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Decompositions of fuzzy *g-continuity

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Abstract - The aim of this paper is to give decompositions of a weaker form of fuzzy continuity, namely fuzzy *g-continuity, by providing the concepts of fuzzy *g$_t$-sets, fuzzy *g$_\alpha$$*$-sets, fuzzy *g$_t$-continuity and fuzzy *g$_\alpha$$*$-continuity.

Keywords - *fuzzy *g-closed set, fuzzy $\hat{\eta}$-closed set, fuzzy *g_t-set, fuzzy *$g_\alpha$$*$-set, fuzzy *$g_t$-continuity, fuzzy *$g_\alpha$$*$-continuity.*

1 Introduction

Levine [12], Mashhour et. al. [13] and Njastad [15] introduced semi-open sets, preopen sets and α-open sets respectively. In 1961, Levine [11] obtained a decomposition of continuity which was later improved by Rose [22]. Tong [24] decomposed continuity into α-continuity and A-continuity and showed that his decomposition is independent of Levine's. Hatir et. al. [8] also obtained a decomposition of continuity. The concept of ω-closed sets was introduced and studied by Sheik John and Sundaram [23]. Veerakumar [27] and Abd El-Monsef et. al. [2] introduced $\hat{g}$-closed sets and $\alpha\hat{g}$-closed sets in topological spaces respectively. It is known that ω-closed sets and $\hat{g}$-closed sets are same. Veerakumar [29] introduced and studied α^*g-closed sets in topological spaces. Benchalli et. al. [5] introduced and studied the notion of $\omega\alpha$-closed sets. It is known that $\omega\alpha$-closed sets, α^*g-closed sets and $\alpha\hat{g}$-closed sets are all same. Palaniappan et. al. [17] introduced and studied $\hat{\eta}$-closed sets in topological spaces. In this paper we introduce *g$_t$-continuity and *g$_\alpha$$*$-continuity to obtain decompositions of *g-continuity in fuzzy topological spaces.

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2 Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (simply, X and Y) denote fuzzy topological spaces on which no separation axioms are assumed. Let A be a subset of a space X. The closure of A and the interior of A are denoted by cl(A) and int(A), respectively.

The following definitions are useful in the sequel.

Definition 2.1. *A subset A of a fuzzy topological space (X, τ) is said to be fuzzy semi-open [3] (resp. fuzzy preopen [6], fuzzy α-open [6]) if A $\leq$ cl(int(A)) (resp. A $\leq$ int(cl(A)), A $\leq$ int(cl(int(A)))). The complement of fuzzy semi-open (resp. fuzzy preopen, fuzzy α-open) set is called fuzzy semi-closed (resp. fuzzy preclosed, fuzzy α-closed) set.*

Definition 2.2. *A subset A of a fuzzy topological space (X, τ) is said to be*

1. *a fuzzy t-set [19] if int(A) = int(cl(A)).*
2. *an fuzzy α*-set [18] if int(A) = int(cl(int(A))).*

Remark 2.3. *[18]*

1. *Every fuzzy t-set is an fuzzy α*-set, but not conversely.*
2. *An fuzzy open set need not be an fuzzy α*-set.*
3. *The union of two fuzzy α*-sets need not be an fuzzy α*-set.*
4. *Arbitrary intersection of fuzzy α*-sets is an fuzzy α*-set.*

Definition 2.4. *A subset A of a fuzzy topological space (X, τ) is called*

1. *a fuzzy g-closed [4] if cl(A) $\leq$ U, whenever A $\leq$ U and U is fuzzy open in X.*
2. *a fuzzy $\hat{g}$-closed [1] if cl(A) $\leq$ U, whenever A $\leq$ U and U is fuzzy semi-open in X.*

The complement of a fuzzy g-closed (resp. fuzzy $\hat{g}$-closed) set is called fuzzy g-open (resp. fuzzy $\hat{g}$-open).

