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Almost α - γ -Irresolute Functions

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Abstract - The present note introduces new classes of functions called almost γ -irresolute functions and almost α - γ -irresolute functions in topological spaces. We obtain several characterizations of these classes and study their properties and investigate the relationships with the known non-continuous functions.

Keywords - α -open set, semi-open set, preopen set, β -open set, γ -open set, almost α - γ -irresolute function.

1 Introduction

Crossley and Hildebrand (1972) introduced the notion of irresolute functions in topological spaces. The class of almost irresolute functions (Dube, Lee and Panwar 1983) is stronger than β -continuity (Abd El-Monsef, El-Deeb and Mahmoud 1983). Recently, the class of semi α -irresolute functions and almost α -irresolute functions were introduced by Beceren (2000b) and Beceren (2000c) respectively. The purpose of this note is to introduce and investigate the concept of almost α - γ -irresolute functions and give several characterizations and its properties. Relations between this class and other classes of functions are obtained. The class of almost α - γ -irresolute functions, which is stronger than almost α -irresolute functions (Beceren 2000c) and β -continuity (Abd El-Monsef et al. 1983), is a generalization of both almost γ -irresolute functions and semi α -irresolute functions (Beceren 2000b).

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2 Preliminaries

Throughout this note, spaces always mean topological spaces and $f : X \rightarrow Y$ denotes a function of a space X into a space Y . Let S be a subset of a space X . The closure and the interior of S are denoted by $\text{cl}(S)$ and $\text{int}(S)$, respectively.

Definition 2.1. A subset S of a space X is said to be α -open (Njåstad 1965) (resp. semi-open (Levine 1963), preopen (Mashhour, Abd El-Monsef and El-Deeb 1982), β -open (Abd El-Monsef et al. 1983), γ -open (El-Atik 1997)) if $S \subset \text{int}(\text{cl}(\text{int}(S)))$ (resp. $S \subset \text{cl}(\text{int}(S))$, $S \subset \text{int}(\text{cl}(S))$, $S \subset \text{cl}(\text{int}(\text{cl}(S)))$, $S \subset \text{cl}(\text{int}(S)) \cup \text{int}(\text{cl}(S))$).

The family of all α -open (resp. semi-open, preopen, β -open, γ -open) sets in a space X is denoted by τ^α (resp. $\text{SO}(X)$, $\text{PO}(X)$, $\beta\text{O}(X)$, $\gamma\text{O}(X)$). It is shown in 1965 (Njåstad) that τ^α is a topology for X and $\tau \subset \tau^\alpha \subset \text{SO}(X)$. Moreover, $\text{SO}(X) \cup \text{PO}(X) \subset \beta\text{O}(X)$ (Noiri and Popa 1990). The complement of an α -open (resp. β -open, γ -open) set is said to be α -closed (resp. β -closed, γ -closed). The intersection of all α -closed sets containing S is called the α -closure of S and is denoted by $\alpha\text{cl}(S)$; the union of all α -open sets contained in S is called the α -interior of S and is denoted by $\alpha\text{int}(S)$.

Definition 2.2. A function $f : (X, \tau) \rightarrow (Y, \nu)$ is said to be α -irresoluble (Maheshwari and Thakur 1980) (resp. semi α -irresolute (Beceren 2000b)) if $f^{-1}(V)$ is α -open (resp. semi-open) in X for every α -open set V of Y .

Definition 2.3. A function $f : X \rightarrow Y$ is said to be strongly α -continuous (Beceren 2000a) (resp. irresolute (Crossley and Hildebrand 1972), almost irresolute (Dube et al. 1983 or Cammaroto et al. 1989)) if $f^{-1}(V)$ is α -open (resp. semi-open, β -open) in X for every semi-open set V of Y .

Definition 2.4. A function $f : X \rightarrow Y$ is said to be β -continuous (Abd El-Monsef et al. 1983) (resp. β -irresolute (Mahmoud and Abd El-Monsef 1990)) if $f^{-1}(V)$ is β -open in X for every open (resp. β -open) set V of Y .

