p_k)b_0 J_m + \frac{1}{4}b_0^2 J_m^2}}{b_0}$$

If L_s is below 198 m, J_{max} will be the smallest value and consequently used for J_m. Thus J_k will be constant for all vessels below 198 m (Djupvik, 2014).
In the cases where $L_s > L^*$ the two factors J_m^* and J_k^* are used as the number of damages for vessels with a length above L^* (260 m) are low, causing deviations in the distribution functions.

As a solution, the distribution functions only yield for vessels with length less than L^*. In cases where the ship length is greater than L^*, the factors J_m^* and J_k^* are used and converted to J_m and J_k according to the following calculations (Olufsen and Hjort, 2013).

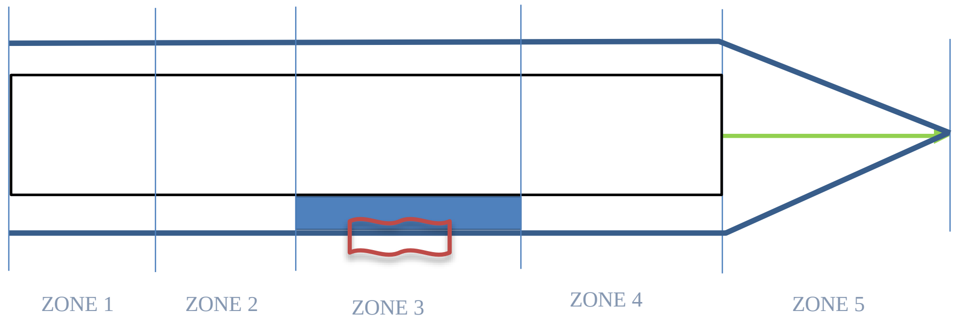
$$J_m^* = \min\left(J_{\max}, \frac{l_{\max}}{L^*}\right)$$

$$J_m^* = \frac{l_{\max}}{L^*}$$

$$J_m = \frac{J_m^* \cdot L^*}{L_S}$$

$$J_k^* = \frac{J_m^*}{2} + \frac{\sqrt{1 + (1-2p_k)b_0 J_m^* + \frac{1}{4}b_0^2 J_m^{*2}}}{b_0}$$

$$J_k = \frac{J_k^* \cdot L^*}{L_S}$$



$$b_{12} = 2\left(\frac{p_k}{J_k} - \frac{1-p_k}{J_m - J_k}\right)$$

When J_m, J_k and b_{12} are found, the normalized damage length, J_n can be calculated. J_n is used to calculate p(x1, x2) (IMO, 2014).

$$J = \frac{(x2 - x1)}{L_S} \qquad\qquad J_n = \min\,(J, J_m)$$

J – The non-dimensional damage length
J_n – The normalized length of a compartment or group of compartments

Which equation for calculating p(x1, x2) is dependent in the damage case considered.
The three different alternatives are presented (IMO, 2014):

1. In cases where neither limits of the compartment or group of compartments under consideration coincides with the aft or forward terminal. In other words, if the damage under consideration is not located at the aft or forward end of the vessel, p(x1, x2) should be calculates using equation 11.

$$J \le J_k : p(x1, x2) = p_1 \quad \Rightarrow \quad p_1 = \frac{1}{6} J^2 (b_{11} J + 3b_{12}) \tag{11}$$

$$J > J_k : p(x1, x2) = p_2$$

$$p_2 = -\frac{1}{3} b_{11} J_k^3 + \frac{1}{2}(b_{11} J - b_{12}) J_k^2 + b_{12} J J_k - \frac{1}{3} b_{21}\left(J_n^3 - J_k^3\right) + \frac{1}{2}(b_{21} J - b_{22})\left(J_n^2 - J_k^2\right)$$
$$+ b_{22} J (J_n - J_k)$$

2. In cases where one of the sides, forward or aft, of the compartment or group of compartments coincides with the forward or aft terminal. In other words, if the damage under consideration is located either at the aft end or at the forward end of the vessel, p(x1, x2) should be calculated according to equation 10, where p1 and p2 are calculated as in equation 12.

