ISSN: **1304-7981** Number: **8**, Year:**2015**, Pages: **57-71** **http://jnrs.gop.edu.tr**

Gaziosmanpasa University
Graduate School of Natural and Applied Sciences
Journal of New Results in Science

Received: 12.12.2014 *Editors-in-Chief*: *Bilge Hilal Cadırcı*
Accepted: 14.01.2015 *Area Editor*: *Serkan Demiriz*

A Decision Making Method via TOPSIS on Soft Sets

Selim Eraslan[a,1] (seraslan4@yahoo.com)

[a]*Kırıkkale MYO, Kırıkkale University, 71200 Kırıkkale, Turkey*

Abstract - In this paper, we first present TOPSIS (technique for order performance by similarity to ideal solution) that is a multi-criteria decision making (MCDM) technique. TOPSIS is a practical and useful technique for ranking and selection of a number of externally determined alternatives through distance measures. We then give a decision making method by suing TOPSIS on soft set theory. Finally an application is given for this new method.

Keywords - *Soft sets; TOPSIS; multi-attribute decision making; normalization.*

1 Introduction

Multi-criteria or multi-attribute decision making (MCDM/MADM) is to choose the prior one among the several alternatives; shortly it means evaluation, ordering and choosing [15]. MCDM may be described as the choice made by using at least two criteria from decision makers. Considering the contrast among the criteria in MCDM, it is aimed to make the best decision [36]. The basic steps of MCDM methods are as [25]: (1) to determine the evaluation criteria of the system, (2) to determine the alternatives, (3) to evaluate the alternatives according to the criteria, (4) to apply the MCDM method, and (5) to choose an alternative according to the essentials of the method.

It is rather difficult to choose the best alternative for the decision makers. When making a choice among the several alternatives or there are some problems that are related to find the most suitable one among the conflicting alternatives, a lot of decision makers apply the MCDM methods. The most important of them is the seven methods which

[1]Corresponding Author

is known as MCDM methods as: (1) ELECTRE, (2) PROMETHEE, (3) VIKOR, (4) AHP, (5) Fuzzy AHP, (6) TOPSIS and (7) Fuzzy TOPSIS.

TOPSIS is a useful method in dealing with MCDM or MADM problems in the real world [15]. It helps decision maker(s) organize the problems to be solved, and carry out analysis, comparisons and rankings of the alternatives. TOPSIS method was initially presented by Hwang and Yoon [15]. It has been deemed one of the major decision making methods within the world [33]. In recent years, it has been successfully applied to the areas of human resources management [12], location analysis [39], quality control [38], water management [35], manufacturing [1, 22, 26], product design [19], purchasing and outsourcing [17, 34], financial performance measurement [14]) and transportation [16]. In addition, the concept of TOPSIS has also been connected to multi-objective decision making [20] and group decision making [32].

The basic idea of TOPSIS is rather simple. It originates from the concept of a displaced ideal point from which the compromise solution has the shortest distance [4, 20, 40, 43]. A relative advantage of TOPSIS is its ability to identify the best alternative quickly [28].
Hwang and Yoon [15] proposed that the ranking of alternatives will be based on the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS) to determine the best alternative. This method simultaneously considers the distances to both PIS and NIS, and a preference order is ranked according to their relative closeness. While the PIS is to maximize benefit criteria and minimize cost criteria, the NIS is to maximize cost criteria and minimize benefit criteria. Distance is the degree or amount of separation between two points, lines, surfaces, or objectives. Originally TOPSIS utilized Euclidean distances to measure the alternatives with their PIS and NIS [33].

According to Kim et al. [18] TOPSIS has four advantages: (1) a scalar value that accounts for both the best and worst alternatives simultaneously; (2) a sound logic that represents the rationale of human choice; (3) the performance measures of all alternatives on attributes can be visualized on a polyhedron, at least for any two dimensions; and (4) a simple computation process that can be easily programmed into a spreadsheet [17, 32]. These advantages make TOPSIS a major MCDM method as compared with other related methods such AHP and ELECTRE [15].

There are several variations of TOPSIS in the MCDM literature; conventional TOPSIS [15], adjusted TOPSIS (A-TOPSIS) [13] and modified TOPSIS (M-TOPSIS) [31].

