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Relations on Interval Valued Neutrosophic Soft Sets

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Abstract – Mukherjee [34] introduced the concept of interval valued intuitionistic fuzzy soft relation. In this paper we will extend this concept to the case of interval valued neutrosophic soft relation (IVNSS relation for short) which can be discussed as a generalization of soft relations, fuzzy soft relation, intuitionistic fuzzy soft relation, interval valued intuitionistic fuzzy soft relations and neutrosophic soft relations. Basic operations are presented and the various properties like reflexivity, symmetry, transitivity of IVNSS relations are also studied.

Keywords – *Soft sets, Neutrosophic soft sets, Neutrosophic soft cartesian product, Neutrosophic soft relation.*

1. Introduction

In 1999, Smarandache introduced the theory of neutrosophic set (NS) [37], which is the generalization of the classical sets, conventional fuzzy set [44], intuitionistic fuzzy set [7] and interval valued fuzzy set [38]. This concept has been successfully applied to many fields such as databases [4,5], medical diagnosis problem [6], decision making problem [26], topology [27], control theory [1] etc. The concept of neutrosophic set handle indeterminate data whereas fuzzy set theory, and intuitionistic fuzzy set theory failed when the relation are indeterminate.

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Presently works on the neutrosophic set theory is progressing rapidly. Bhowmik and Pal [10,11] defined intuitionistic neutrosophic set. Later on Salam and Alblowi [36] introduced another concept called Generalized neutrosophic set. Wang et al. [39] proposed another extension of neutrosophic set which is single valued neutrosophic. Also Wang et al. [40] introduced the notion of interval valued neutrosophic set which is an instance of neutrosophic set. It is characterized by an interval membership degree, interval indeterminacy degree and interval non-membership degree. Geogiev [24] explored some properties of the Neutrosophic logic and proposes a general simplification of the neutrosophic sets into a subclass of theirs, comprising of elements of R^3. Ye [42,43] defined similarity measures between interval neutrosophic sets and their multicriteria decision-making method. Majumdar and Samanta [31] proposed some types of similarity and entropy of neutrosophic sets, Broumi and Smarandache [16,17,18] proposed several similarity measures of neutrosophic sets. Chi and Peid [19] extended TOPSIS to interval neutrosophic set, and so on.

In 1999 a Russian researcher, Molodotsov proposed an new mathematical tool called soft set theory [33], for dealing with uncertainty and how soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory.

Although there many authors [2,12,20,21,28,35,41] have contributed a lot towards fuzzification which leads to a series of mathematical models such as fuzzy soft set, generalized fuzzy soft set, possibility fuzzy soft set, fuzzy parameterized soft set and so on, intuitionistic fuzzy soft set which is based on a combination of the intuitionistic fuzzy sets and soft set models. Later a lot of extensions of intuitionistic fuzzy soft [29] are appeared such as generalized intuitionistic fuzzy soft set [8], possibility intuitionistic fuzzy soft set [9] and so on. Few studies are focused on neutrosophication of soft set theory. In [30] Maji, first proposed a new mathematical model called "Neutrosophic Soft Set" and investigate some properties regarding neutrosophic soft union, neutrosophic soft intersection, complement of a neutrosophic soft set, De Morgan law etc. Furthermore, in 2013, Broumi and Smarandache [13] combined the intuitionistic neutrosophic and soft set which lead to a new mathematical model called" intuitionistic neutrosophic soft set". They studied the notions of intuitionistic neutrosophic soft set union, intuitionistic neutrosophic soft set intersection, complement of intuitionistic neutrosophic soft set and several other properties of intuitionistic neutrosophic soft set along with examples and proofs of certain results. Also, in [14] S.Broumi presented the concept of "generalized neutrosophic soft set" by combining the generalized neutrosophic sets [15] and soft set models, studied some properties on it, and presented an application of generalized neutrosophic soft set [14] in decision making problem.

Recently, Deli [22] introduced the concept of interval valued neutrosophic soft set as a combination of interval neutrosophic set and soft set. This concept generalizes the concept of the soft set [33], fuzzy soft set [28], intuitionistic fuzzy soft set [29], interval valued intuitionistic fuzzy soft set [25], the concept of neutrosophic soft set [30] and intuitionistic neutrosophic soft set [13].

This paper is an attempt to extend the concept of interval valued intuitionistic fuzzy soft relation (IVIFSS-relations) introduced by Mukherjee et al. [34] to IVNSS relation.

The organization of this paper is as follow: In section 2, we briefly present some basic definitions and preliminary results are given which will be used in the rest of the paper. In section 3, relation interval neutrosophic soft relation is presented. In section 4 various type of interval valued neutrosophic soft relations. In section 5, we conclude the paper.

2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U. We now recall some basic notions of neutrosophic set, interval neutrosophic set, soft set, neutrosophic soft set and interval neutrosophic soft set.
For more details, the reader may refer to [22, 30, 33, and 40]

Definition 1. [30] Let U be a universe of discourse then the neutrosophic set A is an object having the form

$$A = \{< x: \mu_{A(x)}, \nu_{A(x)}, \omega_{A(x)}>, x \in U\}$$

where the functions $\mu, \nu, \omega : U \to]^{-}0,1^{+}[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set A with the condition.

$$^{-}0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^{+}. \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]{-}0,1^{+}[$. So instead of $]^{-}0,1^{+}[$ we need to take the interval [0,1] for technical applications, because $]^{-}0,1^{+}[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2. [30] A neutrosophic set A is contained in another neutrosophic set B i.e.

$$A \subseteq B \text{ if } \forall x \in U,\ \boldsymbol{\mu}_A(x) \leq \boldsymbol{\mu}_B(x),\ \boldsymbol{\nu}_A(x) \geq \boldsymbol{\nu}_B(x),\ \boldsymbol{\omega}_A(x) \geq \boldsymbol{\omega}_B(x).$$

Definition 3. [40] Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $\boldsymbol{\mu}_{\mathbf{A}}(\mathbf{x})$, indeteminacy-membership function $\boldsymbol{\nu}_{\mathbf{A}}(\mathbf{x})$ and falsity-membership function $\boldsymbol{\omega}_{\mathbf{A}}(\mathbf{x})$. For each point x in X, we have that $\boldsymbol{\mu}_{\mathbf{A}}(\mathbf{x}), \boldsymbol{\nu}_{\mathbf{A}}(\mathbf{x}), \boldsymbol{\omega}_{\mathbf{A}}(\mathbf{x}) \in [0,1]$.

For two IVNS,

$$A_{\text{IVNS}} = \{<x, [\mu_A^L(x), \mu_A^U(x)], [\nu_A^L(x), \nu_A^U(x)], [\omega_A^L(x), \omega_A^U(x)] > | x \in X\}$$

and

$$B_{\text{IVNS}} = \{<x, [\mu_B^L(x), \mu_B^U(x)], [\nu_B^L(x), \nu_B^U(x)], [\omega_B^L(x), \omega_B^U(x)]> | x \in X\}$$

Then,

1. $A_{\mathrm{IVNS}} \subseteq B_{\mathrm{IVNS}}$ if and only if

$$\mu_A^L(x) \le \mu_B^L(x), \mu_A^U(x) \le \mu_B^U(x), \nu_A^L(x) \ge \nu_B^L(x), \omega_A^U(x) \ge \omega_B^U(x), \omega_A^L(x) \ge F\omega_B^L(x), \omega_A^U(x) \ge \omega_B^U(x).$$

2. $A_{\mathrm{IVNS}} = B_{\mathrm{IVNS}}$ if and only if ,

$$\mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x), \omega_A(x) = \omega_B(x) \text{ for any } x \in X.$$

3. The complement of A_{IVNS} is denoted by A^o_{IVNS} and is defined by

   A^o_{IVNS}={<x, $[\omega_A^L(x), \omega_A^U(x)]$>, $[1-\nu_A^U(x), 1-\nu_A^L(x)]$, $[\mu_A^L(x), \mu_A^U(x)]$ | x ∈ X }

4. A∩B = { <x, [min($\mu_A^L(x)$,$\mu_B^L(x)$), min($\mu_A^U(x)$,$\mu_B^U(x)$)], [max($\nu_A^L(x)$,$\nu_B^L(x)$), max($\nu_A^U(x)$,$\nu_B^U(x)$], [max($\omega_A^L(x)$,$\omega_B^L(x)$), max($\omega_A^U(x)$,$\omega_B^U(x)$)] >: x ∈ X }

5. A∪B = {<x, [max($\mu_A^L(x)$,$\mu_B^L(x)$), max($\mu_A^U(x)$,$\mu_B^U(x)$)], [min($\nu_A^L(x)$,$\nu_B^L(x)$), min($\nu_A^U(x)$,$\nu_B^U(x)$], [min($\omega_A^L(x)$,$\omega_B^L(x)$), min($\omega_A^U(x)$,$\omega_B^U(x)$)] >: x ∈ X }

As an illustration, let us consider the following example.

