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Upper and Lower Rarely m_X-m_Y Continuous Multifunction

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Abstract – In 1979, Popa introduced the notion of rare continuity. $Sharmistha$ Bhattacharya $(Halder)$ introduced the concept of rare m_X set and rarely m_X continuous functions. This paper is devoted to the study of upper and lower rarely $m_X - m_Y$ - continuous $multifunctions$.

Keywords – m_X-*open, upper rarely* m_X- continuous, lower rarely m_X- continuous

1. Introduction

The spaces $(X, m_X) and\ (Y, m_Y) implies\ m_X - structure\ \ on$ which no separation axioms are assumed unless explicitly stated. m_X open and m_Y open sets play an important role in generalizations of continuity in m_X structure. By using these sets many authors introduce and studied various types of generalizations of continuity**.** In year 2000, Popa and Nori [13] introduced on M-continuous functions. One of the most significant of those notions is continuous $multifunction$s. Continuity and multifunction are basic topics in several branches of mathematics such as in general topology. In 1989, $Popa[16]$ introduced the notion of rarely $continuous\ multifunction$. In year 2000, Noiri and Popa [11] introduced and investigated on upper and lower M-continuous $multifunctions,$ in year 2010, Bhattacharya (Halder) [8] introduced and investigated rarely m_X continuous function. Recently (2013) Ekici, Jafari [3] introduced and investigated rare s-$precontinuity$ for multifunction. The purpose of the present paper is to introduce the concept of rare m_X-m_Y –continuous multifunctions i.e the notion of rare m_X-m_Y – continuity, as a generalization of rare continuous multifunction and study some of its properties.

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In the next section, some of the important preliminaries required are cited. These are required to proceed further through this paper.

2. Preliminary

In this section some of the important required preliminaries are given.

Definition 2.1. [17] A subfamily m_X of P(X) is called a minimal structure on X if $\phi \in m_X$ and $X \in m_X$. Each member of m_X is said to be an m_X open set, and the complement of an m_X open set is said to be *an* m_X closed set. We denote by (X, m_X) a nonempty set X with a minimal structure m_X on X.

Definition 2.2. [17] Let X *be* a nonempty set and m_X a minimal structure on X. For a subset A of X, m_X -ClA and $m_X - Int(A)$ are defined as follows:

$$m_X - Cl(A) = \cap\{F : A \subseteq F, X \setminus F \in m_X\},$$
$$m_X - Int(A) = \cup\{G : G \subseteq A, G \in m_X\}.$$

Definition 2.3. [5] A rare set is a set S such that $Int(S) = \phi$, and a dense set is a set S such that $Cl(S) = X$.

Definition 2.4. [12] A function f: X → Y is called a rarely continuous function if for each $x \in X$ and each $G \in O(Y, f(x))$, there exist a rare set R_G with $G \cap Cl(R_G) = \phi$ and $U \in O(X, x)$ *such that* $f(U) \subseteq G \cup R_G$.

Definition 2.5. [8] Let $\in m_X$ be a minimal structure on X. A subset A of X is said to be an open m_X [resp., m_X] set if $m_X - Int(A)$ = A [resp., $m_X - Cl(A)$ = A].

Definition 2.6. [8] A subset A of X is said to be a rare [resp., dense] m_X set if $m_X - Int(A)$ = ϕ [resp., $m_X - Cl(A)$ = X].

Definition 2.7. [8] A function f : $(X, m_X) \rightarrow (Y, m_Y)$ is called a rarely m_X-m_Y continuous function if for each open m_Y set G containing f(x) there exists a rare m_Y set R_G with G ∩ m_Y -$Cl(R_G)$ = ϕ and an open m_X set U containing x such that $f(U) \subseteq G \cup R_G$.

