ISSN: **1304-7981** Number: **6**, Year: **2014**, Pages: **32-43** **http://jnrs.gop.edu.tr**

Gaziosmanpasa University
Graduate School of Natural and Applied Sciences
Journal of New Results in Science

Received: 01.06.2014
Accepted: 26.06.2014

Editors-in-Chief: *Naim Çağman*
Area Editor: *Oktay Muhtaroğlu*

On Some New Paranormed Sequence Spaces and Their Topological Properties

Osman Duyar (osman-duyar@hotmail.com)

Anatolian High School, 60200 Tokat, Turkey

Abstract - In this study, we define new paranormed sequence spaces $c_0(u, v; p, \widehat{G})$ and $c(u, v; p, \widehat{G})$ by combining a generalized weighted mean and a generalized difference operator $\widehat{B} = B(r, s, t)$. Furthermore, we compute the $\alpha-$ and $\beta-$ duals and obtain bases for these sequence spaces. Finally, we characterize the classes of matrix mappings from the new paranormed sequence spaces to the spaces $\mu(q)$ for $\mu \in \{c, \ell, \ell_\infty\}$.

Keywords - *Matrix domain of a sequence space, paranormed sequence spaces, weighted mean matrix, Matrix transformations, Schauder basis, $\alpha-$ and $\beta-$ duals.*

1 Introduction

By ω, we shall denote the space of all real valued sequences. Any vector subspace of ω is called as a *sequence space*. We shall write ℓ_∞, c and c_0 for the spaces of all bounded, convergent and null sequences, respectively. Also by bs, cs, ℓ_1 and ℓ_p ; we denote the spaces of all bounded, convergent, absolutely and $p-$ absolutely convergent series, respectively; $1 < p < \infty$.

A linear topological space X over the real field $\mathbb{R}$ is said to be a paranormed space if there is a subadditive function $g : X \to \mathbb{R}$ such that $g(\theta) = 0, g(x) = g(-x)$ and scalar multiplication is continuous, i.e., $|\alpha_n - \alpha| \to 0$ and $g(x_n - x) \to 0$ imply $g(\alpha_n x_n - \alpha x) \to 0$ for all α's in $\mathbb{R}$ and all x's in X, where θ is the zero vector in the linear space X.

Assume here and after that (p_k) be a bounded sequences of strictly positive real numbers with $\sup p_k = H$ and $M = \max\{1, H\}$. Then, the linear spaces $c(p), c_0(p)$ $\ell_\infty(p)$ and $\ell(p)$ were defined by Maddox [36, 37] (see also Simons [39] and Nakano [38]) as follows:

$$\begin{aligned} c(p) &= \left\{x=(x_k)\in\omega:\lim_{k\to\infty}|x_k-l|^{p_k}=0 \text{ for some } l\in\mathbb{C}\right\},\\ c_0(p) &= \left\{x=(x_k)\in\omega:\lim_{k\to\infty}|x_k|^{p_k}=0\right\},\\ \ell_\infty(p) &= \left\{x=(x_k)\in\omega:\sup_{k\in\mathbb{N}}|x_k|^{p_k}<\infty\right\}\end{aligned}$$

which are the complete spaces paranormed by

$$g_1(x)=\sup_{k\in\mathbb{N}}|x_k|^{p_k/M} \text{ iff } \inf_{k\in\mathbb{N}}p_k>0,$$

and the space

$$\ell(p)=\left\{x=(x_k)\in\omega:\sum_k|x_k|^{p_k}<\infty\right\}$$

is the complete paranormed by

$$g_2(x)=\left(\sum_k|x_k|^{p_k}\right)^{1/M}.$$

We shall assume throughout that $p_k^{-1}+(p_k^{'})^{-1}=1$ provided $1<\inf p_k<H<\infty$ and use the convention that any term with negative subscript is equal to zero. For simplicity in notation, here and in what follows, the summation without limits runs from 0 to ∞. By $\mathcal{F}$ and $\mathbb{N}_k$, we shall denote the collection of all finite subsets of $\mathbb{N}$ and the set of all $n\in\mathbb{N}$ such that $n\geq k$, respectively.

Let (X,h) be a paranormed space. A sequence (b_k) of the elements of X is called a basis for X if and only if, for each $x\in X$, there exists a unique sequence (α_k) of scalars such that

$$h\left(x-\sum_{k=0}^{n}\alpha_k b_k\right)\to 0 \quad as \quad n\to\infty.$$

The series $\sum\alpha_k b_k$ which has the sum x is then called the expansion of x with respect to (b_n) and written as $x=\sum\alpha_k b_k$.