In many fields, such as economics, engineering, environment, involve data that contain uncertainties. To understand and manipulate the uncertainties, there are many approaches such as probability theory, fuzzy set theory Zadeh [42], intuitionistic fuzzy sets Atanassov [3], rough set theory Pawlak [27], etc. Each of these theories has its own difficulties as pointed out in Molodtsov [23]. To address these difficulties, Molodtsov [23] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from difficulties. Soft set theory is a newly emerging mathematical tool to deal with uncertain problems.

The main advantage of soft set theory in data analysis is that it does not need any grade of membership as in the fuzzy set theory. Therefore, the theory of soft sets has advanced in a variety of ways and in many disciplines. Molodtsov [23], Molodtsov et al. [24] applied the soft sets to fields such as game theory, operations research, Riemann-integration, Perron integration, probability and so on.

After Molodtsov [23], different types of operations of soft sets have been defined. First operations of soft sets and their properties were given by Maji et al. [21]. Ali et al. [2], Çağman and Enginoğlu [8], Zhu and Wen [44] and Çağman [11] also made some contributions to the operations of soft sets. By using these operations, works on soft set theory and its applications have been progressing rapidly.

In this paper, we extend the concept of TOPSIS to develop a new method for solving MCDM problems using by soft set. Hence, TOPSIS will be extended on the soft set theory. Then, we illustrate the proposed method with a numerical example.

The study is organized as follows. In the next section, a simplified description of soft set is presented. The main procedure for the conventional TOPSIS is described in a series of steps in Section 3. In Section 4, will focus on the proposed soft TOPSIS model in a step by step. Afterwards, an example is provided to demonstrate the computational efficiency of the proposed new model. Finally, conclusion is pointed out.

2 Soft Sets and their Operations

In this section, we present basic definitions of soft sets and briefly their operations. For more detail of the soft sets, we refer to the earlier studies [5, 6, 7, 8, 9, 10, 11, 21, 23].

Definition 2.1. [23] Let U and X be two non empty set and $P(U)$ is the power set of U. Then, a soft set f over U is a function defined by

$$f : X \to P(U)$$

where U refer to an initial universe and X is a set of parameters.

In other words, the soft set is a parameterized family of subsets of the set U. Every set $f(x)$, $x \in X$, from this family may be considered as the set of x-elements of the soft set f, or as the set of x-approximate elements of the soft set.

As an illustration, let us consider the following examples.

A soft set f describes the attractiveness of the houses which Mr.X is going to buy.

U - is the set of houses under consideration.

X - is the set of parameters. Each parameter is a word or a sentence.

X = {expensive; beautiful; wooden; cheap; in the green surroundings; modern; in good repair; in bad repair }

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on.

It is worth noting that the sets $f(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection.

A soft set over U can be represented by the set of ordered pairs

$$f = \{(x, f(x)) : x \in X\}$$

Note that the set of all soft sets defined from X to $P(U)$ will be denoted by S_X^U.

Definition 2.2. [7] Let $f \in S_X^U$. Then, f is an empty soft set, denoted by Φ_X, if $f(x) = \emptyset$ for all $x \in X$. f is a universal soft set, denoted by $f_{\tilde{X}}$, if $f(x) = U$ for all $x \in X$.

Definition 2.3. [7] Let $f, g \in S_X^U$. Then, f is a soft subset of g, denoted by $f \widetilde{\subseteq} g$, if $f(x) \subseteq g(x)$ for all $x \in X$. f and g are soft equal, denoted by $f = g$, if and only if $f(x) = g(x)$ for all $x \in X$.

Definition 2.4. [7] Let $f, g \in S_X^U$. Then, union of f and g, denoted by $f \widetilde{\cup} g$, if $(f \widetilde{\cup} g)(x) = f(x) \cup g(x)$ for all $x \in X$. Intersection of f and g, denoted by $f \widetilde{\cap} g$, if $(f \widetilde{\cap} g)(x) = f(x) \cap g(x)$ for all $x \in X$. Complement of f, denoted by $f^{\tilde{c}}$, if $f^{\tilde{c}}(x) = U \setminus f(x)$ for all $x \in X$.