Definition 2.5. A function $f : (X, \tau) \rightarrow (Y, \nu)$ is said to be almost α -irresolute (Beceren 2000c) if $f^{-1}(V)$ is β -open in X for every α -open set V of Y .

Remark 2.6. (Noiri and Popa 2010) In a topological space, the following hold:

1. $\tau \subset \tau^\alpha \subset \text{SO}(X) \subset \gamma\text{O}(X) \subset \beta\text{O}(X)$.
2. $\tau \subset \tau^\alpha \subset \text{PO}(X) \subset \gamma\text{O}(X) \subset \beta\text{O}(X)$.

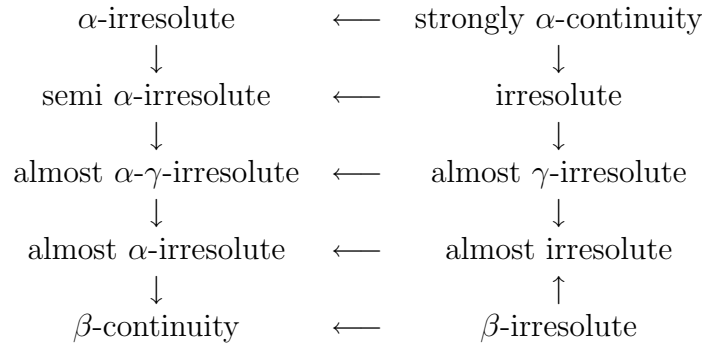
Definition 2.7. A function $f : X \rightarrow Y$ is said to be γ -continuous (resp. γ -irresolute) (El-Atik 1997) if $f^{-1}(V)$ is γ -open in X for every open (resp. γ -open) set V of Y .

3 Almost α - γ -Irresolute Functions

Definition 3.1. A function $f : (X, \tau) \rightarrow (Y, \nu)$ is said to be

1. almost α - γ -irresolute if $f^{-1}(V)$ is γ -open in X for every α -open set V of Y .
2. almost γ -irresolute if $f^{-1}(V)$ is γ -open in X for every semi-open set V of Y .

From the definitions, we obtain the following diagram:



The Examples given below show that the converses of these implications are not true in general.

Example 3.2. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is α -irresolute but it is not strongly α -continuous.

Example 3.3. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{b\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is semi α -irresolute but it is not α -irresolute.

Example 3.4. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is irresolute but it is not strongly α -continuous.

Example 3.5. Let X and τ be as in Example 3.2. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is semi α -irresolute but it is not irresolute.

Example 3.6. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, d\}, \{a, b, d\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is almost α - γ -irresolute but it is not semi α -irresolute.

Example 3.7. Let X and τ be as in Example 3.6. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is almost γ -irresolute but it is not irresolute.

Example 3.8. Let $X = Y = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \{a, b, c, d\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is almost α - γ -irresolute but it is not almost γ -irresolute.

Example 3.9. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\}$ and $\sigma = \{\emptyset, \{b, d\}, \{a, b, d\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is almost α -irresolute but it is not almost α - γ -irresolute.

Example 3.10. Let X and τ be as in Example 3.8. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is almost irresolute but it is not almost γ -irresolute.

Example 3.11. Let X and τ be as in Example 3.2. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is almost α -irresolute but it is not almost irresolute.

Example 3.12. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is β -continuous but it is not almost α -irresolute.

Example 3.13. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is almost irresolute but it is not β -irresolute.

Example 3.14. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{b\}, X\}$ and $\sigma = \{\emptyset, \{b, d\}, \{a, b, d\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is β -continuous but it is not β -irresolute.

