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Influence of Wall Properties on Peristaltic Transport of a Micropolar Fluid in an Inclined Channel

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Abstract – Peristaltic transport of an incompressible micropolar fluid in an inclined two dimensional channel with the influence of wall properties has been studied. The equations governing the flow have been linearised under long wave length approximation and a perturbation method of solution has been obtained in terms of wall slope parameter, under dynamic boundary conditions. Analytical expression has been derived for the time average velocity and the effects of pertinent parameters on time average velocity have been studied. It has been observed that time average velocity increases with rigidity and stiffness in the wall. Further, the time average velocity increases with the inclination α.

Keywords – *Peristaltic transport, Micropolar fluid, Inclined channel, Wall Properties.*

1 Introduction

Peristaltic motion is a mechanism for fluid transport which is achieved when progressive waves of area contraction or expansion propagate along the walls of a distensible channel (or tube) containing the fluid. Peristalsis is known to be the main mechanism for fluid transport in many physiological situations such as transport of urine through ureter, food mixing and chyme movement in the intestines, blood flow in cardiac chambers etc. Mechanical devices like finger pumps and roller pumps also operate on this principle. Peristaltic pumping is used in biomedical devices like heart lung machine to pump blood.

Since the first investigation of Latham [1], several researchers [2, 3, 4, 5, 6, 7] have studied the peristaltic transport of Newtonian and non Newtonian fluids in different situations using analytical, numerical and experimental methods.

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The theory of micropolar fluids and the application of some technical flows of these fluids have been presented in the works by Eringen [8] and Ariman et al. [9]. In the micropolar fluid theory, apart from the classical velocity field, the microrotation vector $\vec{g}$ and the gyration parameter J are introduced to investigate the kinematics of micro rotations and provide a good mathematical model for the non-Newtonian behavior observed in certain man made liquids such as polymeric fluids and in naturally occurring liquids such as animal blood.

Hence, the peristaltic transport of micropolar fluid has received some attention in last few decades. Several attempts [10, 11, 12, 13] have been made to understand the peristaltic motion of a micropolar fluid under various conditions.

However, the interaction of the wall properties with fluid flow in peristaltic transport has not received much attention. Mittra and Prasad [14] studied peristaltic transport in a two-dimensional channel considering the elasticity of the walls. They solved this problem under the approximation of small amplitude ratio with dynamic boundary conditions. Muthu et al. [15] extended the analysis of Mittra and Prasad [14] to micropolar fluids.

But, it is known that many ducts in physiological systems are not horizontal but have some inclinations with the axis. However, the effect of inclination on fluid flows has not received much attention. Hence, Vajravelu et al. [16] studied the peristaltic transport of a Herschel-Bulkley fluid in an inclined tube. Srinivas and Pushparaj [17] analyzed non-linear peristaltic transport in an inclined asymmetric channel. Nadeem and Akbar [18] considered the influence of heat transfer on peristaltic transport of Herschel-Bulkley fluid in a non-uniform inclined tube. Rami Reddy et al. [19] studied peristaltic transport of conducting fluid in an inclined asymmetric channel. However, no attempt has been made to study the influence of wall properties on peristaltic transport of a micropolar fluid in an inclined channel.

In view of this, the peristaltic transport of a micropolar fluid in an inclined channel with dynamic boundary conditions has been studied. A perturbation method of solution has been obtained in terms of wall slope parameter assuming that the wave length of peristaltic wave is large in comparison to the mean half width of the channel. Expressions for the stream function and average velocity have been derived. The effects of various parameters on time average velocity have been studied.

2 Formulation of the Problem

Consider the flow of an unsteady incompressible micropolar fluid in an inclined two dimensional channel with flexible walls. It is assumed that traveling sinusoidal waves with speed c, amplitude a and wave length λ are imposed on the walls of the channel. The channel is inclined at an angle α with the horizontal line (Fig.1). Cartesian coordinate system (x, y) is chosen with the x-axis aligned with the central line of the channel and in the direction of propagation of waves. The wall deformation due to the propagation of an infinite train of peristaltic waves is given by

$$y = \eta(x,t) = d + a\sin\frac{2\pi}{\lambda}(x - ct) \tag{1}$$

where d is the mean half width of the channel.

The governing equation of motion of the flexible wall may be expressed as

$$L(\eta) = p - p_0 \tag{2}$$

where L is an operator, which is used to represent the motion of stretched membrane with damping forces such that

$$L = -T\frac{\partial^2}{\partial x^2} + m\frac{\partial^2}{\partial t^2} + C\frac{\partial}{\partial t} \tag{3}$$

Here, T is the tension in the membrane, m is the mass per unit area and C is the coefficient of viscous damping force.

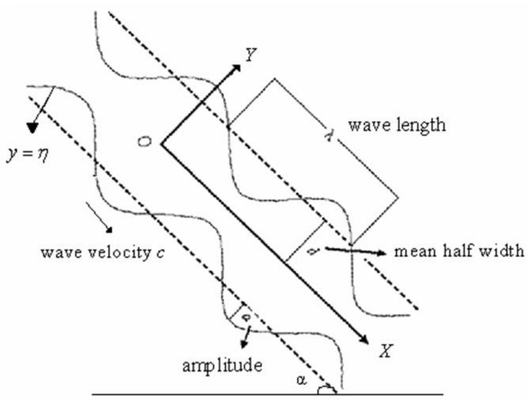


Figure 1: Geometry of two dimensional peristaltic transport in an inclined tube

The equations governing the flow of an incompressible micropolar fluid in cartesian form for the present problem are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \left(\frac{2\mu+\kappa}{2}\right)\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + \kappa\frac{\partial g}{\partial y} + \rho g^* \sin\alpha \tag{5}$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \left(\frac{2\mu+\kappa}{2}\right)\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] - \kappa\frac{\partial g}{\partial x} - \rho g^* \cos\alpha \tag{6}$$

$$\rho J\left[\frac{\partial g}{\partial t}+u\frac{\partial g}{\partial x}+v\frac{\partial g}{\partial y}\right]=-2\kappa g+\gamma\left[\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial y^2}\right]+\kappa\left[\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right] \tag{7}$$

where u and v are the velocity components in the x and y directions respectively, g is the microrotation component, ρ is the density, p is the pressure, J is the micro inertia constant, g^* is acceleration due to gravity, μ is the coefficient of viscosity and κ and γ are the viscosity coefficients for the micropolar fluid and $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$.