Example 1. Assume that the universe of discourse U={x_1,x_2,x_3}, where x_1 characterizes the capability, $x_2$characterizes the trustworthiness and x3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an interval neutrosophic set (INS) of U, such that,

A = {< x_1, [0.3 0.4], [0.5 0.6], [0.4 0.5] >, < x_2, [0.1 0.2], [0.3 0.4], [0.6 0.7]>,
< x_3, [0.2 0.4], [0.4 0.5], [0.4 0.6] >}

where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

Definition 4. [33] Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (K, A) is called a soft set over U, where K is a mapping given by K: A → P(U).

As an illustration, let us consider the following example.

Example 2. Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . . , h_5}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_6}, where e_1, e_2, . . ., e_6 stand for the attributes "beautiful", "cheap", "green", "costly", "in the green surroundings'", "moderate", respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K, A) that describes the “attractiveness of the houses” in the opinion of a buyer, says Thomas, may be defined like this:

A= {e_1, e_2, e_3, e_4, e_5},

K(e_1) = {h_2, h_3, h_5},
K(e_2) = {h_2, h_4},
K(e_3) = {h_1},
K(e_4) = U,
K(e_5) = {h_3, h_5}.

Definition 5. [22] Let U be an initial universe set and A ⊂ E be a set of parameters. Let IVNS (U) denotes the set of all interval neutrosophic subsets of U. The collection (K, A) is termed to be the soft interval neutrosophic set over U, where F is a mapping given by K: A → IVNS(U).

The interval neutrosophic soft set defined over a universe is denoted by INSS.

Here, if ϒ and Ψ be two INSS then,

1. ϒ is an ivn-soft subset of Ψ, denoted by $\Upsilon \Subset \Psi$, if K(e) ⊆L(e) for all e∈E.
2. ϒ is an ivn-soft equals toΨ, denoted by $\Upsilon = \Psi$, if K(e)=L(e) for all e∈E.
3. The complement of ϒ is denoted by Υ^c, and is defined by Υ^c= {(x, K^o (x)): x∈E}
4. The union of ϒ and Ψ is denoted by $\Upsilon \cup'' \Psi$, if K(e) ∪L(e) for all e∈E.
5. The intersection of ϒand Ψ is denoted by $\Upsilon \cap'' \Psi$, if K(e) ∪L(e) for all e∈E.

To illustrate let us consider the following example:

Let U be the set of houses under consideration and E is the set of parameters (or qualities). Each parameter is a interval neutrosophic word or sentence involving interval neutrosophic words. Consider E = {beautiful, costly, in the green surroundings, moderate, expensive}. In this case, to define an interval neutrosophic soft set means to point out beautiful houses, costly houses, and so on. Suppose that, there are five houses in the universe U given by, U = {h_1, h_2, h_3, h_4, h_5} and the set of parameters A = {e_1, e_2, e_3, e_4}, where each e_i is a specific criterion for houses:

e_1 stands for ‘beautiful’,
e_2 stands for ‘costly’,
e_3 stands for ‘in the green surroundings’,
e_4 stands for ‘moderate’,

Suppose that,

K(beautiful)={<h_1, [0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>, <h_2, [0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, <h_3, [0.6, 0.7], [0.2, 0.3], [0.3, 0.5] >, <h_4, [0.7, 0.8], [0.3, 0.4], [0.2, 0.4] >, <h_5, [0.8, 0.4], [0.2, 0.6], [0.3, 0.4] >}.

K(costly)={< h_1, [0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>, <h_2, [0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, <h_3, [0.6, 0.7], [0.2, 0.3], [0.3, 0.5] >, <h_4, [0.7, 0.8], [0.3, 0.4], [0.2, 0.4] >, <h_5, [0.8, 0.4], [0.2, 0.6], [0.3, 0.4] >}.

K(in the green surroundings)= {< h_1, [0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2, [0.4, 0.5], [0.7, 0.8], [0.2, 0.3]>, < h_3, [0.6, 0.7], [0.2, 0.3], [0.3, 0.5]>, < h_4, [0.7, 0.8], [0.3, 0.4], [0.2, 0.4] >, < h_5, [0.8, 0.4], [0.2, 0.6], [0.3, 0.4] >},

K(moderate)={< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7, 0.8], [0.2, 0.3] >, < h_3, [0.6, 0.7], [0.2, 0.3], [0.3, 0.5] >, < h_4, [0.7, 0.8], [0.3, 0.4], [0.2, 0.4] >, < h_5, [0.8, 0.4], [0.2, 0.6], [0.3, 0.4] >}.

3. Relations on Interval Valued Neutrosophic Soft Sets

In this section, we define relations on interval valued neutrosophic soft sets and study some desired properties of interval valued neutrosophic soft sets.

Definition 6. Let U be an initial universe and (F, A) and (G, B) be two interval valued neutrosophic soft set. Then cartesian product of (F, A) and (G, B) denoted by (H, AxB), where H is mapping given by H: AxB→IVNS (U) is defined as

$$H\,(a,b)=F(a)\cap \boldsymbol{G}(\boldsymbol{b})$$

Let (F, A) and (G, B) be two interval valued neutrosophic soft sets. Then an interval valued neutrosophic soft relation (IVNSS-relation for short) from (F, A) to (G, B) is an interval valued neutrosophic soft subset of (H, AxB).

The collection of relations on interval valued neutrosophic soft sets on interval valued neutrosophic soft universe (U, Ax B) is denoted by $\sigma_U(A\text{x}\, B)$.

Example 3. (i) Let us consider an interval valued neutrosophic soft set (F, A) which describes the ‘attractiveness of the houses’ under consideration. Let the universe set U={ h_1, h_2, h_3, h_4, h_5 } and the set of parameter A={beautiful(e_1), in the green surroundings (e_3)}.

Then the tabular representation of the interval valued neutrosophic soft set (F, A) is given below:

Table 1: The tabular representation of the ivn-soft set of (F, A)

U	Beautiful (e_1)	in the green surroundings (e_3)
h_1	([0.5, 0.6],[0.3 0.8],[0.3,0.4])	([0.2, 0.6],[0.1, 0.3],[0.2,0.8])
h_2	([0.2, 0.5],[0.4, 0.7],[0.5,0.6])	([0.4, 0.5],[0.3, 0.5],[0.2,0.4])
h_3	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.2, 0.3],[0.1, 0.3],[0.4,0.5])
h_4	([0.1, 0.7],[0.2, 0.4],[0.6,0.7])	([0.5, 0.6],[0.4, 0.5],[0.3,0.4])
h_5	([0.4, 0.5],[0.3, 0.5],[0.2,0.4])	([0.3, 0.6],[0.2, 0.3],[0.5,0.6])

(ii) Now Let us consider an interval valued neutrosophic soft set (G, B) which describes the `cost of the houses' under consideration. Let the universe set U={h_1,h_2,h_3,h_4,h_5} and the set of parameter B={costly(e_2), moderate (e_4)}.