Theorem 3.15. The following are equivalent for a function $f : (X, \tau) \rightarrow (Y, \nu)$:

1. f is almost α - γ -irresolute;
2. $f : (X, \tau) \rightarrow (Y, \nu^\alpha)$ is γ -continuous;
3. For each $x \in X$ and each α -open set V of Y containing $f(x)$, there exists a γ -open set U of X containing x such that $f(U) \subset V$;
4. $f^{-1}(V) \subset cl(int(f^{-1}(V))) \cup int(cl(f^{-1}(V)))$ for every α -open set V of Y ;
5. $f^{-1}(F)$ is γ -closed in X for every α -closed set F of Y ;
6. $int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subset f^{-1}(\alpha cl(B))$ for every subset B of Y ;
7. $f(int(cl(A)) \cap cl(int(A))) \subset \alpha cl(f(A))$ for every subset A of X .

Proof. (1) \Rightarrow (2). Let $x \in X$ and let V be any α -open set of Y containing $f(x)$. By Definition 3.1, $f^{-1}(V)$ is γ -open in X and contains x . Hence the function $f : (X, \tau) \rightarrow (Y, \nu^\alpha)$ is γ -continuous.

(2) \Rightarrow (3). Let $x \in X$ and let V be any α -open set of Y containing $f(x)$. Set $U = f^{-1}(V)$, then by (2), U is a γ -open subset of X containing x and $f(U) \subset V$.

(3) \Rightarrow (4). Let V be any α -open set of Y and $x \in f^{-1}(V)$. By (3), there exists a γ -open set U of x containing x such that $f(U) \subset V$. Thus, we have $x \in U \subset cl(int(U)) \cup int(cl(U)) \subset cl(int(f^{-1}(V))) \cup int(cl(f^{-1}(V)))$ and hence $f^{-1}(V) \subset cl(int(f^{-1}(V))) \cup int(cl(f^{-1}(V)))$.

(4) \Rightarrow (5). Let F be any α -closed subset of Y . Set $V = Y - F$, then V is α -open in Y . By (4), we obtain $f^{-1}(V) \subset cl(int(f^{-1}(V))) \cup int(cl(f^{-1}(V)))$ and hence $f^{-1}(F) = X - f^{-1}(Y - F) = X - f^{-1}(V)$ is γ -closed in X .

(5) \Rightarrow (6). Let B be any subset of Y . Since $\alpha cl(B)$ is an α -closed subset of Y , then $f^{-1}(\alpha cl(B))$ is γ -closed in X and hence $int(cl(f^{-1}(\alpha cl(B)))) \cap cl(int(f^{-1}(\alpha cl(B)))) \subset f^{-1}(\alpha cl(B))$. Therefore, we obtain $int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subset f^{-1}(\alpha cl(B))$.

(6) \Rightarrow (7). Let A be any subset of X . By (6), we have $int(cl(A)) \cap cl(int(A)) \subset int(cl(f^{-1}(f(A)))) \cap cl(int(f^{-1}(f(A)))) \subset f^{-1}(\alpha cl(f(A)))$ and hence $f(int(cl(A)) \cap cl(int(A))) \subset \alpha cl(f(A))$.

(7) \Rightarrow (1). Let V be any α -open subset of Y . Since $f^{-1}(Y - V) = X - f^{-1}(V)$ is a subset of X and by (7), we obtain $f(int(cl(f^{-1}(Y - V))) \cap cl(int(f^{-1}(Y - V)))) \subset \alpha cl(f^{-1}(Y - V)) \subset \alpha cl(Y - V) = Y - \alpha int(V) = Y - V$ and hence

$X - [\text{cl}(\text{int}(f^{-1}(V))) \cup \text{int}(\text{cl}(f^{-1}(V)))]$
 $= \text{int}(\text{cl}(X - f^{-1}(V))) \cap \text{cl}(\text{int}(X - f^{-1}(V)))$
 $= \text{int}(\text{cl}(f^{-1}(Y - V))) \cap \text{cl}(\text{int}(f^{-1}(Y - V)))$
 $\subset f^{-1}[\text{f}[(\text{int}(\text{cl}(f^{-1}(Y - V))) \cap \text{cl}(\text{int}(f^{-1}(Y - V)))] \subset f^{-1}(Y - V) = X - f^{-1}(V)$. Therefore, we have $f^{-1}(V) \subset \text{cl}(\text{int}(f^{-1}(V))) \cup \text{int}(\text{cl}(f^{-1}(V)))$ and hence $f^{-1}(V)$ is γ -open in X . Thus, f is almost α - γ -irresolute.