3 TOPSIS

The operations within the TOPSIS process include: decision matrix normalization, distance measures, and aggregation operators [33]. A decision matrix is usually required prior to the beginning of the process. The decision matrix contains competitive alternatives row-wise, with their attributes ratings or scores column-wise. Suppose that the available data being completed in the given decision matrix, including quantitative and qualitative information. Normalization is an operation to make these scores conform to or reduced to a norm or standard. The normalization of qualitative data or linguistic data could be first transformed to a linear scale, e.g., 1-10. To compare the alternatives on each attribute, the normalized process is usually made column-wise, and the normalized value will be a positive value between 0 and 1. In this way, computational problems, resulting from different measurements in the decision matrix, are eliminated [33, 40]. At the same time, Yoon and Hwang partition attributes into three groups: benefit attributes, cost attributes, and non-monotonic attributes. In addition, on the basis of the works of Hwang and Yoon [15], Milani et al. [22] and Yoon and Hwang [40] a few common normalization methods are classified as vector normalization, linear normalization, and non-monotonic normalization to fit real-world situations under different circumstances [33].

The main procedure of TOPSIS can be described in a series of steps [15, 29, 30, 37, 41]:

Throughout this paper, $I_n = \{1, 2, ..., n\}$ for all $n \in \mathbb{N}$.

Step 1. Defining the problem (purpose of determination and assessment criteria identification)

For this step, alternatives and evaluation criteria are determined. Let us assume that $DM = \{D_p,\ p \in I_n\}$ be set of p decision makers; A_i $(i \in I_m)$ denotes the alternative and C_j $(j \in I_n)$ represents the criterion (alternatives); with quantitative and qualitative data respectively.

Step 2. Determining scorecard and criteria weights and construct decision matrix D for each decision makers.

In this step, in line the criteria values of alternatives, A_i, and in column the evaluation criteria, C_j, decision matrix $D = (d_{ij})_{n\times m}$ is constructed. The structure of the matrix can be expressed as

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & & \vdots \\ d_{i1} & d_{i2} & \cdots & d_{in} \\ \vdots & \vdots & & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix} \end{array} = [d_{ij}]_{m\times n}$$

where, m is the number of alternatives, n is the number of the criteria, d_{ij} is the criterion value of i.th alternatives received from the j.th criterion.

Step 3. Creating the weighted standard (normalized) decision matrix, R.

Process in this step, converting values to different criteria in interval [0,1], in the unit (normalized) is intended to provide opportunities for comparisons between the recognition criteria. After the decision matrix is created, using the vector normalization formula

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{k=1}^{m} d_{kj}^2}} \quad , \quad \forall d_{ij} \neq 0 \;\; and \;\; \forall i \in I_m \quad , \quad \forall j \in I_n$$

so decided each row vector in the matrix, it is achieved by dividing the value of the norm of the vector r_{ij}. So normalized decision matrix can be represented as

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} = [r_{ij}]_{m\times n}$$

where, r_{ij} is the score of i.th alternatives as per j. criterion.

Step 4. Creating the weighted normalized decision matrix, V.

In this step, the first to be assigned to each evaluation criterion relative weight values indicating the importance is determined so that the condition (w_j). So, the R matrix of elements in accordance with the criteria strongly weighted.

Generally total weight is $\sum_{i=1}^{n} w_i = 1$. Hence $0 \leq w_j \leq 1$ is. Determine the importance weight of the criterion vector as

$$W = \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix} \end{array} = [w_j]_{1\times n}$$

where, w_j is j.th relative importance of criteria (weight).

R matrix elements are multiplied by the weight vector W (as $v_{ij} = w_j \cdot r_{ij}$) is another expression of the R matrix is multiplied by the weight normalized weighted value for each column in the decision matrix is generated as follows.

$$V = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix} = [v_{ij}]_{m\times n}$$

where v_{ij} is the dimensionless criterion value.

Step 5. Determining the positive and negative ideal solutions A^+ (PIS) and A^- (NIS), respectively.

At this stage, from the V matrix, produced two different solution set, positive ideal solution and negative ideal solution according to the weighted normal value largest (best) choice can alternatively be at least (worst) value is preferable to determine alternative.

The evaluation criteria of each column value that is composed of the largest of the best value of the matrix V is the ideal solution in terms of benefits; negative ideal solution is comprised of the lowest value. If the evaluation criteria in terms of cost in this case is the ideal solution composed of the smallest value of column V matrix is composed of negative ideal solution for maximum value.

