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Soft Connectedness Via Soft Ideals

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Abstract - The notion of soft topological spaces with soft ideal was studied by Kandil et al. [15]. Applications to various fields were further investigated by Kandil et al. [13, 14]. The notion of connectedness in soft topological spaces was initiated by Lin in [19]. The purpose of this paper is to introduce and study the notion of connectedness to soft topological spaces with soft ideals. We study the notions of $\star$-soft connected sets, $\star$-soft separated sets and $\star_s$-soft connected sets in soft topological spaces with soft ideals.

Keywords - *Soft sets, Soft topological spaces, Soft connected, Soft hyperconnected, $\star$-soft separated, $\star$-soft connected, $\star$-soft hyperconnected.*

1 Introduction

The concept of soft sets was first introduced by Molodtsov [25] in 1999 as a general mathematical tool for dealing with uncertain objects. In [25, 26], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on.

After presentation of the operations of soft sets [23], the properties and applications of soft set theory have been studied increasingly [4, 18, 26, 29]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1, 3, 8, 21, 22, 23, 24, 26, 27, 35]. To develop soft set theory, the operations

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of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [9].

Recently, in 2011, Shabir and Naz [30] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Hussain and Ahmad [10] investigated the properties of open (closed) soft, soft nbd and soft closure. They also defined and discussed the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces. Kandil et al.[12] introduced a unification of some types of different kinds of subsets of soft topological spaces using the notions of γ-operation. Kandil et al.[16] generalize this unification of some types of different kinds of subsets of soft topological spaces using the notions of γ-operation to supra soft topological spaces. The notion of soft ideal is initiated for the first time by Kandil et al.[15]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, \tilde{I})$. Applications to various fields were further investigated by Kandil et al. [13, 14]. In this paper we introduce and study the notion of connectedness to soft topological spaces with soft ideals and give basic definitions and theorems about it. This paper, not only can form the theoretical basis for further applications of topology on soft set, but also lead to the development of information systems.

2 Preliminary

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1. *[25] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \phi$ i.e*
$F_A = \{F(e) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2.2. *[23] Let $F_A,\ G_B \in SS(X)_E$. Then F_A is soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if*

(1) *$A \subseteq B$, and*

(2) *$F(e) \subseteq G(e), \forall e \in A$.*

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$.

Definition 2.3. *[23] Two soft subset F_A and G_B over a common universe set X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.*

Definition 2.4. *[4] The complement of a soft set (F, A), denoted by $(F, A)'$, is defined by $(F, A)' = (F', A)$, $F' : A \to P(X)$ is a mapping given by $F'(e) = X - F(e)$, $\forall e \in A$ and F' is called the soft complement function of F.*
Clearly $(F')'$ is the same as F and $((F, A)')' = (F, A)$.

Definition 2.5. *[30] The difference of two soft sets (F, E) and (G, E) over the common universe X, denoted by $(F, E) - (G, E)$ is the soft set (H, E) where for all $e \in E$, $H(e) = F(e) - G(e)$.*

Definition 2.6. *[30] Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$.*

Definition 2.7. *[30] Let $x \in X$. Then x_E denote the soft set over X for which $x_E(e) = \{x\}$ $\forall e \in E$ and called the singleton soft point.*

Definition 2.8. *[23] A soft set (F, A) over X is said to be a NULL soft set denoted by $\tilde{\phi}$ or ϕ_A if for all $e \in A$, $F(e) = \phi$ (null set).*

Definition 2.9. *[23] A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A$, $F(e) = X$. Clearly we have $X'_A = \phi_A$ and $\phi'_A = X_A$.*

Definition 2.10. *[23] The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e), \ e \in A - B, \\ G(e), \ e \in B - A, \\ F(e) \cup G(e), \ e \in A \cap B \end{cases}.$$

Definition 2.11. *[23] The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F, E) over a universe X in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.*

Definition 2.12. *[36] Let I be an arbitrary indexed set and $L = \{(F_i, E), i \in I\}$ be a subfamily of $SS(X)_E$.*

(1) *The union of L is the soft set (H, E), where $H(e) = \bigcup_{i\in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcup}_{i\in I}(F_i, E) = (H, E)$.*

(2) *The intersection of L is the soft set (M, E), where $M(e) = \bigcap_{i\in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcap}_{i\in I}(F_i, E) = (M, E)$.*

Definition 2.13. *[30] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if*

(1) *$\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X$, $\forall e \in E$,*

(2) *the union of any number of soft sets in τ belongs to τ,*

(3) *the intersection of any two soft sets in τ belongs to τ.*

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.14. *[10] Let (X, τ, E) be a soft topological space. A soft set (F, A) over X is said to be closed soft set in X, if its relative complement $(F, A)'$ is open soft set.*

Definition 2.15. *[10] Let(X, τ, E) be a soft topological space. The members of τ are said to be open soft sets in X. We denote the set of all open soft sets over X by $OS(X, \tau, E)$, or when there can be no confusion by $OS(X)$ and the set of all closed soft sets by $CS(X, \tau, E)$, or $CS(X)$.*

Definition 2.16. *[30] Let (X, τ, E) be a soft topological space and $(F, A) \in SS(X)_E$. The soft closure of (F, A), denoted by $cl(F, A)$ is the intersection of all closed soft super sets of (F, A). Clearly $cl(F, A)$ is the smallest closed soft set over X which contains (F, A) i.e*
$cl(F, A) = \tilde{\cap}\{(H, C) : (H, C) \text{ is closed soft set and } (F, A)\tilde{\subseteq}(H, C)\})$.

Definition 2.17. *[36] Let (X, τ, E) be a soft topological space and $(F, A) \in SS(X)_E$. The soft interior of (G, B), denoted by $int(G, B)$ is the union of all open soft subsets of (G, B). Clearly $int(G, B)$ is the largest open soft set over X which contained in (G, B) i.e*
$int(G, B) = \tilde{\cup}\{(H, C) : (H, C) \text{ is an open soft set and } (H, C)\tilde{\subseteq}(G, B)\})$.

Definition 2.18. *[36] The soft set $(F, E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$, and the soft point (F, E) is denoted by x_e.*

Proposition 2.19. *[31] The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it.*

Definition 2.20. *[36] The soft point x_e is said to be belonging to the soft set (G, A), denoted by $x_e\tilde{\in}(G, A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.*

Definition 2.21. *[36] A soft set (G, B) in a soft topological space (X, τ, E) is called a soft neighborhood (briefly: nbd) of the soft point $x_e\tilde{\in}X_E$ if there exists an open soft set (H, C) such that $x_e\tilde{\in}(H, C)\tilde{\subseteq}(G, B)$.*
A soft set (G, B) in a soft topological space (X, τ, E) is called a soft neighborhood of the soft (F, A) if there exists an open soft set (H, C) such that $(F, A)\tilde{\in}(H, C)\tilde{\subseteq}(G, B)$. The neighborhood system of a soft point x_e, denoted by $N_\tau(x_e)$, is the family of all its neighborhoods.

Theorem 2.22. *[19] Let (X, τ, E) be a soft topological space. For any soft point x_e, $x_e\tilde{\in}cl(F, A)$ if and only if each soft neighborhood of x_e intersects (F, A).*

Definition 2.23. *[30] Let (X, τ, E) be a soft topological space, $(F, E) \in SS(X)_E$ and Y be a non null subset of X. Then the sub soft set of (F, E) over Y denoted by (F_Y, E), is defined as follows:*

$$F_Y(e) = Y \cap F(e) \ \forall e \in E.$$

In other words $(F_Y, E) = \tilde{Y}\tilde{\cap}(F, E)$.

Definition 2.24. *[30] Let (X, τ, E) be a soft topological space and Y be a non null subset of X. Then*

$$\tau_Y = \{(F_Y, E) : (F, E) \in \tau\}$$

is said to be the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

Theorem 2.25. *[30] Let (Y, τ_Y, E) be a soft subspace of a soft topological space (X, τ, E) and $(F, E) \in SS(X)_E$. Then*

(1) *If (F, E) is open soft set in Y and $\tilde{Y} \in \tau$, then $(F, E) \in \tau$.*

(2) *(F, E) is open soft set in Y if and only if $(F, E) = \tilde{Y} \tilde{\cap} (G, E)$ for some $(G, E) \in \tau$.*

(3) *(F, E) is closed soft set in Y if and only if $(F, E) = \tilde{Y} \tilde{\cap} (H, E)$ for some (H, E) is τ-closed soft set.*

Definition 2.26. *[12] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. If $(F, E) \tilde{\subseteq} cl(int(F, E))$, then (F, E) is called semi-open soft set. We denote the set of all semi-open soft sets by $SOS(X, \tau, E)$, or $SOS(X)$ and the set of all semi-closed soft sets by $SCS(X, \tau, E)$, or $SCS(X)$.*

Definition 2.27. *[2] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets, $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a mapping. Then;*

(1) *If $(F, A) \in SS(X)_A$. Then the image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that*
$$f_{pu}(F)(b) = \begin{cases} \cup_{x \in p^{-1}(b) \cap A} \; u(F(a)), & p^{-1}(b) \cap A \neq \phi, \\ \phi, & otherwise. \end{cases}$$
for all $b \in B$.