Let X,Y be any two sequence spaces and $A=(a_{nk})$ be an infinite matrix of real numbers a_{nk}, where $n,k\in\mathbb{N}$. Then, we say that A defines a matrix mapping from X into Y, and we denote it by writing $A:X\to Y$, if for every sequence $x=(x_k)\in X$ the sequence $Ax=((Ax)_n)$, the A-transform of x, is in Y, where

$$(Ax)_n=\sum_k a_{nk}x_k, \quad (n\in\mathbb{N}). \tag{1}$$

For a sequence space X, the matrix domain X_A of an infinite matrix A is defined by

$$X_A=\{x=(x_k)\in\omega: \quad Ax\in X\}. \tag{2}$$

By $(X:Y)$, we denote the class of all matrices A such that $A:X\to Y$. Thus, $A\in(X:Y)$ if and only if the series on the right-hand side of (1) converges for each $n\in\mathbb{N}$ and every $x\in X$, and we have $Ax=\{(Ax)_n\}_{n\in\mathbb{N}}\in Y$ for all $x\in X$. A sequence

x is said to be A- summable to α if Ax converges to α which is called as the A- limit of x.

Let r, s and t be non-zero real numbers, and define the generalized difference matrix $\widehat{B} = B(r,s,t) = \{b_{nk}(r,s,t)\}$ by

$$b_{nk}(r,s,t) = \begin{cases} r, & (k = n) \\ s, & (k = n-1) \\ t, & (k = n-2) \\ 0, & (0 \leq k < n-1 \text{ or } k > n) \end{cases} \tag{3}$$

for all $n, k \in \mathbb{N}$.

We write by $\mathcal{U}$ for the set of all sequences $u = (u_n)$ such that $u_n \neq 0$ for all $n \in \mathbb{N}$. For $u \in \mathcal{U}$, let $1/u = (1/u_n)$. Let $u, v \in \mathcal{U}$ and let us define the matrix $G(u,v) = (g_{nk})$ and $\Delta = (\delta_{nk})$ as follows:

$$g_{nk} = \begin{cases} u_n v_k, & (0 \leq k \leq n), \\ 0, & (k > n), \end{cases} \qquad \delta_{nk} = \begin{cases} (-1)^{n-k}, & (n-1 \leq k \leq n), \\ 0, & (0 \leq k < n-1 \text{ or } k > n), \end{cases}$$

for all $n, k \in \mathbb{N}$, where u_n depends only on n and v_k only on k. The matrix $G(u,v)$, defined above, is called as generalized weighted mean or factorable matrix.

The main purpose of this study is to introduce the sequence spaces $c_0(u,v;p,\widehat{G})$ and $c(u,v;p,\widehat{G})$ which is the set of all sequences whose $G(u,v;\widehat{B})$-transforms are in the spaces $c_0(p)$ and $c(p)$, respectively, where $G(u,v;\widehat{B})$ denotes the matrix $G(u,v;\widehat{B}) = G(u,v)\widehat{B} = \widehat{G} = (\widehat{g}_{nk})$ defined by

$$\widehat{g}_{nk} = \begin{cases} u_n v_k r + u_n v_{k+1} s + u_n v_{k+2} t, & (k < n-1) \\ u_n v_{n-1} r + u_n v_n s, & (k = n-1) \\ u_n v_n r, & (k = n) \\ 0, & (\text{otherwise}) \end{cases} \tag{4}$$

for all $k, n \in \mathbb{N}$. Also, we have investigated some topological structures, which have completeness, the $\alpha-$ and $\beta-$ duals, and the basis of these sequence spaces. Finally, we characterize some matrix mappings on these spaces.

2 The Paranormed Sequence Spaces $c_0(u,v;p,\widehat{G})$ and $c(u,v;p,\widehat{G})$

In this section, we define the new sequence spaces $c_0(u,v;p,\widehat{G})$ and $c(u,v;p,\widehat{G})$ derived by using the generalized weighted mean and generalized difference operator, and prove that these sequence spaces is the complete paranormed linear metric spaces and compute their $\alpha-$ and $\beta-$ duals. Also, we give the basis for these spaces.

Let r and s be non-zero real numbers, and define the double-band matrix $B(r,s) = \{b_{nk}(r,s)\}$ by

$$b_{nk}(r,s) = \begin{cases} r, & (k = n), \\ s, & (k = n-1), \\ 0, & (0 \leq k < n-1 \text{ or } k > n), \end{cases}$$

for all $k, n \in \mathbb{N}$.

