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New Bootstrap Methods for the Hypothesis Tests of the Population Mean in Ranked Set Sampling

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Abstract: Ranked Set Sampling is an efficient technique when it is difficult to measure sampling units in respect to cost or time. Although this technique can be used for every sample sizes, the small sample sizes are preferred for better ranking. However, when the sample sizes are small, it is very difficult to obtain distribution of the statistic for the statistical inference such as hypothesis test. In this case, resampling techniques like bootstrap can be used to construct pseudo distribution of the statistics. In this study, the bootstrap methods for hypothesis test about population mean under ranked set sampling is given. A simulation study is also performed to examine the performance of these methods.

Sıralı Küme Örneklemesinde Yığın Ortalamasına İlişkin Hipotez Testi İçin Yeni Bootstrap Metotları

Anahtar Kelimeler
Sıralı küme örneklemesi,
Bootstrap,
Testin gücü,
Monte Carlo simülasyonu

Özet: Sıralı küme örneklemesi, örnekleme birimlerini ölçmenin maliyet ve zaman bakımından zor olduğu durumda kullanılan etkin bir örnekleme tekniğidir. Bu teknik, her çapta örnek için kullanılabilir olmasına rağmen sıralama hatasını minimuma indirmek için küçük örnek çaplarında daha çok tercih edilir. Ancak, hipotez testi gibi istatistiksel çıkarsama yaparken küçük örnek çapı durumunda, istatistiğin kesin ya da asimptotik dağılımını elde etmek oldukça zordur. Bu durumda, istatistiğin yapay dağılımını elde etmek için Bootstrap gibi yeniden örnekleme teknikleri kullanılabilir. Bu çalışmada, sıralı küme örneklemesi altında yığın ortalamasına ilişkin hipotez testi için Bootstrap metotları verilmiştir. Ayrıca, verilen metotların performansını değerlendirmek için simülasyon çalışması yapılmıştır.

1. Introduction

Ranked Set Sampling (RSS) is a sampling technique when it is difficult to measure sampling units but it is easy to rank them by means of techniques that do not require high cost and/or time. RSS was proposed by McIntyre [1] to estimate pasture yields. Takahasi and Wakimoto [2] studied mathematical theory of this technique. They demonstrated that when ranking is perfect, the ranked set sample mean is an unbiased estimator of the population mean and the variance of the ranked set sample mean is always smaller than the variance of simple random sampling on the same sample size.

Dell and Clutter [3] studied imperfect ranking case. As long as ranking is not random, the estimator obtained by RSS is more efficient than Simple Random Sampling (SRS). Besides, it is possible to obtain effective estimators of the population parameters under RSS. For example, Shen [4] examined the estimation of the population mean for the log-normal distribution under RSS. Bhoj and Absanullah [5] estimated the population parameters of generalized geometric distribution under RSS. They obtained the best linear estimators of and parameters when the sample selection was made according to RSS. Abu-Dayyeh, Assrhani and Ibrahim [6] estimated the shape and scale parameters of pareto distribution using RSS. In this study, in which SRS was also used, the estimators were compared in terms of their biases and mean square errors. Furthermore, Albatineh et al. [7] studied the confidence interval estimation for the population coefficient of variation under RSS. They compared SRS to RSS using confidence interval width and coverage probabilities. Some recent studies about RSS were given by Öztürk [8], Öztürk and Demirel [9].

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Additionally, hypothesis testing for population parameters under RSS were performed in some other papers. Muttlak and Abu-Dayyeh [10] studied hypothesis testing for the population parameters using RSS. They compared RSS with SRS according to the powers of test for the population mean and variance. Özdemir and Gökpınar [11] studied hypothesis testing for the population mean under different RSS designs. They observed that the powers of test values for all RSS designs are higher than those of SRS. Moreover, Özdemir, Ebegil and Gökpınar [12] derived a test statistic for the difference of two population means using RSS. The type I error rates and the powers of the test were examined by the Monte Carlo simulation study under normality and non-normality. It was shown that the powers of test obtained by RSS were higher than obtained by the SRS.

Most of the statistical inference procedures about parameters require the distributional information of the statistic. In many cases, the exact distribution of this statistic cannot be determined. Asymptotic methods are alternatives for these situations. However, using asymptotic methods may not be feasible in practice because these methods do not work very well in small sample sizes. In small sample cases, resampling techniques such as Bootstrap can be preferred to asymptotic techniques. The Bootstrap technique is one of the most popular methods of resampling methods if the distributions of the statistic could not be obtained analytically, this method would be preferred in practice. Bootstrap was first introduced by Bradley Efron [13]. This method is an alternative way to obtain empirical sampling distribution of the statistic. Since the basic bootstrap method does not require any distributional assumption, it is very useful for statistical inference. In the bootstrap method, B bootstrap samples are generated with n sample size resamples from the original sample by replacing. The Bootstrap method is a widely used method of constructing the confidence intervals, the standard error of the estimator, and testing hypothesis.