(2) *If $(G, B) \in SS(Y)_B$. Then the inverse image of (G, B) under f_{pu}, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that*
$$f_{pu}^{-1}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B, \\ \phi, & otherwise. \end{cases}$$
for all $a \in A$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective.

Definition 2.28. *[12, 20, 36] Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then The function f_{pu} is called*

(1) *Continuous soft if $f_{pu}^{-1}(G, B) \in \tau_1 \; \forall \; (G, B) \in \tau_2$.*

(2) *Open soft if $f_{pu}(G, A) \in \tau_2 \forall \; (G, A) \in \tau_1$.*

(3) *Semi open soft if $f_{pu}(G, A) \in SOS(Y) \forall \; (G, A) \in \tau_1$.*

(4) *Semi continuous soft function if $f_{pu}^{-1}(G, B) \in SOS(X) \forall \; (G, B) \in \tau_2$.*

(5) *Irresolute soft if* $f_{pu}^{-1}(G,B) \in SOS(X) \forall\ (G,B) \in SOS(Y)[f_{pu}^{-1}(F,B) \in SCS(X)$ $\forall\ (F,B) \in SCS(Y)]$.

(6) *Irresolute open soft (resp. irresolute closed soft) if* $f_{pu}(G,A) \in SOS(Y) \forall\ (G,A) \in SOS(X)(resp. f_{pu}(F,A) \in SCS(Y) \forall\ (F,A) \in SCS(Y))$.

(7) β*-irresolute soft if* $f_{pu}^{-1}(G,B) \in \beta OS(X) \forall\ (G,B) \in \beta OS(Y)$.

Theorem 2.29. *[36] Let* $SS(X)_A$ *and* $SS(Y)_B$ *be families of soft sets. For the soft function* $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$*, the following statements hold,*

(a) $f_{pu}^{-1}((G,B)') = (f_{pu}^{-1}(G,B))' \forall\ (G,B) \in SS(Y)_B$.

(b) $f_{pu}(f_{pu}^{-1}((G,B))) \tilde{\subseteq} (G,B) \forall\ (G,B) \in SS(Y)_B$. *If* f_{pu} *is surjective, then the equality holds.*

(c) $(F,A) \tilde{\subseteq} f_{pu}^{-1}(f_{pu}((F,A))) \forall\ (F,A) \in SS(X)_A$. *If* f_{pu} *is injective, then the equality holds.*

Definition 2.30. *[6] Let* (X,τ,E) *be a soft topological space and* $x, y \in X$ *such that* $x \neq y$*. Then* (X,τ,E) *is called a soft Hausdorff space or soft* T_2 *space if there exist open soft sets* (F,E) *and* (G,E) *such that* $x \in (F,E)$*,* $y \in (G,E)$ *and* $(F,E) \tilde{\cap} (G,E) = \tilde{\phi}$

Definition 2.31. *[11]. A non-empty collection* I *of subsets of a set* X *is called an ideal on* X*, if it satisfies the following conditions*

(1) $A \in I$ *and* $B \in I \Rightarrow A \cup B \in I$,

(2) $A \in I$ *and* $B \subseteq A \Rightarrow B \in I$,
i.e. I *is closed under finite unions and subsets.*

Definition 2.32. *[15] Let* $\tilde{I}$ *be a non-null collection of soft sets over a universe* X *with a fixed set of parameters* E*, then* $\tilde{I} \subseteq SS(X)_E$ *is called a soft ideal on* X *with a fixed set* E *if*

(1) $(F,E) \in \tilde{I}$ *and* $(G,E) \in \tilde{I} \Rightarrow (F,E) \tilde{\cup} (G,E) \in \tilde{I}$,

(2) $(F,E) \in \tilde{I}$ *and* $(G,E) \tilde{\subseteq} (F,E) \Rightarrow (G,E) \in \tilde{I}$,
i.e. $\tilde{I}$ *is closed under finite soft unions and soft subsets.*

Definition 2.33. *[19] Let* (X,τ,E) *be a soft topological space. A soft separation on* $\tilde{X}$ *is a pair of non null proper open soft sets such that* $(F,E) \tilde{\cap} (G,E) = \tilde{\phi}$ *and* $\tilde{X} = (F,E) \tilde{\cup} (G,E)$.

Definition 2.34. *[19] A non null soft subsets* (F,E)*,* (G,E) *of a soft topological space* (X,τ,E) *are said to be soft separated sets if* $cl(F,E) \tilde{\cap} (G,E) = (F,E) \tilde{\cap} cl(G,E) = \tilde{\phi}$.

Definition 2.35. *[19] A soft topological space* (X,τ,E) *is said to be soft connected if and only if* $\tilde{X}$ *can not expressed as the soft union of two soft separated sets in* (X,τ,E)*. Otherwise,* (X,τ,E) *is said to be soft disconnected.*

Definition 2.36. *[7] Let* (X,τ,E) *be a soft topological space. A soft semi separation on* $\tilde{X}$ *is a pair of non null proper semi open soft sets* $(F,E),(G,E)$ *such that* $(F,E) \tilde{\cap} (G,E) = \tilde{\phi}$ *and* $\tilde{X} = (F,E) \tilde{\cup} (G,E)$.

Definition 2.37. *[7] A soft topological space* (X,τ,E) *is said to be semi-soft connected if and only if there is no soft semi separations on* $\tilde{X}$*. Otherwise,* (X,τ,E) *is said to be soft semi disconnected.*

2.1 Soft Connected Spaces Via Soft Ideals

In this section we extend the notion of soft connectedness mentioned in [19] to soft topological spaces with soft ideal and study some of its basic properties.

Definition 2.38. *Let (X, τ, E) be a soft topological space. Two soft sets (F, E) and (G, E) are said to be disjoint if $F(e) \cap G(e) = \phi \; \forall e \in E$.*

Definition 2.39. *Let (X, τ, E) be a soft topological space. A $\star$-soft separation of $\tilde{X}$ is a pair of non null proper soft sets with $(F, E) \in \tau$ and $(G, E) \in \tau^*$ such that $(F, E)\tilde{\cap}(G, E) = \tilde{\phi}$ and $\tilde{X} = (F, E)\tilde{\cup}(G, E)$.*

Definition 2.40. *A soft topological space with soft ideal $(X, \tau, E, \tilde{I})$ is said to be $\star$-soft connected if and only if there is no $\star$-soft separations on $\tilde{X}$. If $(X, \tau, E, \tilde{I})$ has such $\star$-soft separations, then $(X, \tau, E, \tilde{I})$ is said to be $\star$-soft disconnected.*

Theorem 2.41. *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and (X, τ^*, E) be a its $\star$-soft topological space, then the following are equivalent:*

(1) *$\tilde{X}$ is $\star$-soft connected.*

(2) *$\tilde{X}$ can not be expressed as a soft union of two non null disjoint soft subsets (F, E), (G, E) of $\tilde{X}$ which are τ-open soft and τ^*-open soft respectively.*

(3) *$\tilde{X}$ can not be expressed as a soft union of two non null disjoint soft subsets (F, E), (G, E) of $\tilde{X}$ which are τ-closed soft and τ^*-closed soft respectively.*

(4) *There is no proper soft subset (F, E) of $\tilde{X}$, which is neither τ^*-open soft and τ-closed soft, nor τ^*-closed soft and τ-open soft.*

Proof.

(1) $\Leftrightarrow$ (2) It is obvious from Definition 2.40.

(2) $\Rightarrow$ (3) Suppose that $\tilde{X} = (F, E)\tilde{\cup}(G, E)$ for some τ-closed soft (F, E) and τ^*-closed soft (G, E) such that $(F, E)\tilde{\cap}(G, E) = \tilde{\phi}$. Then $(F, E) = (G, E)'$ which is τ-open soft and τ^*-open soft, $\tilde{X} = (G, E)\tilde{\cup}(G, E)'$ and $(G, E)\tilde{\cap}(G, E)' = \tilde{\phi}$, which is a contradiction with (2).

(3) $\Rightarrow$ (4) Suppose that there is a proper soft subset (F, E) of $\tilde{X}$, which either τ^*-open soft and τ-closed soft, or τ^*-closed soft and τ-open soft. Then $(F, E)'$ is τ^*-closed soft and (F, E) is τ-closed soft with $\tilde{X} = (F, E)\tilde{\cup}(F, E)'$ and $(F, E)\tilde{\cap}(F, E)' = \tilde{\phi}$, which is a contradiction with (3).