Altay and Başar [6] have examined topological properties of the space $\ell(u, v; p)$ which is defined by

$$\ell(u, v, p) = \{x = (x_k) \in \omega : y = \left(\sum_{j=0}^{k} u_k v_j x_j\right) \in \ell(p)\}.$$

Başarır and Kara have recently defined the sequence space $\ell(u, v; p, \widehat{B})$ in [26], which consists of all sequences such that GB-transforms are in $\ell(p)$, where $G = G(u, v)$ is the weighted mean transform and $B = B(r, s)$ is the generalized difference transform.

Following Altay and Başar [6] and Başarır and Kara [26] we define the sequence spaces $\lambda(u, v; p, \widehat{B})$ by

$$\lambda(u, v; p, \widehat{G}) = \left\{x = (x_k) \in \omega : \left(\sum_{i=0}^{k} u_k v_i (r x_i + s x_{i-1} + t x_{i-2})\right) \in \lambda(p)\right\}$$

for $\lambda \in \{c_0, c\}$. We may redefine the spaces $c_0(u, v; p, \widehat{G})$ and $c(u, v; p, \widehat{G})$ using the notation (2) as follows:

$$c_0(u, v; p, \widehat{G}) = \{c_0(p)\}_{\widehat{G}} \quad \text{and} \quad c(u, v; p, \widehat{G}) = \{c(p)\}_{\widehat{G}}.$$

If p_k and r, s, t are selected as suitable, this definition includes the special cases in the articles [6, 7, 8, 15, 16, 24, 26, 30, 31].

Now, we define the sequence $y = (y_k)$ as the $\widehat{G}$-transform of a sequence $x = (x_k)$, i.e.

$$y_k = u_k \sum_{i=0}^{k-2} (r v_i + s v_{i+1} + t v_{i+2}) x_i + u_k (r v_{k-1} + s v_k) x_{k-1} + u_k v_k r x_k \tag{5}$$

for all $k \in \mathbb{N}$.

Theorem 2.1. *The sequence spaces $c_0(u, v; p, \widehat{G})$ and $c(u, v; p, \widehat{G})$ are the complete linear metric spaces paranormed by g, defined by*

$$g(x) = \sup_{k \in \mathbb{N}} \left| \sum_{j=0}^{k} u_k v_j (r x_j + s x_{j-1} + t x_{j-2}) \right|^{p_k/M}.$$

Proof: The proof of this theorem follows from the similar arguments as in the Theorem3.1 in [26]. So we omit the detail.

Theorem 2.2. *The sequence spaces $c(u, v; p, \widehat{G})$ and $c_0(u, v; p, \widehat{G})$ are linearly isomorphic to the spaces $c(p)$ and $c_0(p)$, respectively, where $0 < p_k \le H < \infty$.*

Proof: We establish this for the space $c(u, v; p, \widehat{G})$. To prove the theorem, we should show the existence of a linear bijection between the spaces $c(u, v; p, \widehat{G})$ and $c(p)$ for $0 < p_k \le H < \infty$. With the notation of (5), define the transformations T from $c(u, v; p, \widehat{G})$ to $c(p)$ by $x \mapsto y = Tx$. The linearity of T is trivial. Further, it is obvious that $x = \theta$ whenever $Tx = \theta$ and hence T is injective.

Let $y=(y_k)\in c(p)$ and define the sequence $x=(x_k)$ by

$$x_k=\sum_{j=0}^{k} d_{kj}\sum_{i=j-1}^{j}(-1)^{j-i}\frac{1}{v_j u_i}y_i \tag{6}$$

for $k\in\mathbb{N}$ where $d_{nk}=0$ for $k>n$ and

$$d_{nk}=\frac{1}{r}\sum_{v=0}^{n-k}\left(\frac{-s+\sqrt{s^2-4tr}}{2r}\right)^{n-k-v}\left(\frac{-s-\sqrt{s^2-4tr}}{2r}\right)^{v} \tag{7}$$

for $0\leq k\leq n$. Then, we get that

$$\begin{aligned} g(x) &= \sup_{k\in\mathbb{N}}\left|u_k\sum_{i=0}^{k-2}(rv_i+sv_{i+1}+tv_{i+2})x_i+u_k(rv_{k-1}+sv_k)x_{k-1}+u_kv_krx_k\right|^{p_k/M} \\ &= \sup_{k\in\mathbb{N}}|y_k|^{p_k/M}=g_1(y)<\infty. \end{aligned}$$

Thus, we deduce that $x\in c(u,v;p,\widehat{G})$ and consequently T is surjective and is paranorm preserving. Hence, T is a linear bijection and this says us that the spaces $c(u,v;p,\widehat{G})$ and $c(p)$ are linearly isomorphic, as desired.