In RSS, small set sizes are preferred to achieve minimum ranking error. For this reason, Hui, Modarres and Zheng [14] used the Bootstrap method to construct the confidence interval for the regression estimator of the population mean under RSS. Moreover, they showed that the performance of the proposed method under the assumption of linearity is very well. In this study, we interested in testing against given in Eq.(1)

$$H_0\colon \mu = \mu_0 \qquad (1)$$
$$H_0\colon \mu > \mu_0$$

In statistical theory, for testing against given in Eq.(1) classical t test is used. Even when underlying distribution is not normal, this method can be used for large sample sizes according to central limit theory. This method can also be applied to hypothesis tests under RSS for large samples. However, in practice, it is a well-known fact that RSS is appropriate especially for small sample cases. Thus, the approximation cannot be valid for hypothesis tests using RSS in many cases. For this reason, in this study, the resampling procedures given by Modarres, Hui and Zheng [15] were adapted to the hypothesis test about the population mean which was given in Eq.(1). Modarres, Hui and Zheng [15] suggested three different resampling techniques with Bootstrap for the confidence interval of population mean under RSS. They studied coverage probabilities and confidence interval widths of the population mean under RSS using three different Bootstrap resampling techniques.

The sample selection procedure and properties of the estimator for the population mean under RSS was explained and three different Bootstrap resampling method was introduced in section 2. Then, type I error rates and powers of tests were investigated under different distributions using Bootstrap methods with a simulation study in Section 3. Finally, in Section 4, conclusions were presented.

2. Bootstrap Hypothesis Testing for the Population Mean

In this section, the sample selection process in the RSS was considered as follows:

Firstly, m set with size m are selected with r cycle. Thus, we have m^2r units which was given in Table 1.

X_{ikj} denotes the value of k^{th} sample unit in i^{th} set and j^{th} cycle (Step 1 in Table 1). In i^{th} set and j^{th} cycle sampe units are ordered using auxiliary information or visual ranking methods without actual measurement as $X_{i(1)j} \leq X_{i(2)j} \leq \cdots \leq X_{i(m)j}$, *i=1, 2,..., m; j=1, 2,..., r* (Step 2 in Table 1). In j^{th} cycle, i^{th} unit are selected from i^{th} set. This selected unit is measured and the measurement is denoted as $X_{(i)j}$ (Step 3 in Table 1).

Finally, the ranked set sample with size *n=mr* are obtained as illustreted in Table 2. Here $\boldsymbol{X_{(i)j}}$ represents *i.* order statistics of *i.* set in cycle *j (i=1, 2,..., m; j=1, 2,..., r)* under the assumption of perfect ranking.

Remark: In SRS, the random sample $X_1, X_2, \dots, X_n$ drawn from an unknown distribution F with population mean μ and variance σ^2 and all sample units are independent and identically distributed.

However, a ranked set sample has more complex structure. In RSS, the random sample $X_{(i)1}, X_{(i)2}, \dots, X_{(i)r} \sim F_i$, (*i=1, 2,... ,m*) where $F_{(i)}$ denotes the distribution function of the i^{th} order statistic. The sample units are independent but not-identically distributed.