For a subset A of a fuzzy topological space X, the fuzzy α-closure (resp. fuzzy semi-closure, fuzzy pre-closure) of A, denoted by αcl(A) (resp. scl(A), pcl(A)), is the intersection of all fuzzy α-closed (resp. fuzzy semi-closed, fuzzy preclosed) subsets of X containing A. Dually, the fuzzy α-interior (resp. fuzzy semi-interior, fuzzy pre-interior) of A, denoted by αint(A) (resp. sint(A), pint(A)), is the union of all fuzzy α-open (resp. fuzzy semi-open, fuzzy preopen) subsets of X contained in A.

Proposition 2.5. *[18] Let A and B be fuzzy subsets of a fuzzy topological space X. If B is an fuzzy α*-set, then αint(A$\wedge$B) = αint(A)$\wedge$int(B).*

Definition 2.6. *A subset A of a fuzzy topological space (X, τ) is called*

1. *a fuzzy *g-closed [21] if cl(A) $\leq$ U, whenever A $\leq$ U and U is fuzzy $\hat{g}$-open in (X, τ).*

2. an fuzzy $\alpha\hat{g}$-closed [21] if $\alpha cl(A) \leq U$, whenever $A \leq U$ and U is fuzzy $\hat{g}$-open in (X, τ).

3. a fuzzy $\hat{\eta}$-closed [21] if $pcl(A) \leq U$, whenever $A \leq U$ and U is fuzzy $\hat{g}$-open in (X, τ).

*The complement of fuzzy *g-closed set (resp. fuzzy $\alpha\hat{g}$-closed set, fuzzy $\hat{\eta}$-closed set) is fuzzy *g-open (resp. fuzzy $\alpha\hat{g}$-open, fuzzy $\hat{\eta}$-open).*

Remark 2.7. *The following hold in any fuzzy topological spaces:*

1. Every fuzzy α-closed set is fuzzy $\alpha\hat{g}$-closed, but not conversely.[21]
2. Every fuzzy $\alpha\hat{g}$-closed set is fuzzy $\hat{\eta}$-closed, but not conversely.[21]
3. Every fuzzy *g-closed set is fuzzy $\alpha\hat{g}$-closed, but not conversely.[21]
4. Every fuzzy closed set is fuzzy α-closed, but not conversely.[6]
5. Every fuzzy closed set is fuzzy *g-closed, but not conversely.[21]

Definition 2.8. *[21] A subset S of a fuzzy topological space (X, τ) is said to be*

1. fuzzy $\hat{g}lc^*$-set if $S = U \wedge F$, where U is fuzzy $\hat{g}$-open and F is fuzzy closed in (X, τ).
2. fuzzy $D\eta^*$-set if $S = U \wedge F$, where U is fuzzy $\hat{g}$-open and F is fuzzy α-closed in (X, τ).
3. fuzzy $D\eta^{**}$-set if $S = U \wedge F$, where U is fuzzy $\alpha\hat{g}$-open and F is a fuzzy t-set in (X, τ).

Definition 2.9. *A fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is said to be*

1. fuzzy α-continuous [6] if for each $V \in \sigma$, $f^{-1}(V)$ is an fuzzy α-open set in (X, τ).
2. fuzzy $\alpha\hat{g}$-continuous [21] if for each $V \in \sigma$, $f^{-1}(V)$ is an fuzzy $\alpha\hat{g}$-open set in (X, τ).
3. fuzzy $\hat{\eta}$-continuous [21] if for each $V \in \sigma$, $f^{-1}(V)$ is fuzzy $\hat{\eta}$-open set in (X, τ).
4. fuzzy $D\eta^*$-continuous [21] if for each $V \in \sigma$, $f^{-1}(V)$ is fuzzy $D\eta^*$-set in (X, τ).
5. fuzzy $D\eta^{**}$-continuous [21] if for each $V \in \sigma$, $f^{-1}(V)$ is fuzzy $D\eta^{**}$-set in (X, τ).
6. fuzzy $D^*\eta^*$-continuous [21] if for each $V^c \in \sigma$, $f^{-1}(V)$ is fuzzy $D\eta^*$-set in (X, τ).
7. fuzzy *g-continuous [21] if for each $V^c \in \sigma$, $f^{-1}(V)$ is fuzzy *g-closed set in (X, τ).
8. fuzzy $\hat{G}LC^*$-continuous [21] if for each $V^c \in \sigma$, $f^{-1}(V)$ is fuzzy $\hat{g}lc^*$-set in (X, τ).