Let $\lambda\in\{c_0,c\}$. Because of the isomorphism T between the sequence spaces $\lambda(u,v;p,\widehat{G})$ and $\lambda(p)$ is onto, the inverse image of the basis of the space $\lambda(p)$ is the basis of the space $\lambda(u,v;p,\widehat{G})$. Therefore, we may give a corollary with respect to Schauder basis of the new sequence spaces $\lambda(u,v;p,\widehat{G})$:

Corollary 2.3. *Let $\alpha_k=\widehat{G}_k(x)$ for all $k\in\mathbb{N}$ and $\lim_{k\to\infty}\widehat{G}_k(x)=l$. Define the sequence $b^{(k)}=\{b_n^{(k)}\}_{n\in\mathbb{N}}$ for every fixed $k\in\mathbb{N}$ by*

$$b_n^{(k)}=\begin{cases}\dfrac{d_{nk}}{u_kv_k}-\dfrac{d_{n,k+1}}{u_kv_{k+1}}, & (n>k)\\ \dfrac{1}{ru_kv_k}, & (n=k)\\ 0, & (n<k).\end{cases} \tag{8}$$

Then, the following statements hold:

(i) The sequence $\{b^{(k)}\}_{k\in\mathbb{N}}$ is a basis for the space $c_0(u,v;p,\widehat{G})$ and any $x\in c_0(u,v;p,\widehat{G})$ has a unique representation of the form $x=\sum_k\alpha_kb^{(k)}$.

(ii) The sequence $\{b,b^{(0)},b^{(1)},b^{(2)},...\}$ is a basis for the space $c(u,v;p,\widehat{G})$, where $b=(b_k)=\left(\sum_{j=0}^{k}d_{kj}\right)$, and any $x\in c(u,v;p,\widehat{G})$ has a unique representation of the form

$$x=lb+\sum_k[\alpha_k-l]b^{(k)}.$$

3 The $\alpha-$ and $\beta-$ Dual of The Spaces $c(u,v;p,\widehat{G})$ and $c_0(u,v;p,\widehat{G})$

For the sequence spaces X and Y, define the set $S(X,Y)$ by

$$S(X,Y)=\{z=(z_k):xz=(x_kz_k)\in Y \ \text{ for all } \ x\in X\}. \tag{9}$$

With the notation of (9), the $\alpha-$ and $\beta-$ duals of a sequence space X, which are respectively denoted by X^α and X^β are defined by

$$X^\alpha=S(X,\ell_1) \text{ and } X^\beta=S(X,cs)$$

We shall quote some lemmas which are needed in proving our theorems.

Lemma 3.1. *[33, Theorem 5.1.1 with $q=1$] $A\in(c_0(p):\ell_1)$ if and only if*

$$\sup_{K\in\mathcal{F}}\sum_n\left|\sum_{k\in K}a_{nk}B^{-1/p_k}\right|<\infty, \quad (\exists B\in\mathbb{N}_2). \tag{10}$$

Lemma 3.2. *[34, Corollary 2] $A\in(c_0(p):c)$ if and only if*

$$\sup_{n\in\mathbb{N}}\sum_k|a_{nk}|B^{-1/p_k}<\infty, \quad (\exists B\in\mathbb{N}_2), \tag{11}$$

$$\lim_{n\to\infty}a_{nk}=\alpha_k \quad \textit{exists for every fixed } k. \tag{12}$$

Theorem 3.3. *Let $K^*=\{k\in\mathbb{N}:0\le k\le n\}\cap K$ for $K\in\mathcal{F}$ and $B\in\mathbb{N}_2$. Define the sets $G_1(p),G_2(p),G_3(p),G_4(p),G_5(p)$ and $G_6(p)$ as follows:*