$$J \le J_k \quad \Rightarrow \quad p(x1, x2) = \frac{1}{2}(p_1 + J) \tag{12}$$

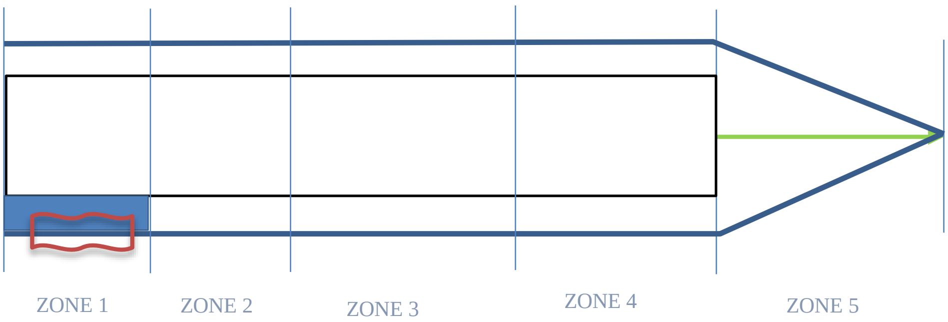
$$J > J_k \quad \Rightarrow \quad p(x1, x2) = \frac{1}{2}(p_2 + J)$$

3. In cases where one of the compartment or group of compartments considered extends over the entire subdivision length Ls, p(x1, x2) should be calculated using equation 13.

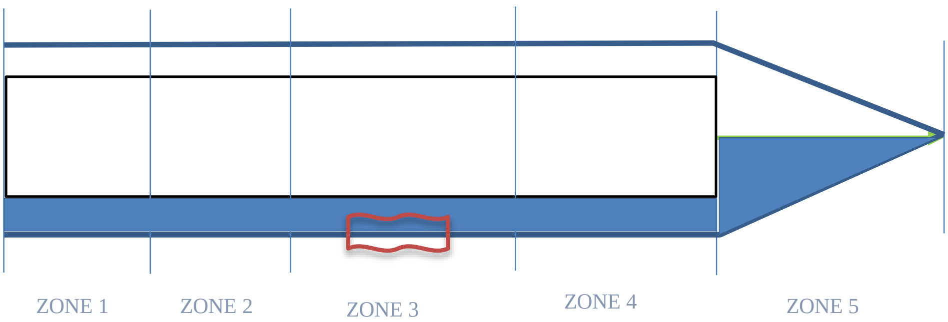
$$p(x1, x2) = 1 \qquad (13)$$

2.3.1 Calculation of the r(x_{1j}, x_{2j}, b_k) factor

The factor r is the probability that a penetration is less than a given transverse breadth, b. The factor is based in damage statistics using the same approach as the p(x1, x2) factor. Equations for calculating r(x1j, x2j, bk) are derived from damage statistics of more than 400 cases collected by the HARDER project. The data presented as damage penetrations as a function of the ships' breadth. The line dividing the penetrations at the B/5 limit is implemented for comparison. The B/5 limit is used in the deterministic regulations (Olufsen and Hjort, 2013).



The non-dimensional penetration depth b, is measured from the deepest subdivision draught as a transverse distance from the ship side, normal to the centreline, to the longitudinal barrier. In cases where the longitudinal barrier is not parallel to the ship hull, an assumed line should determine the distance b. The distance b from the hull to a longitudinal boundary can be found according to



SOLAS Resolution MSC.281(85) – Explanatory Notes. Examples of how this is done are shown in the figure 4 below.

Calculation of the r factor is done using equation 14 (IMO, 2014):

$$r(x1, x2, b) = 1 - (1 - C) \cdot \left[1 - \frac{G}{p(x1, x2)}\right] \tag{14}$$