By a multifunction $F: X \rightarrow Y$ we mean a point to set correspondence from X to Y and assume that $F(x) \neq \phi$ for all $x \in X$. Let $F: X \rightarrow Y$ be a multifunction. We denote the upper and lower inverse of a subset K of a space Y by F^+(K) and F^-(K), respectively, which are defind as follows :

$$F^+(K) = \{x \in X : F(x) \subset N\} \text{ and } F^-(K) = \{x \in X : F(x) \cap N \neq \phi\}$$

Definition 2.8. [15] A multifunction $F: X \rightarrow Y$ is called

(1) *lower s − precontinuous* if for each $x \in X$ and each open set M having connected complement such that $x \in F^-(M)$, there exists a preopen set N containing x such that $N \subset F^-(M)$,

(2) $upper\ s-precontinuous$ if for each $x\in X$ and each open set M having connected complement such that $x\in F^{+}(M)$, there exists a preopen set N containing x such that $N\subset F^{+}(M)$,

$\boldsymbol{Definition\ 2.9[4]}$: A multifunction $F: X\to Y$ is called

(1) $lower\ \ weakly\ s-precontinuous$ at $x\in X$ and if for each open set M of Y having connected complement such that x∈ $F^{-}(M)$, there exists a preopen set N containing x such that N⊂ F^{-}(Cl(M)),

(2)) $upper\ weakly\ s-precontinuous$ at $x\in X$ and if for each open set M of Y having connected complement such that x∈ $F^{+}(M)$, there exists a preopen set N containing x such that $N\subset F^{+}(Cl\ (M))$,

(3) $upper(lower)\ weakly\ s-precontinuous$ if it is upper (resp. lower) weakly s-pre continuous at each $x\in X$.

$\boldsymbol{Definition\ 2.10[3]}$: A multifunction $F: X\to Y$ is called

(1) upper rarely s-$precontinuous$ at $x\in X$ if for each open set M having connected complement such that $F(x)\subset M$, there exist a rare set R_M with $M\cap Cl(R_M)=\phi$ and a preopen set N containing x such that $F(N)\subset M\cup R_M$,

(2) lower rarely s-$precontinuous$ at $x\in X$ if for each open set M having connected complement with $F(x)\cap M=\phi$, there exist a rare set R_M with $M\cap Cl(R_M)=\phi$ and a preopen set N containing x such that $F(n)\cap(M\cup R_M)\neq\phi$ for every $n\in N$.

(3) $upper\ \ (resp. lower\ )\ rarely\ s-precontinuous\ if\ it\ i$s upper(resp. lower) rarely s-pre continuous at each $x\in X$.

$\boldsymbol{Theorem\ 2.11[8]}$: Let $g:(X,m_X)\to(Y,m_Y)$ be m_X-m_Y continuous and injective. Then g preserves rare m_X sets.

$\boldsymbol{Theorem\ 2.12}$ **[8]**. The following statements are equivalent for a function f : X → Y .

(1) The function f is rarely m_X-m_Y continuous at $x\ \in\ X$.

(2) For each open m_Y set G containing f(x) there exists a open m_X set U containing x such that

$$m_Y\ -\ Int\ f(U)\ \cap\ m_Y\ -\ Int\ (Y\setminus G)\ =\ \phi.$$

(3) For each open m_Y set G containing f(x) there exists a open m_X set U containing x such that

$$m_Y-\ Int\ f(U)\ \subseteq\ m_Y-Cl\ (G).$$

$\boldsymbol{Theorem\ 2.13[8]}$. Let $f:(X,m_X)\to(Y,m_Y)$ be a rarely m_X-m_Y continuous function. Then the graph function $g: X\ \to\ X\ \times\ Y$, defined by g(x) = (x, f(x)) for every x in X, is a rarely m_X-$m_Y \times m_Y$ continuous function.

***Theorem* 2.14**[8]. Let $f: (X, m_X) \rightarrow (Y, m_Y)$ be a rarely m_X-m_Y continuous function and $g: (Y, m_Y) \rightarrow (Z, m_Z)$ a one to one m_Y-m_Z continuous function. Then $gof: (X, m_X) \rightarrow (Z, m_Z)$ is rarely m_X-m_Z continuous.

3. *Rarely* $m_X - m_Y$ *Continuous Multifunction's*

In this section we introduced rarely m_X-m_Y continuous multifunction and also investigate some of its property.

***Definition* 3.1** The multifunction $F: (X, m_X) \rightarrow (Y, m_Y)$ is called

(1) Upper rarely m_X-m_Y continuous (briefly $u.r.\ m_X$-m_Y.c.) if for each $x \in X$ and each m_Y open set $V \subseteq Y$ with $F(x) \subseteq V$, there exist a rare m_Y set R_V disjoint from V and an open m_X set U containing x such that $F(U) \subseteq V \cup R_V$

(2) The multifunction $F: X \rightarrow Y$ is called lower rarely m_X-m_Y.-continuous (briefly $l.r.\ m_X$-m_Y..c.) if for each $x \in X$ and each m_Y open set $V \subseteq Y$ *with* $F(x) \cap V \neq \phi$; there exist a rare m_Y set R_V disjoint from V and $U \in m_X(X;\ x)$ *such that* $F(u) \cap (V \cup R_V) \neq \phi$;*for each* $u \in U$.