$$\begin{aligned}
G_1(p) &= \bigcup_{B>1}\left\{a=(a_k)\in\omega:\sup_{K\in\mathcal{F}}\sum_n\left|\sum_{k\in K^*}c_{nk}B^{-1/p_k}\right|<\infty\right\},\\
G_2(p) &= \left\{a=(a_k)\in\omega:\sum_n\left|\sum_k c_{nk}\right|<\infty\right\},\\
G_3(p) &= \bigcup_{B>1}\left\{a=(a_k)\in\omega:\sup_{n\in\mathbb{N}}\sum_k|\widehat{g}_k(n)|B^{-1/p_k}<\infty\right\},\\
G_4(p) &= \bigcup_{B>1}\left\{a=(a_k)\in\omega:\left\{\frac{a_k}{ru_kv_k}B^{-1/p_k}\right\}\in\ell_\infty\right\},\\
G_5(p) &= \left\{a=(a_k)\in\omega:\lim_{n\to\infty}\widehat{g}_k(n)=\alpha_k, \quad \textit{exists for every fixed } k\right\},\\
G_6(p) &= \left\{a=(a_k)\in w:\sup_{k\in\mathbb{N}}\left|\frac{1}{r}\frac{a_k}{u_kv_k}\right|<\infty \textit{ and } \sum_{j=k}^\infty d_{jk}a_j \textit{ exists for each } k\in\mathbb{N}\right\}
\end{aligned}$$

Then,

$$\{c_0(u,v;p,\widehat{G})\}^\alpha=G_1(p), \ \{c(u,v;p,\widehat{G})\}^\alpha=G_1(p)\cap G_2(p),$$

$$\{c_0(u,v;p,\widehat{G})\}^\beta=\bigcap_{i=3}^6G_i(p), \ \{c(u,v;p,\widehat{G})\}^\beta=\{c_0(u,v;p,\widehat{G})\}^\beta\cap cs,$$

Proof: We give the proof for the space $c_0(u,v;p,\widehat{G})$. Let us take any $a=(a_n)\in\omega$ and define the matrix $C=(c_{nk})$ via the sequence $a=(a_n)$ by

$$c_{nk}=\begin{cases}\dfrac{d_{nk}a_n}{u_kv_k}-\dfrac{d_{n,k+1}a_n}{u_kv_{k+1}}, & (k<n)\\ \dfrac{a_n}{ru_nv_n}, & (k=n)\\ 0, & (k>n)\end{cases}$$

where $n,k\in\mathbb{N}$. Bearing in mind (5) we immediately derive that

$$\begin{aligned} a_nx_n &= \sum_{k=0}^{n} d_{nk}\sum_{j=k-1}^{k}(-1)^{k-j}\frac{1}{v_ku_j}a_ny_j\\ &= \sum_{k=0}^{n-1}\left(\frac{d_{nk}}{v_k}-\frac{d_{n,k+1}}{v_{k+1}}\right)\frac{a_n}{u_k}y_k+\frac{a_n}{ru_nv_n}y_n\\ &= C_n(y) \end{aligned} \tag{13}$$

for all $n\in\mathbb{N}$. We therefore observe by (13) that $ax=(a_nx_n)\in\ell_1$ whenever $x\in c_0(u,v;p,\widehat{G})$ if and only if $Cy\in\ell_1$ whenever $y\in c_0(p)$. This means that $a=(a_n)\in\{c_0(u,v;p,\widehat{G})\}^{\alpha}$ whenever $x=(x_n)\in c_0(u,v;p,\widehat{G})$ if and only if $C\in(c_0(p):\ell_1)$. Then, we derive by Lemma 3.1 that

$$\{c_0(u,v;p,\widehat{G})\}^{\alpha}=G_1(p).$$

We show that now $\beta-$dual of the space $\{c_0(u,v;p,\widehat{G})\}^{\beta}$. For this purpose we use the following equation;

$$\begin{aligned} \sum_{k=0}^{n}a_kx_k &= \sum_{k=0}^{n}\left[\sum_{j=0}^{k}d_{kj}\sum_{i=j-1}^{i}(-1)^{j-i}\frac{1}{v_ju_i}y_i\right]a_k\\ &= \sum_{k=0}^{n}\left(\frac{y_k}{u_k}-\frac{y_{k-1}}{u_{k-1}}\right)\left(\frac{1}{v_k}\sum_{j=k}^{n}d_{jk}a_j\right)\\ &= \sum_{k=0}^{n-1}\frac{1}{u_k}\left[\frac{\sum_{j=k}^{n}d_{jk}a_j}{v_k}-\frac{\sum_{j=k+1}^{n}d_{j,k+1}a_j}{v_{k+1}}\right]y_k+\frac{1}{r}\frac{a_n}{u_nv_n}y_n\\ &= \sum_{k=0}^{n-1}\widehat{g}_k(n)y_k+\frac{1}{r}\frac{a_n}{u_nv_n}y_n\\ &= E_n(y);\quad (n\in\mathbb{N}) \end{aligned} \tag{14}$$

where $E=(e_{nk})$ is defined by

$$e_{nk}=\begin{cases}\widehat{g}_k(n), & (k<n)\\ \dfrac{a_n}{ru_nv_n}, & (k=n)\\ 0, & (k>n)\end{cases}$$

for all $k,n\in\mathbb{N}$. Thus, we deduce from Lemma 3.2 with (14) that $ax=(a_kx_k)\in cs$ whenever $x=(x_k)\in c_0(u,v;p,\widehat{G})$ if and only if $Ey\in c$ whenever $y=(y_k)\in c_0(p)$.