It is assumed that $p_0=0$ and the channel walls are inextensible so that only their lateral motions normal to the undeformed positions occur. The horizontal displacement thus is assumed to be zero.

Thus the no-slip boundary conditions for the velocity and microrotation are

$$u=0,\ g=0 \quad \text{at} \quad y=\pm\eta(x,t) \tag{8}$$

The equation of motion of the flexible walls, following Mittra and Prasad [14], is

$$\frac{\partial L(\eta)}{\partial x}=-\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]+\left[\frac{2\mu+\kappa}{2}\right]\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\kappa\frac{\partial g}{\partial y}-\rho g^*\sin\alpha$$

$$\text{at } y=\pm\eta(x,t),$$

where

$$\frac{\partial L(\eta)}{\partial x}=\frac{\partial p}{\partial x}=-T\frac{\partial^3\eta}{\partial x^3}+m\frac{\partial^3\eta}{\partial t^2\partial x}+C\frac{\partial^2\eta}{\partial t\partial x} \tag{9}$$

Defining the stream function ψ by

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x} \tag{10}$$

and eliminating the pressure between (5) and (6), equations (5) - (7), become

$$\rho\left[\frac{\partial}{\partial t}\nabla^2\psi + \frac{\partial \psi}{\partial y}\nabla^2\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial x}\nabla^2\frac{\partial \psi}{\partial y}\right] = \left(\frac{2\mu+\kappa}{2}\right)\nabla^4\psi + \kappa\nabla^2 g \tag{11}$$

$$\rho J\left[\frac{\partial g}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial g}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial g}{\partial y}\right] = \gamma\ \nabla^2 g - \kappa\left[\nabla^2\psi + 2g\right] \tag{12}$$

Introducing the following non-dimensional quantities

$$x' = \frac{x}{\lambda}, y' = \frac{y}{d}, t' = \frac{ct}{\lambda}, \psi' = \frac{\psi}{cd}, \eta' = \frac{\eta}{d}, g' = \frac{dg}{c}, \tag{13}$$

equations (11), (12), (8) and (9), after dropping the primes, can be written as

$$\delta R_e\left[\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi_t + \psi_y\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi_x - \psi_x\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi_y\right] = \left(\frac{2+\mu_1}{2}\right)\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2\psi + \mu_1\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)g \tag{14}$$

$$\delta R_l\left[\frac{\partial g}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial g}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial g}{\partial y}\right] = 2\left(1-N^2\right)\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)g - N^2M^2\left[\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi + 2g\right] \tag{15}$$

The boundary conditions are

$$\psi_y = 0,\ g = 0\ \ at\ \ y = \pm\eta = \pm\left(1 + \varepsilon\sin 2\pi(x-t)\right) \tag{16}$$

$$-\delta\left[\frac{\partial^2\psi}{\partial y\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right] + \left(\frac{2+\mu_1}{2R_e}\right)\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi_y + \frac{\mu_1}{R_e}\frac{\partial g}{\partial y} + \frac{\sin\alpha}{F}$$
$$= E_1\frac{\partial^3\eta}{\partial x^3} + E_2\frac{\partial^3\eta}{\partial x\partial t^2} + E_3\frac{\partial^2\eta}{\partial t\partial x}\ \text{ at }\ \ y = \pm\eta(x,t), \tag{17}$$

where $R_e\left(=\frac{\rho cd}{\mu}\right)$ is the Reynolds number, $\mu_1\left(=\frac{\kappa}{\mu}\right)$ denotes non-dimensional quantity for micropolar fluid, $\varepsilon\left(=\frac{a}{d}\right)$ and $\delta\left(=\frac{d}{\lambda}\right)$ are geometric parameters,

$$N = \sqrt{\frac{\mu_1}{2+\mu_1}}, \quad M = 2d\sqrt{\frac{\mu}{\gamma}}, \quad K_1 = \frac{Cd^3}{\mu\lambda^2}, K_2 = \frac{Td^4}{\mu^2\lambda^3}, \quad m_1 = \frac{md^2}{\rho\lambda^3}, \quad R_l = \frac{4\rho c d\mu J}{\gamma(2\mu+\kappa)},$$

$$E_1 = -\frac{K_2}{R_e{}^2}, \; E_2 = m_1, \; E_3 = \frac{K_1}{R_e}, \; F = \frac{\mu c}{\rho g^* d^3}.$$

The non-dimensional quantities E_1, E_2 and E_3 are related to the wall motion through the dynamic boundary condition given in equation (17). The parameters E_1 and E_2 respectively represent the rigidity and stiffness of the wall. The viscous damping force in the wall is represented by E_3. In particular, $E_3 = 0$ implies that the walls move up and down with no damping force on them and hence indicates the case of elastic walls (i.e. $E_3 = 0$). Note that R_l is modified Reynolds number, which involves the square of a length of typical microstructure J, and it is reasonable to assume that R_l is much less than unity.