Table 1. Sample selection procedure in RSS with sample size *n=mr*

Cycle	*Step 1*	*Step 2*	*Step 3*
1	$X_{111}, X_{121}, \dots, X_{1m1}$ $X_{211}, X_{221}, \dots, X_{2m1}$ $\vdots \quad \vdots \quad \vdots$ $X_{m11}, X_{m21}, \dots, X_{mm1}$	$\boldsymbol{X_{1(1)1}} \le X_{1(2)1} \le \cdots \le X_{1(m)1}$ $X_{2(1)1} \le \boldsymbol{X_{2(2)1}} \le \cdots \le X_{2(m)1}$ $\vdots \quad \vdots \quad \vdots$ $X_{m(1)1} \le X_{m(2)1} \le \cdots \le \boldsymbol{X_{m(m)1}}$	$\boldsymbol{X}_{(\boldsymbol{1})\boldsymbol{1}}$ $\boldsymbol{X}_{(\boldsymbol{2})}$ $\vdots$ $\boldsymbol{X}_{(\boldsymbol{m})\boldsymbol{1}}$
2	$X_{112}, X_{122}, \dots, X_{1m2}$ $X_{212}, X_{222}, \dots, X_{2m2}$ $\vdots \quad \vdots \quad \vdots$ $X_{m12}, X_{m22}, \dots, X_{mm2}$	$\boldsymbol{X_{1(1)2}} \le X_{1(2)2} \le \cdots \le X_{1(m)2}$ $X_{2(1)2} \le \boldsymbol{X_{2(2)2}} \le \cdots \le X_{2(m)2}$ $\vdots \quad \vdots \quad \vdots$ $X_{m(1)2} \le X_{m(2)2} \le \cdots \le \boldsymbol{X_{m(m)2}}$	$\boldsymbol{X}_{(\boldsymbol{1})\boldsymbol{2}}$ $\boldsymbol{X}_{(\boldsymbol{2})\boldsymbol{2}}$ $\vdots$ $\boldsymbol{X}_{(\boldsymbol{m})\boldsymbol{2}}$
j	$X_{11j}, X_{12j}, \dots, X_{1mj}$ $X_{21j}, X_{22j}, \dots, X_{2mj}$ $\vdots \quad \vdots \quad \vdots$ $X_{m1j}, X_{m2j}, \dots, X_{mmj}$	$\boldsymbol{X_{1(1)j}} \le X_{1(2)j} \le \cdots \le X_{1(m)j}$ $X_{2(1)j} \le \boldsymbol{X_{2(2)j}} \le \cdots \le X_{2(m)j}$ $\vdots \quad \vdots \quad \vdots$ $X_{m(1)j} \le X_{m(2)j} \le \cdots \le \boldsymbol{X_{m(m)j}}$	$\boldsymbol{X}_{(\boldsymbol{1})\boldsymbol{j}}$ $\boldsymbol{X}_{(\boldsymbol{2})\boldsymbol{j}}$ $\vdots$ $\boldsymbol{X}_{(\boldsymbol{m})\boldsymbol{j}}$
$\vdots$	…	…	
r	$X_{11r}, X_{12r}, \dots, X_{1mr}$ $X_{21r}, X_{22r}, \dots, X_{2mr}$ $\vdots \quad \vdots \quad \vdots$ $X_{m1r}, X_{m2r}, \dots, X_{mmr}$	$\boldsymbol{X_{1(1)r}} \le X_{1(2)r} \le \cdots \le X_{1(m)r}$ $X_{2(1)r} \le \boldsymbol{X_{2(2)r}} \le \cdots \le X_{2(m)r}$ $\vdots \quad \vdots \quad \vdots$ $X_{m(1)r} \le X_{m(2)r} \le \cdots \le \boldsymbol{X_{m(m)r}}$	$\boldsymbol{X}_{(\boldsymbol{1})\boldsymbol{r}}$ $\boldsymbol{X}_{(\boldsymbol{2})\boldsymbol{r}}$ $\vdots$ $\boldsymbol{X}_{(\boldsymbol{m})\boldsymbol{r}}$

Table 2. Ranked set sample with size *n*

	Cycle			
Order statistics	1	2	…	r
1	$X_{(1)1}$	$X_{(1)2}$	…	$X_{(1)r}$
2	$X_{(2)1}$	$X_{(2)2}$		$X_{(2)r}$
$\vdots$	$\vdots$	$\vdots$	…	$\vdots$
m	$X_{(m)1}$	$X_{(m)2}$		$X_{(m)r}$

Estimator of the population mean obtained from ranked set sample is defined as follows;

$$\bar{X}_{RSS} = \frac{1}{mr}\sum_{i=1}^{m}\sum_{j=}^{r} X_{(i)j} \qquad (2)$$

In RSS, the sample units are generally ranked with visually ranking or using personal judgements. Thus, small set sizes are preferred to achieve minimum ranking error. However, the exact or asymptotic distribution of the statistic is quite hard to obtain in small sample sizes. In this case, bootstrap technique is a good alternative to these methods.

Before proceeding to the bootstrap resampling methods, we need to give the test statistics for testing the H_0 against H_1 in Eq.(1).

The test statistic is

$$T_{RSS} = \frac{\bar{X}_{RSS}-\mu_0}{\sqrt{\hat{V}(\bar{X}_{RSS})}} \qquad (3)$$

where

$$\hat{V}(\bar{X}_{RSS}) = \frac{\hat{\sigma}^2_{RSS}}{mr} \qquad (4)$$

$\hat{\sigma}^2_{RSS}$ is the unbiased estimator of σ^2 under RSS and it can defined as follows:

$$\hat{\sigma}^2_{RSS} = \frac{1}{mr}\{(m-1)MST + (mr-m+1)MSE\} \quad (5)$$

where MSE and MST are the mean-square error and mean-square treatment, respectively [16].

MSE and MST can be defined as follows:

$$MST = \frac{1}{m-1}\sum_{i=1}^{m}\sum_{j=1}^{r}(X_{(i)j} - \bar{X}_{RSS})^2 - \frac{1}{m-1}\sum_{i=1}^{m}\sum_{j=1}^{r}(X_{(i)j} - \bar{X}_{(i).})^2 \quad (6)$$

$$MSE = \frac{1}{m(r-1)}\sum_{i=1}^{m}\sum_{j=1}^{r}(X_{(i)j} - \bar{X}_{(i).})^2 \quad (7)$$

where $\bar{X}_{(i).} = \frac{1}{r}\sum_{j=1}^{r} X_{(i)j}$

Using these definitions, the modified methods for testing H_0 against H_1 in Eq. (1) are given as follows.