Recently, the following decompositions have been established in [21].

Theorem 2.10. *A fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is fuzzy α-continuous if and only if it is both fuzzy $\alpha\hat{g}$-continuous and fuzzy $D^*\eta^*$-continuous.*

Theorem 2.11. *A fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is fuzzy $\alpha\hat{g}$-continuous if and only if it is both fuzzy $\hat{\eta}$-continuous and fuzzy $D\eta^{**}$-continuous.*

3 On fuzzy *g_t-sets and fuzzy *$g_\alpha$$*$-sets

Definition 3.1. *A subset S of a fuzzy topological space (X, τ) is called*

1. *fuzzy *g_t-set if $S = U \wedge F$, where U is fuzzy *g-open in X and F is a fuzzy t-set in X,*
2. *fuzzy *$g_\alpha$$*$-set if $S = U \wedge F$, where U is fuzzy *g-open in X and F is a fuzzy α*-set in X.*

*The family of all fuzzy *g_t-sets (resp. fuzzy *$g_\alpha$$*$-sets) in a fuzzy topological space (X, τ) is denoted by f*g_t(X, τ)(resp. f*$g_\alpha$$*$(X, τ)).*

Proposition 3.2. *Let S be a subset of a fuzzy topological space (X, τ).*

1. *If S is a fuzzy t-set, then S$\in$f*g_t(X, τ).*
2. *If S is a fuzzy α*-set, then S$\in$f*$g_\alpha$$*$(X, τ).*
3. *If S is a fuzzy *g-open set in X, then S$\in$f*g_t(X, τ) and S$\in$f*$g_\alpha$$*$(X, τ).*

Proof. The proof is obvious.

Proposition 3.3. *In a fuzzy topological space X, every fuzzy *g_t-set is fuzzy *$g_\alpha$$*$-set but not conversely.*

Example 3.4. *Let $X = \{a, b\}$ and $\tau = \{0_X, \lambda, 1_X\}$ where λ is fuzzy set in X defined by $\lambda(a) = 0.3$, $\lambda(b) = 0.5$. Then (X, τ) is a fuzzy topological space. Clearly λ_1 defined as $\lambda_1(a) = 0.7$, $\lambda_1(b) = 0.6$ is fuzzy *$g_\alpha$$*$-set but it is not fuzzy *g_t-set.*

Remark 3.5. *The following examples show that*

1. *the converse of Proposition 3.2 need not be true.*
2. *the concepts of fuzzy *g_t-sets and fuzzy $\hat{\eta}$-open sets are independent.*
3. *the concepts of fuzzy *$g_\alpha$$*$-sets and fuzzy $\alpha\hat{g}$-open sets are independent.*

Example 3.6. *Let $X = \{a, b\}$ with $\tau = \{0_X, \beta, 1_X\}$ where β is fuzzy set in X defined by $\beta(a) = 0.3$, $\beta(b) = 0.6$. Then (X, τ) is a fuzzy topological space. Clearly*

1. *β is fuzzy *g_t-set but not a fuzzy t-set*
2. *β is fuzzy *$g_\alpha$$*$-set but not a fuzzy α*-set.*

Example 3.7. *Let $X = \{a, b\}$. Consider the fuzzy topology τ as in Example 3.4, where λ_2 is fuzzy set in X defined by $\lambda_2(a) = 0.7$, $\lambda_2(b) = 0.5$. Clearly λ_2 is both fuzzy *g_t-set and fuzzy *$g_\alpha$$*$-set, but it is not a fuzzy *g-open set.*