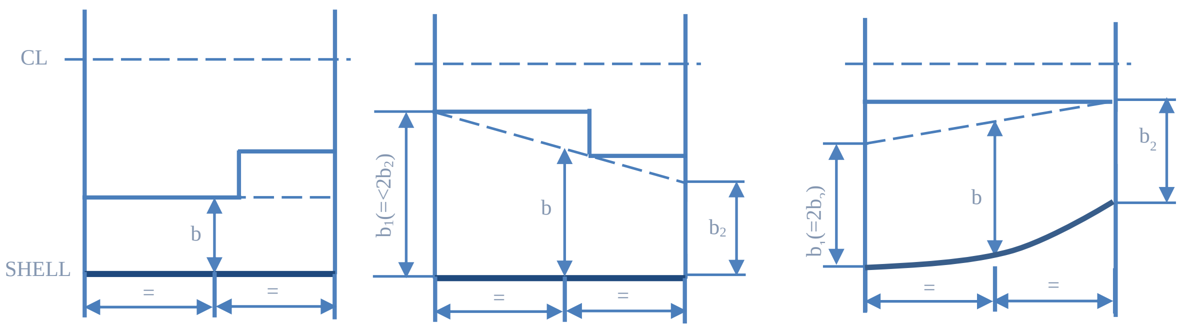


Figure 4. Calculation of the penetration depth b according to SOLAS Resolution MSC.281(85).

$$C = 12 \cdot J_b \cdot (-45 \cdot J_b + 4)$$

$$J_b = \frac{b}{15B}$$

b – Penetration depth

B – Maximum ship beam at deepest draught
When calculating G in r(x1j, x2j, bk), the same conditions are used as for the selection of p(x1,x2) in chapter 4.3.3 (IMO, 2014):
In the case where the compartment or groups of compartments considered extends over the entire subdivision length, G should be calculated using equation 15.

$$G = G_1 = \frac{1}{2} b_{11} J_b{}^2 + b_{12} J_b \tag{15}$$

In the case where neither limit of the compartment or group of compartments under consideration coincides with the aft or forward terminals, G should be calculated using equation 16.

$$G = G_2 = -\frac{1}{3} b_{11} J_0{}^3 + \frac{1}{2}(b_{11} J - b_{12}) J_0{}^2 + Jb_{12} J_0 \tag{16}$$

$$J_0 = \min(J, J_b)$$

In the case where the aft limit of the compartment or group of compartments under consideration coincides with the aft terminal, or the forward limit of the compartment of group of compartments under consideration coincides with the forward terminal, G should be calculated using equation 17.

$$G = \frac{1}{2}(G_2 + G_1 J) \tag{17}$$

A more thorough explanation and derivation of the formulas for p(x1, x2) and the r factor can be found in Marie Lützen's PhD thesis, ''Ship Collision Damage''.

2.4 Calculation of the S_i factor (Reg. 7–2)

The Si factor is dependent on the survivability of the vessel after a specific damage has occurred. A survivability factor of 1 denote that the vessel will survive flooding of the specific damage case, while a factor of 0 denotes that the vessel will not survive. The factor is calculated using equation 18 (IMO, 2014):

$$S_i = \min\{S_{intermediate,i}, S_{final,i} \cdot S_{mom,i}\} \tag{18}$$

$S_{intermediate,i}$ – Probability to survive all intermediate flooding stages until the final equilibrium stage
$S_{final,i}$ – Probability to survive in the final equilibrium stage of flooding
$S_{mom,i}$ – Probability to survive heeling moments

To collect data, the HARDER project investigated the wave height distributions at the time of the accidents in the casualty database. The project suggested that within a sea state range between 0 to 4 m, the proposed GZ criteria would be rather accurate for prediction of the vessel's survival. GZ is the distance of the righting arm that gives a righting moment on the vessel using the buoyancy force. There are no requirements for stability in the intermediate stage for cargo ships, i.e. for cargo ship $S_{intermediate,i}$ is set equal to 1. The heeling angle GZmax, range of positive GZ and the equilibrium of the heeling angle make the foundation for the calculation of S. Several of the criteria in regulation 7- 2 appear as deterministic. The probabilistic element enters with the probability of successful evacuation that will increase if the static heeling angle is low and if the evacuation route will not be impeded by water. The S factor is highly related to the distribution of residual buoyancy, it is therefore combined with the probability that the watertight decks will remain intact.