2.1. Method 1. Bootstrap RSS by rows (BRSSR)

1. Compute $\boldsymbol{T_{RSS}}$ using Eq.(4) for obtained ranked set sample.
2. Assign to each element of the i^{th} row in Table 2, a probability of $\frac{1}{r}$ and select *r* elements randomly with replacement to obtain $X^*_{(i)1}, \dots, X^*_{(i)r}$.
3. Perform step 2, for *i=1,2,...,m*, to obtain a bootstrap ranked set sample $\{X^*_{(i)j}\}$
4. Generate B bootstrap sample $\{X^*_{(i)j}, i = 1,2,\dots,m, j = 1,2,\dots,r\}$ and from these samples obtain T^*_i using Eq. (4).
5. p value is estimated as follows

$$\hat{p} = \frac{\#(T^*_i > T_{RSS})}{B}$$

6. If $\hat{p} < \alpha$ then H_0 is rejected.

2.2. Method 2. Bootstrap RSS (BRSS)

1. Assign to each element of the ranked set sample a probability of $\frac{1}{mr}$.

2. Randomly draw *m* elements in Table 2, $y_1, y_2, \dots, y_m \sim F_n$ sort them in ascending order $y_{(1)} \le y_2 \le \cdots y_{(m)}$ and retain $X^*_{(i)1} = y_{(i)}$. Where $y_{(1)} \le y_2 \le \cdots y_{(m)}$ denote ordered statistics.
3. Perform step 2 for *i=1,2,...,m*.
4. Repeat step 2 and step 3 *r* times to obtain $\{X^*_{(i)j}\}$.
5. B bootstrap samples are generated and $T^*_1, T^*_2, \dots, T^*_B$ are calculated.
6. p value is estimated as follows

$$\hat{p} = \frac{\#(T^*_i > T_{RSS})}{B}$$

7. If $\hat{p} < \alpha$ then H_0 is rejected

2.3. Method 3. Mixed row bootsrap RSS (MRBRSS)

1. Assign to each element of the i^{th} row in Table 2, a probability of $\frac{1}{r}$ for *i=1, 2,..., m*
2. Order $y^*_1, y^*_2, \dots, y^*_m$ in ascending order to get $y^*_1 \le y^*_2 \le \cdots \le y^*_m$ and retain $X^*_{(i)1} = y^*_{(i)}$
3. Perform step 1 and step 2 for *i=1, 2,..., m* to obtain $X^*_{(1)1}, X^*_{(2)1}, \dots, X^*_{(m)1}$.
4. Repeat step 1 to step 3 *r* times to obtain $\{X^*_{(i)j}\}$
5. B bootstrap samples are generated and $T^*_1, T^*_2, \dots, T^*_B$ are calculated.
6. p value is estimated as follows

$$\hat{p} = \frac{\#(T^*_i > T_{RSS})}{B}$$

7. If $\hat{p} < \alpha$ then H_0 is rejected.

3. Simulation Study

In this section, the hypothesis given in Eq. (1) was considered based on RSS with Bootstrap methods. We generate random numbers using different distributions with shift parameter μ^0 (i.e. the mean of the distribution is as $\mu_1 = \mu_0 + \mu^0$) to obtain type I error rates and powers of test. When μ^*=0 (i.e. H_0 is true), the reject rate of H_0 corresponds the type I error rates. The reject rates in the other cases of μ^* correspond to the power rates of the tests. The distributions using in simulation study are as Standard Normal (0, 1), Uniform (0, 1), Exponential (1) and Gamma (4, 1). The value of μ_0 is well known that $\mu_0 = 0$ for Standard Normal distribution, $\mu_0 = 0.5$ for Uniform distribution, $\mu_0 = 1$ for Exponential distribution and $\mu_0 = 4$ for Gamma distribution. The distributions and values of the shift parameters were given in Table 3.

Table 3. Distributions and μ^* values

	Standard Normal (0, 1)	**Uniform (0, 1)**	**Exponential (1)**	**Gamma (4, 1)**
μ^*	0	0	0	0
	0.1	0.1	0.1	0.2
	0.2	0.13	0.17	0.4
	0.3	0.16	0.23	0.6
	0.4	0.2	0.3	0.8

We use the nominal level α =0. 05. Monte Carlo simulation was performed using MATLAB R2017a. To estimate the p value of the test, B=2000 bootstrap iteration was utilized. Besides, T=2000 repetition was used to obtain Type I error and powers of the test.

To compare the type I error and powers of the tests of the methods, m was taken as 2,3,4,5,6 and r was taken as 2,4,6,8,10.