This means that $a = (a_n) \in \{c_0(u, v; p, \widehat{G})\}^{\beta}$ whenever $x = (x_n) \in c_0(u, v; p, \widehat{G})$ if and only if $E \in (c_0(p) : c)$. Therefore we derive from Lemma 3.2 and (14) that

$$\{c_0(u, v; p, \widehat{G})\}^{\beta} = \bigcap_{i=3}^{6} G_i(p).$$

4 Some Matrix Mappings on the Sequence Spaces $c_0(u, v; p, \widehat{G})$ and $c(u, v; p, \widehat{G})$

In this final section, we state some results which characterize various matrix mappings on the spaces $c_0(u, v; p, \widehat{G})$ and $c(u, v; p, \widehat{G})$. We shall write throughout for brevity that

$$\widehat{a}_{nk}(m) = \frac{1}{u_k}\left[\frac{\sum_{j=k}^{m} d_{jk}a_{j,k}}{v_k} - \frac{\sum_{j=k+1}^{m} d_{j,k+1}a_{j,k+1}}{v_{k+1}}\right] \qquad \text{for } k < m$$

and

$$\widehat{a}_{nk} = \frac{1}{u_k}\left[\frac{\sum_{j=k}^{\infty} d_{jk}a_{j,k}}{v_k} - \frac{\sum_{j=k+1}^{\infty} d_{j,k+1}a_{j,k+1}}{v_{k+1}}\right]$$

for all $k, m, n \in \mathbb{N}$ provided the series on the right hand to be convergent.

Theorem 4.1. *Let λ be any given sequence space and $\mu \in \{c_0, c\}$. Then, $A = (a_{nk}) \in (\mu(u, v; p, \widehat{G}) : \lambda)$ if and only if $B \in (\mu : \lambda)$ and*

$$B^{(n)} \in (\mu : c) \tag{15}$$

for every fixed $n \in \mathbb{N}$, where $b_{nk} = \widehat{a}_{nk}$ and $B^{(n)} = (b^{(n)}_{mk})$

$$b^{(n)}_{mk} = \begin{cases} \widehat{a}_{nk}(m), & (k < n) \\ \dfrac{a_{nm}}{ru_m v_m}, & (k = n) \\ 0, & (k > n) \end{cases}$$

for all $k, m \in \mathbb{N}$.

Proof: This result can be proved similarly as the proof of Theorem 3.1 in [8].

Now, we may quote our corollaries on the characterization of some matrix classes concerning with the sequence spaces $c_0(u, v; p, \widehat{G})$ and $c(u, v; p, \widehat{G})$. Prior to giving the corollaries, let us suppose that (q_n) is a non-decreasing bounded sequence of positive real numbers and consider the following conditions:

$$\sup_{n\in\mathbb{N}}\left[\sum_k |a_{nk}|\, B^{-1/p_k}\right]^{q_n} < \infty, \quad (\exists B\in\mathbb{N}), \tag{16}$$

$$\sup_{n\in\mathbb{N}}\left|\sum_k a_{nk}\right|^{q_n} < \infty, \tag{17}$$

$$\sup_{K\in\mathcal{F}}\sum_n\left|\sum_{k\in K} a_{nk}B^{-1/p_k}\right|^{q_n} < \infty, \quad (\exists B\in\mathbb{N}), \tag{18}$$

$$\sum_n\left|\sum_k a_{nk}\right|^{q_n} < \infty, \tag{19}$$

$$\exists\alpha\in\mathbb{R}\ni\lim_{n\to\infty}\left|\sum_k a_{nk}-\alpha\right|^{q_n} = 0, \tag{20}$$

$$\exists(\alpha_k)\subset\mathbb{R}\ni\lim_{n\to\infty}|a_{nk}-\alpha_k|^{q_n} = 0, \quad (\forall k\in\mathbb{N}), \tag{21}$$

$$\exists(\alpha_k)\subset\mathbb{R}\ni\sup_{n\in\mathbb{N}}K^{1/q_n}\sum_k |a_{nk}-\alpha_k|\, B^{-1/p_k} < \infty, \quad (\forall K,\exists B\in\mathbb{N}) \tag{22}$$

$$\sup_{n\in\mathbb{N}}\sum_k |a_{nk}|B^{-1/p_k} < \infty, \quad (\exists B\in\mathbb{N}) \tag{23}$$