S_{final} is calculated using equation 19 (IMO, 2014):

$$S_{final} = K \cdot \left[\frac{GZ_{\max}}{0.12} \cdot \frac{Range}{16}\right]^{\frac{1}{4}} \tag{19}$$

$$K = \sqrt{\frac{\theta_{\max} - \theta_e}{\theta_{\max} - \theta_{\min}}}$$

$K = 1 \quad if \quad \theta_e \leq \theta_{\min} \ , K = 0 \quad if \quad \theta_e \geq \theta_{\max}$

θ_e - Equilibrium heeling angle after damage

$\theta_{\min}$ - Minimum heeling angle

$\theta_{\max}$ - Maximum heeling angle

Passenger Ships: $\theta_{\min} = 7 \ \deg rees \ and \ \theta_{\max} = 15 \ \deg rees$

Cargo Ships: $\theta_{\min} = 15 \ \deg rees \ and \ \theta_{\max} = 30 \ \deg rees$

Range – The range with positive righting arm

The K value is based on the obtained heeling angle and is applied to give satisfactory heeling angles for the different ship types. The equation above shows that if the vessel heel more than 15 degrees for passenger vessels and 30 degrees for cargo vessels, the value of Si will be equal to 0. The ship designer has to be cautious when designing the arrangement in order to prevent larger heeling angles than the maximum values. The damage states for such scenarios would not contribute to the attained index. It is a common design measure to leave out longitudinal bulkheads in the double bottom in order to get symmetrical damages and thus avoid excessive heeling (Djupvik, 2014).
GZmax is measured in meters and is the maximum righting arm. The value should be between θe and θv, where θv is the angle where GZ gets negative or when a nonwatertight opening is submerged. Range is measured in degrees and is the distance between θe and θv. A typical GZ curve is illustrated in figure 16, where the different parameters mentioned are displayed. Figure 5a and 5b illustrate a case where GZmax is reached before the actual GZmax for the vessel, caused by an opening being submerged when the heeling reaches the θv value. Designers locate all

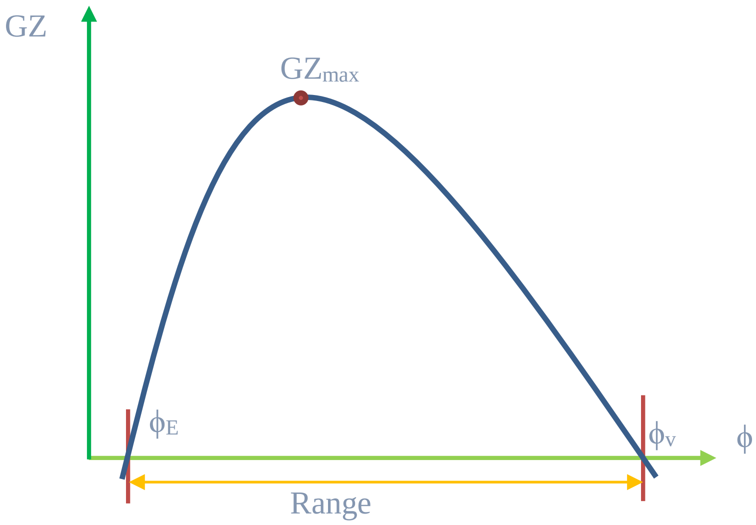


Figure 5a. GZ curve (Djupvik,2014)

openings a certain distance above first deck to avoid this scenario of cutting the GZ curve before reaching maximum value (Djupvik, 2014).

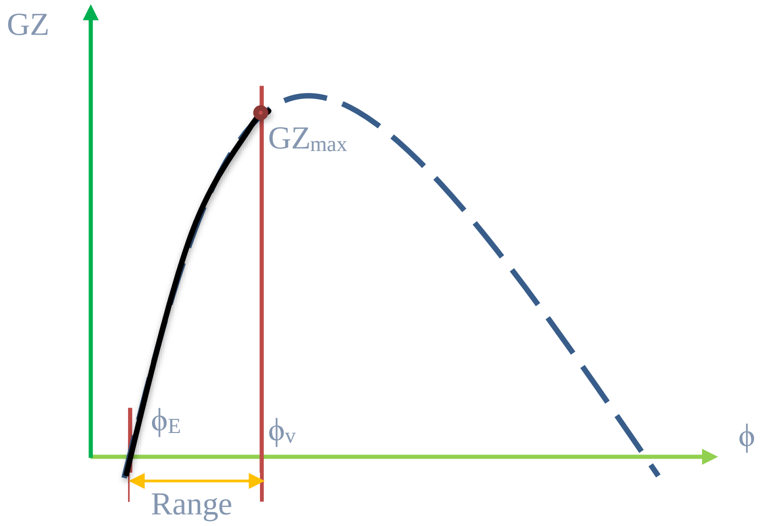


Figure 5b. GZ curve submerged opening(Djupvik,2014)