The power of the tests were ignored and indicated with (*), when the type I error rates of these tests were greater than %6. Results are given in Tables 4-7. Tables 4-7 in Appendix A.

As far as the results in Table 4 is concerned, type I error rates of method 3 are far from its nominal level for all the cases. On the other hand, type I error rates of method 2 are close to its nominal level in all the cases. Nevertheless, type I error rates of method 1 have a great variation according to cycle sizes and set sizes. However, in small set sizes *(e.g. m=2)* type I error rates of method 1 is getting closer to its nominal level when cycle sizes are equal or greater than 6. When set sizes are getting larger, even great cycle sizes are not sufficient to make type I error rates of method 1 getting closer to its nominal level. Besides, power rates of method 1 is greater than powers of method 2 in all the cases when type I error rates are close to its nominal level. Thus, under normality, when set sizes are small and cycle sizes are moderate, method 1 is preferable than method 2. As expected, as μ^* increases, the powers of the test increase for method 1 and method 2.

The type I error rates and powers of the test under the uniform distribution were shown in Table 5. As it is evident in this table, type I error rates and powers of the test results are quite similar to those of standard normal distribution in Table 4. However, the type I error rates of method 1 is getting closer its nominal level faster than those of the standard normal distribution. For example; when *m=2,* type I error rates of method 1 is close to its nominal level even in small cycle sizes *(r=4).* The remaining results are similar to the standard normal distribution.

The type I error rates and powers of the test under the exponential distribution were shown in Table 6.

Based on the results presented in this table, type I error rates of method 1 are similar to those of the standard normal distribution. However, type I error rates of the test under the exponential distribution are getting slower to its nominal level than those of the standard normal distribution. For example, type I error rates of the test under standard normal distribution is close to its nominal level when *m=4* or *5* and *r=10*. However, type I error rates of the test under exponential distribution never gets close to its nominal level when *m>3*.

The type I error rates and powers of test under the gamma distribution were pointed out in Table 7. As seen from this table, type I error rates of method 1 is getting close to its nominal level faster than the exponential distribution. However, the difference between type I error rates and its nominal level under gamma distribution is not as small as those of under uniform distribution.

Generally, for all the distributions, the powers of method 1 is greater than the power of method 2 when type I error rates are close to its nominal level.

4. Discussion and Conclusion

In this study, the hypothesis testing was examined for the population mean under ranked set sampling using different bootstrap resample selection methods. A simulation study was performed under some symmetric and non-symmetric distributions. In the light of the simulation study, type I error rates of method 2 are close to its nominal level in all of the considered cases and distributions. Unlike the previous method, method 3 is far from its nominal level in all the cases and distributions. However, type 1 error rates of method 1 are close to its nominal level when set sizes are small and cycle sizes are large. Moreover, in these cases, the powers of method 1 are greater than those of method 2. Further study could be carried out on the hypothesis testing for the difference of two population means under RSS using bootstrap resample selection methods.

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Appendicies

Appendix A. Tables 4-7

Table 4: Type I error rates and powers of the test under standard normal (0,1) distribution