Maximum values which help determine the ideal solution, which helps to determine the minimum values of (v_j^+) and negative ideal solution (v_j^-) using the following formulas calculated.

$$A^+ = \{v_1^+, \cdots, v_j^+, \cdots, v_n^+\} = \{(\underbrace{max}_{i} v_{ij}|j \in J_1)\ ,\ (\underbrace{min}_{i} v_{ij}|j \in J_2)\ ,\ i \in I_m\}$$

$$A^- = \{v_1^-, \cdots, v_j^-, \cdots, v_n^-\} = \{(\underbrace{min}_{i} v_{ij}|j \in J_1)\ ,\ (\underbrace{max}_{i} v_{ij}|j \in J_2)\ ,\ i \in I_m\}$$

where

A^+: set showing the most suitable alternative for each criterion may be preferred (ideal solution),

A^-: set showing at least preferable alternative for each criterion (negative ideal solution),

J_1 : set showing the benefits of criteria,

J_2 : set showing the cost (loss) of criteria,

$J_1 \cap J_2 = \emptyset$ and $J_1 \cup J_2 = \{1, \cdots, n\}$

v_j^+: among all the alternatives are the best value for j.th criteria and if J_1 the benefits criteria, $= \{\underbrace{max}_{i}(v_{ij})\}$, if J_2 the cost criteria $= \{\underbrace{min}_{i}(v_{ij})\}$.

v_j^+: among all the alternatives are the worst value for j.th criteria and if J_1 the benefits criteria, $= \{\underbrace{min}_{i}(v_{ij})\}$, if J_2 the cost criteria $= \{\underbrace{max}_{i}(v_{ij})\}$.

where J_1 and J_2 are associated with the benefit and cost attribute sets, respectively.

Step 6. For each alternative calculating the separation measurement from the ideal (S_i^+) and the negative ideal (S_i^-) solutions.

At this stage, calculate the separation measures of for each alternative simulator from the PIS and NIS is calculated by the Euclidean distance formula as

$$S_i^+ = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^+)^2} \quad , \quad \forall i \in I_m$$

and

$$S_i^- = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2} \quad , \quad \forall i \in I_m$$

where

S_i^+: represents distance of i.th alternative from the ideal solution.

S_i^-: represents distance of i.th alternative from the negative ideal solution.

Step 7. Calculation the relative closeness of alternative to the ideal solution

In this step, the relative closeness of a particular alternative simulator to the ideal simulator, C_i^+, can be expressed as

$$C_i^+ = \frac{S_i^+}{(S_i^+ + S_i^-)} \quad , \quad 0 \leq C_i^+ \leq 1 \quad , \quad \forall i \in I_m$$

where, C_i^+, represents the success of i.th alternative higher values in the industry.

An alternative is closer to the ideal alternative, value of the alternative approaches 1. $C_i^+ = 1$ is the relevant alternative is the ideal solution shows the absolute closeness to the alternative about the negative ideal solution is $C_i^+ = 0$. Options available in the most close to the ideal solution is determined as the most appropriate option.

Step 8. Ranking the preference order.

In this step, a set of alternatives can now be preference ranked according to the descending order of the value of C_i^+. Obtained in the previous step C_i^+ values using alternatives to decreasing C_i^+ values with the largest selection of the best when it is ranked (most preferred) option that is determined.

4 A Decision Making Method on Soft Sets

In this section, we then give a decision making method by using TOPSIS on soft set theory. The detailed procedure, with a few options within each step, is illustrated in the following.

Step 1. Defining the problem.

Let us assume that $DM = \{D_p,\ p \in I_n\}$ is set of decision makers, $U = \{u_i,\ i \in I_m\}$ denotes set of alternatives and $X = \{x_j,\ j \in I_n\}$ is a set of all parameters (criterion). Then, a soft set f over U is a function defined by

$$f : X \to P(U)$$

Step 2. Construct decision matrix D for each decision makers.

$$D = \begin{array}{c} \\ u_1 \\ u_2 \\ \vdots \\ u_i \\ \vdots \\ u_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & & \vdots \\ d_{i1} & d_{i2} & \cdots & d_{in} \\ \vdots & \vdots & & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix} \end{array} = [d_{ij}]_{m\times n}$$

where $D = \bigcup_{i,j\in I_n} d_{ij},\ \ d_{ij} = f_{X_i}(x_j)$ is the criterion value of i.th alternatives received from the j.th criterion, X_i is the parameter sets of decision makers D_p and f_{X_i} is the soft set which was construct by D_p.