(4) $\Rightarrow$ (1) Suppose that $\tilde{X} = (F, E)\tilde{\cup}(G, E)$ for some τ-open soft (F, E) and τ^*-open soft (G, E) such that $(F, E)\tilde{\cap}(G, E) = \tilde{\phi}$. Then $(F, E) = (G, E)'$ and $(G, E) = (F, E)'$. Then (F, E) is τ^*-closed soft and τ-open soft and (G, E) is τ^*-open soft and τ-closed soft, which is a contradiction with (4).

Theorem 2.42. *If $(X, \tau, E, \tilde{I})$ is $\star$-soft connected topological space with soft ideal, then (X, τ, E) is soft connected.*

Proof. Immediate.

Remark 2.43. *The converse of Theorem 2.42 is not true in general, as in the following example.*

Example 2.44. *Let* $X = \{h_1, h_2, h_3\}$, $E = \{e\}$, $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$, *where* $(F_1, E), (F_2, E)$ *are soft sets over* X *defined by* $F_1(e) = \{h_2\}$ *and* $F_2(e) = \{h_1, h_2\}$ *and* $\tilde{I} = \{\phi, (I_1, E), (I_2, E), (I_3, E)\}$ *be a soft ideal over* X *where* $(I_1, E), (I_2, E), (I_3, E)$ *are soft sets over* X *defined by* $I_1(e) = \{h_1\}$, $I_2(e) = \{h_2\}$ *and* $I_3(e) = \{h_1, h_2\}$. *Then* $\tau^* = SS(X)_E$ *is a soft topology finer than* τ. *Hence* (X, τ, E) *is soft connected but* $(X, \tau, E, \tilde{I})$ *is* $\star$*-soft disconnected.*

Theorem 2.45. *Let* $(X_1, \tau_1, A, \tilde{I})$ *and* $(X_2, \tau_2, B, \tilde{J})$ *be soft topological spaces with soft ideal and* $f_{pu} : (X_1, \tau_1, A, \tilde{I}) \rightarrow (X_2, \tau_2, B, \tilde{J})$ *be a surjective continuous soft function. If* $(X_1, \tau_1, A, \tilde{I})$ *is* $\star$*-soft connected, then* $(X_2, \tau_2, B, \tilde{J})$ *is* $\star$*-soft connected.*

Proof. Suppose that $\tilde{X_2}$ is $\star$-soft disconnected. Then there exists a $\star$-soft separation $(F, B), (G, B)$ of $\tilde{X_2}$ with $(F, B) \in \tau_2$ and $(G, B) \in \tau_2^*$ such that $(F, B)\tilde{\cap}(G, B) = \tilde{\phi_B}$ and $\tilde{X_2} = (F, B)\tilde{\cup}(G, B)$ from Theorem 2.41. So we have $f_{pu}^{-1}[(F, B)\tilde{\cap}(G, B)] = f_{pu}^{-1}(F, B)\tilde{\cap} f_{pu}^{-1}(G, B) = f_{pu}^{-1}(\tilde{\phi_B}) = \tilde{\phi_A}$ and $f_{pu}^{-1}[(F, B)\tilde{\cup}(G, B)] = f_{pu}^{-1}(F, B)\tilde{\cup} f_{pu}^{-1}(G, B) = f_{pu}^{-1}(\tilde{X_2}) = \tilde{X_1}$ from Theorem 2.29. Thus, $f_{pu}^{-1}(F, B)$ and $f_{pu}^{-1}(G, B)$ are $\star$-soft separation of $\tilde{X_1}$, which is a contradiction with $\star$-soft connectedness of $\tilde{X_1}$. Hence $(X_2, \tau_2, B, \tilde{J})$ is $\star$-soft connected.

Definition 2.46. *Let* (X, τ, E) *be a soft topological space and* $(Z, E)\tilde{\subseteq}\tilde{X}$ *with* $x \in (Z, E)$. *Then the soft component of* (Z, E) *w.r.t.* x *is the maximal of all soft connected subspaces of* (Z, τ_Z, E) *containing* x *and denoted by* $\tilde{C}[(Z, E), x]$ *or* $\tilde{C}(Z_E, x)$ *for short, i.e*

$$\tilde{C}(Z_E, x) = \tilde{\cup}\{Y_E \tilde{\subseteq} Z_E : x \in Y_E,\ Y_E \text{ is soft connected}\}.$$

Theorem 2.47. *Every soft component of a soft topological space* (X, τ, E) *is a maximal soft connected subset of* $\tilde{X}$.

Proof. It is obvious from Definition 2.46.

Theorem 2.48. *Every soft component of a soft topological space* (X, τ, E) *is closed soft set.*

Proof. It is obvious from Definition 2.46 and from the fact that the soft closure of a soft connected set is a soft connected.

Theorem 2.49. *Let* (X, τ, E) *be a soft topological space. Then:*

(1) *Each element* $x \in X$ *is contained in exactly one component of* $\tilde{X}$.

(2) *Any two soft components w.r.t. two different elements of* X *are either disjoint or identical.*

Proof.

(1) Let $x \in X$ and consider the collection

$$\tilde{C} = \{(Z, E)\tilde{\subseteq}\tilde{X} : x \in (Z, E),\ (Z, E) \text{ is soft connected}\}$$

Then we have,

(a) $\tilde{C} \neq \tilde{\phi}$, for the singleton soft point x_E is a soft connected subset of $\tilde{X}$ containing x.Then $x_E \in \tilde{C}$.

(b) $\tilde{\cap}\{(Z,E)\tilde{\subseteq}\tilde{X} : x \in (Z,E) : (Z,E)\ is\ soft\ connected\} \neq \tilde{\phi}$. Since $x \in (Z,E)\ \forall (Z,E) \in \tilde{C}$.

(c) The soft set $\tilde{\cup}\{(Z,E)\tilde{\subseteq}\tilde{X} : x \in (Z,E),\ (Z,E)\ is\ soft\ connected\}$, having a non null soft intersection, is soft connected subset of $\tilde{X}$ containing x.

(d) $\tilde{\cup}\{(Z,E)\tilde{\subseteq}\tilde{X} : x \in (Z,E),\ (Z,E)\ is\ soft\ connected\}$ is the largest soft connected subset of $\tilde{X}$ containing x, which is the soft component $\tilde{C}(\tilde{X},x)$ of X w.r.t x and containing x from Definition 2.46.

Now, suppose $\tilde{C}^*(\tilde{X},x)$ be another soft component containing x, then $\tilde{C}^*(\tilde{X},x)$ is a soft connected subset of $\tilde{X}$ containing x, but $\tilde{C}(\tilde{X},x)$ is a soft component, then $\tilde{C}(\tilde{X},x)$ is the largest soft connected subset of $\tilde{X}$ containing x, consequently, $\tilde{C}^*(\tilde{X},x)\tilde{\subseteq}\tilde{C}(\tilde{X},x)$.Similarly $\tilde{C}(\tilde{X},x)\tilde{\subseteq}\tilde{C}^*(\tilde{X},x)$, and hence x is contained in exactly one soft component of $\tilde{X}$.

(2) Let $\tilde{C}(\tilde{X},x_1)$, $\tilde{C}(\tilde{X},x_2)$ be the soft components of X w.r.t two different elements x_1, x_2 of X with $x_1 \neq x_2$ respectively. If $\tilde{C}(\tilde{X},x_1)\tilde{\cap}\tilde{C}(\tilde{X},x_2) = \tilde{\phi}$, then we get the proof. So, let $\tilde{C}(\tilde{X},x_1)\tilde{\cap}\tilde{C}(\tilde{X},x_2) \neq \phi$. We may choose a $x \in \tilde{C}(\tilde{X},x_1)\tilde{\cap}\tilde{C}(\tilde{X},x_2)$. Clearly, $x \in \tilde{C}(\tilde{X},x_1)$ and $x \in \tilde{C}(\tilde{X},x_2)$, which mean that $\tilde{C}(\tilde{X},x_1)$ is the largest soft connected subset of $\tilde{X}$ containing x , $\tilde{C}(\tilde{X},x_2)$ is the largest soft connected subset of $\tilde{X}$ containing x. Therefore, $\tilde{C}(\tilde{X},x_1) = \tilde{C}(\tilde{X},x_2)$, and hence $\tilde{C}(\tilde{X},x_1)$ and $\tilde{C}(\tilde{X},x_2)$ are identical. This completes the proof.