The micropolar parameter μ_1 denotes the ratio of the viscosity coefficient for the micropolar fluids and classical viscosity coefficient, which characterizes the coupling of (14) and (15). Another micropolar parameter M can be thought of as a fluid property depending upon the size of microstructure. It can be seen that when κ and γ are zero, that is, when μ_1 becomes zero and M tends to infinity, (14) and (15) reduce to the classical Navier-Stokes equations.

3 Method of Solution

We seek perturbation solution in terms of wall slope parameter δ ($\delta << 1$) as follows:

$$S = S_0 + \delta S_1 + \delta^2 S_2 + \cdots \tag{18}$$

where S represents any flow variable.

Substituting (18) in (14) to (17) and collecting the coefficients of various powers of δ, we get the following sets of equations:

Zeroth Order:

$$\left(\frac{2+\mu_1}{2}\right)\frac{\partial^4\psi_0}{\partial y^4} + \mu_1\frac{\partial^2 g_0}{\partial y^2} = 0, \tag{19}$$

$$2(1-N^2)\frac{\partial^2 g_0}{\partial y^2} - N^2M^2\left(\frac{\partial^2\psi_0}{\partial y^2} + 2g_0\right) = 0, \tag{20}$$

Boundary Conditions:

$$\psi_{0y} = 0,\ g_0 = 0 \text{ at } y = \pm\eta(x,t) \tag{21}$$

$$\left(\frac{2+\mu_1}{2R_e}\right)\frac{\partial^3\psi_0}{\partial y^3} + \frac{\mu_1}{R_e}\frac{\partial g_0}{\partial y} + \frac{\sin\alpha}{F} = E_1\frac{\partial^3\eta}{\partial x^3} + E_2\frac{\partial^3\eta}{\partial t^2\partial x} + E_3\frac{\partial^2\eta}{\partial t\partial x}. \tag{22}$$

First order:

$$\left(\frac{2+\mu_1}{2}\right)\frac{\partial^4\psi_1}{\partial y^4} + \mu_1\frac{\partial^2 g_1}{\partial y^2} = R_e\left[\frac{\partial^2}{\partial y^2}\frac{\partial\psi_0}{\partial t} + \frac{\partial\psi_0}{\partial y}\frac{\partial^2}{\partial y^2}\frac{\partial\psi_0}{\partial x} - \frac{\partial\psi_0}{\partial x}\frac{\partial^2}{\partial y^2}\frac{\partial\psi_0}{\partial y}\right], \tag{23}$$

$$2(1-N^2)\frac{\partial^2 g_1}{\partial y^2} - N^2M^2\left(\frac{\partial^2\psi_1}{\partial y^2} + 2g_1\right) = R_l\left[\frac{\partial g_0}{\partial t} + \frac{\partial\psi_0}{\partial y}\frac{\partial g_0}{\partial x} - \frac{\partial\psi_0}{\partial x}\frac{\partial g_0}{\partial y}\right], \tag{24}$$

Boundary Conditions:

$$\frac{\partial\psi_1}{\partial y} = 0,\quad g_1 = 0 \quad \text{at} \quad y = \pm\eta(x,t) \tag{25}$$

$$-\left(\frac{\partial^2\psi_0}{\partial y\partial t} + \frac{\partial\psi_0}{\partial y}\frac{\partial^2\psi_0}{\partial x\partial y} - \frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_0}{\partial y^2}\right) + \left(\frac{2+\mu_1}{2R_e}\right)\frac{\partial^3\psi_1}{\partial y^3} + \frac{\mu_1}{R_e}\frac{\partial^2 g_1}{\partial y} = 0. \tag{26}$$

Solving equations (19), (20), (23) and (24) under the boundary conditions (21), (22), (25) and (26), we finally get

$$\psi_0 = \frac{1}{1-N^2}\left(\frac{A_1 y^3}{6}\right) + A_2 y - \frac{N^2}{1-N^2}\left(\frac{A_1\sinh NMy}{N^2M^2\sinh NM\eta}\right), \tag{27}$$

$$g_0 = A_1\left(\frac{\eta\sinh NMy - y\sinh NM\eta}{2(1-N^2)\sinh NM\eta}\right), \tag{28}$$

$$\psi_1 = -2N^2 I + R_m\left\{\frac{1}{70}A\frac{\partial A}{\partial x}y^7 + K_{11}\frac{y^5}{20} + K_{12}\left(\frac{\sinh NMy}{N^2M^2}\right) + \right.$$

$$K_{13}\left(\frac{y^2\sinh NMy}{N^2M^2} - \frac{4y\cosh NMy}{N^3M^3} + \frac{6\sinh NMy}{N^4M^4}\right) - K_{14}\left(\frac{y\cosh NMy}{N^2M^2} - \frac{2\sinh NMy}{N^3M^3}\right)$$

$$-NML_1\frac{\partial A}{\partial x}\left(\frac{y^3\cosh NMy}{N^2M^2}-\frac{6y^2\sinh NMy}{N^3M^3}+\frac{18y\cosh NMy}{N^4M^4}-\frac{24\sinh NMy}{N^5M^5}\right)+B_3\frac{y^3}{6}+B_4y\,, \tag{29}$$

$$g_1=P_1\left(y-\frac{\eta\sinh NMy}{\sinh NM\eta}\right)+P_2\left(y^3-\frac{\eta^3\sinh NMy}{\sinh NM\eta}\right)+P_3\left(y^5-\frac{\eta^5\sinh NMy}{\sinh NM\eta}\right)+$$