$S_{intermediate,i}$ is only calculated for passenger vessels and is calculated using equation 20 (IMO, 2014):

$$S_{intermediate} = K \cdot \left[\frac{GZ_{max}}{0.05} \cdot \frac{Range}{7} \right]^{\frac{1}{4}} \quad (20)$$

GZ_{max} - is not to be taken as more than 0.05 m

Range is not to be taken more than 7 degrees

$S_{intermediate} = 0$, if the heeling angle exceeds 15 degrees

$S_{intermediate,i}$ – is calculated similar to the calculation of S_{final} for all intermediate stages of flooding

$S_{mom,i}$ is the probability to withstand heeling moments from wind, movement of passengers or movement of survival crafts. The calculations for $S_{mom,i}$ are based on the vessel's displacement, GZ_{max} and M_{heel}. For cargo vessels $S_{mom,i}$ is always set equal 1. For passenger vessels, the factor is calculated using equation 21 (IMO, 2014):

$$S_{mom,i} = \frac{(GZ_{max} - 0.04) \cdot Displ}{M_{heel}} \quad (21)$$

$M_{heel} = \max(M_{passenger}, M_{wind}, M_{survival\ craft})$

$S_{mom} \leq 1$, the factor can never have a value larger than 1

$$M_{passenger} = (0.075 \cdot N_p) \cdot (0.45 \cdot B)$$

N_p – Maximum number of passengers permitted

B – Ship Beam

$$M_{wind} = \frac{P \cdot A \cdot Z}{9.806}$$

P = 120 N/m^2
A – Projected wind area
Z – Distance from projected wind area to T/2
T – Ship draught

$M_{survivalcraft}$ is the maximum assumed heeling moment from launching a fully loaded survival craft on one side of the ship. After calculating the three moments, the maximum of the values is used as M_{heel}.

2.5 Calculation of the V_i factor (Reg. 7–2)

The Vi factor is the probability that a deck above the waterline will not be breached after an arbitrary ship has struck the ship. Vi is implemented in order to account for contributions from the horizontal divisions, as the buoyancy above the waterline will affect the residual ship stability. If a compartment above the waterline is submerged, it will influence the buoyancy, thus influencing the GZ-curve, and thus affecting the S factor. The Vi factor is calculated using equation 22 (IMO, 2014):

$$V_i = v(H_m, d) - v(H_{(m-1)}, d) \tag{22}$$

$H_m - Least\ height\ to\ first\ horizontal\ boundary\ above\ the\ waterline,\ measured\ from\ the\ baseline.\ The\ horizontal\ boundary\ must\ limit\ the\ extent\ of\ flooding\ vertically\ and\ and\ be\ within\ the\ longitudinal\ range\ of\ the\ damage$
$H_{m-1} - Least\ height\ to\ (m-1) horizontal\ boundary\ above\ the\ waterline,\ measured\ from\ the\ basline.\ The\ horizontal\ boundary\ must\ limit\ the\ extent\ of\ flooding\ vertically\ and\ be\ within\ the\ longitudinal\ range\ of\ the\ damage$
$m - Horizontal\ boundary\ upwards\ from\ the\ waterline$
$d - Draught$
$V_i\ should\ in\ no\ cases\ be\ taken\ as\ less\ than\ zero\ or\ more\ than\ one$
v(H_m, d) and v($H_{(m-1)}$, d) are calculated as follows (IMO, 2014):

$$For\quad (H-d) < 7.8:\quad v(H,d) = 0.8\frac{(H-d)}{7.8}$$

$$For\quad (H-d) \geq 7.8:\quad v(H,d) = 0.8 + 0.2\frac{(H-d)-7.8}{4.7}$$

$v(H_m,d) = 1$ *if* H_m *coincides with the uppermost waterthight boundary of the ship within the longitudinal range of the damage*
$v(H_0,d) = 0$

Equation 22 is developed using the statistics collected from the HARDER Project presented in figure 6 below. The red line in figure 6 represents the formula for Vi where (H – d) is the distance between the initial waterline and the horizontal limit above the damage. The damage extent is limited to 12.5 m above the waterline.