Theorem 3.16. *A function $f : X \rightarrow Y$ is almost α - γ -irresolute if the graph function $g : X \rightarrow X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, is almost α - γ -irresolute.*

Proof. Let $x \in X$ and V be any α -open set of Y containing $f(x)$. Then, by Lemma 3.1 of Chae et al. (1986), $X \times V$ is an α -open set of $X \times Y$ containing $g(x)$. Since g is almost α - γ -irresolute, there exists a γ -open set U of X containing x such that $g(U) \subset X \times V$ and hence $f(U) \subset V$. Thus, the function f is almost α - γ -irresolute.

Theorem 3.17. *If a function $f : X \rightarrow \prod Y_\lambda$ is almost α - γ -irresolute, then $P_\lambda \circ f : X \rightarrow Y_\lambda$ is almost α - γ -irresolute for each $\lambda \in \Lambda$, where P_λ is the projection of $\prod Y_\lambda$ onto Y_λ .*

Proof. Let V_λ be any α -open set of Y_λ . Since P_λ is continuous and open, it is α -irresolute by Theorem 3.2 of (Mashhour et al. 1983) and hence $P_\lambda^{-1}(V_\lambda)$ is α -open in $\prod Y_\lambda$. Since f is almost α - γ -irresolute, then $f^{-1}(P_\lambda^{-1}(V_\lambda)) = (P_\lambda \circ f)^{-1}(V_\lambda)$ is γ -open in X . Hence $P_\lambda \circ f$ is almost α - γ -irresolute for each $\lambda \in \Lambda$.

Theorem 3.18. *If the product function $f : \prod X_\lambda \rightarrow \prod Y_\lambda$ is almost α - γ -irresolute, then $f_\lambda : X_\lambda \rightarrow Y_\lambda$ is almost α - γ -irresolute for each $\lambda \in \Lambda$.*

Proof. Let $\lambda_0 \in \Lambda$ be an arbitrary fixed index and V_{λ_0} be any α -open set of Y_{λ_0} . Then, $\prod Y_\gamma \times V_{\lambda_0}$ is α -open in $\prod Y_\lambda$ by Lemma 3.1 of (Chae et al. 1986), where $\lambda_0 \neq \gamma \in \Lambda$. Since f is almost α - γ -irresolute, then $f^{-1}(\prod Y_\gamma \times V_{\lambda_0}) = \prod X_\gamma \times f_{\lambda_0}^{-1}(V_{\lambda_0})$ is γ -open in $\prod X_\lambda$ and hence, $f_{\lambda_0}^{-1}(V_{\lambda_0})$ is γ -open in X_{λ_0} . This implies that f_{λ_0} is almost α - γ -irresolute.

Theorem 3.19. *If $f : (X, \tau) \rightarrow (Y, \nu)$ is almost α - γ -irresolute and A is an α -open subset of X , then the restriction $f/A : A \rightarrow Y$ is almost α - γ -irresolute.*

Proof. Let V be any α -open set of Y . Since f is almost α - γ -irresolute, then $f^{-1}(V)$ is γ -open in X . By Lemma 3.9(i) of Rajesh(2007) and since A is α -open in X , then $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is γ -open in A and hence f/A is almost α - γ -irresolute.