$$\frac{b_{11}}{2N^2M^2}(y\cosh NMy-\coth NM\eta\sinh NMy)+\frac{b_{12}}{4NM}\left\{\frac{2}{3}y^3\cosh NMy-\frac{y^2\sinh NMy}{NM}+\frac{y\cosh NMy}{N^2M^2}-\right.$$
$$\left.\left(\frac{2}{3}\eta^3\cosh NM\eta-\frac{\eta^2\sinh NM\eta}{NM}+\frac{\eta\cosh NM\eta}{N^2M^2}\right)\frac{\sinh NMy}{\sinh NM\eta}\right\}+\frac{b_{13}}{4NM}\left\{y^2\sinh NMy-\frac{y\cosh NMy}{NM}-\right.$$
$$\left.(\eta^2\sinh NM\eta-\frac{\eta\cosh NM\eta}{NM})\frac{\sinh NMy}{\sinh NM\eta}\right\}+\frac{b_{14}}{4NM}\left\{\frac{1}{2}y^4\sinh NMy-\frac{y^3\cosh NMy}{NM}+\frac{3y^2\sinh NMy}{2N^2M^2}-\right.$$
$$\left.\frac{3y\cosh NMy}{2N^3M^3}-\left(\frac{\eta^4\sinh NM\eta}{2}-\frac{\eta^3\cosh NM\eta}{NM}+\frac{3\eta^2\sinh NM\eta}{2N^2M^2}-\frac{3\eta\cosh NM\eta}{2N^3M^3}\right)\frac{\sinh NMy}{\sinh NM\eta}\right\}+$$
$$\frac{R_n}{6NM}K_{15}\left[\sinh 2NMy-2\sinh NMy\cosh NM\eta\right], \tag{30}$$

where

$$I=P_1\left(\frac{y^3}{6}-\frac{\eta\sinh NMy}{N^2M^2\sinh NM\eta}\right)+P_2\left(\frac{y^5}{20}-\frac{\eta^3\sinh NMy}{N^2M^2\sinh NM\eta}\right)+P_3\left(\frac{y^7}{42}-\frac{\eta^5\sinh NMy}{N^2M^2\sinh NM\eta}\right)+$$

$$\frac{b_{11}}{2N^2M^2}\left(\frac{y\cosh NMy}{N^2M^2}-\frac{2\sinh NMy}{N^3M^3}-\frac{\coth NM\eta\sinh NMy}{N^2M^2}\right)+\frac{b_{12}}{4NM}\left\{\frac{2y^3\cosh NMy}{3N^2M^2}-\right.$$
$$\frac{5y^2\sinh NMy}{N^3M^3}+\frac{17y\cosh NMy}{N^4M^4}-\frac{24\sinh NMy}{N^5M^5}-\frac{2}{3}\left(\eta^3\cosh NM\eta-\frac{\eta^2\sinh NM\eta}{NM}+\frac{\eta\cosh NM\eta}{N^2M^2}\right)\times$$
$$\left.\frac{\sinh NMy}{N^2M^2\sinh NM\eta}\right\}+\frac{b_{13}}{4NM}\left\{\frac{y^2\sinh NMy}{N^2M^2}-\frac{5y\cosh NMY}{N^3M^3}+\frac{8\sinh NMy}{N^4M^4}-\left(\eta^2\sinh NM\eta-\frac{\eta\cosh NM\eta}{NM}\right)\times\right.$$
$$\left.\frac{\sinh NMy}{N^2M^2\sinh NM\eta}\right\}+\frac{b_{14}}{4NM}\left\{\frac{y^4\sinh NMy}{2N^2M^2}-\frac{5y^3\cosh NMy}{N^3M^3}+\frac{27y^2\sinh NMy}{N^4M^4}-\frac{153y\cosh NMy}{2N^5M^5}+\frac{99\sinh NMy}{N^6M^6}\right.$$
$$\left.-\left(\frac{\eta^4\sinh NM\eta}{2}-\frac{\eta^3\cosh NM\eta}{NM}+\frac{3\eta^2\sinh NM\eta}{2N^2M^2}-\frac{3\eta\cosh NM\eta}{2N^3M^3}\right)\frac{\sinh NMy}{N^2M^2\sinh NM\eta}\right\}+$$
$$\frac{R_n}{6N^2M^2}K_{15}\left(\frac{\sinh 2NMy}{4NM}-\frac{2\cosh NM\eta\sinh NMy}{NM}\right),$$

$$G = P_1\left(\frac{y^2}{2} - \frac{\eta\cosh NMy}{NM\sinh NM\eta}\right) + P_2\left(\frac{y^4}{4} - \frac{\eta^3\cosh NMy}{NM\sinh NM\eta}\right) + P_3\left(\frac{y^6}{6} - \frac{\eta^5\cosh NMy}{NM\sinh NM\eta}\right) +$$

$$\frac{b_{11}}{2N^2M^2}\left(\frac{y\sinh NMy}{NM} - \frac{\cosh NMy}{N^2M^2} - \frac{\coth NM\eta\cosh NMy}{NM}\right) + \frac{b_{12}}{4NM}\left\{\frac{2y^3\sinh NMy}{3NM} -\right.$$

$$\frac{3y^2\cosh NMy}{N^2M^2} + \frac{7y\sinh NMy}{N^3M^3} - \frac{7\cosh NMy}{N^4M^4} - \frac{2}{3}\left(\eta^3\cosh NM\eta - \frac{\eta^2\sinh NM\eta}{NM} + \frac{\eta\cosh NM\eta}{N^2M^2}\right)\times$$

$$\left.\frac{\cosh NMy}{NM\sinh NM\eta}\right\} + \frac{b_{13}}{4NM}\left\{\frac{y^2\cosh NMy}{NM} - \frac{3y\sinh NMY}{N^2M^2} + \frac{3\cosh NMy}{N^3M^3} - \left(\eta^2\sinh NM\eta - \frac{\eta\cosh NM\eta}{NM}\right)\times\right.$$