***Theorem* 3.2** *For a multifunction* $F: X \rightarrow Y$ *the following are equivalent* :

(1) F *is* $u.r.m_X - m_Y$ $.c$ *at* $x \in X$,

(2) For each open m_Y set $V \subseteq Y$ with $F(x) \subseteq V$ there exists $U \in m_X(X, x)$ such that $m_Y - Int\ F(U)) \cap m_Y - Int\ (Y - V)] = \phi$.

(3) For each open m_Y set $V \subseteq Y$ with $F(x) \subseteq V$ there exists $U \in m_X(X, x)$ such that m_Y-$Int[F(U)] \subseteq m_Y - Cl(V)$.

Proof: (1) ⇒ (2): Let V be an open m_Y set containing $F(x)$. Since V is an open m_Y set, $V = m_Y$ -$IntV \subseteq m_Y$-Int (m_Y-ClV).

Also, m_Y - Int ($m_Y - Cl(V)$) is an open m_Y set containing $F(x)$. Since F is a upper rarely m_X-m_Y continuous function there exists a rare m_Y set $R_V$with m_Y - Int (m_Y-ClV) ∩ m_Y - Cl(R_G) = ϕ, and an m_X open set U containing x so *that* $F(U) \subseteq m_Y$-Int (m_Y -ClV) $\cup R_V$. So, we have

m_Y-$Int\ F(U \cap m_Y$ - $Int\ (Y - V)$
$\subseteq m_Y$-Int [m_Y-Int (m_Y -$Cl(V)) \cup R_V$] ∩ [$Y - m_Y$-$Cl\ (V)$]
$\subseteq$ [m_Y-Int (m_Y-$Cl(V)) \cup m_Y$-$Int(R_V)$] ∩ [$Y - m_Y$-$Cl\ (V)$]
$\subseteq m_Y$-$ClV \cap (Y - m_Y$-$Cl(G)) = \phi$.

(2)⇒(3): From (3), m_Y - $Int\ F(U) \cap m_Y$ - $Int\ (Y - V) = \phi$, so m_Y - $Int\ F(U) \subseteq Y$ - m_Y - $Int\ (Y - V) = m_Y - Cl(V)$.

(3)⇒(1): Let there exist an open m_Y set V containing $F(x)$ with the properties given in (3). Then there exists an open m_X set U containing x such that m_Y - $Int\ F(U) \subseteq m_Y - Cl(V)$. We have

$F(U) = [F(U)$ - m_Y-$Int\ F(U)] \cup m_Y$-$Int\ F(U)$
$\subseteq [F(U)$ - m_Y-$Int\ F(U)] \cup m_Y$-$Cl(V)$
$= [F(U)$ - m_Y-$Int\ F(U)] \cup G \cup (m_Y$-$ClG \setminus G)$
$= [(F(U) - m_Y$- $Int\ F(U)) \cap (Y - V)] \cup G \cup (m_Y$-$ClV - V)$.

Let $R_1 = [F(U)$ - m_Y-$Int\ (F(U))] \cap (Y - G)$ and $R_2 = m_Y$-$Cl(V) - V$. Then, $R_V = R_1 \cup R_2$ is a rare set such that m_Y-$Cl(R_V) \cap V = \phi$ and $F(U) \subseteq V \cup R_V$. Therefore from Definition 2.7, F is a rarely m_X-m_Y continuous function.

***Theorem* 3.3.** For a multifunction $F: X \to Y$ the following are equivalent:

(1) F is $l.r.$ m_X-m_Y.c at $x \in X$

(2) For *each open* m_Y*.set* $V \subseteq Y$ such that $F(x) \cap V \neq \phi$, there exists a rare m_Y set R_V with $V \cap m_Y$-$Cl\ (R_V) = \phi$ *such that* $x \in m_X$-$Int\ (F^-(V \cup R_V))$.

(3) For each open m_Y set V such that $F(x) \cap V \neq \phi$, *there exists* a rare m_Y set R_V with m_Y-$Cl(V) \cap R_V = \phi$ *such that* $x \in m_X$ - $Int\ (F^-(m_Y - Cl(V) \cup R_V)$.