Example 3.8. *Let $X = \{a, b\}$. Consider the fuzzy topology τ as in Example 3.6, where β_1 and β_2 are fuzzy sets in X defined by $\beta_1(a) = 0.7$, $\beta_1(b) = 0.4$ and $\beta_2(a) = 0.7$, $\beta_2(b) = 0.5$. Clearly β_1 is fuzzy *g_t-set but not a fuzzy $\hat{\eta}$-open set and β_2 is a fuzzy $\hat{\eta}$-open set but not fuzzy *g_t-set.*

Example 3.9. *Let $X = \{a,\ b\}$. Consider the fuzzy topology τ as in Example 3.4, where λ_2 is fuzzy set in X defined by $\lambda_2(a) = 0.7$, $\lambda_2(b) = 0.5$. Clearly λ_2 is fuzzy $^*g_\alpha*$-set but not a fuzzy $\alpha\hat{g}$-open set.*

Example 3.10. *Let $X = \{a,\ b\}$. Consider the fuzzy topology τ as in Example 3.6, where β_3 is fuzzy set in X defined by $\beta_3(a) = 0.7$, $\beta_3(b) = 0.6$. Clearly β_3 is a fuzzy $\alpha\hat{g}$-open set but not fuzzy $^*g_\alpha*$-set.*

Remark 3.11.

1. *The union of two fuzzy $^*g_t$-sets need not be fuzzy $^*g_t$-set.*
2. *The union of two fuzzy $^*g_\alpha*$-sets need not be fuzzy $^*g_\alpha*$-set.*

Example 3.12. *Let $X = \{a,\ b\}$. Consider the fuzzy topology τ as in Example 3.6, where β_3 and β_4 are fuzzy sets in X defined by $\beta_3(a) = 0.7$, $\beta_3(b) = 0.6$ and $\beta_4(a) = 0.7$, $\beta_4(b) = 0.3$. Clearly β and β_4 are both fuzzy $^*g_t$-set and fuzzy $^*g_\alpha*$-set but $\beta \vee \beta_4 = \beta_3$ is neither a fuzzy $^*g_\alpha*$-set nor a fuzzy $^*g_t$-set.*

Lemma 3.13. *[21]*

1. *A subset S of X is fuzzy *g-open if and only if $F \leq int(S)$ whenever $F \leq S$ and F is fuzzy $\hat{g}$-closed in X.*
2. *A subset S of X is fuzzy $\alpha\hat{g}$-open if and only if $F \leq \alpha int(S)$ whenever $F \leq S$ and F is fuzzy $\hat{g}$-closed in X.*
3. *A subset S of X is fuzzy $\hat{\eta}$-open if and only if $F \leq pint(S)$ whenever $F \leq S$ and F is fuzzy $\hat{g}$-closed in X.*

Theorem 3.14. *A subset S is fuzzy *g-open in $(X,\ \tau)$ if and only if it is both fuzzy $\alpha\hat{g}$-open and fuzzy $^*g_\alpha*$-set in $(X,\ \tau)$.*

Proof. Necessity. The proof is obvious.

Sufficiency. Let S be a fuzzy $\alpha\hat{g}$-open set and fuzzy $^*g_\alpha*$-set. Since S is fuzzy $^*g_\alpha*$-set, $S = A \wedge B$, where A is fuzzy *g-open and B is a fuzzy α^*-set. Assume that $F \leq S$, where F is fuzzy $\hat{g}$-closed in X. Since A is fuzzy *g-open, by Lemma 3.13(1), $F \leq \mathrm{int}(A)$. Since S is fuzzy $\alpha\hat{g}$-open in X, by Lemma 3.13(2),
$F \leq \alpha\mathrm{int}(S) = S \wedge \mathrm{int}(\mathrm{cl}(\mathrm{int}(S))) = (A \wedge B) \wedge \mathrm{int}(\mathrm{cl}(\mathrm{int}(A \wedge B)))$
$\leq A \wedge B \wedge \mathrm{int}(\mathrm{cl}(\mathrm{int}(A))) \wedge \mathrm{int}(\mathrm{cl}(\mathrm{int}(B))) = A \wedge B \wedge \mathrm{int}(\mathrm{cl}(\mathrm{int}(A))) \wedge \mathrm{int}(B) \leq \mathrm{int}(B)$.
Therefore, we obtain $F \leq \mathrm{int}(B)$ and hence $F \leq \mathrm{int}(A) \wedge \mathrm{int}(B) = \mathrm{int}(S)$. Hence S is fuzzy *g-open.