Now, we can give the corollaries:

Corollary 4.2. *$A=(a_{nk})$ be any infinite matrix. Then the following statements hold:*
(i) $A=(a_{nk})\in(c_0(u,v;p,\widehat{G}):\ell(q)$ if and only if (18) holds with $\widehat{a}_{nk}$ instead of a_{nk} and (15) also holds with $\lambda=c_0$.
(ii) $A=(a_{nk})\in(c_0(u,v;p,\widehat{G}):c(q))$ if and only if (21), (22) and (23) hold with $\widehat{a}_{nk}$ instead of a_{nk} and (15) also holds with $\lambda=c_0$.
(iii) $A=(a_{nk})\in(c_0(u,v;p,\widehat{G}):\ell_\infty(q))$ if and only if (16) and (17) hold with $\widehat{a}_{nk}$ instead of a_{nk} and (15) also holds $\lambda=c_0$.

Corollary 4.3. *$A=(a_{nk})$ be any infinite matrix. Then the following statements hold:*
(i) $A=(a_{nk})\in(c(u,v;p,\widehat{G}):\ell(q))$ if and only if (18) and (19) hold with $\widehat{a}_{nk}$ instead of a_{nk} and (15) also holds with $\lambda=c$.
(ii) $A=(a_{nk})\in(c(u,v;p,\widehat{G}):c(q))$ if and only if (20)-(23) hold with $\widehat{a}_{nk}$ instead of a_{nk} and (15) also holds $\lambda=c$.
(iii) $A=(a_{nk})\in(c(u,v;p,\widehat{G}):\ell_\infty(q))$ if and only if (16) and (17) hold with $\widehat{a}_{nk}$ instead of a_{nk} and (15) also holds $\lambda=c$.

References

[1] C.-S. Wang, On Nörlund sequence spaces, Tamkang J. Math. 9(1978) 269-274.

[2] P.-N. Ng, P.-Y. Lee, Cesàro sequence spaces of non-absolute type, Comment. Math. (Prace Mat.) 20(2)(1978) 429-433.

[3] E. Malkowsky, Recent results in the theory of matrix transformations in sequence spaces, Mat. Vesnik 49(1997) 187-196.

[4] A. M. Jarrah and E. Malkowsky, Ordinary, absolute and strong summability and matrix transformations,Filomat 17(2003), 59-78

[5] B. Altay, F. Başar, Some Euler sequence spaces of non-absolute type, Ukrainian Math. J. 57(1)(2005) 1-17.

[6] B. Altay, F. Başar, Some paranormed sequence spaces of non-absolute type derived by weighted mean, J. Math. Anal. Appl. 319 (2006), 494-508.

[7] H. Polat, V. Karakaya and N. Şimşek, Difference sequence spaces derived by generalized weighted mean, Appl. Math. Lett. 24(2011), no.5, 608-614.

[8] B. Altay, F. Başar, Generalization of the space $\ell(p)$ derived by weighted mean , J. Math. Anal. Appl 330 (2007) 174-185.

[9] E. Malkowsky, E. Savaş, Matrix transformations between sequence spaces of generalized weighted means, Appl. Math. Comput. 147(2004) 333-345.

[10] M. Başarır, On some new sequence spaces and related matrix transformations, Indian J. Pure Appl. Math. 26(10) (1995) 1003-1010.

[11] F. Başar, B. Altay, M. Mursaleen, Some generalizations of the space bv_p of $p-$ bounded variation sequences, Nonlinear Anal. 68(2)(2008) 273-287.

[12] B. Altay, On the space of $p-$ summable difference sequences of order m, $(1 \leq p < \infty)$, Studia Sci. Math. Hungar. 43(4)(2006) 387-402.

[13] E. Malkowsky, M. Mursaleen, S. Suantai, The dual spaces of sets of difference sequences of order m and matrix transformations, Acta Math. Sin. Eng. Ser. 23(3)(2007) 521-532.

[14] Demiriz, S., Çakan, C., On some new paranormed Euler sequence spaces and Euler core , Acta Math. Sin. Eng. Ser. **26**(7)(2010), 1207-1222.