Definition 2.50. *A soft topological space (X,τ,E) is said to be soft hyperconnected if and only if every pair of non null proper open soft sets $(F,E),(G,E)$, has a non null soft intersection, i.e (X,τ,E) is said to be soft hyperconnected iff $\forall (F,E),(G,E) \in \tau$ we have $(F,E)\tilde{\cap}(G,E) \neq \tilde{\phi}$.*

Theorem 2.51. *Every soft hyperconnected soft topological space is soft connected.*

Proof. Suppose that (X,τ,E) be a soft disconnected soft topological space, then there exists a proper soft set (F,E), which is τ-open soft and τ-closed soft. Then $(F,E) \in \tau$ and $(F,E)' \in \tau$ such that $(F,E)\tilde{\cap}(F,E)' = \tilde{\phi}$. Hence $\tilde{X}$ is not soft hyperconnected, which is a contradiction. Thus, (X,τ,E) is is soft connected.

Remark 2.52. *The converse of Theorem 2.51 is not true in general, as shown in the following example.*

Example 2.53. *Let $X = \{h_1,h_2,h_3,h_4\}$, $E = \{e_1,e_2\}$ and $\tau = \{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E)\}$ where $(F_1,E),(F_2,E),(F_3,E)$ are soft sets over X defined as follows:*
$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_2\}$,
$F_2(e_1) = \{h_2,h_3\}$, $F_2(e_2) = \{h_1,h_3\}$,
$F_3(e_1) = \{h_1,h_2,h_3\}$, $F_3(e_2) = \{h_1,h_2,h_3\}$.
Then τ defines a soft topology on X. Hence the space (X,τ,E) is soft connected but not soft hyperconnected.

Definition 2.54. *A soft subset (F, E) of a soft topological space with ideal $(X, \tau, E, \tilde{I})$ is said to be $\star$-soft dense if $Cl^*(F, E) = \tilde{X}$. A soft topological space with soft ideal $(X, \tau, E, \tilde{I})$ is said to be a $\star$-soft hyperconnected if every non null τ-open soft set (F, E) is $\star$-soft dense.*

Theorem 2.55. *Every $\star$-soft hyperconnected soft topological space is $\star$-soft connected.*

Proof. Suppose that $(X, \tau, E, \tilde{I})$ be a $\star$-soft disconnected soft topological space with soft ideal. Then $\exists$ two non null proper soft sets $(F, E), (G, E)$ with $(F, E) \in \tau$ and $(G, E) \in \tau^*$ such that $(F, E)\tilde{\cap}(G, E) = \tilde{\phi}$ and $\tilde{X} = (F, E)\tilde{\cup}(G, E)$. Then $(G, E) = \tilde{\phi}$, which is a contradiction. Hence $(X, \tau, E, \tilde{I})$ is $\star$-soft connected.

Theorem 2.56. *If $(X, \tau, E, \tilde{I})$ is $\star$-soft hyperconnected soft topological space with soft ideal, then (X, τ, E) is soft hyperconnected.*

Proof. Immediate.

Remark 2.57. *The converse of Theorem 2.56 not true in general, as shown in the following example.*

Example 2.58. *In Example 2.44, the soft topological space (X, τ, E) is soft hyperconnected and the soft topological space with soft ideal $(X, \tau, E, \tilde{I})$ is not $\star$-soft hyperconnected.*

Theorem 2.59. . *If $(X, \tau, E, \tilde{I})$ be a $\star$-soft hyperconnected soft topological space with soft ideal, then (X, τ, E) is connected.*

Proof. Immediate by Theorem 2.42 and Theorem 2.55.

On accounting of Theorems 2.42, 2.51, 2.55 and Theorem 2.56, we have the following corollary.

Corollary 2.60. *The following implications hold from Theorems 2.42, 2.51, 2.55 and Theorem 2.56 for a soft topological space (X, τ, E) and a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$*

$(X, \tau, E, \tilde{I})$ *is $\star$-soft hyperconnected* $\Rightarrow$ (X, τ, E) *is soft hyperconnected*
$\Downarrow$ $\Downarrow$
$(X, \tau, E, \tilde{I})$ *is $\star$-soft connected* $\Rightarrow$ (X, τ, E) *is soft connected*

2.2 $\star_s$-Soft Connected Spaces Via Soft Ideals

Definition 2.61. *A non null soft subsets (F, E), (G, E) of a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$ are said to be $\star$-soft separated sets if $cl^*(F, E)\tilde{\cap}(G, E) = (F, E)\tilde{\cap}cl(G, E) = \tilde{\phi}$. Here we may interchange the roles of (F, E) and (G, E). Indeed, if (F, E), (G, E) are $\star$-soft separated sets, then $cl^*(G, E)\tilde{\cap}(F, E) = (G, E)\tilde{\cap}cl(F, E) = \tilde{\phi}$.*

Theorem 2.62. *Let $(F, E)\tilde{\subseteq}(G, E)$, $(H, E)\tilde{\subseteq}(K, E)$ and (G, E), (K, E) are soft $\star$-soft separated subsets of a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$. Then (F, E), (H, E) are $\star$-soft separated sets.*

Proof. Since $(F,E)\tilde{\subseteq}(G,E)$, then $cl(F,E)\tilde{\subseteq}cl(G,E)$. It follows that, $cl(F,E)\tilde{\cap}(H,E)\tilde{\subseteq}cl(F,E)\tilde{\cap}(K,E)\tilde{\subseteq}cl(H,E)\tilde{\cap}(K,E)=\tilde{\phi}$.
Also, since $(H,E)\tilde{\subseteq}(K,E)$.
Then $cl^*(H,E)\tilde{\subseteq}cl^*(K,E)$.
Hence $(F,E)\tilde{\cap}cl^*(H,E)\tilde{\subseteq}(F,E)\tilde{\cap}cl^*(K,E)\tilde{\subseteq}cl^*(K,E)\tilde{\cap}(G,E)=\tilde{\phi}$.
Thus, (F,E), (H,E) are $\star$-soft separated sets sets.

Theorem 2.63. *Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal. If (F,E), (G,E) are $\star$-soft separated sets and $(F,E)\tilde{\cup}(G,E)\in\tau$. Then (F,E), (G,E) are τ^*-open soft and τ-open soft, respectively.*

Proof. Suppose that (F,E), (G,E) be a $\star$-soft separated sets such that $(F,E)\tilde{\cup}(G,E)\in\tau$, then $((F,E)\tilde{\cup}(G,E)$ is τ^*-open soft set. Since $cl^*((G,E))$ is τ^*-closed soft set, then $(cl^*((G,E)))'$ is τ^*-open soft set. It follows that, $((F,E)\tilde{\cup}(G,E))\tilde{\cap}(cl^*((G,E)))'=(F,E)$ is τ^*-open soft set. Then $(F,E)=[(F,E)\tilde{\cap}(cl^*((G,E)))']\tilde{\cup}[(G,E)\tilde{\cap}(cl^*((G,E)))']$ is τ^*-open soft set. The rest of the proof by similar way.

Theorem 2.64. *Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal. If (F,E), (G,E) are $\star$-soft separated sets and $(F,E)\tilde{\cup}(G,E)$ is τ-closed soft set. Then (F,E), (G,E) are τ^*-closed soft and τ-closed soft respectively.*

Proof. It is similar to the proof of Theorem 2.63

Theorem 2.65. *Two τ-closed soft subsets of soft topological space with soft ideal $(X,\tau,E,\tilde{I})$ are $\star$-soft separated sets if and only if they are disjoint.*

Proof. Let (F,E), (G,E) are $\star$-soft separated sets. Then $cl^*(G,E)\tilde{\cap}(F,E)=(G,E)\tilde{\cap}cl(F,E)=\tilde{\phi}$. Since (F,E), (G,E) are closed soft sets. Then $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$. Conversely, let (F,E), (G,E) are τ-closed soft sets such that $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$. Then $(G,E)\tilde{\cap}cl(F,E)=\tilde{\phi}$ and $cl^*(G,E)\tilde{\cap}(F,E)\tilde{\subseteq}cl(G,E)\tilde{\cap}(F,E)=(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$ from [[15] Theorem 3.2]. It follows that (F,E), (G,E) are $\star$-soft separated sets.