Theorem 3.15. *A subset S is fuzzy *g-open in $(X,\ \tau)$ if and only if it is both fuzzy $\hat{\eta}$-open and fuzzy $^*g_t$-set in $(X,\ \tau)$.*

Proof. Similar to Theorem 3.14.

Remark 3.16. *We obtain the following diagram by the above discussions:*

$$\begin{array}{ccccc}
\textit{fuzzy closed} & \longrightarrow & \textit{fuzzy } {}^*g\textit{-closed} & \longrightarrow & \textit{fuzzy } {}^*g_t\textit{-set} \\
 & & \downarrow & & \\
\downarrow & & \textit{fuzzy } {}^*g_\alpha*\textit{-set} & & \ddagger \\
 & & \ddagger & & \\
\textit{fuzzy } \alpha\textit{-closed} & \longrightarrow & \textit{fuzzy } \alpha\hat{g}\textit{-closed} & \longrightarrow & \textit{fuzzy } \hat{\eta}\textit{-closed}
\end{array}$$

None of the above implications is reversible as shown by the following Examples.

Example 3.17. *Let $X = \{a, b\}$. Consider the fuzzy topology τ as in Example 3.6, where β_5 is fuzzy set in X defined by $\beta_5(a) = 0.3$, $\beta_5(b) = 0.4$. Clearly β_5 is fuzzy α-closed but it is neither a fuzzy *g-closed set nor a fuzzy closed set.*

Example 3.18. *Let $X = \{a, b\}$. Consider the fuzzy topology τ as in Example 3.4, where λ_3 is fuzzy set in X defined by $\lambda_3(a) = 0.3$, $\lambda_3(b) = 0.6$. Clearly λ_3 is a fuzzy $\alpha\hat{g}$-closed but not a fuzzy α-closed set.*

Example 3.19. *Let $X = \{a, b\}$ with $\tau = \{0_X, A, 1_X\}$ where A is fuzzy set in X defined by $A(a) = 0.6$, $A(b) = 0.5$. Then (X, τ) is a fuzzy topological space. Clearly A_1 defined by $A_1(a) = 0.6$, $A_1(b) = 0.6$ is a fuzzy g-closed set but not a fuzzy $\alpha\hat{g}$-closed set.*

Example 3.20.

1. *Let $X = \{a, b\}$. Consider the fuzzy topology τ as in Example 3.6, where β_5 is a fuzzy set in X defined by $\beta_5(a) = 0.3$, $\beta_5(b) = 0.4$. Clearly β_5 is a fuzzy $\alpha\hat{g}$-closed set but not a fuzzy g-closed set.*

2. *Let $X = \{a, b\}$. Consider the fuzzy topology τ as in Example 3.4, clearly λ is fuzzy $^*g_\alpha*$-set but it is neither a fuzzy *g-closed nor a fuzzy $\alpha\hat{g}$-closed set.*

Example 3.21.