Then the tabular representation of the interval valued neutrosophic soft set (G, B) is given below:

Table 2: The tabular representation of the ivn-soft set of (G, B)

U	costly(e_2)	moderate (e_4)
h_1	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])
h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])
h_5	([0.2, 0.6],[0.2, 0.4],[0.3,0.5])	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])

Let us consider the Cartesian product (H, AxB) of P and Q as;

Table 3: The tabular representation of Cartesian product (H, AxB) P⋐ (H, AxB)

U	(e_1 ,e_2)	(e_1 ,e_4)	(e_3 ,e_2)	(e_3 ,e_4)
h_1	([0.3, 0.4],[0.7, 0.9], [0.3,0.4])	([0.4, 0.6],[0.7, 0.8],[0.3,0.4])	([0.2, 0.4],[0.7, 0.9],[0.2,0.8])	([0.2, 0.6],[0.7, 0.8],[0.2,0.8])
h_2	([0.2, 0.5],[0.4, 0.7],[0.5,0.7])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.4, 0.5],[0.3, 0.5],[0.2,0.7])	([0.1, 0.5],[0.4, 0.7],[0.50.6])
h_3	([0.3,0. 4],[0.7, 0.9],[0.3,0.4])	([0.3, 0.4],[0.7, 0.9],[0.2,0.4])	([0.2, 0.3],[0.2, 0.7],[0.4,0.5])	([0.2, 0.3],[0.1, 0.3],[0.4, 0.5]
h_4	([0.1, 0.7],[0.3, 0.4],[0.6,0.7])	([0.1, 0.4],[0.7, 0.9],[0.6,0.7])	([0.5, 0.6],[0.4, 0.5],[0.3,0.4])	([0.3, 0.4],[0,7, 0.9],[0.3,0.4])
h_5	([0.2, 0.5],[0.3, 0.5],[0.3,0.5])	([0.4, 0.5],[0.6, 0.7],[0.3,0.4])	([0.2, 0.6],[0.2, 0.4],[0.5,0.6])	([03, 0.6],[0.6, 0.7],[0.5,0.6])

Then, we consider the two IVNSS-relations P and Q on the two given interval valued neutrosophic soft sets given below:

Table 4: The tabular representation of P⋐ (H,AxB)

U	(e_1 ,e_2)	(e_1 ,e_4)	(e_3 ,e_2)	(e_3 ,e_4)
h_1	([0.2, 0.4],[0.3, 0.4],[0.1,0.2])	([0.3, 0.4],[0.3, 0.5],[0.3,0.4])	([0.3, 0.5],[0.3, 0.4],[0.3,0.5])	([0.4, 0.5],[0.3, 0.6],[0.2, 1])
h_2	([0.1,0. 3],[0.4, 0.5],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.4])	([0.3, 0.5],[0.2, 0.4],[0.4, 0.5]
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.3,0.6])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.3, 0.4],[0.3, 0.4],[0.4,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Table 5: The tabular representation of Q⋐(J,A xB)

U	(e_1 ,e_2)	(e_1 ,e_4)	(e_3 ,e_2)	(e_3 ,e_4)
h_1	([0.2, 0.4],[0.3, 0.4],[0.1,0.2])	([0.3, 0.4],[0.3, 0.5],[0.3,0.4])	([0.3, 0.5],[0.3, 0.4],[0.3,0.5])	([0.4, 0.5],[0.3, 0.6],[0.2, 1])
h_2	([0.1,0. 3],[0.4, 0.5],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.4])	([0.3, 0.5],[0.2, 0.4],[0.4, 0.5]
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.3,0.6])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.3, 0.4],[0.3, 0.4],[0.4,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

The tabular representations of P and Q are called relational matrices for P and Q respectively. From above we have, $\mu_{H(e_1,e_2)}(h_1)$=[0.2,0.3] ,$\upsilon_{H(e_1,e_2)}(h_2)$ =[0.3,0.4] and $\omega_{H(e_i,e_j)} = [0.1,0.2]$. But this intervals lie on the 1st row-1st column and 2nd row-1st column respectively. So we denote $\mu_{H(e_1,e_2)}(h_1)|_{(1,1)}$ = [0.2, 0.3] and $\upsilon_{H(e_1,e_2)}(h_2)|_{(1,1)}$ = [0.3, 0.4] and $\omega_{H(e_i,e_j)}|_{(1,1)} = [0.3, 0.4]$ etc to make the clear concept about what are the positions of the intervals in the relational matrices.

Definition 7. The order of the relational matrix is ($\boldsymbol{\theta}, \boldsymbol{\lambda}$), where $\boldsymbol{\theta}$ = number of the universal points and $\boldsymbol{\lambda}$ = number of pairs of parametrers considered in the relational matrix. In example 3 both the relational matrix for P and Q are of order (5,4). If $\boldsymbol{\theta} = \boldsymbol{\lambda}$, then the relational matrix is called a square matrix

Definition 8. Let P, Q ∈ $\boldsymbol{\sigma_U(Ax\ B)}$, P= (H, AxB), Q = (J, AxB) and the order of their relational matrices are same. Then we define

$$P \cup Q = (H \blacksquare J, AxB)$$

where H■ J :AxB →IVNS(U) is defined as

$$(H \blacksquare J)(e_{i,}e_j) = H(e_{i,}e_j) \vee J(e_{j,}e_j) \text{ for } (e_{i,}e_j) \in AxB,$$

where ∨ denotes the interval valued neutrosophic union.

$$P \cap Q = (H \diamond J, AxB)$$

where H⋄J :AxB →IVNS(U) is defined as

$$(H \diamond J)(e_{i,}e_j) = H\,(e_{i,}e_j) \wedge J(e_{i,}e_j) \text{ for } (e_{j,}e_j) \in A \text{ x } B$$

where ∧ denotes the interval valued neutrosophic intersection

$$P^c = (\sim H, AxB)$$

where ~H :AxB →IVNS(U) is defined as ~H $(e_{i,}e_j)$ =[H$(e_{i,}e_j)$] ^{c} for $(e_{i,}e_j)$ ∈ A x B,

where c denotes the interval valued neutrosophic complement.

Example 4. Consider the interval valued neutrosophic soft sets (F,A) and (G,B) given in example 3. Let us consider the two IVNSS-relations P_1 and Q_1 given below:

Table 6: The tabular representation of P_1, P_1=(J,AxB)

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.2, 0.4],[0.3, 0.4],[0.1,0.2])	([0.3, 0.4],[0.3, 0.5],[0.3,0.4])	([0.3, 0.5],[0.3, 0.4],[0.3,0.5])	([0.4, 0.5],[0.3, 0.6],[0.2, 1])
h_2	([0.1,0. 3],[0.4, 0.5],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.4])	([0.3, 0.5],[0.2, 0.4],[0.4, 0.5]
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.3,0.6])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.3, 0.4],[0.3, 0.4],[0.4,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Table 7: The tabular representation of $Q_{1,}Q_1$=(J,AxB):

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.5, 0.8],[0.1, 0.2],[0.1,0.2])	([0.2, 0.3],[0.3, 0.6],[0.3,0.4])	([0.2 0.5],[0.3, 0.5],[0.2,0.4])	([0.2, 0.4],[0.2, 0.3],[1, 1])
h_2	([0.4, 0.5],[0.2, 0.4],[1, 1])	([0.4, 0.6],[0.2, 0.3],[0.2,0.4])	([0.4, 0.5],[0.4, 0.5],[0.2,0.5])	([0.4, 0.5],[0.1, 0.2],[1, 1])
h_3	([0.2, 0.3],[0.5, 0.6],[0.2,0.4])	([0.3, 0.4],[0.4, 0.5],[1, 1])	([0.7, 0.8],[0.1, 0.2],[0.2,0.5])	([0.3, 0.5],[0.3, 0.4],[0,0.4])
h_4	([0.3, 0.5],[0.3, 0.4],[0, 1])	([0.3, 0.5],[0.2, 0.4],[0.1,0.2])	([0.2, 0.4],[0.2, 0.3],[0,0.5])	([0.3, 0.7],[0.1, 0.3],[0.6,0.7])

Then $P_1 \cup Q_1$:

Table 8: The tabular representation of $P_1 \cup Q_1$

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.5, 0.8],[0.1, 0.2],[0.1,0.2])	([0.3, 0.4],[0.3, 0.5],[0.3,0.4])	([0.3 0.5],[0.3, 0.4],[0.2,0.4])	([0.4, 0.5],[0.2, 0.3],[0.2, 1])
h_2	([0.4, 0.5],[0.2, 0.4],[1, 1])	([0.4, 0.6],[0.2, 0.3],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.4])	([0.4, 0.5],[0.1, 0.2],[0.4, 0.5])
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.3, 0.6],[0.1, 0.3],[1, 1])	([0.7, 0.8],[0.1, 0.2],[0.2,0.5])	([0.3, 0.5],[0.3, 0.4],[0,0.4])
h_4	([0.3, 0.5],[0.3, 0.4],[0, 1])	([0.3, 0.5],[0.2, 0.4],[0.1,0.2])	([0.3, 0.4],[0.2, 0.3],[0,0.5])	([0.3, 0.7],[0.1, 0.3],[0.6,0.7])

$P_1 \cap Q_1$:

Table 9: The tabular representation of $P_1 \cap Q_1$

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.2, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.3],[0.3, 0.6],[0.3,0.4])	([0.2 0.5],[0.3, 0.5],[0.3,0.5])	([0.2, 0.4],[0.3, 0.6],[1, 1])
h_2	([0.1, 0.3],[0.4, 0.5],[1, 1])	([0.1, 0.2],[0.2, 0.3],[0.2,0.4])	([0.4, 0.5],[0.4, 0.5],[0.2,0.5])	([0.3, 0.5],[0.2, 0.4],[1, 1])
h_3	([0.2, 0.3],[0.5, 0.6],[0.2,0.4])	([0.2, 0.4],[0.4, 0.5],[1, 1])	([0.7, 0.8],[0.1, 0.3],[0.3,0.6])	([0.2, 0.5],[0.3, 0.4],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.2, 0.4],[0.3, 0.4],[0.4,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

${P_1}^c$

Table 10: The tabular representation of ${P_1}^c$

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.1, 0.2],[0.6, 0.7],[0.2,0.4])	([0.3, 0.4],[0.5, 0.7],[0.3,0.4])	([0.3 0.5],[0.6, 0.7],[0.3,0.5])	([0.2, 1],[0.4, 0.7],[0.4, 0.5])
h_2	([1, 1],[0.5, 0.6],[0.1, 0.3])	([0.2, 0.4],[1, 1],[0.1,0.2])	([0.2, 0.4],[0.7, 0.9],[0.4,0.5])	([0.4, 0.5],[0.6, 0.8],[0.3, 0.5])
h_3	([0.2, 0.4],[0.6, 0.9],[0.2,0.6])	([1, 1],[0.7, 0.9],[0.2, 0.6])	([0.3, 0.6],[0.7, 0.9],[0.2,0.3])	([0, 0.4],[0.7, 0.8],[0.2,0.5])
h_4	([0, 0.1],[0.5, 0.7],[0.2, 0.4])	([0.1, 0.2],[0.5, 0.6],[0.3,0.4])	([0.4, 0.5],[0.6, 0.7],[0.3,0.4])	([0.6, 0.7],[0.5, 0.6],[0,0.2])

Theorem 1: Let P, Q, R ∈ $\boldsymbol{\sigma_U(Ax\ B)}$ and the order of their relational matrices are same.Then the following properties hold:

a) $(P \cup Q)^c = P^c \cap Q^c$
b) $(P \cap Q)^c = P^c \cup Q^c$
c) P ∪ (Q ∪ R)=(P ∪ Q) ∪ R
d) P ∩ (Q ∩ R)=(P ∩ Q) ∩ R
e) P ∩ (Q ∪ R)=(P ∩ Q) ∪ (P ∩ R
f) P ∪ (Q ∩ R)=(P ∪ Q) ∩ (P ∪ R)

Proof. Let P= (H, AxB), Q =(J,AxB) .then P ∪ Q = (H ■J ,AxB), where

a) H ■J: Ax B →IVNS(U) is defined as

$$(H \blacksquare J)(e_i, e_j) = H(e_i, e_j) \vee J(e_i, e_j) \text{ for } (e_i, e_j) \in A \times B.$$

So $(P \cup Q)^c$= (~H ■J, AxB), where ~H ■J:A xB→IVNS(U) is defined as

$$\begin{aligned}(\sim H \blacksquare J)(e_i, e_j) &= [H(e_i, e_j) \vee J(e_i, e_j)]^c \\ &= \Big[\big\{< h_k, \mu_{H(e_i,e_j)}(h_k), \upsilon_{H(e_i,e_j)}(h_k), \omega_{H(e_i,e_j)}(h_k) >: h_k \in U\big\} \vee \\ &\quad \big\{< h_k, \mu_{J(e_i,e_j)}(h_k), \upsilon_{J(e_i,e_j)}(h_k), \omega_{J(e_i,e_j)}(h_k) >: h_k \in U\big\}\Big]^c \\ &= \{< h_k, [\max(\inf \mu_{H(e_i,e_j)}(h_k), \inf\mu_{J(e_i,e_j)}(h_k)), \\ &\quad \max(\sup \mu_{H(e_i,e_j)}(h_k), \sup\mu_{J(e_i,e_j)}(h_k))], \\ &\quad [\min(\inf \upsilon_{H(e_i,e_j)}(h_k), \inf\upsilon_{J(e_i,e_j)}(h_k)),\end{aligned}$$

$$\min(\sup \upsilon_{H(e_i,e_j)}(h_k), \sup\upsilon_{J(e_i,e_j)}(h_k)],$$
$$[\min(\inf \omega_{H(e_i,e_j)}(h_k), \inf\omega_{J(e_i,e_j)}(h_k),$$
$$\min(\sup \omega_{H(e_i,e_j)}(h_k), \sup\omega_{J(e_i,e_j)}(h_k)]>: h_k \in U\}^c$$
$$=\{<, [\min(\inf \omega_{H(e_i,e_j)}(h_k), \inf\omega_{J(e_i,e_j)}(h_k),$$
$$\min(\sup \omega_{H(e_i,e_j)}(h_k), \sup\omega_{J(e_i,e_j)}(h_k)],$$
$$[1\text{-} \min(\sup \upsilon_{H(e_i,e_j)}(h_k), \sup\upsilon_{J(e_i,e_j)}(h_k),$$
$$1 - \min(\inf \upsilon_{H(e_i,e_j)}(h_k), \inf\upsilon_{J(e_i,e_j)}(h_k)],$$
$$[\max(\inf \mu_{H(e_i,e_j)}(h_k), \inf\mu_{J(e_i,e_j)}(h_k),$$
$$\max(\sup \mu_{H(e_i,e_j)}(h_k), \sup\mu_{J(e_i,e_j)}(h_k)] >: h_k \in U\}$$

Now;

$$P^c \cap Q^c = (\sim H, A \times B) \cap (\sim J, A \times B),$$

where ~H , ~J: A xB→IVNS(U) are defined as

$\sim H(e_i, e_j) = [H(e_i, e_j)]^c$ and $\sim J(e_i, e_j) = [J(e_i, e_j)]^c$ for $(e_j, e_j) \in A \times B$,

we have

$$(\sim H, A \times B) \cap (\sim J, A \times B) = (\sim H \diamond \sim J, A \times B)(e_i, e_j)$$

Now for $(e_i, e_j) \in A \times B$,

$(\sim H \diamond \sim J)(e_i, e_j) = \sim H(e_j, e_j) \wedge \sim J(e_i, e_j) =$

$$\{<h_k, \left[\inf \omega_{H(e_i,e_j)}(h_k), \mathrm{Sup}\omega_{H(e_i,e_j)}(h_k)\right], \left[1 - \mathrm{Sup}\, \upsilon_{H(e_i,e_j)}(h_k), 1 - \inf \upsilon_{H(e_i,e_j)}(h_k)\right], [\inf \mu_{H(e_i,e_j)}(h_k), \mathrm{Sup}\mu_{H(e_i,e_j)}(h_k)] >: h_k \in U\}$$
$$\wedge\{<h_k, \left[\inf \omega_{J(e_i,e_j)}(h_k), \mathrm{Sup}\omega_{J(e_i,e_j)}(h_k)\right], [1 - \mathrm{Sup}\, \upsilon_{J(e_i,e_j)}(h_k), 1 - \inf \upsilon_{J(e_i,e_j)}(h_k)], [\inf \mu_{J(e_i,e_j)}(h_k), \mathrm{Sup}\mu_{J(e_i,e_j)}(h_k)] >: h_k \in U\}$$

$$=<h_k, [\min(\inf \omega_{H(e_i,e_j)}(h_k), \inf \omega_{H(e_i,e_j)}(h_k)), \min(\mathrm{Sup}\omega\mu_{H(e_i,e_j)}(h_k), \mathrm{Sup}\omega_{J(e_i,e_j)}(h_k)],$$
$$[\max((1 - \mathrm{Sup}\, \upsilon_{H(e_i,e_j)}(h_k)), (1 - \mathrm{Sup}\, \upsilon_{J(e_i,e_j)}(h_k))), \max((1 - \inf\upsilon_{H(e_i,e_j)}(h_k)),$$
$$(1 - \inf \upsilon_{J(e_i,e_j)}(h_k)))], [\max(\inf \mu_{H(e_i,e_j)}(h_k), \inf \mu_{J(e_i,e_j)}(h_k)), \max(\mathrm{Sup}\, \mu_{H(e_i,e_j)}(h_k),$$
$$\mathrm{Sup}\, \mu_{J(e_i,e_j)}(h_k))]>: h_k \in U\}$$

$$=\{<h_k, [\min(\inf \omega_{H(e_i,e_j)}(h_k), \inf\omega_{J(e_i,e_j)}(h_k), \min(\sup \omega_{H(e_i,e_j)}(h_k), \sup\omega_{J(e_i,e_j)}(h_k)],$$
$$[1\text{-}\min(\sup \upsilon_{H(e_i,e_j)}(h_k), \sup\upsilon_{J(e_i,e_j)}(h_k), 1\text{-}$$
$$\min(\inf \upsilon_{H(e_i,e_j)}(h_k), \inf\upsilon_{J(e_i,e_j)}(h_k)],$$
$$[\max(\inf \mu_{H(e_i,e_j)}(h_k), \inf\mu_{J(e_i,e_j)}(h_k), \max(\sup \mu_{H(e_i,e_j)}(h_k), \sup\mu_{J(e_i,e_j)}(h_k)] >: h_k \in U\}$$