Definition 2.66. *A soft topological space with soft ideal $(X,\tau,E,\tilde{I})$ is said to be $\star_s$-soft connected if and only if $\tilde{X}$ can not expressed as the soft union of two $\star$-soft separated sets in $(X,\tau,E,\tilde{I})$.*

Theorem 2.67. *Let $(Z,\tau_Z,E,\tilde{I}_Z)$ be a soft subspace of a soft topological space with soft ideal $(X,\tau,E,\tilde{I})$ and $(F_1,E),(F_2,E)\tilde{\subseteq}(Z,E)\tilde{\subseteq}\tilde{X}$. Then $(F_1,E),(F_2,E)$ are $\star$-soft separated on τ_Z if and only if $(F_1,E),(F_2,E)$ are $\star$-soft separated on τ, where τ_Z is the soft relative topology with soft ideal on (Z,E) and $\tilde{I}_Z=\{(Z,E)\tilde{\cap}(I,E):(I,E)\in\tilde{I}\}$.*

Proof. Suppose that $(F_1,E),(F_2,E)$ are $\star$-soft separated on τ_Z $\Leftrightarrow$ $cl^*_{\tau_Z}(F_1,E)\tilde{\cap}(F_2,E)=\tilde{\phi}$ and $(F_1,E)\tilde{\cap}cl_{\tau_Z}(F_2,E)=\tilde{\phi}$ $\Leftrightarrow$ $[cl^*_\tau(F_1,E)\tilde{\cap}(Z,E)]\tilde{\cap}(F_2,E)=cl^*_\tau(F_1,E)\tilde{\cap}(F_2,E)=\tilde{\phi}$ and $[cl_\tau(F_2,E)\tilde{\cap}(Z,E)]\tilde{\cap}(F_1,E)=cl_\tau(F_2,E)\tilde{\cap}(F_1,E)=\tilde{\phi}$ $\Leftrightarrow$ $(F_1,E),(F_2,E)$ are $\star$-soft separated on τ.

Theorem 2.68. *Let (Z,E) be a soft subset of a soft topological space with soft ideal $(X,\tau,E,\tilde{I})$. Then (Z,E) is $\star_s$-soft connected w.r.t $(X,\tau,E,\tilde{I})$ if and only if (Z,E) is $\star_s$-soft connected w.r.t $(Z,\tau_Z,E,\tilde{I}_Z)$.*

Proof. Suppose that (Z, E) is not $\star_s$-soft connected w.r.t $(Z, \tau_Z, E, \tilde{I}_Z) \Leftrightarrow (Z, E) = (F_1, E)\tilde{\cup}(F_2, E)$, where (F_1, E) and (F_2, E) are $\star$-soft separated on $\tau_Z \Leftrightarrow (Z, E) = (F_1, E)\tilde{\cup}(F_2, E)$, where (F_1, E) and (F_2, E) are $\star$-soft separated on τ_Z from Theorem 2.67 $\Leftrightarrow$ (Z, E) is not $\star_s$-soft connected w.r.t $(X, \tau, E, \tilde{I})$.

Theorem 2.69. *Let $(Z, \tau_Z, E, \tilde{I}_Z)$ be a $\star_s$-soft connected subspace of a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$ and (F, E), (G, E) are $\star$-soft separated of $\tilde{X}$ with $(Z, E)\tilde{\subseteq}(F, E)\tilde{\cup}(G, E)$, then either $(Z, E)\tilde{\subseteq}(F, E)$ or $(Z, E)\tilde{\subseteq}(G, E)$.*

Proof. Let $(Z, E)\tilde{\subseteq}(F, E)\tilde{\cup}(G, E)$ for some $\star$-soft separated sets (F, E), (G, E) on τ. Since $(Z, E) = ((Z, E)\tilde{\cap}(F, E))\tilde{\cup}((Z, E)\tilde{\cap}(G, E))$.

Then $((Z, E)\tilde{\cap}((F, E))\tilde{\cup}cl^*_\tau((Z, E)\tilde{\cap}(G, E))\tilde{\subseteq}(F, E)\tilde{\cap}cl^*_\tau(G, E) = \tilde{\phi}$.

Also, $cl_\tau((Z, E)\tilde{\cap}((F, E))\tilde{\cup}((Z, E)\tilde{\cap}(G, E))\tilde{\subseteq}cl_\tau(F, E)\tilde{\cap}(G, E) = \tilde{\phi}$. If $(Z, E)\tilde{\cap}(F, E)$ and $(Z, E)\tilde{\cap}(G, E)$ are non null soft sets. Then (Z, E) is not $\star_s$-soft connected w.r.t $(Z, \tau_Z, E, \tilde{I}_Z)$ from Theorem 2.68, which is a contradiction. Thus, either $(Z, E)\tilde{\cap}(F, E) = \tilde{\phi}$ or $(Z, E)\tilde{\cap}(G, E) = \tilde{\phi}$. It follows $(Z, E) = (Z, E)\tilde{\cap}(F, E)$ or $(Z, E) = (Z, E)\tilde{\cap}(G, E)$. This implies that, $(Z, E)\tilde{\subseteq}(F, E)$ or $(Z, E)\tilde{\subseteq}(G, E)$.

Theorem 2.70. *Let $(Z, \tau_Z, E, \tilde{I}_Z)$ be a $\star_s$-soft connected subspace of a $\star_s$-soft connected topological space with soft ideal $(X, \tau, E, \tilde{I})$ such that $(Z, E)'$ is the soft union of two $\star$-soft separated sets (F, E), (G, E) of $\tilde{X}$, then $(Z, E)\tilde{\cup}(F, E)$ and $(Z, E)\tilde{\cup}(G, E)$ are $\star_s$-soft connected.*

Proof. Suppose that $(Z, E)\tilde{\cup}(F, E)$ is not $\star_s$-soft connected w.r.t $(X, \tau, E, \tilde{I})$. Then, there exist two non null $\star$-soft separated sets (K, E) and (H, E) of $\tilde{X}$ such that $(Z, E)\tilde{\cup}(F, E) = (K, E)\tilde{\cup}(H, E)$.

Since (Z, E) is $\star_s$-soft connected and $(Z, E)\tilde{\subseteq}(Z, E)\tilde{\cup}(F, E) = (K, E)\tilde{\cup}(H, E)$. It follows that, either $(Z, E)\tilde{\subseteq}(K, E)$ or $(Z, E)\tilde{\subseteq}(H, E)$ from Theorem 2.69. Suppose $(Z, E)\tilde{\subseteq}(K, E)$. Since $(Z, E)\tilde{\cup}(F, E) = (K, E)\tilde{\cup}(H, E)$ and $(Z, E)\tilde{\subseteq}(K, E)$, implies that $(Z, E)\cup(F, E)\tilde{\subseteq}(K, E)\cup(F, E)$. So $(K, E)\tilde{\cup}(H, E)\tilde{\subseteq}(K, E)\tilde{\cup}(F, E)$. Hence $(H, E)\tilde{\subseteq}(F, E)$, $(F, E)\tilde{\subseteq}(F, E)$. By Theorem 2.62, $(H, E), (G, E)$ are $\star$-soft separated.

Now, $cl^*_\tau(H, E)\tilde{\cap}[(K, E)\tilde{\cup}(G, E)] = [cl^*_\tau(H, E)\tilde{\cap}(K, E)]\tilde{\cup}[cl^*_\tau(H, E)\tilde{\cap}(G, E)] = \tilde{\phi}$ and $(H, E)\tilde{\cap}cl_\tau\ [(K, E)\tilde{\cup}(G, E)] = (H, E)\tilde{\cap}\ [cl_\tau(K, E)\tilde{\cup}\ cl_\tau(G, E)] = [(H, E)\tilde{\cap}cl_\tau(K, E)]\tilde{\cup}$ $[(H, E)\tilde{\cap}\ cl_\tau(G, E)] = \tilde{\phi}$. It follows that, (H, E) and $[(K, E)\tilde{\cup}(G, E)]$ are $\star$-soft separated on τ. Since $(Z, E)' = (F, E)\tilde{\cup}(G, E)$.

Then $\tilde{X} = (Z, E)\tilde{\cup}(Z, E)' = (Z, E)\tilde{\cup}[(F, E)\tilde{\cup}(G, E)] = [(Z, E)\tilde{\cup}(F, E)]\tilde{\cup}(G, E) = [(K, E)\tilde{\cup}(H, E)]\tilde{\cup}(G, E) = (H, E)\tilde{\cup}[(K, E)\tilde{\cup}(G, E)]$. Thus, $\tilde{X}$ is the soft union of two non null $\star$-soft separated sets (H, E) and $(K, E)\tilde{\cup}(G, E)$, which is a contradiction. A similar contradiction will arise if $(Z, E)\tilde{\subseteq}(H, E)$. Hence, $(Z, E)\tilde{\cup}(F, E)$ is $\star_s$-soft connected. Similarly, we can prove that$(Z, E)\tilde{\cup}(G, E)$ is $\star_s$-soft connected.

If $\tilde{I} = \{\tilde{\phi}\}$ in Theorem 2.70, we get the following corollary.

Corollary 2.71. *Let (Z, τ_Z, E) be a soft connected subspace of a soft topological space (X, τ, E) such that $(Z, E)'$ is the soft union of two soft separated sets (F, E), (G, E) of $\tilde{X}$, then $(Z, E)\tilde{\cup}(F, E)$ and $(Z, E)\tilde{\cup}(G, E)$ are soft connected.*

The following theorem shows that the soft union of two $\star_s$-soft connected sets is a $\star_s$-soft connected set, if none of them is $\star_s$-soft separated.