(4) For each regular open m_Y set V *such that* $F(x) \cap V \neq \phi$, there exists a rare m_Y set R_V with $V \cap m_Y - Cl\ (R_V) = \phi$ such that $x \in m_X$ - $Int\ (F^-(V \cup R_V))$.

Proof: **(1) ⇒ (2)** *Let open* m_Y set $V \subseteq Y$ such *that* $F(x) \cap V \neq \phi$. By (i) ,there exist a rare m_Y set R_V with $V \cap m_Y$-$Cl\ (R_V) = \phi$ and open m_X set U containing x such that $F(x) \cap (V \cup R_V) \neq \phi$ for each $u \in U$. Therefore, $u \in F^-(V \cup R_V)$ for each $u \in U$ and hence $U \subseteq F^-(U \cup R_V)$. Since $U \in m_X$ (X, x), we obtain $x \in U \subset m_X$-$Int\ (F^-(V \cup R_V))$.

(2)⇒(3): Suppose that V is an open m_Y set containing $F(x)$. *Then* there exist a rare m_Y set R_V with $V \cap m_Y$-$Cl\ (R_V) = \phi$ such that $x \in m_X$-$Int\ (F^-(V \cup R_V))$. Since $V \cap m_Y$-$Cl\ (R_V) = \phi$, $R_V \subseteq (Y \setminus m_Y$-ClV$) \cup (m_Y$ -ClV$\setminus$ V). Now, we have $R_V \subseteq (R_V \cup (Y \setminus m_Y - Cl(V)) \cup (m_Y - ClV \setminus V)$. Let $R_1 = (R_V \cup (Y \setminus m_Y$ -$ClV))$. It follows that R_1 *is* a rare m_Y set with m_Y-$ClV \cap R_1 = \phi$. Therefore, $x \in m_X$-$Int\ (F^-(V \cup R_V)) \subseteq m_X$ - Int $(F^-(m_Y$-$Cl(V) \cup R_1)$.

(3)⇒(4): Let V be any regular open m_X set of Y such that $F(x) \cap V \neq \phi$. By (iii), there exists a rare set R_V with $m_Y - Cl$(V) $\cap R_V = \phi$ such that $x \in m_X$ -$Int\ (F^-(m_Y$-$Cl(V) \cup R_V)$. Put $P = R_V \cup (Cl(V) - V)$,then P is a rare set and $V \cap m_X Cl(P) = \phi$ .Moreover, we have $x \in m_X$ - Int $(F^-(m_Y$ -$ClV \cup RV) = m_X$- Int $(F^-(R_V \cup (m_Y - Cl(V) - V) \cup V)) = m_X$ - Int $(F^-(P \cup V)$.

(4)⇒(1): Let open m_Y set $V \subseteq Y$ such that $F(x) \cap V \neq \phi$.Then $F(x) \cap m_Y$Int$(m_Y Cl(V)) \neq \phi$ and m_YInt$(m_Y Cl(V))$ is regular open m_X- set in Y. By (iv), there exists a rare set R_V with $V \cap m_Y Cl(R_V) = \phi$ such that $x \in m_X$ - Int $(F^-(V \cup R_V))$. Therefore, there exists

U$\in m_X$ (X, x) such that $U\subset(F^{-}(V \cup R_V))$;$hence\ F(u)\cap(V\cup R_V)\neq\phi\ for\ each\ u\in U$. This shows that F is lower rarely m_X- m_Y continuous at x.

Corollary 3.4: (**Theorem 2.12**) The following statements are equivalent for a function $f : X \to Y$.

(1) The function f is rarely m_X-m_Y continuous at $x \in X$.

(2) For each open m_Y set V containing f(x) there exists a open m_X set U containing x such that m_Y- $Int\ f(U) \cap m_Y$ - Int (Y \ V) = ϕ.

(3) For each open m_Y set V containing f(x) there exists a openm_Y set U containing x such that m_Y-$Int\ f(U) \subseteq m_Y$-$Cl\ (V)$.

Theorem 3.5: *If* $F : X \to Y$ *is an* $u.r.m_X - m_Y.c.$ *multifunction* then for any open m_X set $U \subseteq X$ *containing* x and any open m_Y set $V\subseteq Y$ *with* $F(x) \subseteq V$ there exists a rare set R_V disjoint from V and a nonempty open m_X set $W \subseteq U$ such that $F(W) \subseteq V \cup R_V$.