Step 3. Obtaining the weighted normalized decision matrix, V.

The weighted normalized decision matrix is generated as follows.

$$V = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix} = [v_{ij}]_{m\times n}$$

where

$$v_{kt} = \sum_{i=1}^{n} \gamma_{f_{X_i}(x_k)}(u_t) \quad , \quad \forall i,k,t \in I_n$$

and

$$\gamma_{f_{X_i}(x_k)}(u_t) = \begin{cases} 1, & u_t \in f_{X_i}(x_k) \\ 0, & u_t \notin f_{X_i}(x_k) \end{cases}$$

Step 4. Creating the decision matrix (vector), R.

$$k(u_j) = \sum_{i=1}^{n} v_{ji}$$

where $k(u_j)$ is decision values of u_j. Thus the decision matrix of each alternative values for the deciders is expressed as

$$R = [k(u_1) \quad \cdots \quad k(u_n)]$$

Step 5. Ranking the preference order.

5 An Application

In this section, we have presented an application for the soft TOPSIS-decision making method. Now, by using the algorithm of the soft TOPSIS-decision making method we can solve the following example (problem) step by step as follows:

Step 1. Defining the problem.

Assume that a real estate agent has a set of different types of houses (universal set-alternatives) $U = \{u_1, u_2, u_3\}$ which may be characterized by a set of all parameters $X = \{x_1, x_2, x_3\}$. For $j = 1, 2, 3$ the parameters x_j stand for "cheap", "modern", "large", respectively. Then we can give the following examples.

Suppose that three decision-makers come to the real estate agent to buy a house. Firstly, each decision-maker has to consider their own set of parameters. Then, they can construct their soft sets. Next, by using the soft TOPSIS-decision making method we select a house on the basis for the sets of decision-makers parameters.

i. Assume that first decision-maker (D_1) choose the set of parameters as $X_1 = \{x_1, x_2\} \subseteq X$ and construct soft set as follows;

$$f_{X_1} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$$

In a similar way,

ii. Assume that second decision-maker (D_2) choose the set of parameters as $X_2 = \{x_1\} \subseteq X$ and construct soft set as

$$f_{X_2} = \{(x_1, \{u_1, u_3\})\}$$

iii. Assume that third decision-maker (D_3) choose the set of parameters as $X_3 = \{x_2, x_3\} \subseteq X$ and construct soft set as

$$f_{X_3} = \{(x_2, \{u_1, u_3\}), (x_3, \{u_1\})\}$$

Step 2. Construct decision matrix D for each decision-makers.

We can represent f_{X_i} soft sets in a tabular form as shown below.

	x_1	x_2	x_3
f_{X_1}	$\{u_1\}$	$\{u_1, u_2\}$	$\emptyset$
f_{X_2}	$\{u_1, u_3\}$	$\emptyset$	$\emptyset$
f_{X_3}	$\emptyset$	$\{u_1, u_3\}$	$\{u_1\}$

We can construct soft sets of decision-makers,D_i, in a tabular form respectively as

	x_1	x_2	x_3
u_1	1	1	0
u_2	0	1	0
u_3	0	0	0

	x_1	x_2	x_3
u_1	1	0	0
u_2	0	0	0
u_3	1	0	0

	x_1	x_2	x_3
u_1	0	1	1
u_2	0	0	0
u_3	0	1	0

Then decision matrix D is constructed as

$$D = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc} f_{X_1}(x_1) & f_{X_1}(x_2) & f_{X_1}(x_3) \\ f_{X_2}(x_1) & f_{X_2}(x_2) & f_{X_2}(x_3) \\ f_{X_3}(x_1) & f_{X_3}(x_2) & f_{X_3}(x_3) \end{array} \right] \end{array} = \left[\begin{array}{ccc} \{u_1\} & \{u_1, u_2\} & \emptyset \\ \{u_1, u_3\} & \emptyset & \emptyset \\ \emptyset & \{u_1, u_3\} & \{u_1\} \end{array} \right]$$

Step 3. Creating the weighted normalized decision matrix, V.