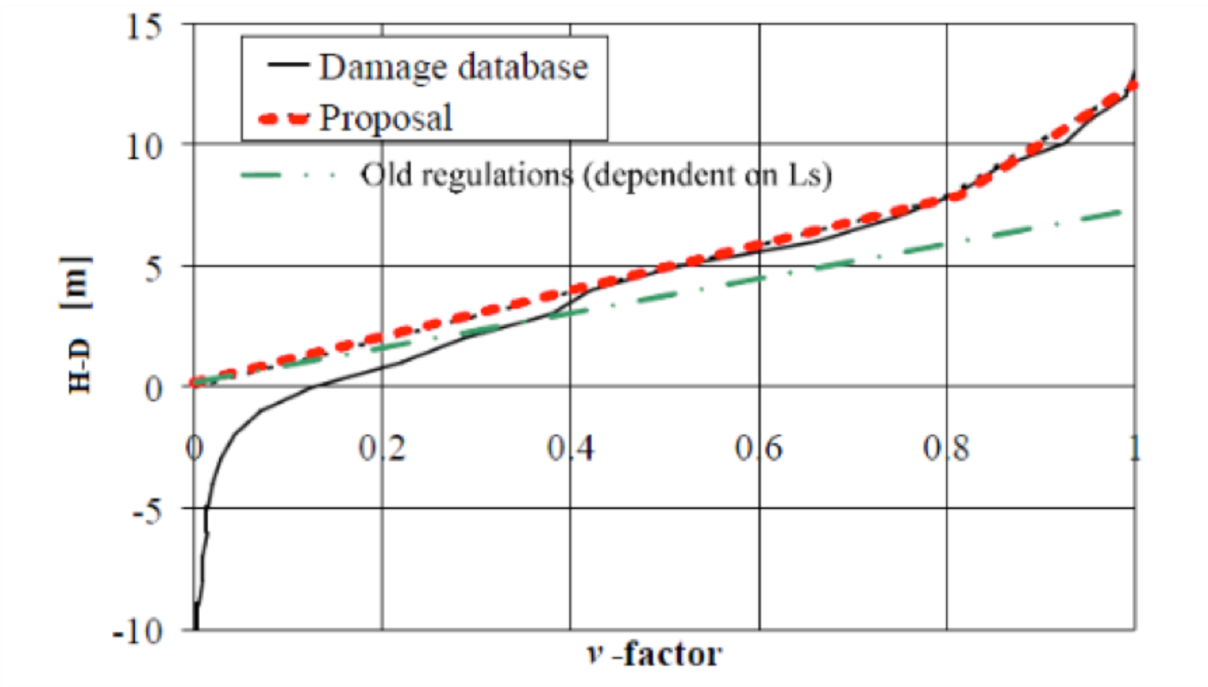


Figure 6. Vertical damage distribution (Olufsen and Hjort, 2013)

When calculating the Vi factor for a specific damage case, the decks that must be considered are the ones affected by the damage, located above the waterline. Affected is meant by the decks that are connected to the damage, i.e. the decks that have been breached by the damage and the top deck that limits the damage.
As an example, Figure 7 shows a scenario where both the wing ballast tank and U-tank is damaged. It furthermore displays how the H_m and H_{m-1} values should be taken for this damage case.

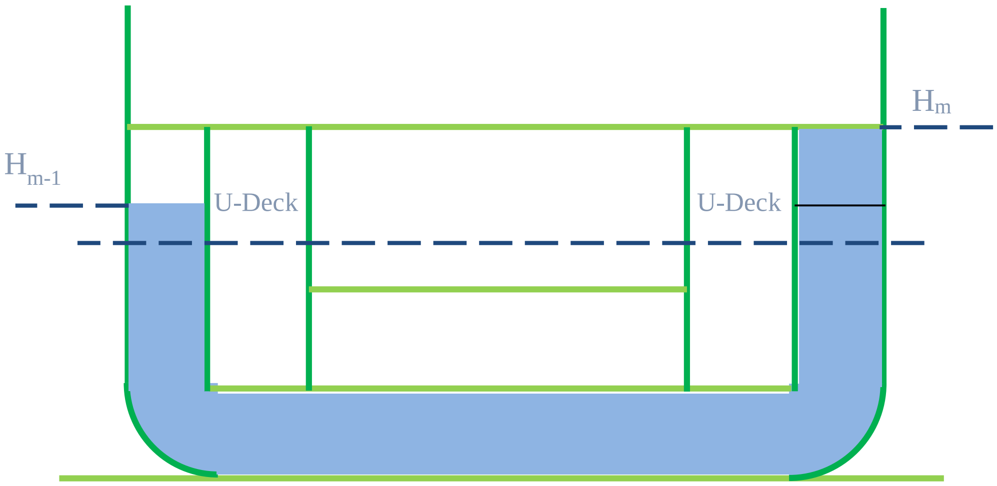


Figure 7. Wing ballast tank and U-tank damaged.