1. *Let $X = \{a, b\}$. Consider the fuzzy topology τ as in Example 3.6, where β_3 is a fuzzy set in X defined by $\beta_3(a) = 0.7$, $\beta_3(b) = 0.6$. Clearly β_3 is a fuzzy $\alpha\hat{g}$-closed set but not fuzzy $^*g_\alpha*$-set.*

2. *Let $X = \{a, b\}$. Consider the fuzzy topology τ as in Example 3.4, clearly λ is fuzzy $\hat{\eta}$-closed set but not a fuzzy $\alpha\hat{g}$-closed set.*

Example 3.22. *Let $X = \{a, b\}$. Consider the fuzzy topology τ as in Example 3.4, where λ_3 is a fuzzy set in X defined by $\lambda_3(a) = 0.3$, $\lambda_3(b) = 0.6$. Clearly λ_3 is fuzzy *g-closed, but it is neither a fuzzy α-closed nor a fuzzy closed set.*

Example 3.23. *Let $X = \{a, b\}$. Consider the fuzzy topology τ as in Example 3.6, where β_3 is a fuzzy set in X defined by $\beta_3(a) = 0.7$, $\beta_3(b) = 0.6$. Clearly*

1. *β is fuzzy $^*g_t$-set but not a fuzzy *g-closed set.*

2. *β_3 fuzzy $\hat{\eta}$-closed set but not fuzzy $^*g_t$-set.*

3. *β is fuzzy $^*g_t$-set but not a fuzzy $\hat{\eta}$-closed set.*

Remark 3.24. *The concepts of fuzzy g-closed sets and fuzzy $\alpha\hat{g}$-closed sets are independent by the Examples 3.19 and 3.20.*

Remark 3.25. *The concepts of fuzzy *g-closed sets and fuzzy α-closed sets are independent by the Examples 3.17 and 3.22.*

Proposition 3.26. *Let (X, τ) be a fuzzy topological space. Then a subset A of X is fuzzy closed if and only if it is both fuzzy *g-closed and fuzzy $\hat{g}$lc*-set.*

Proof. Necessity is trivial. To prove the sufficiency, assume that A is both fuzzy *g-closed and fuzzy $\hat{g}$lc*-set. Then A= U$\wedge$V, where U is fuzzy $\hat{g}$-open and V is fuzzy closed in X. Therefore A $\leq$ U and A $\leq$ V and so by hypothesis, cl(A) $\leq$ U and cl(A) $\leq$ V, thus cl(A) $\leq$ U$\wedge$V= A and hence cl(A)= A. Therefore A is fuzzy closed in X.

Remark 3.27. *The following Examples show that the concepts of fuzzy *g-closed sets and fuzzy $\hat{g}lc^*$-sets are independent.*

Example 3.28. *Let $X = \{a,\ b\}$. Consider the fuzzy topology τ as in Example 3.4, clearly λ_3 defined by $\lambda_3(a) = 0.3$, $\lambda_3(b) = 0.6$ is fuzzy *g-closed set but not fuzzy $\hat{g}lc^*$-set.*

Example 3.29. *Let $X = \{a,\ b\}$. Consider the fuzzy topology τ as in Example 3.4, clearly λ is fuzzy $\hat{g}lc^*$-set but not fuzzy *g-closed set.*

4 Decompositions of fuzzy *g-continuity

Definition 4.1. *A fuzzy function $f : (X,\ \tau) \to (Y,\ \sigma)$ is said to be*

1. *fuzzy $^*g_t$-continuous if for each $V \in \sigma$, $f^{-1}(V) \in f^*g_t(X,\ \tau)$.*

2. *fuzzy $^*g_\alpha *$-continuous if for each $V \in \sigma$, $f^{-1}(V) \in f^*g_\alpha *(X,\ \tau)$.*

Proposition 4.2. *For a fuzzy function $f : (X,\ \tau) \to (Y,\ \sigma)$, the following implications hold:*

1. *fuzzy *g-continuity $\Rightarrow$ fuzzy $^*g_t$-continuity;*

2. *fuzzy *g-continuity $\Rightarrow$ fuzzy $^*g_\alpha *$-continuity;*

3. *fuzzy *g-continuity $\Rightarrow$ fuzzy $\alpha\hat{g}$-continuity $\Rightarrow$ fuzzy $\hat{\eta}$-continuity.*

The reverse implications in Proposition 4.2 are not true as shown in the following Examples.