Then, $(P \cup Q)^c = P^c \cap Q^c$

b) Proof is similar to a)

b) let P= (H, AxB), Q =(J,AxB) and R= (K,AxB). Then,

$$P \cup Q = (H \blacksquare J, A\, XB),$$

where H ■J : A xB→IVNS(U) is defiend as

(H■ J)$(e_{i,}e_j)$=H(e_j, e_j) ∨ J(e_i, e_j) for ($e_{j,}e_j$) ∈ A x B .

So (P ∪ Q) ∪ R=((H ■J) ■K, AxB),

where (H ■J) ■K : A xB→IVNS(U) is defined as for ($e_{j,}e_j$) ∈ A x B

(H ■J) ■K) ($e_{j,}e_j$)= H(e_i, e_j) ∨ J(e_i, e_j) ∨ K(e_i, e_j)

Now as
(H(e_i, e_j) ∨ J(e_i, e_j)) ∨ K(e_i, e_j) = H(e_i, e_j) ∨ (J(e_i, e_j) ∨ K(e_i, e_j)).

Therefore; (H ■J) ■K) ($e_{j,}e_j$)=((H ■ (J ■K)) ($e_{j,}e_j$), also we have

P∪(Q∪R)=(P∪Q)∪R=(H■(J■K),AxB). Consequently, P∪(Q∪R)=(P∪Q)∪R

d) Proof is similar to c)

c) let P= (H,AxB), Q=(J,AxB) and R=(K,AxB). Then Q ∪ R = (J■K,AXB),

where; J ■K: A xB→IVNS(U) is defiend as

(J ■K)$(e_{i,}e_j)$ = J(e_j, e_j) ∨ K(e_i, e_j) for ($e_{j,}e_j$) ∈ A x B .

Then,

P ∩ (Q ∪ R)=((H⋄ (J ■K), AxB),

where H⋄ (J ■K): A xB→IVNS(U) is defined as for ($e_{j,}e_j$) ∈ A x B,

(H⋄ (J ■K)) ($e_{j,}e_j$)= H(e_i, e_j) ∧(J(e_i, e_j) ∨ K(e_i, e_j))

since
H(e_i, e_j) ∧(J(e_i, e_j) ∨ K(e_i, e_j)) =(H(e_i, e_j) ∧ (J(e_i, e_j))∨(H(e_i, e_j) ∧ K(e_i, e_j)).

We have

(H⋄ (J ■K)) ($e_{j,}e_j$)= (H(e_i, e_j) ∧ (J(e_i, e_j)) ∨(H(e_i, e_j) ∧ K(e_i, e_j))

Also we have

(P∩Q) ∪(P∩R)= (H⋄J, AxB) ∪ (H ⋄ K, AxB)=((H⋄ J) ■(H⋄K),A xB)

Now for ($e_{j,}e_j$) ∈ A x B ,
((H⋄ J) ■(H⋄K)) ($e_{j,}e_j$) = (H⋄ J)($e_{j,}e_j$) ∨ (H⋄ K) ($e_{j,}e_j$) =(H(e_i, e_j) ∧J(e_i, e_j)) ∨
(H(e_i, e_j) ∧ K(e_i, e_j)) =(H⋄ (J ■K)) ($e_{j,}e_j$).

Consequently,
P ∩ (Q ∪ R)=(P ∩ Q) ∪ (P ∩ R).

g) Proof is similar to e)

Definition 9. Let P, Q ∈ $\sigma_U(Ax\ B)$ and the order of their relational matrices are same. Then
P ⊆ Q if H $(e_{j,}e_j)$ ⊆ J $(e_{j,}e_j)$ for $(e_{j,}e_j)$ ∈ A x B where P= (H, A x B) and Q=(J, A x B)

Definition 10. Let U be an initial universe and (F, A) and (G, B) be two interval valued neutrosophic soft sets. Then a null relation between them is denoted by $\mathbf{O_U}$ and is defined as $\mathbf{O_U}$ =($\mathbf{H_O}$, A xB) where $\mathbf{H_O}\left(\boldsymbol{e_{i,}e_j}\right)$={<$\mathbf{h_k}$,[0, 0],[1, 1],[1, 1]>;$\mathbf{h_k}$ ∈ U} for $\left(\boldsymbol{e_{i,}e_j}\right)$ ∈ AxB.

Example 5. Consider the interval valued neutrosophic soft sets (F, A) and (G, B) given in example 3. Then a null relation between them is given by

Table 11: The tabular representation of $\mathrm{O_U}$

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_2	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_3	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_4	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])

Remark 2. It can be easily seen that P ∪ $\mathrm{O_U}$ =P and P ∩ $\mathrm{O_U}$ =$\mathrm{O_U}$ for any P ∈ $\sigma_U(Ax\ B)$

Definition 11. Let U be an initial universe and (F, A) and (G, B) be two interval valued neutrosophic soft sets. Then an absolute relation between them is denoted by $\mathbf{I_U}$ and is defineded as $\mathbf{I_U}$ =($\mathbf{H_I}$, AxB) where $\mathbf{H_I}\left(\boldsymbol{e_{i,}e_j}\right)$ = {<$\mathbf{h_k}$,[1, 1],[0, 0],[0, 0]>; $\mathbf{h_k}$ ∈ U} for $\left(\boldsymbol{e_{i,}e_j}\right)$ ∈ A xB.

Example 6. Consider the interval valued neutrosophic soft sets (F, A) and (G, B) given in example 3. Then an absolute relation between them is given by

Table 12: The tabular representation of $\mathrm{I_U}$

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_2	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_3	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_4	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])

Remark 3. It can be easily seen that P ∪ $\mathrm{I_U}$ =$\mathrm{I_U}$ and P ∩ $\mathrm{I_U}$ =P for any P ∈ $\sigma_U(Ax\ B)$

Definition 12. Let $\boldsymbol{\tau}$ be a sub-collection of interval valued neutrosophic soft set relations of the same order belonging to $\boldsymbol{\sigma_U(Ax\ B)}$.Then $\boldsymbol{\tau}$ is said to form a relational topology over $\boldsymbol{\sigma_U(Ax\ B)}$ if the following conditions are satisfied:

(i) $\mathrm{O_U}$,$\mathrm{I_U}$ ∈ τ

(ii) $\cup_{\mathrm{P_i}} \in \tau$, $\mathrm{P_i} \in \tau$

(iii) If $\mathrm{P_1}$,$\mathrm{P_2}$ ∈ τ ,then $\mathrm{P_1} \cap \mathrm{P_2} \in \tau$

Then we say that $(\sigma_U(Ax\ B),\tau)$ is a conditional relational topological space

Example 7: Consider example 3. Then the collection $\tau = \{0_U, I_U, P, Q\}$ forms a relational topology on $\sigma_U(Ax\ B)$.

4. Various type of interval valued neutrosophic soft relation

In this section, we present some basic properties of IVNSS relation. Let P $\in \sigma_U(Ax\ B)$and P=(H, A xB) and Q=(J, A xB) whose relational matrix is a square matrix.