3. GM limiting curve

Metacentric height (GM) is the distance from the vessel's centre of gravity to its metacentre. A large GM value implies great initial stability, as the ability to return to upright position after being exposed for an external force causing the vessel to heel is great. The GM value affects the natural period of roll where large values are associated with short roll periods, which can be uncomfortable for passengers and crew. Passenger vessels are therefore usually designed with GM values that are sufficiently high, but not as high that it will cause rapid roll motions under operation.

There is a requirement that the ship has to operate within the GM limiting curve. The minimum curve defines the vessel's acceptable operational area, and is dependent on draught, trim and the vertical centre of gravity. The limiting curve should be used when determining the loading conditions for the vessel under consideration. Loading conditions with a GM above the limit curves ensures that the vessel operates under compliance with the stability criteria. The GM values used for calculation of the attained index are the basis for the limiting curves that the vessel has to operate within. Figure 7 present an example for a GM limiting curve.

The calculations on the deepest and partial draught are normally done for level trim. At the lightest service draught the actual trim may be used (Olufsen and Hjort, 2013).

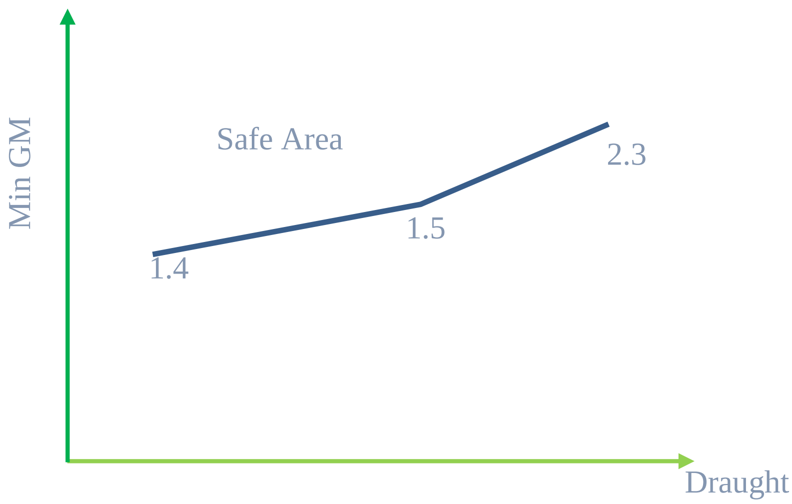


Figure 7. Example of GM limiting curve (Olufsen and Hjort, 2013)

4. Conclusions

This paper presents PDS calculations in terms of knowledge and understanding. Generally:

- It can be concluded that the height of U-deck certainly influences the A-index.
- The main contributions to the A-index come from the Si-factor, most likely due to heeling moments that causes changes in the equilibrium heel angle. However, the Vi-factor is also influential, but the impact is not as consistent for all U-deck heights as compared to the Si-factor.
- The Pi-factor does not contribute to changes in A-index, since changes to the arrangement only are made in vertical direction.
- The A-index increases almost proportionally with the GM values according to the 'PDS criterion'.
- The only factor that contributes to changes in A-index is the Si factor; in such cases the A-index is heavily dependent on the heeling moment caused by potential damage. The heeling moment causes a change in equilibrium heel angle, which in turn affects the Si-factor.
- The most important uncertainties would be related to the Si-factor. The development of the Si-factor for different GM values are clear. The Si-factor changes when the heeling moment changes; a larger heeling moment causes a reduction in the Si-factor.
- The A-index does in general obtain a larger value when the size of the wing ballast tanks is reduced. However, it is not considered relevant to install smaller wing ballast tanks than the size corresponding to this U-deck height.
- The A-index is generally better for larger initial GM values, because the increase in GM values results in smaller heeling moments in case of damage.
- There are some uncertainties related to the analysis of the Vi-factor based on different U-deck heights. It should be investigated more damage zones in general and damage cases penetrating longitudinal bulkheads within each zone.
- Additionally, it could be investigated why the A_S-index or the A_L-index seems to be most critical.