Proof. Let V $\subseteq$Y be open m_Y *set with* $F(x) \subseteq V$, and let U $\subseteq$X be an open m_X set containing x. By Theorem 3.2, there exists G $\in m_X$ (X; x) such that $F(G) \cap (Y \setminus V)$ has empty interior *and is therefore a rare* m_X *set*, say R_V. In addition, R_V is disjoint from V. Then $U\setminus G$ is an m_X-open set containing x. If $W = m_X\ int(U \setminus G)$ then W is a non-empty m_X open set contained in U.

Consequently, $F(W) \subseteq F(G) \subseteq V \cup (F(G) \cap (Y \setminus V)) \subseteq V \cup R_V$.

Definition 3.6. For a multifunction $F : (X,m_X)\to(Y,m_Y)$ the graph multifunction $GF : (X,m_X)\to(X,m_X)\times(Y,m_Y)$ is defined as *follows*: $G_F\ (x) = \{(x;\ y) : y \in F(x)\}$ for each $x \in X$.

Theorem 3.7. Let $F : (X,m_X) \to (Y,m_Y)$ *be a* $u.r.m_X - m_Y.c$ *multifunction*. Then the graph function $G_F : X \to X\times Y$, defined by $G_F\ (x) = \{(x;\ y) : y \in F(x)\}$ for each $x \in X$ *is a* $u.r.m_X$-$m_Y\times m_Y$. c multifunction.

Proof. Suppose that $x \in X$ and that W is any open $m_X\times m_Y$ set containing G_F (x). It follows that there exists an open m_X set U and an open m_Y set V such that (x, F(x)) $\in$ $U\times V \subseteq W$. Since F *is* $u.r.m_X$-m_Y.c multifunction, from **Theorem 3.2** there exists an open m_X set D containing x such that $m_X \times m_Y$ - Int F(D) $\subseteq m_Y$ -Cl V . *Let* $E = U \cap D$. It follows that E is an open $m_X$set containing x for which

$$m_X\times m_Y\text{-}Int[G_F(E)]\subseteq m_X\times m_Y\text{-}Int\ (U\times F(D)) \subseteq U\times m_Y-Cl(V) \subseteq m_X\times m_Y\text{-}Cl(W)$$

Therefore, G_F *is a* $u.r.\ m_X$-$m_Y\times m_Y$.c multifunction.

Corollary 3.8 (**Theorem 2.13**) : Let f : (X, m_X) → (Y, m_Y) be a rarely m_X-m_Y continuous function. Then the graph *function* $g : X \to X\times Y$, defined by $g(x) = (x, f(x))$ for every x in X, is a rarely m_X-$m_Y\times m_Y$ continuous function.

Theorem 3.9: *Let* $F: (X, m_X) \to (Y, m_Y)$ *be a u.r.* m_X-m_Y.c multifunction and $G: (Y, m_Y) \to (Z, m_Z)$ a one to one u. m_X-m_Y.*c multifunction*. Then $GoF: (X, m_X) \to (Y, m_Y)$ *is u.r.* $m_X - m_Y$.*c multifunction.*

Proof: Suppose that $x \in X$ *and* $G \; o \; F(x) \in$ V, where V is an open m_Z set in Z. By *hypothesis,* G is u. m_X-m_Y.c *multifunction*, therefore there exists an m_Y open set D ⊆Y *containing* $F(x)$ *such that* $G(D) \subseteq V$. *Since* F *is u.r.* $m_X - m_Y$.*c multifunction*, there exists a rare m_Y set R_D with D ∩ m_Y -Cl $(R_D) = \phi$, and an m_X open set U containing x such that F(U) ⊆ D ∪R_D. It follows from Theorem 2.11 that G(R_D) is a rare m_Z set in Z. Since R_D is a subset of $Y \setminus G$, and G is injective, we have m_Y -Cl $(G(R_D))$ ∩V=ϕ. This implies that $G \; o \; F(U) \subseteq V \cup G(R_D)$, *hence the* result.