$$\left.\frac{\cosh NMy}{NM\sinh NM\eta}\right\} + \frac{b_{14}}{4NM}\left\{\frac{y^4\cosh NMy}{2NM} - \frac{3y^3\sinh NMy}{N^2M^2} + \frac{12y^2\cosh NMy}{N^3M^3} - \frac{45y\sinh NMy}{2N^4M^4} + \frac{45\cosh NMy}{2N^5M^5}\right.$$

$$\left.-\left(\frac{\eta^4\sinh NM\eta}{2} - \frac{\eta^3\cosh NM\eta}{NM} + \frac{3\eta^2\sinh NM\eta}{2N^2M^2} - \frac{3\eta\cosh NM\eta}{2N^3M^3}\right)\frac{\cosh NMy}{NM\sinh NM\eta}\right\} +$$

$$\frac{R_n}{6N^2M^2}K_{15}\left(\frac{\cosh 2NMy}{2} - 2\cosh NM\eta\cosh NMy\right),$$

$$A_1 = -\frac{2R_e\varepsilon}{2+\mu_1}\left[(2\pi)^3\cos 2\pi(x-t)(E_1+E_2) - (2\pi)^2E_3\sin 2\pi(x-t)\right] + \left(\frac{2R_e\varepsilon}{2+\mu_1}\right)\frac{\cos\alpha}{F},$$

$$A_2 = \left(\frac{-A_1}{1-N^2}\right)\left(\frac{\eta^2}{2} - \frac{N\eta}{M}\coth NM\eta\right), A = \frac{A_1}{6(1-N^2)}, L_1 = -\frac{N^2A_1\eta}{(1-N^2)N^2M^2\sinh NM\eta},$$

$$B_1 = \frac{A_1\eta}{2(1-N^2)\sinh NM\eta}, B_2 = -\frac{A_1}{2(1-N^2)}, R_m = \frac{2R_e}{2+\mu_1}, R_n = \frac{R_l}{2(1-N^2)},$$

$$P_1 = \frac{1}{N^2M^2}\left[-\frac{6R_m}{1-N^2}\left(\frac{6A}{N^2M^2}\frac{\partial A}{\partial x} + \frac{1}{2}K_{16} - \frac{R_n}{N^2M^2}K_{17}\right) - R_nK_{17}\right],$$

$$P_2 = -\frac{6R_m}{1-N^2}\left(\frac{6A}{N^2M^2}\frac{\partial A}{\partial x} + \frac{1}{2}K_{11} - \frac{R_n}{N^2M^2}K_{17}\right), \quad P_3 = -\frac{3R_mA}{10(1-N^2)}\frac{\partial A}{\partial x},$$

$$b_{11} = \frac{N^2M^2R_m}{2(1-N^2)}K_{12} + R_nK_{18}, \qquad b_{12} = \frac{N^2M^2R_m}{2(1-N^2)}K_{13} + 3R_nA\frac{\partial B_1}{\partial x},$$

$$b_{13} = -\frac{N^2M^2R_m}{2(1-N^2)}K_{14} + R_nNMK_{19}, \quad b_{14} = -\left(\frac{N^3M^3R_m}{2(1-N^2)}L_1 + R_nB_1NM\right)\frac{\partial A}{\partial x},$$

$$K_{10}=\left(\frac{\partial L_1}{\partial t}+A_2\frac{\partial L_1}{\partial x}+L_1\frac{\partial A_2}{\partial x}\right),K_{11}=\left(\frac{\partial A}{\partial t}+A_2\frac{\partial A}{\partial x}-A\frac{\partial A_2}{\partial x}\right),$$

$$K_{12}=\left(\frac{\partial L_1}{\partial t}+A_2\frac{\partial L_1}{\partial x}+\frac{12A}{N^2M^2}\frac{\partial L_1}{\partial x}+\frac{12L_1}{N^2M^2}\frac{\partial A}{\partial x}+2L_1\frac{\partial A_2}{\partial x}\right),$$

$$K_{13}=\left(3A\frac{\partial L_1}{\partial x}+6L_1\frac{\partial A}{\partial x}\right),\ K_{14}=\left(\frac{6A}{NM}\frac{\partial L_1}{\partial x}+\frac{18L_1}{NM}\frac{\partial A}{\partial x}+NML_1\frac{\partial A_2}{\partial x}\right),$$

$$K_{15}=\left(L_1\frac{\partial B_1}{\partial x}-B_1\frac{\partial L_1}{\partial x}\right),\ K_{16}=\left(\frac{\partial A}{\partial t}+A_2\frac{\partial A}{\partial x}-A\frac{\partial A_2}{\partial x}\right),K_{17}=\left(3A\frac{\partial B_1}{\partial x}-B_2\frac{\partial A}{\partial x}\right),$$

$$K_{18}=\left(\frac{\partial B_2}{\partial t}+A_2\frac{\partial B_2}{\partial x}-B_2\frac{\partial L_1}{\partial x}\right),K_{19}=\left(L_1\frac{\partial B_2}{\partial x}-B_1\frac{\partial A_2}{\partial x}\right).\ K_{20}=\left(A\frac{\partial L_1}{\partial x}+L_1\frac{\partial A}{\partial x}\right),$$

$$F=\left(\frac{\mu_1}{R_e}-2N^2\right)\left(\frac{1-\eta NM\coth NM\eta}{2(1-N^2)}\right)-\left(\frac{2+\mu_1}{R_e}\right).$$