		$\mu^* = 0$			$\mu^* = 0.1$			$\mu^* = 0.2$			$\mu^* = 0.3$			$\mu^* = 0.4$		
		Method			Method			Method			Method			Method		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
m=2	**r=2**	0.1330	0.0525	0.1550	*	0.0910	*	*	0.1405	*	*	0.1870	*	*	0.2160	*
	r=4	0.0800	0.0570	0.0980	*	0.0970	*	*	0.1520	*	*	0.2195	*	*	0.3415	*
	r=6	0.0595	0.0490	0.0775	0.1255	0.1045	*	0.2305	0.1955	*	0.3600	0.3140	*	0.5000	0.4450	*
	r=8	0.0590	0.0505	0.0790	0.1255	0.1110	*	0.2665	0.2345	*	0.4185	0.3830	*	0.5975	0.5530	*
	r=10	0.0530	0.0490	0.0740	0.1565	0.1440	*	0.3020	0.2730	*	0.5125	0.4800	*	0.6870	0.6610	*
m=3	**r=2**	0.1310	0.0465	0.1665	*	0.1035	*	*	0.1545	*	*	0.2460	*	*	0.3350	*
	r=4	0.0815	0.0560	0.1105	*	0.1240	*	*	0.2305	*	*	0.3675	*	*	0.5740	*
	r=6	0.0710	0.0560	0.0975	*	0.1390	*	*	0.2940	*	*	0.5155	*	*	0.7360	*
	r=8	0.0580	0.0510	0.0985	0.1880	0.1515	*	0.4295	0.3780	*	0.6565	0.6215	*	0.8470	0.8215	*
	r=10	0.0555	0.0435	0.0900	0.2075	0.1820	*	0.4585	0.4345	*	0.7310	0.7040	*	0.9095	0.8945	*
m=4	**r=2**	0.1325	0.0430	0.1600	*	0.1100	*	*	0.0905	*	*	0.3280	*	*	0.1825	*
	r=4	0.0845	0.0530	0.1275	*	0.1655	*	*	0.1455	*	*	0.5330	*	*	0.3315	*
	r=6	0.0705	0.0515	0.1105	*	0.1680	*	*	0.1970	*	*	0.6880	*	*	0.4750	*
	r=8	0.0700	0.0495	0.1080	*	0.2035	*	*	0.2120	*	*	0.8045	*	*	0.5840	*
	r=10	0.0540	0.0475	0.0905	0.2880	0.2595	*	0.2780	0.2470	*	0.8930	0.8765	*	0.6485	0.6155	*
m=5	**r=2**	0.1275	0.0495	0.1725	*	0.1265	*	*	0.2620	*	*	0.4545	*	*	0.6230	*
	r=4	0.0890	0.0540	0.1350	*	0.1770	*	*	0.4385	*	*	0.7025	*	*	0.8840	*
	r=6	0.0690	0.0500	0.1180	*	0.2405	*	*	0.5535	*	*	0.8355	*	*	0.9715	*
	r=8	0.0725	0.0545	0.1215	*	0.2660	*	*	0.6580	*	*	0.9155	*	*	0.9925	*
	r=10	0.0550	0.0435	0.1065	0.3480	0.3170	*	0.7745	0.7435	*	0.9690	0.9610	*	0.9970	0.9965	*
m=6	**r=2**	0.1325	0.0500	0.1810	*	0.1520	*	*	0.3280	*	*	0.5470	*	*	0.7265	*
	r=4	0.0780	0.0470	0.1395	*	0.2015	*	*	0.5330	*	*	0.8115	*	*	0.9505	*
	r=6	0.0790	0.0555	0.1375	*	0.2665	*	*	0.6750	*	*	0.9280	*	*	0.9925	*
	r=8	0.0605	0.0480	0.1140	*	0.3360	*	*	0.7730	*	*	0.9735	*	*	0.9990	*
	r=10	0.0665	0.0550	0.1265	*	0.4065	*	*	0.8550	*	*	0.9935	*	*	1	*

Table 5: Type I error rates and powers of the test under uniform (0,1) distribution

		$\mu^* = 0$			$\mu^* = 0.1$			$\mu^* = 0.13$			$\mu^* = 0.16$			$\mu^* = 0.2$		
		Method			Method			Method			Method			Method		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
m=2	**r=2**	0.0995	0.0555	0.1180	*	0.0810	*	*	0.1070	*	*	0.1105	*	*	0.1385	*
	r=4	0.0525	0.0335	0.0620	0.1350	0.0735	*	0.1685	0.1150	*	0.2155	0.1315	*	0.2855	0.1775	*
	r=6	0.0500	0.0340	0.0675	0.1720	0.1200	*	0.2275	0.1650	*	0.3030	0.2350	*	0.4130	0.3225	*
	r=8	0.0450	0.0345	0.0605	0.2005	0.1625	*	0.2770	0.2360	*	0.3710	0.3150	*	0.5070	0.4515	*
	r=10	0.0500	0.0380	0.0690	0.2435	0.2080	*	0.3340	0.2940	*	0.4415	0.4045	*	0.5855	0.5485	*
m=3	**r=2**	0.1205	0.0340	0.1405	*	0.0750	*	*	0.1140	*	*	0.1375	*	*	0.1660	*
	r=4	0.0595	0.0335	0.0970	*	0.1595	*	*	0.2220	*	*	0.2975	*	*	0.4565	*
	r=6	0.0565	0.0380	0.0900	0.2915	0.2195	*	0.4190	0.3385	*	0.5360	0.4770	*	0.6933	0.6270	*
	r=8	0.0525	0.0365	0.0910	0.3480	0.2965	*	0.4715	0.4190	*	0.6305	0.5755	*	0.7913	0.7610	*
	r=10	0.0565	0.0405	0.0975	0.4030	0.3630	*	0.5530	0.5135	*	0.7110	0.6835	*	0.8610	0.8385	*
m=4	**r=2**	0.1410	0.0285	0.1835	*	0.1200	*	*	0.1470	*	*	0.2290	*	*	0.3430	*
	r=4	0.0585	0.0320	0.1025	*	0.2440	*	*	0.3560	*	*	0.4915	*	*	0.6825	*
	r=6	0.0575	0.0415	0.1115	0.4035	0.3380	*	0.5665	0.4965	*	0.7240	0.6695	*	0.8705	0.8330	*
	r=8	0.0575	0.0435	0.1115	0.4880	0.4180	*	0.6705	0.6245	*	0.8125	0.7815	*	0.9390	0.9235	*
	r=10	0.0515	0.0416	0.1045	0.5480	0.5120	*	0.7495	0.7210	*	0.8760	0.8500	*	0.9705	0.9680	*
m=5	**r=2**	0.1440	0.0340	0.1995	*	0.1720	*	*	0.2525	*	*	0.5140	*	*	0.5140	*
	r=4	0.0545	0.0445	0.1475	*	0.3365	*	*	0.5055	*	*	0.8270	*	*	0.8270	*
	r=6	0.0585	0.0430	0.1285	*	0.4630	*	*	0.6570	*	*	0.9525	*	*	0.9525	*
	r=8	0.0590	0.0435	0.1300	*	0.5795	*	*	0.7670	*	*	0.9880	*	*	0.9880	*
	r=10	0.0540	0.0460	0.1170	0.7190	0.6885	*	0.8975	0.8740	*	0.9925	0.9925	*	0.9925	0.9925	*
m=6	**r=2**	0.1375	0.0385	0.2000	*	0.2410	*	*	0.3485	*	*	0.7870	*	*	0.6685	*
	r=4	0.0875	0.0410	0.1545	*	0.4545	*	*	0.6360	*	*	0.7945	*	*	0.9340	*
	r=6	0.0600	0.0450	0.1435	*	0.5950	*	*	0.7970	*	*	0.9200	*	*	0.9885	*
	r=8	0.0610	0.0390	0.1260	*	0.7090	*	*	0.8975	*	*	0.9725	*	*	0.9985	*
	r=10	0.0535	0.0390	0.1355	0.8340	0.8120	*	0.9510	0.9435	*	0.9950	0.9950	*	1	1	*