Corollary 3.10(Theorem 2.14) *Let* f : (X, m_X) → (Y, m_Y) be a rarely m_X-m_Y *continuous function* and g: $(Y, m_Y) \to (Z, m_Z)$ a one to one m_Y -m_Z continuous function. Then $gof: (X, m_X) \to (Z, m_Z)$ *is rarely* m_Y -m_Z *continuous*

Theorem 3.11: Let X be a nonempty set having two minimal structures m_X^1 and m_X^2 such that U∩V∈ m_X^2 whenever U∈ m_X^1 and V∈ m_X^2 .If a multifunction f : (X, m_X) → (Y, m_Y) satisfies the following two conditions:

(1) F : (X, m_X^1) → (Y, m_Y) is upper $m_X - m_Y$ rarely continuous and

(2) For each open set G containing F(x) and having connected complement, $F^-(m_X$ Cl(R_G)) is a m_X^2 closed set of X, where R_G is m_Y rare then F : (X, m_X^2) → (Y, m_Y) is upper $m_X - m_Y$ continuous.

(3) Let $x \in X$ and G be an open set of Y containing F(x) . Since $F: (X, m_X^1) \to (Y, m_Y)$ upper $m_X - m_Y$ rarely continuous , there exist V∈ m_X^1 containing x and a m_Y rare set R_G with $m_Y - Cl(R_G)$∩G = ϕ such that $F(V) \subset G \cup R_G$. If we suppose that x∈F^-(m_Y–Cl(R_G)) then $F(x) \cap mY - Cl(R_G) \neq \phi$, but

$$F(x) \subset G \text{ and } G \cap m_Y - Cl(R_G) = \phi.$$

This is a contradiction. Thus x∉F^-($m_Y - Cl(R_G)$) . Let U =V∩ (X- m_Y –Cl(R_G)) .Then U∈ m_X^2 and x∈U since x∈V and $x \in X$ - F^- (m_Y –$Cl(R_G)$). Let $s \in U$, then $F(s) \subset G \cup R_G$ and F(s) ∩ m_Y – $Cl(R_G)$ = ϕ. Therefore, we have F(s) ∩ R_G =ϕ and hence F(s) ⊂ G . Since U∈ m_X^2 containing x , it follows that F : $(X, m_X^2) \to (Y, m_Y)$ is upper m_X –m_Y continuous.

Definition 3.12: Let f: $(X, m_X) \to (Y, m_Y)$ be a function. $f: (X, m_X) \to (Y, m_Y)$ is called regular m_X open if the image of every m_X open subset of X is m_Y open set in Y.

Definition 3.13: Let (X, m_X) be a m_X structure. X is called m_X -T_2 , if for each pair of *distinct points* x and y in X, there exist disjoint m_X open sets U and V such that x∈U and y∈V.

Theorem 3.14: If F : (X, m_X)→(Y,m_Y) is an r m_X -open upper m_X –m_Y rarely continuous multifunction and Y is m_Y-T_2 , then F : (X, m_X)→(Y, m_Y) is m_Y pre closed.

Proof **:** Suppose that $y \in Y$ and $\{x \in X : f(x) \neq y\}$. Since Y is m_Y-T_2 then there exist disjoint rare m_Y open set R_G and G such that F(x)∈ R_G and and y∈G. Since $m_Y - Cl(R_G)$)∩G = ϕ implies y∉ m_Y –Cl(R_G)). Since F : (X, m_X)→(Y, m_Y) r open upper m_X –m_Y rarely continuous multifunction then there exists a m_X open set A containing x such that m_X-Int(F(G)) = F(G) ⊂ m_Y –$Cl(R_G)$). Suppose that G is not contained in $\{x \in X : F(x) \neq y\}$.Then there exist a point a∈G such that F(a) = y. Since $F(A) \subset m_Y - Cl(R_G))$, then we have $y = F(a) \in m_Y - Cl(R_G))$. *This* is a contradiction. Therefore, $A \subset \{x \in X : F(x) \neq y\}$ and $\{x \in X : F(x) \neq y\}$ is m_X open in X. Thus, $\{x \in X : F(x) \neq y\}$ is m_X open in X. Hence F $^-$(y) = $\{x \in X : F(x) = y\}$ is $m_X$closed in X.

***Theorem* 3.15**: Let $F : X \to Y$ be a upper m_X –m_Y rarely continuous function and Y is a m_X –Hausdorff, then there exist a m_X open set U of X containing x and an m_Y open set V of Y containing y such that F(U) ∩(m_YCl(V) = ϕ for each $(x, y) \notin G_f$.