Using (10), (27) and (29), we get the expressions for velocity as

$$u_0=\frac{A_1}{2(1-N^2)}\left(y^2-\eta^2\right)+\frac{A_1\eta N^2}{NM(1-N^2)}\left(\frac{\cosh NM\eta-\cosh NMy}{\sinh NM\eta}\right), \tag{31}$$

$$u_1=-2N^2G+R_m\left\{\frac{1}{10}A\frac{\partial A}{\partial x}y^6+K_{11}\frac{y^4}{4}+K_{12}\frac{\cosh NMy}{NM}+\right.$$
$$K_{13}\left(\frac{y^2\cosh NMy}{NM}-\frac{2y\sinh NMy}{N^2M^2}+\frac{2\cosh NMy}{N^3M^3}\right)-K_{14}\left(\frac{y\sinh NMy}{NM}-\frac{\cosh NMy}{N^2M^2}\right)$$
$$\left.NML_1\frac{\partial A}{\partial x}\left(\frac{y^3\sinh NMy}{NM}-\frac{3y^2\cosh NMY}{N^2M^2}+\frac{6y\sinh NMy}{N^3M^3}-\frac{6\cosh NMy}{N^4M^4}\right)\right\}+B_3\frac{y^2}{2}+B_4\ . \tag{32}$$

(The expressions for B_3 and B_4 are not given for the sake of brevity).

The average velocity $\bar{u}$, over one period of the motion, is defined by

$$\bar{u}=\int_0^1 u\,dt=\bar{u}_0+\delta\bar{u}_1+--- \tag{33}$$

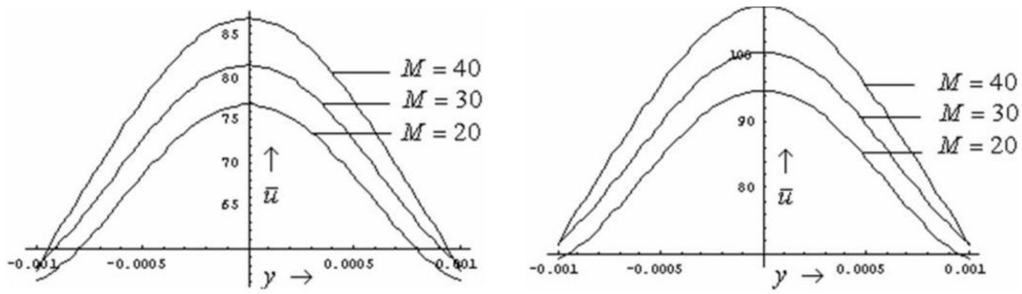
Using (31) and (32) in (33), the expression for average velocity can be obtained.

4 Results and Discussion

In order to observe the effects of various parameters M, μ_1, E_1, E_2, E_3 and α on the flow variables, the time average velocity $\bar{u}$ has been calculated for various values of these parameters. Mathematica software has been used for the numerical evaluation of the analytical results and some important results are graphically presented in Figs. 2-14.

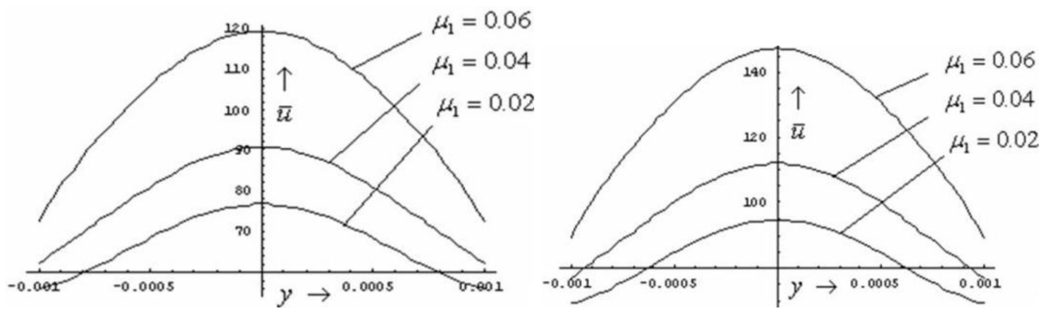
It may be noted that when the micropolar parameter M is large, the viscous effects are dominant than the couple stress effects and hence $M \to \infty$ indicates the case of pure viscous fluid effect. It can be seen from Figs. (2) and (3) that the time average velocity $\bar{u}$ decreases with micropolar parameter M *i.e.*, when the viscous effects are dominant, the average velocity decreases. Further, the time average velocity increases with inclination α.

The effect of cross viscosity parameter μ_1 on the time average velocity $\bar{u}$ is shown in Figs. (4) and (5). The time average velocity $\bar{u}$ decreases with cross viscosity parameter μ_1 i.e., as micropolar effects increase, the average velocity decreases. Further, as in the earlier case, the average velocity increases with inclination α.



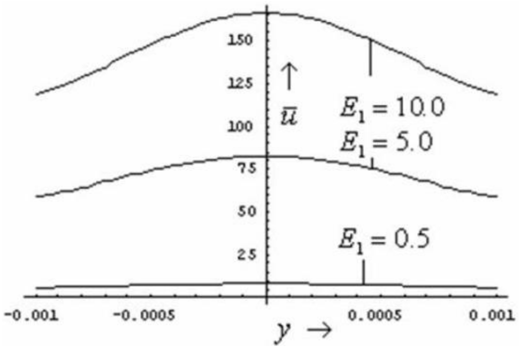
(μ_1=0.02,R_e=10.0,ε=0.2,E_1=1.0,E_2=4.0,E_3=0.06,R_l=0.1,δ=0.2, F=0.1,α=π/4) **(μ_1=0.02,R_e=10.0,ε=0.2,E_1=1.0,E_2=4.0,E_3=0.06,R_l=0.1,δ=0.2, F=0.1,α=π/3)**