Definition 13. An IVNSS-relation P is said to be reflexive if for $(\boldsymbol{e_i}, \boldsymbol{e_j}) \in$ A x B and $\mathbf{h_k} \in$ U, such that $\boldsymbol{\mu}_{\mathbf{H}(e_i,e_j)}(\mathbf{h_k})|_{(\mathbf{m},\mathbf{n})} = [1, 1]$, $\boldsymbol{\upsilon}_{\mathbf{H}(e_i,e_j)}(\mathbf{h_k})|_{(\mathbf{m},\mathbf{n})} = [0, 0]$ and $\boldsymbol{\omega}_{\mathbf{H}(e_i,e_j)}(\mathbf{h_k})|_{(\mathbf{m},\mathbf{n})} = [\mathbf{0\ 0}]$ form= n= k

Example 7. Let U = $\{h_1, h_2, h_3, h_4\}$ be a universe. Then, us consider the interval valued neutrosophic soft sets (F, A) and (G, B) where A= $\{e_1, e_3\}$ and B =$\{e_2, e_4\}$ then a reflexive IVNSS-relation between them is

Table 13: The tabular representation of reflexive IVNSS-relation

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([1, 1],[0, 0],[0, 0])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([1, 1],[0, 0],[0, 0])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])
h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([1, 1],[0, 0],[0, 0])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([1, 1],[0, 0],[0, 0])

Definition 14. An IVNSS-relation P is said to be anti-reflexive if for $(\boldsymbol{e_i}, \boldsymbol{e_j})\in$AxB and $\mathbf{h_k} \in$U, such that$\boldsymbol{\mu}_{\mathbf{H}(e_i,e_j)}(\mathbf{h_k})|_{(\mathbf{m},\mathbf{n})} = [0,0]$,$\boldsymbol{\upsilon}_{\mathbf{H}(e_i,e_j)}(\mathbf{h_k})|_{(\mathbf{m},\mathbf{n})} =[0,0]$ and for m=n=k $\boldsymbol{\omega}_{\mathbf{H}(e_i,e_j)}(\mathbf{h_k})|_{(\mathbf{m},\mathbf{n})} = [\mathbf{1\ 1}]$.

Example 9. Let U = $\{h_1, h_2, h_3, h_4\}$. Then, us consider the interval valued neutrosophic soft sets (F, A) and (G, B) where A= $\{e_1, e_3\}$ an B = $\{e_2, e_4\}$ then an anti-reflexive IVNSS-relation between them is

Table 14: The tabular representation of anti-reflexive IVNSS-relation

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0, 0],[0, 0],[1, 1])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([0, 0],[0, 0],[1, 1])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])
h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0, 0],[0, 0],[1, 1])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0, 0],[0, 0],[1, 1])

Definition 15: An IVNSS-relation P is said to be symmetric if for $(\boldsymbol{e_i}, \boldsymbol{e_j}) \in$AxB and $\mathbf{h_k} \in$U,$\exists$ $(\boldsymbol{e_i}, \boldsymbol{e_j})\in$AxB and $\mathbf{h_l} \in$U such that $\boldsymbol{\mu}_{\mathbf{H}(e_i,e_j)}(\mathbf{h_k})|_{(\mathbf{m},\mathbf{n})} = \boldsymbol{\mu}_{\mathbf{H}(e_p,e_q)}(\mathbf{h_l})|_{(\mathbf{n},\mathbf{m})}$, $\boldsymbol{\upsilon}_{\mathbf{H}(e_i,e_j)}(\mathbf{h_k})|_{(\mathbf{m},\mathbf{n})} = \boldsymbol{\upsilon}_{\mathbf{H}(e_p,e_q)}(\mathbf{h_l})$ and $\boldsymbol{\omega}_{\mathbf{H}(e_i,e_j)}(\mathbf{h_k})|_{(\mathbf{m},\mathbf{n})} = \boldsymbol{\omega}_{\mathbf{H}(e_p,e_q)}(\mathbf{h_l})|_{(\mathbf{n},\mathbf{m})}$.

Example 10. Let U = $\{h_1, h_2, h_3, h_4\}$. Let us consider the interval valued neutrosophic soft sets (F, A) and (G, B) where A= $\{e_1, e_3\}$ an B = $\{e_2, e_4\}$ then a symmetric IVNSS-relation between them is

Table 15: The tabular representation of symmetric IVNSS-relation

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0.4, 0.6],[0.3 0.4],[0.3,0.4])
h_2	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])	([0, 0],[1, 1],[1, 1])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])
h_3	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0.4, 0.6],[0.1, 0.3],[0.2,0.5])	([0.4, 0.5],[0.3, 0.4],[0.1,0.4])
h_4	([0.4, 0.6],[0.3 0.4],[0.3,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.4, 0.5],[0.3, 0.4],[0.1,0.4])	([0.2, 0.7],[0.3, 0.4],[0.6,0.7])

Definition 16. An IVNSS-relation P is said to be anti-symmetric if for each $(\boldsymbol{e_i}, \boldsymbol{e_j}) \in$ A x B and $\mathbf{h_k} \in$ U, $\exists(\boldsymbol{e_i}, \boldsymbol{e_j}) \in$ A x B and $\mathbf{h_l}$ $\in$U such that either
$\mu_{H(e_i, e_j)}(h_k)|_{(m,n)} \neq \mu_{H(e_p, e_q)}(h_l)|_{(n,m)}$, $\upsilon_{H(e_i, e_j)}(h_k)|_{(m,n)} \neq \upsilon_{H(e_p, e_q)}(h_l)|_{(n,m)}$ and
$\omega_{H(e_i, e_j)}(h_k)|_{(m,n)} \neq \omega_{H(e_p, e_q)}(h_l)|_{(n,m)}$ or
$M_{H(e_i, e_j)}(h_k)|_{(m,n)} = \mu_{H(e_p, e_q)}(h_l)|_{(n,m)}$ [0,0], $\upsilon_{H(e_i, e_j)}(h_k)|_{(m,n)}$
$= \upsilon_{H(e_p, e_q)}(h_l)|_{(n,m)}$=[0,0] and $\omega_{H(e_i, e_j)}(h_k)|_{(m,n)} = \omega_{H(e_p, e_q)}(h_l)|_{(n,m)}$=[1 , 1]

Example 11. Let U ={ h_1 , h_2 , h_3 , h_4 }. Let us consider the interval valued neutrosophic soft sets (F, A) and (G, B) where A= $\{e_1, e_3\}$ an B =$\{e_2, e_4\}$ then an anti-symmetric IVNSS-relation between them is

Table 16: The tabular representation of anti-symmetric IVNSS-relation

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0, 0],[0, 0],[1, 1])
h_2	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])	([0, 0],[1, 1],[1, 1])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])
h_3	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0.4, 0.6],[0.1, 0.3],[0.2,0.5])	([0.4, 0.5],[0.3, 0.4],[0.1,0.4])
h_4	([0, 0],[0, 0],[1, 1])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0, 0],[0, 0],[1, 1])	([0.2, 0.7],[0.3, 0.4],[0.6,0.7])

Definition 17. An IVNSS-relation P is said to be perfectly anti-symmetric if for each $(\boldsymbol{e_i}, \boldsymbol{e_j}) \in$ A x B and $\mathbf{h_k} \in$ U, $\exists$ $(\boldsymbol{e_i}, \boldsymbol{e_j}) \in$ A x B and $\mathbf{h_l} \in$ U such that whenever
inf $\mu_{H(e_i, e_j)}(h_k)|_{(m,n)} > 0$, inf $\upsilon_{H(e_i, e_j)}(h_k)|_{(m,n)} > 0$ and
inf $\omega_{H(e_i, e_j)}(h_k)|_{(m,n)} > 0$,$\mu_{H(e_p, e_q)}(h_l)|_{(n,m)}$= [0, 0] , $\upsilon_{H(e_p, e_q)}(h_l)|_{(n,m)}$ =[0 ,0]
and $\omega_{H(e_p, e_q)}(h_l)|_{(n,m)}$=[1, 1].