Proof **:** Suppose that $(x, y) \notin G_f$. *Then* $y \neq f(x)$. Since Y is m_X –Hausdorff, then there exist disjoint m_Y open sets V and R_V such that y∈V and $f(x) \in R_V$, respectively. We have $V \cap m_Y$ -$Cl(R_V) = \phi$ .Since F is upper m_X –m_Y rarely continuous function, then then there exists a m_X open set U containing x such that $F(U) \subset R_V$. It follows from that $F(U) \cap (m_Y\, Cl(V) = \phi$.

***Definition* 3.16**: Let $F : X \to Y$ be a *function* . If $F: X \to Y$ is called r m_X open if $F(A)$ is m_Y open in Y for every m_X open set A of X.

***Theorem* 3.17**: If $F : (X, m_X)$ →(Y, m_Y) upper m_X –m_Y rarely continuous function and r m_X open, then $F : (X, m_X) \to (Y, m_Y)$ is upper weakly m_X –m_Y continuous.
Proof **:** Suppose that $x \in X$ and V is an m_Y open set of Y . Since $F : (X, m_X) \to (Y, m_Y)$ upper m_X –m_Y rarely continuous function then there exists a m_X open set U containing x *and* $F(U) \subset V \cup R_V$such that $m_Y Int(F(U)) \subset m_Y Cl(V)$.*It follows from that* $F(U) \subset m_Y Cl(m_Y Int(F(U))) \subset m_Y Cl$(V).

Hence , F : (X , m_X) →(Y, m_Y) is upper weakly m_X-m_Y continuous.

References

[1] A.Kanibir and I.L.Reilly, Generalized continuity for multifunctions. Acta. math. Hungar, 122(3)(2009)283-292.
[2] C. Berge, Espaces Topologiques. Fonctions Multivoques, Dunod, Paris, 1959.
[3] Erdal Ekici, Saeid Jafari, on rare s-precontinuity for multifunctions , Demonstratio Mathematica,vol.XLVI,no-2(2013),395-403.
[4] E. Ekici, J. H. Park, On weakly s-precontinuous multifunctions, Arabian J. Sci. Engrg. 32(2007), 1a; 83-92
[5] Long.P. E. and Herrington, L. L. Properties of rarely continuous functions, Glasnik Mat.17 (37), 229–236, 1982.
[6] Miguel Caldas and Saeid Jafari , On rarely g-continuous functions, Glasnik Matematicki , Vol. 40(60)(2005), 317 – 322.

[7] Miguel Caldas and Saeid Jafari and T.noiri, Characterizations of rarely g-continuous multifunction, Sarajevo journal of mathematics vol.1(13)(2005),129-133.

[8] Sharmistha Bhattacharya(Halder), A Study on rare m_X sets and rarely m_X continuous function. Hacettepe Journal of Mathematics and Statistics, Vol.39 (3) (2010), 295 – 303.

[9] S.Jafari, Rare α-continuity, Bull, Malays. math. sci. soc(2)28(2)(2005),157-161.

[10] T.Noiri & V. Popa, Slightly m-continuous multifunction, Bulletin of the Institute of Mathematics Academia sinica(New series),vol.1(2006),No-4,pp: 485-505.

[11] T. Noiri & V.Popa On upper and lower M-continuous multifunction, Filomat 14(2000),73-86.

[12] V. Popa , sur certain decomposition de la continuite′ dans les espaces topologiques, Glas.Mat.Ser.III,14(34)(1979),359-362.

[13] V. Popa and T. Noiri, on M-continuous functions, Anal. Univ. “Dunarea de jos” Galati, ser. Mat. Fiz. Mec. Teor.(2),18(23)(2000),31-41.

[14] V. Popa and T. Noiri, On upper and lower weakly α-continuous multifunctions, journal of mathematics, Novi sad 2002,vol.32,br.1,str.7-24

[15] V. Popa and .T. Noiri, On s-$precontinuous\ multifunctions$, Demonstratio Math. 33(3)(2000), 679–687.

[16] V. Popa , Some properties of rarely continuous multifunction, Conf. Nat. Geom. Topologie,Univ.Al.I.Cuza Iasi,(1989),269-274.

[17] V. Popa, and T. Noiri, On M - continuous functions, Anal. Univ. ”Dunarea de Jos” Galati Ser. Mat. Fiz. Mec. Teor. Fasc. II 18 (23), 31–41, 2000.