Figures 2 & 3: Effect of M on time average velocity $\bar{u}$

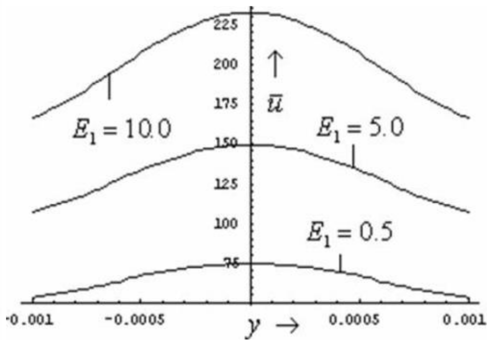


(M=20, R_e=10.0,ε=0.2,E_1=1.0,E_2=4.0,E_3=0.06,R_l=0.1,δ=0.2 , F=0.1,α=π/4) **(M=20, R_e=10.0,ε=0.2,E_1=1.0,E_2=4.0,E_3=0.06,R_l=0.1,δ=0.2, F=0.1,α=π/3)**

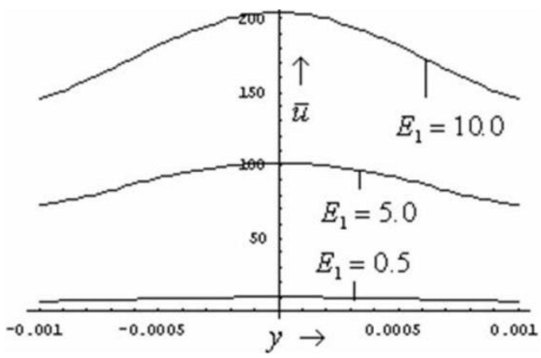
Figures 4 & 5: Effect of μ_1 on time average velocity $\bar{u}$



(*M*=30,μ_1=0.02,*R_e*=10.0,ε=0.2,*E_2*=0.0,*E_3*=0.0,*R_l*=0.1,δ=0.2, *F*=0.1,α=π/4)



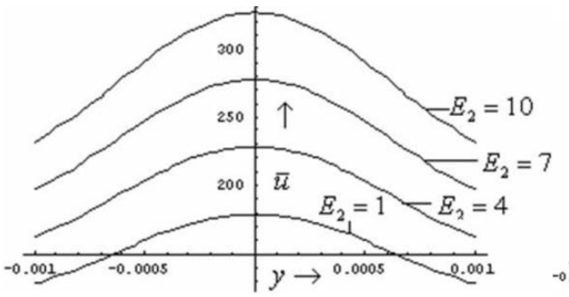
(*M*=30,μ_1=0.02,*R_e*=10.0,ε=0.2,*E_2*=4.0,*E_3*=0.0,*R_l*=0.1,δ=0.2, *F*=0.1,α=π/4)



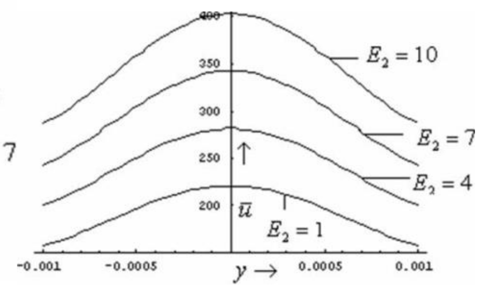
(*M*=30,μ_1=0.02,*R_e*=10.0,ε=0.2,*E_2*=0.0,*E_3*=0.0,*R_l*=0.1,δ=0.2,*F*=0.1,α=π/3)

Figures 6, 7 & 8: Effect of *E_1* on time average velocity $\bar{u}$

Figs. (6) - (8) show that the time average velocity $\bar{u}$ decreases with the rigidity of the membrane (E_1) in absence of dissipative effects (E_3=0) and with stiffness ($E_2 \neq$0) and without stiffness (E_2=0) in the wall.



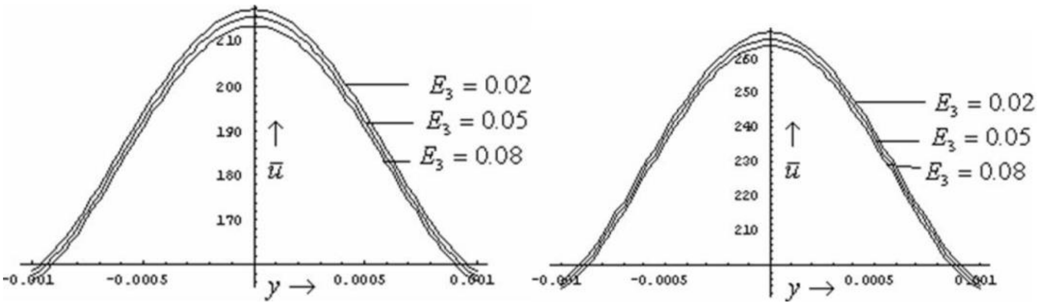
(*M*=30,μ_1=0.02,*R_e*=10.0,ε=0.2,*E_1*=10.0,*E_3*=0.06,*R_l*=0.1, δ=0.2, *F*=0.1,α=π/4)



(*M*=30,μ_1=0.02,*R_e*=10.0,ε=0.2,*E_1*=10.0,*E_3*=0.06,*R_l*=0.1, δ=0.2,*F*=0.1,α=π/3)

Figures 9 & 10: Effect of *E_2* on time average velocity $\bar{u}$

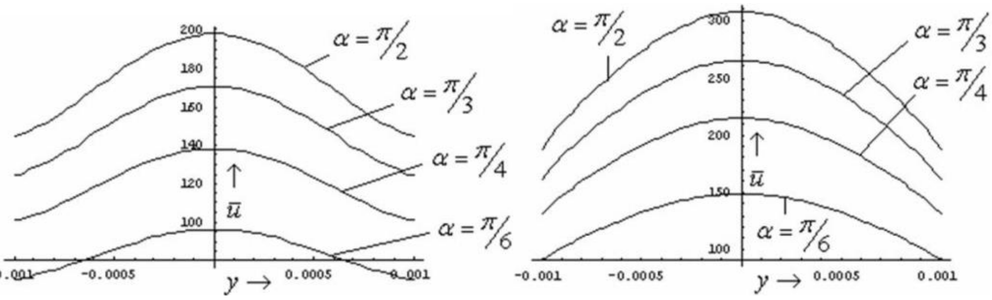
It can be observed that the time average velocity $\bar{u}$ decreases with the stiffness in the wall (E_2) (Figs. 9 and 10) and viscous damping force (E_3) (Figs. 11 and 12).