Theorem 2.72. *If (Z,τ_Z,E) and (Y,τ_Y,E) are $\star_s$-soft connected subspaces of soft topological space with soft ideal $(X,\tau,E,\tilde{I})$ such that none of them is $\star_s$-soft separated, then $(Z,E)\tilde{\cup}(Y,E)$ is $\star_s$-soft connected w.r.t $(X,\tau,E,\tilde{I})$.*

Proof. Let (Z,τ_Z,E) and (Y,τ_Y,E) be $\star_s$-soft connected subspaces of $\tilde{X}$ such that $(Z,E)\tilde{\cup}(Y,E)$ is not $\star_s$-soft connected w.r.t $(X,\tau,E,\tilde{I})$. Then, there exist two non null $\star$-soft separated sets (K,E) and (H,E) of $\tilde{X}$ such that $(Z,E)\tilde{\cup}(Y,E)=(K,E)\tilde{\cup}(H,E)$. Since $(Z,E),(Y,E)$ are $\star_s$-soft connected and $(Z,E),(Y,E)\tilde{\subseteq}(Z,E)\tilde{\cup}(Y,E)=(K,E)\tilde{\cup}$ (H,E).

Then, either $(Z,E)\tilde{\subseteq}(K,E)$ and $(Y,E)\tilde{\subseteq}(H,E)$ or $(Z,E)\tilde{\subseteq}(H,E)$ and $(F,E)\tilde{\subseteq}(K,E)$ Theorem 2.69. If $(Z,E)\tilde{\subseteq}(K,E)$ and $(Y,E)\tilde{\subseteq}(H,E)$.

Then $(Z,E)\tilde{\cap}(H,E)=(K,E)\tilde{\cap}(Y,E)=\tilde{\phi}$. Therefore, $[(Z,E)\tilde{\cup}(Y,E)]\tilde{\cap}(K,E)=[(Z,E)\tilde{\cap}(K,E)]\tilde{\cup}[(Y,E)\tilde{\cap}(K,E)]=[(Z,E)\tilde{\cap}(K,E)]\tilde{\cup}\tilde{\phi}=(Z,E)\tilde{\cap}(K,E)=(Z,E)$. Also, $[(Z,E)\tilde{\cup}(Y,E)]\tilde{\cap}(H,E)=[(Z,E)\tilde{\cap}(H,E)]\tilde{\cup}[(Y,E)\tilde{\cap}(H,E)]=\tilde{\phi}\tilde{\cup}[(Y,E)\tilde{\cap}(H,E)]=(Y,E)\tilde{\cap}(H,E)=(Y,E)$.

Similarly, if $(Z,E)\tilde{\subseteq}(H,E)$ and $(Y,E)\tilde{\subseteq}(K,E)$. Then $[(Z,E)\tilde{\cup}(Y,E)]\tilde{\cap}(K,E)=(Z,E)$ and $[(Z,E)\tilde{\cup}(Y,E)]\tilde{\cap}(H,E)=(Y,E)$.

Now, $[((Z,E)\tilde{\cup}(Y,E))\tilde{\cap}(H,E)]\tilde{\cap}cl_\tau[((Z,E)\tilde{\cup}(Y,E))\tilde{\cap}(K,E)]\tilde{\subseteq}[((Z,E)\tilde{\cup}(Y,E))\tilde{\cap}(H,E)]\tilde{\cap}[cl_\tau((Z,E)\tilde{\cup}(Y,E))\tilde{\cap}cl_\tau(K,E)]=\tilde{\phi}$ and $cl^*_\tau[((Z,E)\tilde{\cup}(Y,E))\tilde{\cap}(H,E)]\tilde{\cap}[((Z,E)\tilde{\cup}(Y,E))\tilde{\cap}(K,E)]\tilde{\subseteq}[cl^*_\tau((Z,E)\tilde{\cup}(Y,E))\tilde{\cap}cl^*_\tau(H,E)]\tilde{\cap}[((Z,E)\tilde{\cup}(Y,E))\tilde{\cap}(K,E)]=\tilde{\phi}$. It follows that $[(Z,E)\tilde{\cup}(Y,E)]\tilde{\cap}(K,E)=(Z,E)$ and $[(Z,E)\tilde{\cup}(Y,E)]\tilde{\cap}(H,E)=(Y,E)$ are $\star_s$-soft separated, which is a contradiction. Hence $(Z,E)\tilde{\cup}(Y,E)$ is $\star_s$-soft connected w.r.t $(X,\tau,E,\tilde{I})$.

Remark 2.73. *The following example shows that the union of two $\star_s$-soft connected subsets of a soft topological space with soft ideal is not a $\star_s$-soft connected set in general.*

Example 2.74. *Let $X=\{h_1,h_2,h_3,h_4\}$, $E=\{e\}$ and $\tau=\{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E)\}$ where $(F_1,E),(F_2,E),(F_3,E)$ are soft sets over X defined as follows:*
$F_1(e)=\{h_2\}, F_2(e)=\{h_2,h_3\},\ F_3(e)=\{h_1,h_2,h_4\}$.
Then τ defines a soft topology on X. Let $\tilde{I}=\{\tilde{\phi},(I_1,E)\}$, where (I_1,E) is a soft set over X defined by $I_1(e)=\{h_2\}$. Hence the soft sets (G,E) and (H,E) which defined by $G(e)=\{h_1,h_2\}$, $H(e)=\{h_1,h_4\}$, are $\star_s$-soft connected. But their soft union $(G,E)\tilde{\cup}(H,E)=(K,E)$, where $K(e)=\{h_1,h_2,h_4\}$. Here $(K,E)=(A,E)\tilde{\cup}(B,E)$, where $A(e)=\{h_2\}$ and $B(e)=\{h_1,h_4\}$. Then, we have $cl^*_\tau(A,E)\tilde{\cap}(B,E)=\tilde{\phi}$ and $(A,E)\tilde{\cap}cl_\tau(B,E)=\tilde{\phi}$. So $(A,E),(B,E)$ are $\star$-soft separated sets. Hence $(A,E)\tilde{\cup}(B,E)$ is not $\star_s$-soft connected.

Theorem 2.75. *Let $(Z,\tau_Z,E,\tilde{I}_Z)$ be a $\star_s$-soft connected soft subspace of a $\star_s$-soft connected topological space with soft ideal $(X,\tau,E,\tilde{I})$ and $(Y,E)\in SS(X)_E$.*

If $(Z,E)\tilde{\subseteq}(Y,E)\tilde{\subseteq}cl^*_\tau(Z,E)$. Then $(Y,\tau_Y,E,\tilde{I}_Y)$ is $\star_s$-soft connected soft subspace of $(X,\tau,E,\tilde{I})$.

Proof. Suppose that $(Y,\tau_Y,E,\tilde{I}_Y)$ is not $\star_s$-soft connected w.r.t $(X,\tau,E,\tilde{I})$.

Then, there exist $\star$-soft separated sets $(F,E)$and (G,E) on τ such that $(Y,E)=(F,E)\tilde{\cup}(G,E)$. Thus, we have (Z,E) is $\star_s$-soft connected subset of a $\star_s$-soft disconnected space. By Theorem 2.69, either $(Z,E)\tilde{\subseteq}(F,E)$ or $(Z,E)\tilde{\subseteq}(G,E)$. If $(Z,E)\tilde{\subseteq}(F,E)$. Then $cl^*_\tau(Z,E)\tilde{\subseteq}cl^*_\tau(F,E)$. It follows that, $cl^*_\tau(Z,E)\tilde{\cap}(G,E)\tilde{\subseteq}cl^*_\tau(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$. Hence $(G,E)=cl^*_\tau(Z,E)\tilde{\cap}(G,E)=\tilde{\phi}$, which is a contradiction. Also, if $(Z,E)\tilde{\subseteq}(G,E)$.

By a similar way, we can get $(F,E)=\tilde{\phi}$, which is a contradiction. Hence $(Y,\tau_Y,E,\tilde{I}_Y)$ is $\star_s$-soft connected w.r.t $(X,\tau,E,\tilde{I})$.

Corollary 2.76. *If $(Z,\tau_Z,E,\tilde{I}_Z)$ is $\star_s$-soft connected soft subspace of a soft topological space with soft ideal $(X,\tau,E,\tilde{I})$. Then $cl^*_\tau(Z,E)$ is $\star_s$-soft connected.*

Proof. It is obvious from Theorem 2.75.