Example 12. Let U = { h_1 , h_2 , h_3 , h_4 }. Then, us consider the interval valued neutrosophic soft sets (F, A) and (G, B) where A= $\{e_1, e_3\}$ an B = $\{e_2, e_4\}$ then a perfectly anti-symmetric IVNSS-relation between them is

Table 17: The tabular representation of perfectly anti-symmetric IVNSS-relation

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0, 0],[0, 0],[1, 1])
h_2	([0, 0],[0, 0],[1, 1])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0.4, 0.6],[0.1, 0.3],[0.2,0.5])	([0, 0],[0, 0],[1, 1])
h_3	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0, 0],[0, 0],[1, 1])	([0.4, 0.6],[0.1, 0.3],[0.2,0.5])	([0, 0.5],[0, 0.4],[0,0.4])
h_4	([0, 0.6],[0, 0.2],[0, 1])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0, 0.6],[0, 0.3],[0,0.5])	([0.2, 0.7],[0.3, 0.4],[0.6,0.7])

In the following, we define two composite of interval valued neutrosophic soft relation.

Definition 18: Let P ,Q ∈ $\boldsymbol{\sigma_U(AxA)}$ and P =(H,AxA), Q=(J,AxA) and the order of their relational matrices are same.Then the compostion of P and Q, denoted by P*Q is defined by P*Q =(H∘J, AxA) where H∘J : AxA → IVNS(U)

is defined as

$(H\circ J)(e_i,e_j) = \{<h_k, \mu_{(H\circ J)(e_i,e_j)}(h_k), \upsilon_{(H\circ J)(e_i,e_j)}(h_k), \omega_{(H\circ J)(e_i,e_j)}(h_k)>: h_k \in U\}$

where

$\mu_{(H\circ J)(e_i,e_j)}(h_k) = [\max_l(\min(\inf \mu_{H(e_i,e_l)}(h_k), \inf\mu_{J(e_l,e_j)}(h_k)))$
$, \max_l(\min(\sup \mu_{H(e_i,e_l)}(h_k), \sup\mu_{J(e_l,e_j)}(h_k)))],$
$\upsilon_{(H\circ J)(e_i,e_j)}(h_k) = [\min_l(\max(\inf \upsilon_{H(e_i,e_l)}(h_k), \inf\upsilon_{J(e_l,e_j)}(h_k)))$
$, \min_l(\max(\sup \upsilon_{H(e_i,e_l)}(h_k), \sup\upsilon_{J(e_l,e_j)}(h_k)))],$

and

$\omega_{(H\circ J)(e_i,e_j)}(h_k) = [\min_l(\max(\inf \omega_{H(e_i,e_l)}(h_k), \inf\omega_{J(e_l,e_j)}(h_k)))$
$, \min_l(\max(\sup \omega_{H(e_i,e_l)}(h_k), \sup\omega_{J(e_l,e_j)}(h_k)))]$

For $(e_i,e_j) \in$ A x A

Example 13. Let U $=\{h_1, h_2, h_3, h_4\}$.let us consider the interval valued neutrosophic soft sets (F,A) and (G,A) where A=$\{e_1, e_2\}$ .Let P ,Q ∈ $\sigma_U(Ax\ A)$ and P =(H,AxA), Q=(J,AxA) where P:

Table 18: The tabular representation of P

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.3, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.4],[0.3, 0.5],[0.3,0.4])	([0.2 0.5],[0.3, 0.4],[0.3,0.4])	([0.2, 0.3],[0.3, 0.6],[0.2, 0.3])
h_2	([1, 1],[0, 0],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.5])	([0.4, 0.5],[0.1, 0.3],[0.3,0.5])	([0.4, 0.7],[0.1, 0.3],[1, 1])
h_3	([0.2, 0.6],[0.1, 0.4],[0.3,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.2,0.5])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.3,0.4])	([0.3, 0.4],[0.2, 0.3],[0,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Q:

Table 19: The tabular representation of Q

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.5, 0.8],[0.1, 0.2],[0.1,0.2])	([0.2, 0.3],[0.3, 0.6],[0.3,0.4])	([0.2 0.5],[0.3, 0.5],[0.2,0.4])	([0.2, 0.4],[0.2, 0.3],[1, 1])
h_2	([0.4, 0.5],[0.2, 0.4],[1, 1])	([0.4, 0.6],[0.2, 0.3],[0.2,0.4])	([0.4, 0.5],[0.4, 0.5],[0.2,0.5])	([0.4, 0.5],[0.1, 0.2],[1, 1])
h_3	([0.2, 0.3],[0.5, 0.6],[0.2,0.4])	([0.3, 0.4],[0.4, 0.5],[1, 1])	([0.7, 0.8],[0.1, 0.2],[0.2,0.5])	([0.3, 0.5],[0.3, 0.4],[0,0.4])
h_4	([0.3, 0.5],[0.3, 0.4],[0, 1])	([0.3, 0.5],[0.2, 0.4],[0.1,0.2])	([0.2, 0.4],[0.2, 0.3],[0,0.5])	([0.3, 0.7],[0.1, 0.3],[0.6,0.7])

Then,

P*Q

Table 20: The tabular representation of P*Q

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.3, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.4],[0.3, 0.5],[0.2,0.3])	([0.2 0.5],[0.3, 0.4],[0.2,0.4])	([0.2, 0.3],[0.2, 0.6],[0.3, 0.4])
h_2	([0.4, 0.5],[0.2, 0.4],[0.3, 0.5])	([0.1, 0.6],[0.1, 0.2],[0.2,0.5])	([0.4, 0.5],[0.2, 0.4],[0.2,0.5])	([0.4, 0.5],[0.1, 0.3],[0.3, 0.5])
h_3	([0.2, 0.6],[0.1, 0.3],[0,0.4])	([0.2, 0.5],[0.3, 0.4],[0.1, 0.4])	([0.2, 0.5],[0.2, 0.3],[0.2,0.4])	([0.2, 0.5],[0.3, 0.4],[0.2,0.5])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 0.2])	([0.3, 0.4],[0.2, 0.5],[0.3,0.4])	([0.3, 0.4],[0.2, 0.4],[0.2,0.5])	([0.3, 0.4],[0.3, 0.4],[0.2,0.5])

Definition 19. Let P ,Q ∈ $\boldsymbol{\sigma_U(AxA)}$ and P =(H,AxA), Q=(J,AxA) and the order of their relational matrices are same.Then the compostion of P and Q, denoted by P∘ Q is defined by

P∘ Q =(H∘ J,AxA) where H∘ J :AxA → IVNS(U)

is defined as $(\mathrm{H}\circ \mathrm{J})(e_{i,}e_j)$
$=\{<\mathrm{h_k}, \mu_{(\mathrm{H}\circ \mathrm{J})(e_{i,}e_j)}(\mathrm{h_k}), \upsilon_{(\mathrm{H}\circ \mathrm{J})(e_{i,}e_j)}(\mathrm{h_k}), \omega_{(\mathrm{H}\circ \mathrm{J})(e_{i,}e_j)}(\mathrm{h_k})> : \mathrm{h_k} \in \mathrm{U}\}$

where
$\mu_{(\mathrm{H}\circ \mathrm{J})(e_{i,}e_j)}(\mathrm{h_k})=[\min_l(\max(\inf \mu_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k}),\inf\mu_{\mathrm{J}(e_{l,}e_j)}(\mathrm{h_k})))$
$,\min_l(\max(\sup \mu_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k}),\sup\mu_{\mathrm{J}(e_{l,}e_j)}(\mathrm{h_k})))],$

$\upsilon_{(\mathrm{H}\circ \mathrm{J})(e_{i,}e_j)}(\mathrm{h_k})=[\max_l(\min(\inf \upsilon_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k}),\inf\upsilon_{\mathrm{J}(e_{l,}e_j)}(\mathrm{h_k})))$
$,\max_l(\min(\sup \upsilon_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k}),\sup\upsilon_{\mathrm{J}(e_{l,}e_j)}(\mathrm{h_k})))],$

and
$\omega_{(\mathrm{H}\circ \mathrm{J})(e_{i,}e_j)}(\mathrm{h_k})= [\max_l(\min(\inf \omega_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k}),\inf\omega_{\mathrm{J}(e_{l,}e_j)}(\mathrm{h_k})))$
$,\max_l(\min(\sup \omega_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k}),\sup\omega_{\mathrm{J}(e_{l,}e_j)}(\mathrm{h_k})))]$, for $(e_{i,}e_j) \in$ A x A.