(M=20,μ_1=0.02,R_e=10.0,ε=0.2,E_1=10.0,E_2=4.0,R_l=0.1, δ=0.2, F=0.1,α=π/4)   (M=20,μ_1=0.02,R_e=10.0,ε=0.2,E_1=10.0,E_2=4.0,R_l=0.1, δ=0.2, F=0.1,α=π/3)

Figures 11 & 12: Effect of E_3 on time average velocity $\bar{u}$

The time average velocity $\bar{u}$ increases with the inclination α (Figs. 13 and 14).



(M=20,μ_1=0.02,R_e=10.0,ε=0.2,E_1=5.0,E_2=4.0, E_3=0.06, R_l=0.1, δ=0.2,F=0.1)   (M=20,μ_1=0.06,R_e=10.0,ε=0.2,E_1=5.0,E_2=4.0, E_3=0.06, R_l=0.1,δ=0.2,F=0.1)

Figures 13 & 14: Effect of α on time average velocity $\bar{u}$

5 Conclusion

Peristaltic transport of a micropolar fluid in an inclined channel with wall properties is investigated under the assumption of long wavelength approximation. Analytical expressions for stream function and time average velocity are obtained. The effects of various relevant parameters on time average velocity $\bar{u}$ have been studied. The following are some of the important observations.

- It is found that time average velocity increases with micropolar parameter, cross viscosity parameter and inclination of the channel.
- The time average velocity decreases with viscous damping force in the channel wall.

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References

[1] T.W. Latham, *Fluid motion in a peristaltic pump*, M.S. Thesis, M.I.T., Cambridge, MA, 1966.
[2] Y.C. Fung, C.S. Yih, *Peristaltic transport*, J. Appl. Mech. Trans. ASME, Volume 35 (1968), pp: 669-675.
[3] A.H. Shapiro, M.Y. Jaffrin, S.L. Weinberg, *Peristaltic pumping with long wavelength at low Reynolds number*, J. Fluid Mech., Volume 37 (1969), pp: 799-825.
[4] G. Radhakrishnamacharya, *Long wave length approximation to peristaltic motion of a power law fluid*, Rheol. Acta, Volume 21 (1982), 30-35.
[5] L. M. Srivastava, *Peristaltic transport of couple stress fluid*, Rheol. Acta Volume 25 (1986), pp: 638-641.
[6] V.P. Srivastava, M. Saxena, *A two-fluid model of non-Newtonian blood flow induced by Peristaltic waves*, Rheol. Acta, Volume 34 (1995), pp: 406-415.
[7] A.M. Sobh, *Interaction of Couple stresses and slip flow on Peristaltic transport in uniform and non uniform channels*, Turkish J. Eng. Env. Sci., Volume 32 (2008), pp: 117-123.
[8] A.C. Eringen, *Theory of microploar fluids*, J. Math. Mech., Volume 16 (1966), pp: 1-18.
[9] T. Ariman, M.A. Turk, N.D. Sylvester, *Microcontinuum fluid mechanics. A review*, Int. J. Engg. Sci., Volume 11 (1973), pp: 905-930.
[10] R. Girija Devi, R. Devanathan, *Peristaltic motion of micropolar fluid*, Proc. Indian Acad. Sci., Volume 81A (1975), pp: 149-163.
[11] D. Philip, Peeyush Chandra, *Peristaltic transport of simple micro fluid*, Proc. Nat. Acad. Sci. India, 65(A) (1995), pp: 63-74.
[12] D. Srinivasacharya, M. Mishra, A. Ramachandra Rao, *Peristaltic Pumping of a Micropolar Fluid in a Tube*, Acta Mech., Volume 161 (2003), pp: 165-178.
[13] T. Hayat, N. Ali, Z. Abbas, *Peristaltic motion of a micropolar fluid in a channel with different wave forms*, Physics Letters A, Volume 370 (2007), pp: 331-334.
[14] T. K. Mittra, S. N. Prasad, *On the influence of wall properties and Poiseuille flow in the peristalsis*, J. Biomechanics, Volume 6 (1973), pp: 681-693.
[15] P. Muthu, B. V. Rathishkumar, P. Chandra, *On the influence of wall properties in the peristaltic motion of miropolar fluid*, ANZIAM J, Volume 45 (2003), pp: 245-260.
[16] K. Vajravelu, S. Sreenadh, V. Ramesh Babu, *Peristaltic transport of Herschel-Bulkley fluid in an inclined tube*, Int. J Non-linear Mech., Volume 40 (2005), pp: 83-90.
[17] S.Srinivas, V. Pusharaj, *Non-linear Peristaltic transport in an inclined asymmetric channel*, Commun Nonlinear Sci Numer Simulat, Volume 13 (2008), pp: 1782-1795.
[18] S. Nadeem, Akbar Noreen Sher, *Influence of heat transfer on a peristaltic transport of Herschel-Bulkley fluid in a non-uniform inclined tube*, Commun Nonlinear Sci Numer Simult Volume 14 (2009), pp: 4100-4113.
[19] G. Rami Reddy, P. V. Satya Narayana, S. Venkataramana, *Peristaltic Transport of a Conducting Fluid in an Inclined Asymmetric Channel*, Applied Mathematical Sciences, Volume 4(35) (2010), pp: 1729 – 1741.