In this step, calculate the weights corresponding to each parameter.

$$v_{11} = \sum_{i=1}^{3} \gamma_{f_{X_i}(x_1)}(u_1) = \gamma_{f_{X_1}(x_1)}(u_1) + \gamma_{f_{X_2}(x_1)}(u_1) + \gamma_{f_{X_3}(x_1)}(u_1) = 1+1+0 = 2$$

$$v_{12} = \sum_{i=1}^{3} \gamma_{f_{X_i}(x_1)}(u_2) = \gamma_{f_{X_1}(x_1)}(u_2) + \gamma_{f_{X_2}(x_1)}(u_2) + \gamma_{f_{X_3}(x_1)}(u_2) = 0+0+0 = 0$$

$$v_{13} = \sum_{i=1}^{3} \gamma_{f_{X_i}(x_1)}(u_3) = \gamma_{f_{X_1}(x_1)}(u_3) + \gamma_{f_{X_2}(x_1)}(u_3) + \gamma_{f_{X_3}(x_1)}(u_3) = 0+1+0 = 1$$

$$v_{21} = \sum_{i=1}^{3} \gamma_{f_{X_i}(x_2)}(u_1) = \gamma_{f_{X_1}(x_2)}(u_1) + \gamma_{f_{X_2}(x_2)}(u_1) + \gamma_{f_{X_3}(x_2)}(u_1) = 1+0+1 = 2$$

$$v_{22} = \sum_{i=1}^{3} \gamma_{f_{X_i}(x_2)}(u_2) = \gamma_{f_{X_1}(x_2)}(u_2) + \gamma_{f_{X_2}(x_2)}(u_2) + \gamma_{f_{X_3}(x_2)}(u_2) = 1+0+0 = 1$$

$$v_{23} = \sum_{i=1}^{3} \gamma_{f_{X_i}(x_2)}(u_3) = \gamma_{f_{X_1}(x_2)}(u_3) + \gamma_{f_{X_2}(x_2)}(u_3) + \gamma_{f_{X_3}(x_2)}(u_3) = 0+1+1 = 2$$

$$v_{31} = \sum_{i=1}^{3} \gamma_{f_{X_i}(x_3)}(u_1) = \gamma_{f_{X_1}(x_3)}(u_1) + \gamma_{f_{X_2}(x_3)}(u_1) + \gamma_{f_{X_3}(x_3)}(u_1) = 0+0+1 = 1$$

$$v_{32} = \sum_{i=1}^{3} \gamma_{f_{X_i}(x_3)}(u_2) = \gamma_{f_{X_1}(x_3)}(u_2) + \gamma_{f_{X_2}(x_3)}(u_2) + \gamma_{f_{X_3}(x_3)}(u_2) = 0+0+0 = 0$$

$$v_{33} = \sum_{i=1}^{3} \gamma_{f_{X_i}(x_3)}(u_3) = \gamma_{f_{X_1}(x_3)}(u_3) + \gamma_{f_{X_2}(x_3)}(u_3) + \gamma_{f_{X_3}(x_3)}(u_3) = 0+0+0 = 0$$

Then the weight matrix is obtained as

$$V = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix}$$

Step 4. Creating the decision matrix (vector), R.

Now, calculate the individual elements of the R matrix.

$$k(u_1) = \sum_{i=1}^{3} v_{1i} = v_{11} + v_{12} + v_{13} = 2 + 2 + 1 = 5$$

$$k(u_2) = \sum_{i=1}^{3} v_{2i} = v_{21} + v_{22} + v_{23} = 0 + 1 + 0 = 1$$

$$k(u_3) = \sum_{i=1}^{3} v_{3i} = v_{31} + v_{32} + v_{33} = 1 + 2 + 0 = 3$$

Thus the decision matrix of each alternative values for the decision-makers is obtained as

$$R = [5 \ \ 1 \ \ 3]$$

Step 5. Ranking the preference order.

Ranking among the alternatives would be created in the order in descending order of the values $k(u_j)$ calculated in the fifth step. So when the fifth step in the calculation of the evaluation of the candidate houses (alternatives) from small to large $k(u_2) < k(u_3) < k(u_1)$, the order form is realized in the form of ranking $u_2 < u_3 < u_1$. In other words, the most suitable house appears to be u_1.