[15] Demiriz, S., Özdemir O., Duyar O., On some new generalized difference sequence spaces of non-absolute type, arXiv:1309.3903, 2013.

[16] Duyar O., Demiriz, S., On some new generalized difference sequence spaces and their topological properties, Journal of New Result in Science, 6(2014) 1-14.

[17] B. Altay, F. Başar, Certain topological properties and duals of the matrix domain of a triangle matrix in a sequence space, J. Math. Anal. Appl. 336(1)(2007) 632-645.

[18] H. Kızmaz, On certain sequence spaces, Canad. Math. Bull. 24(2)(1981) 169-176.

[19] C. Aydın, F. Başar, Some new sequence spaces which include the spaces ℓ_p and ℓ_∞, Demonstratio Math. 38(3)(2005) 641-656.

[20] C. Aydın, F. Başar, Some generalizations of the sequence space a_p^r, Iran. J. Sci. Technol. A Sci. 30(No. A2)(2006) 175-190.

[21] F. Başar, B. Altay, On the space of sequences of $p-$ bounded variation and related matrix mappings, Ukrainian Math. J. 55(1) (2003) 136-147.

[22] E. Malkowsky, M. Mursaleen, Some matrix transformations between the difference sequence spaces $\Delta c_0(p), \Delta c(p)$ and $\Delta \ell_\infty(p)$, Filomat, 15(2001) 353-363.

[23] Z. U. Ahmad, M. Mursaleen, Köthe-Toeplitz duals of some new sequence spaces and their matrix maps, Publ. Inst. Math. (Belgrad) 42(1987) 57-61.

[24] A. Sönmez, F. Başar, Generalized difference spaces of non-absolute type of convergent and null sequences, Abstract and Applied Analysis Volume 2012 , Article ID 435076, (2012) 20 pages.

[25] M. Başarır, E. E. Kara, On the m^{th} order difference sequence space of generalized weighted mean and compact operators, Acta Mathematica Scientia, 33(B3)(2013), 1-18.

[26] M. Başarır, E. E. Kara, On the $B-$ difference sequence space derived by generalized weighted mean and compact operators, Journal of Mathematical Analysis and Applications, 391(2012), 67-81.

[27] E. E. Kara, Some topological and geometrical properties of new Banach sequence space, Journal of Inequalities and Applications, 2013(38) (2013), 15 pages.

[28] M. Başarır, E. E. Kara, On some difference sequence space of weighted means and compact operators, Annals of Functional Analysis, 2(2)(2011), 116-131.

[29] E. E. Kara, M. Başarır, On compact operators and some Euler $B^{(m)}-$ difference sequence spaces, Journal of Mathematical Analysis and Applications, 379(2011), 499-511.

[30] M. Mursaleen, A.K. Noman, On the spaces of $\lambda-$convergent and bounded sequences, Thai Journal of Mahematics, 8(2) (2010), 311-329.

[31] M. Mursaleen, Abdullah K. Noman, On some new sequence spaces of non-absolute type related to the spaces ℓ_p and ℓ_∞ I, Filomat 25:2 (2011), 33-51.

[32] A. Sönmez, Some new sequence spaces derived by the domain of the triple band matrix, Comput. Math. Appl., 62 (2011) 641-650.

[33] K.-G.Grosse-Erdmann, Matrix transformations between the sequence spaves of Maddox, J. Math. Anal. Appl. **180**(1993), 223-238.

[34] C. G. Lascarides, I.J. Maddox, Matrix transformations between some classes of sequences, Proc. Camb. Phil. Soc. **68**(1970), 99-104.

[35] I.J. Maddox, Elements of Functional Analysis, second ed., The Cambridge University Press, Cambridge, 1988.

[36] I.J. Maddox, Paranormed sequence spaces generated by infinite matrices, Proc. Camb. Phios. Soc. **64**(1968), 335-340.

[37] I.J.Maddox, Spaces of strongly summable sequences, Quart. J. Math. Oxford **18**(2)(1967), 345-355.

[38] H. Nakano, Modulared sequence spaces, Proc. Jpn. Acad. **27**(2)(1951), 508-512.

[39] S. Simons, The sequence spaces $\ell(p_v)$ and $m(p_v)$, Proc. London Math. Soc. **15**(3)(1965), 422-436.

[40] F. Başar, Summability Theory and Its Appliactions, Bentham Science Publishers, ISBN:978-1-60805-252-3, 2011.

[41] A. Wilansky, Summability through Functional Analysis, North-Holland Mathematics Studies , Amsterdam, **85** 1984.