Table 6: Type I error rates and powers of the test under exp (1) distribution

		$\mu^* = 0$			$\mu^* = 0.1$			$\mu^* = 0.17$			$\mu^* = 0.23$			$\mu^* = 0.3$		
		Method			Method			Method			Method			Method		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
m=2	r=2	0.1300	0.0260	0.1385	*	0.0355	*	*	0.0545	*	*	0.0770	*	*	0.1150	*
	r=4	0.0710	0.0285	0.0820	*	0.0680	*	*	0.1040	*	*	0.1625	*	*	0.2525	*
	r=6	0.0490	0.0265	0.0630	0.1210	0.0900	*	0.2040	0.1560	*	0.2895	0.2470	*	0.4380	0.3840	*
	r=8	0.0560	0.0360	0.0730	0.1275	0.1045	*	0.2410	0.2075	*	0.3570	0.3220	*	0.5205	0.4855	*
	r=10	0.0550	0.0415	0.0700	0.1510	0.1215	*	0.2665	0.2345	*	0.4015	0.3795	*	0.5905	0.5715	*
m=3	r=2	0.1490	0.0280	0.1720	*	0.0585	*	*	0.0935	*	*	0.1510	*	*	0.2440	*
	r=4	0.0755	0.0355	0.1000	*	0.0950	*	*	0.1820	*	*	0.2940	*	*	0.4570	*
	r=6	0.0720	0.0410	0.0910	*	0.1345	*	*	0.2680	*	*	0.4305	*	*	0.6275	*
	r=8	0.0710	0.0525	0.0940	*	0.1730	*	*	0.3345	*	*	0.5100	*	*	0.7370	*
	r=10	0.0535	0.0405	0.0735	0.2120	0.1860	*	0.4150	0.3785	*	0.6080	0.5930	*	0.8080	0.8055	*
m=4	r=2	0.1340	0.0270	0.1725	*	0.0825	*	*	0.1645	*	*	0.220	*	*	0.3795	*
	r=4	0.0845	0.0395	0.1115	*	0.1295	*	*	0.2705	*	*	0.4375	*	*	0.6435	*
	r=6	0.0700	0.0480	0.1055	*	0.1905	*	*	0.3800	*	*	0.5980	*	*	0.8265	*
	r=8	0.0735	0.0570	0.1060	*	0.2360	*	*	0.4730	*	*	0.6935	*	*	0.8910	*
	r=10	0.0665	0.0485	0.0905	*	0.2395	*	*	0.5220	*	*	0.7720	*	*	0.9435	*
m=5	r=2	0.1375	0.0360	0.1730	*	0.1075	*	*	0.2180	*	*	0.3415	*	*	0.5320	*
	r=4	0.0930	0.0475	0.1280	*	0.1760	*	*	0.3610	*	*	0.5790	*	*	0.8045	*
	r=6	0.0765	0.0425	0.1165	*	0.2380	*	*	0.5010	*	*	0.7335	*	*	0.9295	*
	r=8	0.0665	0.0515	0.0980	*	0.2635	*	*	0.5870	*	*	0.8205	*	*	0.9625	*
	r=10	0.0725	0.0580	0.1120	*	0.3310	*	*	0.6780	*	*	0.8950	*	*	0.9865	*
m=6	r=2	0.1475	0.0395	0.1880	*	0.1355	*	*	0.2800	*	*	0.4255	*	*	0.6595	*
	r=4	0.0900	0.0450	0.1220	*	0.2105	*	*	0.4460	*	*	0.6865	*	*	0.9000	*
	r=6	0.0690	0.0460	0.1045	*	0.2755	*	*	0.5720	*	*	0.8315	*	*	0.9630	*
	r=8	0.0705	0.0550	0.1130	*	0.3355	*	*	0.7045	*	*	0.9055	*	*	0.9900	*
	r=10	0.0690	0.0490	0.1005	*	0.3970	*	*	0.7615	*	*	0.9535	*	*	0.9960	*