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Appendix

List of Symbols

A Attained Subdivision Index ('A-index')

A_P Attained Subdivision Index for a partial loading condition

A_i Attained Subdivision Index for a specific damage case at a particular loading condition

b Mean transverse distance between the shell (hull) and a longitudinal barrier
d Draught in question
dA Contribution to A in the event of horizontal subdivision above the waterline
d_s Deepest subdivision draught
d_P Partial subdivision draught
d_L Light service draught
Δ Displacement of the ship
G Centre of gravity
GZ_{max} Max. GZ value; usually the peak of the GZ curve, but not always
$H_{j,n,m}$ '…the least height above the baseline, in metres, within the longitudinal range of x1(j)...x2(j+n-1) of the mth horizontal boundary which is assumed to limit the vertical extent of flooding for the damaged compartments under consideration…'
$H_{j,n,m-1}$ '… the least height above the baseline, in metres, within the longitudinal range of x1(j)...x2(j+n-1) of the (m-1)th horizontal boundary which is assumed to limit the vertical extent of flooding for the damaged compartments under consideration…'
j The aftmost involved damage zone number, starting with number 1 at the stern
j '…signifies the aft terminal of the damaged compartments under consideration…' (Related to the 'v-factor')
J Non-dimensional damage length
J_{max} Overall normalised maximum damage length = 10/33
J_k / J_{kn} Knuckle point in the damage statistics distribution from the HARDER project
J_m The maximum non-dimensional damage length for the particular ship in question
J_n The normalised length of a compartment or group of compartments
k No. particular longitudinal bulkhead functioning as a barrier for transverse penetration
K Intersection point between the line that goes through GM and the keel
L_S Subdivision length; the 'ship length' used in the 'PDS regulations'
Lmax Maximum absolute damage length = 60 m
L* Length where normalised distribution ends = 260 m
m Represents each horizontal boundary counted upwards from the waterline under consideration
M Metacentric height
M_{heel} Maximum assumed heeling moment
$M_{passenger}$ Moment caused by movement of passengers
M_{wind} Moment caused by wind
$M_{survivalcraft}$ Moment caused by davit-launching of survival crafts
N_1 Number of persons for whom lifeboats are provided
N_2 Number of persons in excess of N1, including officers and crew
N N1 + 2N2
n The number of adjacent damage zones involved in the damage
N_P The maximum number of passengers permitted to be on board in the service condition, corresponding to the deepest subdivision draught
p_i Probability that a specific damage condition occurs
p_k Cumulative probability at Jkn = 11/12
p(x1,x2) Accounts for the probability of the considered longitudinal damage extent
r(x1,x2,b) A factor accounting for the transverse damage extent
Range Distance between θe and θv
R Required Subdivision Index
R_0 The R value calculated for cargo ships with a length above 100 m

S_i Factor accounting for the probability of survival after damage to the ship
$S_{intermediate,i}$ The probability of surviving all intermediate flooding stages until the equilibrium stage
$S_{final,i}$ The probability of surviving in the final equilibrium stage of flooding
$S_{mom,i}$ The probability of surviving heeling moments
S_{min} The least 's-factor'
θ_e Equilibrium angle
θ_v Angle of vanishing stability
θ_{min} Minimum heel angle = 7 degrees for passenger ships; 25 degrees for cargo ships
θ_{max} Maximum heel angle = 15 degrees for passenger ships; 30 degrees for cargo ships
v_i Probability that a watertight deck above the waterline remains intact after damage
x_1 Distance from the aft terminal of the ship to the aft end of the zone in question
x_2 Distance from the aft terminal of the ship to the forward end of the zone in question
Z Distance from the centre of A to T/2, where T is the draught of the ship

List of Acronyms

DDS Deterministic damage stability
ICCL International Convention on Load Lines
IMO International Maritime Organization
MCA Maritime and Coastguard Agency
MSC Maritime Safety Committee
PDS Probabilistic damage stability
pdf Probability distribution function
SOLAS Safety of Life at Sea