Example 14. Let U $=\{h_1, h_2, h_3, h_4\}$.let us consider the interval valued neutrosophic soft sets (F,A) and (G,A) where A=$\{e_1, e_2\}$ .Let P ,Q ∈ $\sigma_U(Ax\ A)$ and P =(H, AxA), Q=(J,AxA) where P:

Table 21: The tabular representation of P

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.3, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.4],[0.3, 0.5],[0.3,0.4])	([0.2 0.5],[0.3, 0.4],[0.3,0.4])	([0.2, 0.3],[0.3, 0.6],[0.2, 0.3])
h_2	([1, 1],[0, 0],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.5])	([0.4, 0.5],[0.1, 0.3],[0.3,0.5])	([0.4, 0.7],[0.1, 0.3],[1, 1])
h_3	([0.2, 0.6],[0.1, 0.4],[0.3,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.2,0.5])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.3,0.4])	([0.3, 0.4],[0.2, 0.3],[0,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Q:

Table 22: The tabular representation of Q

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.5, 0.8],[0.1, 0.2],[0.1,0.2])	([0.2, 0.3],[0.3, 0.6],[0.3,0.4])	([0.2 0.5],[0.3, 0.5],[0.2,0.4])	([0.2, 0.4],[0.2, 0.3],[1, 1])
h_2	([0.4, 0.5],[0.2, 0.4],[1, 1])	([0.4, 0.6],[0.2, 0.3],[0.2,0.4])	([0.4, 0.5],[0.4, 0.5],[0.2,0.5])	([0.4, 0.5],[0.1, 0.2],[1, 1])
h_3	([0.2, 0.3],[0.5, 0.6],[0.2,0.4])	([0.3, 0.4],[0.4, 0.5],[1, 1])	([0.7, 0.8],[0.1, 0.2],[0.2,0.5])	([0.3, 0.5],[0.3, 0.4],[0,0.4])
h_4	([0.3, 0.5],[0.3, 0.4],[0, 1])	([0.3, 0.5],[0.2, 0.4],[0.1,0.2])	([0.2, 0.4],[0.2, 0.3],[0,0.5])	([0.3, 0.7],[0.1, 0.3],[0.6,0.7])

Then P∘Q;

Table 23: The tabular representation of P∘Q

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.2, 0.5],[0.3, 0.4],[0.3,0.4])	([0.2, 0.4],[0.3, 0.4],[0.3,0.4])	([0.2 0.4],[0.3, 0.4],[0.2,0.4])	([0.2, 0.3],[0.2, 0.6],[0.3, 0.4])
h_2	([0.4, 0.5],[0.1, 0.3],[0.2, 0.5])	([0.4, 0.5],[0.1, 0.3],[0.3,0.5])	([0.4, 0.5],[0.1, 0.3],[0.2,0.5])	([0.4, 0.5],[0.1, 0.3],[1, 1])
h_3	([0.2, 0.3],[0.2, 0.3],[1,1])	([0.2, 0.4],[0.2, 0.4],[0.3, 0.5])	([0.2, 0.5],[0.2, 0.4],[0.2,0.5])	([0.2, 0.5],[0.1 0.3],[1,1])
h_4	([0.3, 0.4],[0.3, 0.4],[0.3, 0.7])	([0.2, 0.4],[0.3, 0.5],[0.2,0.5])	([0.2, 0.5],[0.4, 0.5],[0.2,0.5])	([0.2, 0.4],[0.2, 0.3],[0.6,0.7])

Definition 20. Let P ∈ $\boldsymbol{\sigma_U(AxA)}$ and P = (H,AxA).Then P is called transitive IVNSS-relation if P*P ⊆ P, i.e $\cup(\mathbf{H}(\boldsymbol{e_{i,}e_l}) \cap \mathbf{H}(\boldsymbol{e_{l,}e_j})) \subseteq \mathbf{H}(\boldsymbol{e_{i,}e_j})$, i.e,

$\max(\inf\mu_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k}), \inf\mu_{\mathrm{H}(e_{l,}e_j)}(\mathrm{h_k})) \leq \inf\mu_{\mathrm{H}(e_{i,}e_j)}(\mathrm{h_k}),$
$\max(\sup\mu_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k}), \sup\mu_{\mathrm{H}(e_{l,}e_j)}(\mathrm{h_k})) \leq \sup\mu_{\mathrm{H}(e_{i,}e_j)}(\mathrm{h_k}),$

$\min(\inf\upsilon_{H(e_i,e_l)}(h_k), \inf\upsilon_{H(e_l,e_j)}(h_k)) \le \inf\upsilon_{H(e_i,e_j)}(h_k)$,
$\min(\sup\upsilon_{H(e_i,e_l)}(h_k), \sup\upsilon_{H(e_l,e_j)}(h_k)) \le \sup\upsilon_{H(e_i,e_j)}(h_k)$,
$\min(\inf\omega_{H(e_i,e_l)}(h_k), \inf\omega_{H(e_l,e_j)}(h_k)) \le \inf\omega_{H(e_i,e_j)}(h_k)$,
$\min(\sup\omega_{H(e_i,e_l)}(h_k), \sup\omega_{H(e_l,e_j)}(h_k)) \le \sup\omega_{H(e_i,e_j)}(h_k)$,

Example 15. Let U ={h_1 , h_2 ,h_3, h_4}.let us consider the interval valued neutrosophic soft sets (F,A) and (G,A) where A={e_1 , e_2 } .Let P ,Q ∈ $\sigma_U(Ax\ A)$ and P =(H, AxA), Q=(J,AxA) where P:

Table 24: The tabular representation of P

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.3, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.4],[0.3, 0.5],[0.3,0.4])	([0.2 0.5],[0.3, 0.4],[0.2,0.4])	([0.2, 0.3],[0.3, 0.6],[1, 1])
h_2	([1, 1],[0, 0],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.5])	([0.4, 0.7],[0.1, 0.3],[1, 1])
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.2,0.5])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.3, 0.4],[0.2, 0.3],[0,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Then P*P;

Table 25: The tabular representation of transitive IVNSS-relation P*P

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.3, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.4],[0.3, 0.5],[0.3,0.4])	([0.2 0.5],[0.3, 0.4],[0.2,0.4])	([0.2, 0.3],[0.3, 0.6],[1, 1])
h_2	([1, 1],[0, 0],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.5])	([0.4, 0.7],[0.1, 0.3],[1, 1])
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.2,0.5])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.3, 0.4],[0.2, 0.3],[0,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Thus, P*P ⊆ P and so P is a transitive IVNSS-relation.

Definition 21. Let P ∈ $\boldsymbol{\sigma_U(Ax\ A)}$ and P = (H, AxA).Then P is called equivalence IVNSS-relation if P satisfies the following conditions:

1. Reflexivity (see definition 13).
2. Symmetry (see definition 15).
3. Transitivity (see definition 20).

Example 16. Let U={h_1, h_2,h_3}. Then, us consider the interval valued neutrosophic soft sets (F,A) where A={e_1 , e_2}. Let P ,Q ∈ $\sigma_U(Ax\ A)$ and P =(H, AxA), where P: P*P

Table 26: The tabular representation of P*P

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)
h_1	([1, 1],[0, 0],[0, 0])	([0.2, 0.3],[0.2, 0.4],[0.3,0.4])	([0.1, 0.5],[0.2, 0.4],[0.2,0.3])
h_2	([0.2, 0.3],[0.4, 0.6],[0.3,0.4])	([1, 1],[0, 0],[0, 0])	([0.2, 0.3],[0.1, 0.5],[0.2,0.3])
h_3	([0.1, 0.5],[0.2, 0.4],[0.2,0.3])	([0.2, 0.3],[0.1, 0.5],[0.2,0.3])	([1, 1],[0, 0],[0, 0])

Table 27: The tabular representation of equivalence IVNSS-relation

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)
h_1	([1, 1],[0, 0],[0, 0])	([0.2, 0.3],[0.2, 0.4],[0.3,0.4])	([0.1, 0.5],[0.2, 0.4],[0.2,0.3])
h_2	([0.2, 0.3],[0.4, 0.6],[0.3,0.4])	([1, 1],[0, 0],[0, 0])	([0.2, 0.3],[0.1, 0.5],[0.2,0.3])
h_3	([0.1, 0.5],[0.2, 0.4],[0.2,0.3])	([0.2, 0.3],[0.1, 0.5],[0.2,0.3])	([1, 1],[0, 0],[0, 0])

Then P is equivalence IVNSS-relation.

5. Conclusions

In this paper we have defined, for the first time, the notion of interval neutrosophic soft relation. We have studied some properties for interval neutrosophic soft relation. We hope that this paper will promote the future study on IVNSS and IVNSS relation to carry out a general framework for their application in practical life.

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