Table 7: Type I error rates and powers of the test under gamma (4,1) distribution

		$\mu^* = 0$			$\mu^* = 0.2$			$\mu^* = 0.4$			$\mu^* = 0.6$			$\mu^* = 0.8$		
		Method			Method			Method			Method			Method		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
m=2	r=2	0.1065	0.0345	0.1200	*	0.0685	*	*	0.1125	*	*	0.1495	*	*	0.1985	*
	r=4	0.0695	0.0335	0.0900	*	0.0895	*	*	0.1615	*	*	0.2530	*	*	0.3575	*
	r=6	0.0590	0.0375	0.0735	0.1355	0.0940	*	0.2495	0.2065	*	0.4020	0.3520	*	0.5775	0.5395	*
	r=8	0.0593	0.0420	0.0790	0.1380	0.1195	*	0.2885	0.2540	*	0.4885	0.4500	*	0.6990	0.6700	*
	r=10	0.0595	0.0475	0.0790	0.1465	0.1275	*	0.3135	0.2915	*	0.5490	0.5200	*	0.7580	0.7435	*
m=3	r=2	0.1210	0.0330	0.1480	*	0.0745	*	*	0.1240	*	*	0.2140	*	*	0.3325	*
	r=4	0.0940	0.0530	0.1215	*	0.1110	*	*	0.2410	*	*	0.4445	*	*	0.6480	*
	r=6	0.0670	0.0400	0.0885	*	0.1410	*	*	0.3450	*	*	0.5865	*	*	0.8045	*
	r=8	0.0578	0.0450	0.0980	0.2005	0.1655	*	0.4750	0.4310	*	0.7460	0.7170	*	0.9170	0.9010	*
	r=10	0.0530	0.0425	0.0770	0.2140	0.1860	*	0.4890	0.4545	*	0.7805	0.7590	*	0.9600	0.9545	*
m=4	r=2	0.1375	0.0345	0.1705	*	0.0905	*	*	0.1995	*	*	0.3605	*	*	0.5590	*
	r=4	0.0870	0.0445	0.1220	*	0.1455	*	*	0.3545	*	*	0.6325	*	*	0.8380	*
	r=6	0.0810	0.0555	0.1220	*	0.1970	*	*	0.4705	*	*	0.7810	*	*	0.9525	*
	r=8	0.0590	0.0450	0.1110	0.2630	0.2120	*	0.6260	0.5785	*	0.8940	0.8800	*	0.9905	0.9850	*
	r=10	0.0585	0.0545	0.0955	0.2780	0.2470	*	0.6740	0.6520	*	0.9285	0.9240	*	0.9980	0.9955	*
m=5	r=2	0.1540	0.0450	0.2065	*	0.1095	*	*	0.2860	*	*	0.5105	*	*	0.7155	*
	r=4	0.0795	0.0440	0.1240	*	0.1830	*	*	0.4355	*	*	0.7575	*	*	0.9470	*
	r=6	0.0780	0.0490	0.1185	*	0.2415	*	*	0.6060	*	*	0.8980	*	*	0.9920	*
	r=8	0.0590	0.0385	0.0975	0.3280	0.2825	*	0.7300	0.7010	*	0.9560	0.9535	*	0.9990	0.9985	*
	r=10	0.0597	0.0445	0.1025	0.3640	0.3265	*	0.8180	0.8015	*	0.9840	0.9815	*	1	1	*
m=6	r=2	0.1365	0.0385	0.1790	*	0.1405	*	*	0.3210	*	*	0.6105	*	*	0.8295	*
	r=4	0.0765	0.0445	0.1270	*	0.2030	*	*	0.5600	*	*	0.8800	*	*	0.9860	*
	r=6	0.0690	0.0470	0.1200	*	0.2780	*	*	0.7340	*	*	0.9675	*	*	0.9980	*
	r=8	0.0725	0.0530	0.1170	*	0.3390	*	*	0.8230	*	*	0.9895	*	*	1	*
	r=10	0.0587	0.0480	0.1085	0.4210	0.3985	*	0.9115	0.9005	*	0.9990	0.9985	*	1	1	*