6 Conclusion

We have proposed new model for decision making. After checking the aggregations under various circumstances, we can see that the new model is rather simple to use and meaningful for aggregation, and it will not cause more computational burden than the original TOPSIS.

Although some observations are obtained from the given examples, we are confident the results for various examples would give us similar conclusions. However, a large number of examples could be recommended for test in future studies.

References

[1] Agrawal, V.P., Kohli, V. and Gupta, S., Computer aided robot selection: The multiple attribute decision making approach. International Journal of Production Research, 29(8), (1991) 1629-1644.

[2] Ali, M.I., Feng, F., Liu, X., Min, W.K. and Shabir, M., On some new operations in soft set theory, Comput. Math. Appl., 57, (2009) 1547-1553.

[3] Atanassov, K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, (1986) 87-96.

[4] Belenson, S.M. and Kapur, K.C., An algorithm for solving multicriterion linear programming problems with examples, Operational Research Quarterly, 24(1), (1973) 65-77.

[5] Çağman, N., Çıtak, F. and Enginoğlu, S., Fuzzy parameterized fuzzy soft set theory and its applications, Turkish Journal of Fuzzy Systems, 1(1), (2010) 21-35.

[6] Çağman, N., Çıtak, F. and Enginoğlu, S., FP-Soft set theory and its applications, Annals of Fazzy Mathematics and Informatics, 2(2), (2011) 219-226.

[7] Çağman, N. and Enginoğlu, S., Soft set theory and uni-int decision making, European Journal of Operational Research, 207, (2010) 848-855.

[8] Çağman, N. and Enginoğlu, S., Soft matrix theory and its decision makings. Computers and Mathematics with Applications, 59, (2010) 3308-3314.

[9] Çağman, N., Enginoğlu, S. and Çıtak, F., Fuzzy soft set theory and its applications, Iranian Journal of Fuzzy Systems, 8 (3), (2011) 137-147.

[10] Çağman, N. and Enginoğlu, S., Fuzzy Soft Matrix Theory and Its Applications in Decision Making, Iranian Journal of Fuzzy Systems, 9(1), (2012) 109-119.

[11] Çağman, N., Contributions to the theory of soft sets, Journal of New Results in Science, 4, (2014) 33-41.

[12] Chen, M.F. and Tzeng, G.H., Combining gray relation and TOPSIS concepts for selecting an expatriate host country, Mathematical and Computer Modelling 40, (2004) 1473-1490.

[13] Deng, H., Yeh, C.H. and Willis, R. J., Inter-company comparison using modified TOPSIS with objective weights. Computers and Operations Research, 27(10), (2000) 963-973.

[14] Feng, C.M. and Wang, R-T., Considering the financial ratios on the performance evaluation of highway bus industry. Transport Reviews, 21(4), (2001) 449-467.

[15] Hwang, C.L. and Yoon, K., Multiple attribute decision making: Methods and applications. New York: Springer-Verlag, (1981).

[16] Janic, M., Multicriteria evaluation of high-speed rail, transrapid maglev, and air passenger transport in Europe, Transportation Planning and Technology 26(6), (2003) 491-512.

[17] Kahraman, C., Engin, O., Kabak, O. and Kaya, I., Information systems outsourcing decisions using a group decision-making approach. Engineering Applications of Artificial Intelligence, 22(6), (2009) 832-841.

[18] Kim, G., Park, C.S. and Yoon, K.P., Identifying investment opportunities for advanced manufacturing systems with comparative-integrated performance measurement, International Journal of Production Economics 50, (1997) 23-33.

[19] Kwong, C.K. and Tam, S.M., Case-based reasoning approach to concurrent design of low power transformers, Journal of Materials Processing Technology 128, (2002) 136-141.

[20] Lai, Y.J., Liu, T.Y. and Hwang, C. L., TOPSIS for MODM. European Journal of Operational Research, 76(3), (1994) 486-500.

[21] Maji, P.K., Biswas, R. and Roy, A.R., Soft set theory, Computers and Mathematics with Applications, 45, (2003) 555-562.