Theorem 2.77. *If for all pair of elements $x,\ y\in X$ with $x\neq y$ there exists a $\star_s$-soft connected set $(Z,E)\subseteq\tilde{X}$ with $x,\ y\in(Z,E)$, then $\tilde{X}$ is $\star_s$-soft connected.*

Proof. Suppose that $\tilde{X}$ is $\star_s$-soft disconnected. Then $\tilde{X}=(F,E)\tilde{\cup}(G,E)$, where $(F,E),(G,E)$ are $\star$-soft separated sets. It follows $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$. So, $\exists\ x\in(F,E)$ and $y\in(G,E)$. Since $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$. Then $x,\ y\in X$ with $x\neq y$. By hypothesis, there exists a $\star_s$-soft connected set $(Z,E)\subseteq\tilde{X}$ with $x,\ y\in(Z,E)$. Moreover, we have (Z,E) is $\star_s$-soft connected subset of a $\star_s$-soft disconnected space. Then, by Theorem 2.69, either $(Z,E)\tilde{\subseteq}(F,E)$ or $(Z,E)\tilde{\subseteq}(G,E)$ and both cases is a contradiction with the hypothesis. This implies that, $\tilde{X}$ is $\star_s$-soft connected.

Theorem 2.78. *Let $\{(Z_j,\tau_{Z_j},E,\tilde{I}_{Z_j}):j\in J\}$ be a non null family of $\star_s$-soft connected subspaces of soft topological space with soft ideal $(X,\tau,E,\tilde{I})$. If $\tilde{\cap}_{j\in J}(Z_j,E)\neq\tilde{\phi}$, then $(\tilde{\cup}_{j\in J}Z_j,\tau_{\tilde{\cup}_{j\in J}Z_j},E,\tilde{I}_{\tilde{\cup}_{j\in J}Z_j})$ is a $\star_s$-soft connected soft subspace of $(X,\tau,E,\tilde{I})$.*

Proof. Suppose that $(Z,\tau_Z,E,\tilde{I}_Z)=(\tilde{\cup}_{j\in J}Z_j,\tau_{\tilde{\cup}_{j\in J}Z_j},E,\tilde{I}_{\tilde{\cup}_{j\in J}Z_j})$ is $\star_s$-soft disconnected. Then $(Z,E)=(F,E)\tilde{\cup}(G,E)$, where $(F,E),(G,E)$ are $\star$-soft separated sets on τ_Z. Since $\tilde{\cap}_{j\in J}(Z_j,E)\neq\tilde{\phi}$. Then $\exists\ x\in\tilde{\cap}_{j\in J}(Z_j,E)$. It follows that, $x\in(Z,E)$. So either $x\in(F,E)$ or $x\in(G,E)$. Suppose that $x\in(F,E)$. Since $x\in(Z_j,E)\ \forall i\in J$ and $(Z_j,E)\tilde{\subseteq}(Z,E)$. Thus, we have (Z_j,E) is $\star_s$-soft connected subset of a $\star_s$-soft disconnected space. By Theorem 2.69, either $(Z_j,E)\tilde{\subseteq}(F,E)$ or $(Z_j,E)\tilde{\subseteq}(G,E)\ \forall i\in J$. If $(Z_j,E)\tilde{\subseteq}(F,E)\ \forall i\in J$. Then $(Z,E)\tilde{\subseteq}(F,E)$. This implies that, $(G,E)=\tilde{\phi}$, which is a contradiction. Also, if $(Z_j,E)\tilde{\subseteq}(G,E)\ \forall i\in J$. By a similar way, we can get $(F,E)=\tilde{\phi}$, which is a contradiction. Thus, $(Z,\tau_Z,E,\tilde{I}_Z)=(\tilde{\cup}_{j\in J}Z_j,\tau_{\tilde{\cup}_{j\in J}Z_j},E,\tilde{I}_{\tilde{\cup}_{j\in J}Z_j})$ is $\star_s$-soft connected soft subspace of $(X,\tau,E,\tilde{I})$.

Theorem 2.79. *Let $\{(Z_j,\tau_{Z_j},E,\tilde{I}_{Z_j}):j\in J\}$ be a family of $\star_s$-soft connected subspaces of soft topological space with soft ideal $(X,\tau,E,\tilde{I})$ such that one of the members of the family intersects every other members, then $(\tilde{\cup}_{j\in J}Z_j,\tau_{\tilde{\cup}_{j\in J}Z_j},E,\tilde{I}_{\tilde{\cup}_{j\in J}Z_j})$ is $\star_s$-soft connected soft subspace of $(X,\tau,E,\tilde{I})$.*

Proof. Let $(Z,\tau_Z,E,\tilde{I}_Z)=(\tilde{\cup}_{j\in J}Z_j,\tau_{\tilde{\cup}_{j\in J}Z_j},E,\tilde{I}_{\tilde{\cup}_{j\in J}Z_j})$ and $(Z_{jo},E)\in\{(Z_j,E):j\in J\}$ such that $(Z_{jo},E)\tilde{\cap}(Z_j,E)\neq\tilde{\phi}\quad\forall i\in J$. Then $(Z_{jo},E)\tilde{\cup}(Z_j,E)$ is $\star_s$- soft connected $\ \forall j\in J$ from Theorem 2.78. Therefore, the collection $\{(Z_{jo},E)\tilde{\cup}(Z_j,E):j\in J\}$ is a collection of a $\star_s$-soft connected subsets of $\tilde{X}$, which having a non null soft intersection. Thus,$(\tilde{\cup}_{j\in J}Z_j,\tau_{\tilde{\cup}_{j\in J}Z_j},E,\tilde{I}_{\tilde{\cup}_{j\in J}Z_j})$ is $\star_s$-soft connected soft subspace of $(X,\tau,E,\tilde{I})$from Theorem 2.78.

Definition 2.80. *Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal and $(Z,E)\tilde{\subseteq}\tilde{X}$ with $x\in(Z,E)$. Then the $\star_s$-soft component of (Z,E) w.r.t. x is the maximal of all $\star_s$-soft connected soft subspaces of $(Z,\tau_Z,E,\tilde{I}_Z)$ containing x and denoted by $\tilde{C}_s[(Z,E),x]$ or $\tilde{C}_s(Z_E,x)$ $for\ short$, i.e*

$$\tilde{C}_s(Z_E, x) = \tilde{\cup}\{Y_E \tilde{\subseteq} Z_E : x \in Y_E,\ Y_E\ is\ \star_s -soft\ connected\}.$$

Theorem 2.81. *Every $\star_s$-soft component of a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$ is a maximal $\star_s$-soft connected subset of $\tilde{X}$.*

Proof. It is obvious from Definition 2.80.

Corollary 2.82. *Every $\star_s$-soft component of a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$ is τ^*-closed soft set.*

Proof. It obvious from Corollary 2.76.

Theorem 2.83. *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal. Then:*

(1) *Each point in X is contained in exactly one $\star_s$-soft component of $\tilde{X}$.*

(2) *Any two $\star_s$-soft components w.r.t. two different points of X are either disjoint or identical.*

Proof. It is similar to the proof of Theorem 2.49.

Proposition 2.84. *Let $(Z, \tau_Z, E, \tilde{I}_Z)$ be a soft subspace of a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$, $(Y, E)\tilde{\subseteq}(Z, E)\tilde{\subseteq}\tilde{X}$ and $(Z, E) \in \tau$. Then $(Y, E) \in \tau^*_Z$ if and only if $(Y, E) \in \tau^*$.*

Proof. Suppose that $(Y, E) \in \tau^*_Z$ and $(Z, E) \in \tau\tilde{\subseteq}\tau^* \Leftrightarrow (Y, E) = (Y, E)\tilde{\cap}(Z, E) \in \tau^*$.

Theorem 2.85. *Let $(Z, \tau_Z, E, \tilde{I}_Z)$ be a soft open subspace of a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$. Then (Z, E) is $\star_s$-soft connected w.r.t $(X, \tau, E, \tilde{I})$ if and only if (Z, E) is $\star$-soft connected w.r.t $(X, \tau, E, \tilde{I})$.*

Proof. Suppose that (Z, E) is not $\star$-soft connected w.r.t $(X, \tau, E, \tilde{I})$. Then, there exist two non null disjoint τ_Z-open soft (F, E) andτ^*_Z-open soft (G, E) such that $(Z, E) = (F, E)\tilde{\cup}(G, E)$. By Proposition 2.84, (F, E) and (G, E) are two non null disjoint τ-open soft and τ^*-open soft. It follows that, $cl^*_\tau(F, E)\tilde{\cap}(G, E) = (F, E)\tilde{\cap}cl_\tau(G, E) = \tilde{\phi}$. This implies that, (Z, E) is not $\star_s$-soft connected w.r.t $(X, \tau, E, \tilde{I})$, which is a contradiction.