Theorem 3.20. *If a function $f : (X, \tau) \rightarrow (Y, \nu)$ is almost α - γ -irresolute, then $f^{-1}(B)$ is γ -closed in X for any nowhere dense set B of Y .*

Proof. Let B be any nowhere dense subset of Y . Then $Y - B$ is α -open in Y . Since f is almost α - γ -irresolute, then $f^{-1}(Y - B) = X - f^{-1}(B)$ is γ -open in X and hence $f^{-1}(B)$ is γ -closed in X .

Theorem 3.21. *A function $f : (X, \tau) \rightarrow (Y, \nu)$ is almost α - γ -irresolute if and only if, for each $y \in Y$ and each open set V of Y such that $y \in \text{int}(\text{cl}(V))$, the inverse image of $V \cup \{y\}$ is γ -open in X .*

Proof. Necessity. Since $V \subset V \cup \{y\} \subset \text{int}(\text{cl}(V))$, then $V \cup \{y\}$ is an α -open set of Y . Since f is almost α - γ -irresolute, then $f^{-1}(V \cup \{y\})$ is γ -open in X .

Sufficiency. Let V be an α -open set of Y . Then, there exists an open set B of Y such that $B \subset V \subset \text{int}(\text{cl}(B))$. By hypothesis, $f^{-1}(B \cup \{y\})$ is γ -open in X for each $y \in V$. This shows that $f^{-1}(V) = \cup\{f^{-1}(B \cup \{y\}) : y \in V\}$ is γ -open in X and hence f is almost α - γ -irresolute.

Theorem 3.22. *Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be functions. Then the composition $g \circ f : X \rightarrow Z$ is almost α - γ -irresolute if f and g satisfy one of the following conditions :*

1. *If f is almost α - γ -irresolute and g is α -irresolute,*
2. *If f is γ -irresolute and g is almost α - γ -irresolute,*
3. *If f is almost γ -irresolute and g is semi α -irresolute.*

Proof. (1) Let W be any α -open subset of Z . Since g is α -irresolute, then $g^{-1}(W)$ is α -open in Y . Since f is almost α - γ -irresolute, then $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is γ -open in X and hence $g \circ f$ is almost α - γ -irresolute.

The proof of the conditions (2) and (3) is analogous to that of (1); it follows from the definitions.

We recall that a space X is said to be submaximal if every dense subset of X is open in X and extremally disconnected if the closure of each open subset of X is open in X .

Theorem 3.23. *Let (X, τ) be a submaximal and extremally disconnected space. Then the following are equivalent for a function $f : (X, \tau) \rightarrow (Y, \nu)$:*

1. *f is α -irresolute;*
2. *f is semi α -irresolute;*
3. *f is almost α -irresolute;*
4. *f is almost γ -irresolute;*
5. *f is almost α - γ -irresolute.*

Proof. This follows from the fact that if (X, τ) is submaximal and extremally disconnected, then $\tau = \tau^\alpha = \text{SO}(X) = \gamma\text{O}(X) = \beta\text{O}(X)$ (Keskin and Noiri(2009), Nasef et al. (1998) and Janković (1983)).

Recall that a space (X, τ) is said to be resolvable if it has two disjoint dense subsets, otherwise it is called irresolvable.

Recall that a space (X, τ) is called strongly irresolvable if every open subspace of (X, τ) is irresolvable.

Theorem 3.24. *Let (X, τ) be a strongly irresolvable space. Then the following are equivalent for a function $f : (X, \tau) \rightarrow (Y, \nu)$:*

1. *f is irresolute,*
2. *f is almost γ -irresolute.*

Proof. This follows from the fact that if (X, τ) is strongly

4 Conclusion

The field of mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to irresolute functions. Therefore, generalization of irresolute functions is one of the most important subject in topology. On the other hand, topology plays a significant role in quantum physics, high energy physics and superstring theory [13-15, 30]. Thus we speculate that studies on almost α - γ -irresolute functions which is a kind of generalized irresolute functions may have possible applications in quantum physics, high energy physics and superstring theory

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