[22] Milani, A.S., Shanian, A. and Madoliat, R., The effect of normalization norms in multiple attribute decision making models: A case study in gear material selection, Structural Multidisciplinary Optimization 29(4), (2005) 312-318.

[23] Molodtsov, D.A., Soft set theory-first results, Computers and Mathematics with Applications, 37(1), (1999) 19-31.

[24] Molodtsov, D.A., Leonov, V.Y. and Kovkov, D.V., Soft Sets Technique and Its Application, Nechetkie Sistemy i Myagkie Vychisleniya 1/1, (2006) 8-39.

[25] Opricovic, S. and Tzeng, G.H., "Compromise Solution By MCDM Methods: A Comparative Analysis Of VIKOR and TOPSIS", European Journal Of Operational Research, C:CLVI, (2004) 445-455.

[26] Parkan, C. and Wu, M.L., Decision-making and performance measurement models with applications to robot selection. Computers and Industrial Engineering, 36(3), (1999) 503-523.

[27] Pawlak, Z., Rough sets, International Journal of Information and Computer Sciences, 11, (1982) 341-356.

[28] Paxkan, C. and Wu, M.L., On the equivalence of operational performance measurement and multiple attribute decision making. International Journal of Production Research, 35(11), (1997) 2963-2988.

[29] Peters, M.L. and Zelewski, S., TOPSIS als Technik zur Effieienzanalyse, Zeitschrift für Ausbildung und Hochschulkontakt, (2007) 1-9.

[30] Rao, R.V., Evaluation of environmentally conscious manufacturing programs using multiple attribute decision-making methods, Proceedings of the Institution of Mechanical Engineers-Part B-Engineering Manufacture, 222(3), (2008) 441-451.

[31] Ren, L., Zhang, Y., Wang, Y. and Sun, Z., Comparative analysis of a novel MTOPSIS method and TOPSIS. Applied Mathematics Research Express, 10. doi:10.1093/amrx/abm005. Article ID abm005, (2007)

[32] Shih, H.S., Lin, W.Y. and Lee, E.S., Group decision making for TOPSIS, in: Joint 9th IFSA World Congress and 20th NAFIPS International Conference, IFSA/NAFIPS, Vancouver, Canada, (2001) 2712-2717.

[33] Shih, H.S., Shyur, H.J. and Lee, E.S., An extension of TOPSIS for group decision making, Mathematical and Computer Modelling 45, (2007) 801-813.

[34] Shyura, H.J. and Shih, H.S., A hybrid MCDM model for strategic vendor selection. Mathematical and Computer Modelling, 44(7-8), (2006) 749-761.

[35] Srdjevic, B., Medeiros, Y.D.P. and Faria, A.S., An objective multi-criteria evaluation of water management scenarios, Water Resources Management 18, (2004) 35-54.

[36] Stern, Z.S., Mehrez, A. and Hadad, Y., An AHP/DEA methodology for ranking decision making units, Intl. Trans. In Op. Res., 7, (2000) 109-124.

[37] Triantaphyllou, E., Multi-Criteria Decision Making Methods: A Comparative Study, Kluwer Academic Publishers, Netherlands, (2000), 139-140.

[38] Yang, T. and Chou, P. Solving a multiresponse simulation-optimization problem with discrete variables using a multi-attribute decision-making method, Mathematics and Computers in Simulation 68, (2005) 9-21.

[39] Yoon, K. and Hwang, C.L., Manufacturing plant location analysis by multiple attribute decision making: Part I-single-plant strategy, International Journal of Production Research 23, (1985) 345-359.

[40] Yoon, K.P. and Hwang, C.L., Multiple Attribute Decision Making: An Introduction, Sage Pub., Thousand Oaks, CA, (1995).

[41] Yurdakul, M. and İç, Y.T., Development of a performance measurement model for manufacturing companies using the AHP and Topsis approaches, International Journal of Production Research, 43(21), (2005) 4609-4641.

[42] Zadeh, L.A., Fuzzy Sets, Information and Control, 8, (1965) 338-353.

[43] Zeleny, M., A concept of compromise solutions and the method of the displaced ideal. Computers and Operations Research, 1(3-4), (1974) 479-496.

[44] Zhu, P. and Wen, Q., Operations on Soft Sets Revisited, Journal of Applied Mathematics, 1-7, (2013).