Conversely, assume that (Z, E) is not $\star_s$-soft connected w.r.t $(X, \tau, E, \tilde{I})$. Then, there exist $\star$-soft separated subsets $(F, E), (G, E)$ of $\tilde{X}$ such that $(Z, E) = (F, E)\tilde{\cup}(G, E)$. By Theorem 2.63, (F, E), (G, E) are τ^*-open soft and τ-open soft. By Proposition 2.84, (F, E), (G, E) are τ^*_Z-open soft and τ_Z-open soft, respectively. Since $(F, E), (G, E)$ are $\star$-soft separated sets. Then $(F, E), (G, E)$ are two non null disjoint τ-open soft and τ^*-open soft such that $(Z, E) = (F, E)\tilde{\cup}(G, E)$. Therefore, (Z, E) is not $\star$-soft connected w.r.t $(X, \tau, E, \tilde{I})$, which is a contradiction.

Theorem 2.86. *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal. Then, each $\star_s$-soft connected subspace $(Z, \tau_Z, E, \tilde{I}_Z)$ of $\tilde{X}$ which is both open soft and $\star$-closed soft is $\star_s$-soft component of $\tilde{X}$.*

Proof. Let $(Z, \tau_Z, E, \tilde{I}_Z)$ be a $\star_s$-soft connected subspace of $\tilde{X}$ which is both open soft and $\star$-closed soft. Let $x \in (Z, E)$. Since (Z, E) is a $\star_s$-soft connected subset of $\tilde{X}$ containing x, if $\tilde{C}_s(Z_E, x)$ is the $\star_s$-soft component of (Z, E) w.r.t. x. Then $(Z, E)\tilde{\subseteq}\tilde{C}_s(Z_E, x)$. Now, we want to show that $\tilde{C}_s(Z_E, x)\tilde{\subseteq}(Z, E)$, it is equivalent to prove that $\tilde{C}_s(Z_E, x)\tilde{\cap}(Z, E)' = \tilde{\phi}$. So, Suppose that $\tilde{C}_s(Z_E, x)\tilde{\cap}(Z, E)' \neq \tilde{\phi}$. Since (Z, E) is both open soft and $\star$-closed soft. Then $(Z, E)'$ is both closed soft and $\star$-open soft. Also,

$$[\tilde{C}_s(Z_E, x)\tilde{\cap}(Z, E)]\tilde{\cap}[\tilde{C}_s(Z_E, x)\tilde{\cap}(Z, E)'] = \tilde{\phi}$$

and

$$[\tilde{C}_s(Z_E, x)\tilde{\cap}(Z, E)]\tilde{\cup}[\tilde{C}_s(Z_E, x)\tilde{\cap}(Z, E)'] = [(Z, E)\tilde{\cup}(Z, E)']\tilde{\cap}\tilde{C}_s(Z_E, x) = \tilde{C}_s(Z_E, x)$$

Again $(Z, E), (Z, E)'$ are two non null disjoint open soft and $\star$-open soft sets, respectively such that $(Z, E)\tilde{\cap}cl(Z, E)' = \tilde{\phi}$ and $cl^*_\tau(Z, E)\tilde{\cap}(Z, E)' = \tilde{\phi}$, which is a contradiction with the $\star_s$-soft connectedness of $\tilde{C}_s(Z_E, x)$. It follows that $\tilde{C}_s(Z_E, x)\tilde{\subseteq}(Z, E)$,and so $\tilde{C}_s(Z_E, x) = (Z, E)$. This completes the proof.

3 Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied the soft set theory, which is initiated by Molodtsov and easily applied to many problems having uncertainties from social life. In the present work, we have continued to study the properties of soft topological spaces. We introduce and study the notion of connectedness to soft topological spaces with soft ideals. We study the notions of $\star$-soft connected sets, $\star$-soft separated sets and $\star_s$-soft connected sets in soft topological spaces with soft ideals. and have established several interesting properties. Because there exists compact connections between soft sets and information systems [29, 33], we can use the results deducted from the studies on soft topological space to improve these kinds of connections. We see that this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

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References

[1] B. Ahmad and A. Kharal, On fuzzy soft sets, Advances in Fuzzy Systems (2009) 1-6.

[2] B. Ahmad and A. Kharal, Mappings of soft classes, to appear in New Math. Nat. Comput.

[3] H. Aktas and N. Cagman, Soft sets and soft groups, Information Sciences 1 (77) (2007) 2726-2735.

[4] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications 57 (2009) 1547-1553.

[5] A. Aygunoglu and H. Aygun, Introduction to fuzzy soft groups, Computers and Mathematics with Applications 58 (2009) 1279-1286.

[6] Banu Pazar Varol and Halis Aygun, On soft Hausdorff spaces, Annals of Fuzzy Mathematics and Informatics 5 (2013) 15-24.

[7] Bin Chen, Some Local Properties of Soft Semi-Open Sets, Hindawi Publishing Corporation, 2013 (1-6).

[8] N. Çagman, F.Citak and S. Enginoglu, Fuzzy parameterized fuzzy soft set theory and its applications, Turkish Journal of Fuzzy Systems 1(1)(2010) 21-35.

[9] N.Cagman and S. Enginoglu, Soft set theory and uni-int decision making, European Journal of Operational Research 207 (2010) 848-855.

[10] S. Hussain and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl., 62 (2011) 4058-4067.

[11] D. Jankovic and T.R. Hamlet, New topologies from old via ideals, The American Mathematical Monthly 97 (1990) 295-310.

[12] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, γ-operation and decompositions of some forms of soft continuity in soft topological spaces, Annals of Fuzzy Mathematics and Informatics 7 (2014) 181-196.

[13] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, γ-operation and decompositions of some forms of soft continuity of soft topological spaces via soft ideal. To appear in the Jökull Journal.

[14] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Semi-soft compactness via soft ideals. , Appl. Math. Inf. Sci. 8 (5) (2014) 1-10.

[15] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft ideal theory, Soft local function and generated soft topological spaces, Appl. Math. Inf. Sci. 8 (4) (2014) 1-9.

[16] A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Supra soft topological spaces. To appear in the Jökull Journal.

[17] K. Kannan, Soft generalized closed sets in soft topological spaces, Journal of Theoretical and Applied Technology 37 (2012) 17-21.

[18] D. V. Kovkov, V. M. Kolbanov and D. A. Molodtsov, Soft sets theory-based optimization, Journal of Computer and Systems Sciences International 46 (6) (2007) 872-880.

[19] F. Lin, Soft connected spaces and soft paracompact spaces, International Journal of Mathematical Science and Engineering, 7 (2) (2013) 1-7.

[20] J. Mahanta and P.K. Das, On soft topological space via semi open and semi closed soft sets, arXiv:1203.4133v,2012.

[21] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, Journal of Fuzzy Mathematics 9 (3) (2001) 589-602.

[22] P. K. Maji, R. Biswas and A. R. Roy, Intuitionistic fuzzy soft sets, Journal of Fuzzy Mathematics 9 (3) (2001) 677-691.

[23] P. K. Maji,R. Biswas and A. R. Roy, Soft set theory, Computers and Mathematics with Applications 45 (2003) 555-562.

[24] P. Majumdar and S. K. Samanta, Generalised fuzzy soft sets, Computers and Mathematics with Applications 59 (2010) 1425-1432.

[25] D. A. Molodtsov, Soft set theory-firs tresults, Computers and Mathematics with Applications 37 (1999) 19-31.

[26] D.Molodtsov, V. Y. Leonov and D. V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya 1(1)(2006) 8-39.

[27] A. Mukherjee and S. B. Chakraborty, On intuitionistic fuzzy soft relations, Bulletin of Kerala Mathematics Association 5(1)(2008) 35-42.

[28] Sk. Nazmul and S. K. Samanta, Neighbourhood properties of soft topological spaces, Annals of Fuzzy Mathematics and Informatics 6(2012) 1-15.

[29] D. Pei and D. Miao, From soft sets to information systems, in: X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R. R. Yager, B. Zhang (Eds.), Proceedings of Granular Computing, in: IEEE, vol.2, 2005, pp. 617-621.

[30] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011) 1786-1799.

[31] Sujoy Das and S. K. Samanta, Soft metric, Annals of Fuzzy Mathematics and Informatics 6 (2013) 77-94.

[32] Weijian Rong, The countabilities of soft topological spaces, International Journal of Computational and Mathematical Sciences 6 (2012) 159-162.

[33] Z. Xiao, L. Chen, B. Zhong, S. Ye, Recognition for information based on the theory of soft sets, in: J. Chen(Ed.), Proceeding of ICSSSM-05, vol. 2, IEEE, pp. 1104-1106, 2005.

[34] W. Xu, J. Ma, S. Wang and G. Hao, Vague soft sets and their properties, Computers and Mathematics with Applications 59 (2010) 787-794.

[35] Y. Zou and Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowledge-Based Systems 21 (2008) 941-945.

[36] I. Zorlutuna, M. Akdag, W.K. Min and S. Atmaca, Remarks on soft topological spaces, Annals of Fuzzy Mathematics and Informatics 3 (2012) 171-185.