Example 4.3. *Let $X = \{a,\ b\}$. Consider the fuzzy topology τ as in Example 3.6 and $\sigma = \{0_X,\ \mu_1,\ 1_X\}$ where μ_1 defined by $\mu_1(a) = 0.7$, $\mu_1(b) = 0.4$. Then $(X,\ \sigma)$ is fuzzy topological space. Let $f : (X,\ \tau) \to (X,\ \sigma)$ be the fuzzy identity function. Clearly f is fuzzy $^*g_t$-continuous function. However, f is neither fuzzy *g-continuous nor fuzzy $\hat{\eta}$-continuous.*

Example 4.4. *Let $X = \{a,\ b\}$. Consider the fuzzy topological space $(X,\ \tau)$ and $(X,\ \sigma)$ as in Example 4.3. Let $f : (X,\ \tau) \to (X,\ \sigma)$ be the fuzzy identity function. Clearly f is fuzzy $^*g_\alpha *$-continuous function. However, f is neither fuzzy *g-continuous nor fuzzy $\alpha\hat{g}$-continuous.*

Example 4.5. *Let $X = \{a,\ b\}$. Consider the fuzzy topology τ as in Example 3.6 and $\sigma = \{0_X,\ \mu_2,\ 1_X\}$ where μ_2 defined by $\mu_2(a) = 0.7$, $\mu_2(b) = 0.6$. Then $(X,\ \sigma)$ is fuzzy topological space. Let $f : (X,\ \tau) \to (X,\ \sigma)$ be the fuzzy identity function. Clearly f is fuzzy $\alpha\hat{g}$-continuous function but it is neither fuzzy $^*g_\alpha *$-continuous function nor fuzzy *g-continuous.*

Example 4.6. *Let $X = \{a,\ b\}$. Consider the fuzzy topology τ as in Example 3.6 and $\sigma = \{0_X,\ \mu_3,\ 1_X\}$ where μ_3 defined by $\mu_3(a) = 0.7$, $\mu_3(b) = 0.5$. Then $(X,\ \sigma)$ is fuzzy topological space. Let $f : (X,\ \tau) \to (X,\ \sigma)$ be the fuzzy identity function. Clearly f is fuzzy $\hat{\eta}$-continuous function but it is not fuzzy $^*g_t$-continuous.*

Example 4.7. *Let $X = \{a,\ b\}$. Consider the fuzzy topological space $(X,\ \tau)$ and $(X,\ \sigma)$ as in Example 4.6. Let $f : (X,\ \tau) \to (X,\ \sigma)$ be the fuzzy identity function. Clearly f is fuzzy $\hat{\eta}$-continuous function but not a fuzzy $\alpha\hat{g}$-continuous.*

Example 4.8. *Let $X = \{a,\ b\}$. Consider the fuzzy topological space $(X,\ \tau)$ and $(X,\ \sigma)$ as in Example 4.6. Let $f : (X,\ \tau) \to (X,\ \sigma)$ be the fuzzy identity function. Clearly f is fuzzy ${}^*g_\alpha{*}$-continuous function but it is not fuzzy ${}^*g_t$-continuous.*

Example 4.9. *Let $X = \{a,\ b\}$. Consider the fuzzy topological space $(X,\ \tau)$ and $(X,\ \sigma)$ as in Example 4.5. Let $f : (X,\ \tau) \to (X,\ \sigma)$ be the fuzzy identity function. Clearly f is fuzzy $\hat{\eta}$-continuous function but it is not fuzzy ${}^*g$-continuous.*

Remark 4.10. *By the above discussions, we obtain the following diagram.*

$$\begin{array}{ccccc}
\text{fuzzy } \alpha\hat{g}\text{-continuity} & & \longrightarrow & & \text{fuzzy } \hat{\eta}\text{-continuity}\\
& \nwarrow & & \nearrow & \\
\ddagger & & \text{fuzzy } {}^*g\text{-continuity} & & \ddagger\\
& \swarrow & & \searrow & \\
\text{fuzzy } {}^*g_\alpha{*}\text{-continuity} & & \longleftarrow & & \text{fuzzy } {}^*g_t\text{-continuity}
\end{array}$$

None of the above implications is reversible.

Theorem 4.11. *A fuzzy function $f : (X,\ \tau) \to (Y,\ \sigma)$ is fuzzy ${}^*g$-continuous if and only if it is both fuzzy $\alpha\hat{g}$-continuous and fuzzy ${}^*g_\alpha{*}$-continuous.*

Proof. The proof follows immediately from Theorem 3.14.

Theorem 4.12. *A fuzzy function $f : (X,\ \tau) \to (Y,\ \sigma)$ is fuzzy ${}^*g$-continuous if and only if it is both fuzzy $\hat{\eta}$-continuous and fuzzy ${}^*g_t$-continuous.*

Proof. From Theorem 3.15, the proof is immediate.

Corollary 4.13. *A fuzzy function $f : (X,\ \tau) \to (Y,\ \sigma)$ is fuzzy ${}^*g$-continuous if and only if it is fuzzy $\hat{\eta}$-continuous, fuzzy $D\eta^{**}$-continuous and fuzzy ${}^*g_\alpha{*}$-continuous.*

Proof. It follows from Theorem 2.11.

Remark 4.14. *fuzzy ${}^*g$-continuity and fuzzy $\hat{G}LC^*$-continuity are independent of each other.*

Example 4.15. *Let $X = \{a,\ b\}$. Consider the fuzzy topology τ as in Example 3.4 and $\sigma = \{0_X,\ \mu_1,\ 1_X\}$ where μ_1 defined by $\mu_1(a) = 0.7$, $\mu_1(b) = 0.4$. Then $(X,\ \sigma)$ is fuzzy topological space. Let $f : (X,\ \tau) \to (X,\ \sigma)$ be the fuzzy identity function. Clearly f is fuzzy ${}^*g$-continuous function but it is not fuzzy $\hat{G}LC^*$-continuous.*

Example 4.16. *Let $X = \{a,\ b\}$. Consider the fuzzy topology τ as in Example 3.4 and $\sigma = \{0_X,\ \mu_3,\ 1_X\}$ where μ_3 defined by $\mu_3(a) = 0.7$, $\mu_3(b) = 0.5$. Then $(X,\ \sigma)$ is fuzzy topological space. Let $f : (X,\ \tau) \to (X,\ \sigma)$ be the fuzzy identity function. Clearly f is fuzzy $\hat{G}LC^*$-continuous function but it is not fuzzy ${}^*g$-continuous.*

Theorem 4.17. *A fuzzy function $f : (X,\ \tau) \to (Y,\ \sigma)$ is fuzzy continuous if and only if it is both fuzzy ${}^*g$-continuous and fuzzy $\hat{G}LC^*$-continuous.*

Proof. It follows from Proposition 3.26.

5 Conclusion

In the classical paper [31] of 1965, Zadeh generalized the usual notion of a set and introduced the important and useful notion of fuzzy sets. Fuzzy continuity is one of the main topics in fuzzy topology. Various authors have introduced various types of fuzzy continuity. Various types of generalizations of fuzzy continuous functions were introduced and studied by various authors in the recent development of fuzzy topology. The decomposition of fuzzy continuity is one of many problems in fuzzy topology. Tong [26] obtained a decomposition of fuzzy continuity by introducing two weak notions of fuzzy continuity namely, fuzzy strong semi-continuity and fuzzy precontinuity. Rajamani et.al [18, 19] obtained a decomposition of fuzzy continuity. In this paper, we obtained decompositions of fuzzy g*-continuity in fuzzy topological spaces by using some fuzzy generalized continuity.

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