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Decompositions of πg-Continuity via Idealization

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Abstract - In this paper, we introduce the notions of $\pi g\alpha$-$\mathcal{I}$-open sets, πgp-$\mathcal{I}$-open sets, E_r-$\mathcal{I}$-sets and E_r^*-$\mathcal{I}$-sets in ideal topological spaces and investigate some of their properties and using these notions we obtain three decompositions of πg-continuity.

Keywords - *$\pi g\alpha$-$\mathcal{I}$-continuity, πgp-$\mathcal{I}$-continuity and πg-continuity.*

1 Introduction and Preliminaries

In 1968, Zaitsev [25] introduced the concept of π-closed sets and in 1970, Levine [13] initiated the study of so called g-closed sets in topological spaces. The concept of g-continuity was introduced and studied by Balachandran et.al in 1991 [3]. Dontchev and Noiri [5] defined the notions of πg-closed sets and πg-continuity in topological spaces. In 1993, Palaniappan and Rao [18] introduced the notions of regular generalized closed (rg-closed) sets and rg-continuity in topological spaces. In 2000, Sundaram and Rajamani [22] obtained three different decompositions of rg-continuity by providing two types of weaker forms of continuity, namely C_r-continuity and C_r^*-continuity. Recently, Noiri et. al. [16] introduced the notions of αg-$\mathcal{I}$-open sets, gp-$\mathcal{I}$-open sets, gs-$\mathcal{I}$-open sets, $C(S)$-$\mathcal{I}$-sets, C^*-$\mathcal{I}$-sets and S^*-$\mathcal{I}$-sets to obtain three different decompositions of g-continuity via idealization. Recently Ravi et. al. [20] obtained three different decompositions of πg-continuity by providing two types of weaker forms of continuity, namely E_r-continuity and E_r^*-continuity. In this paper, we introduce the notions of $\pi g\alpha$-$\mathcal{I}$-open sets, πgp-$\mathcal{I}$-open sets, E_r-$\mathcal{I}$-sets and E_r^*-$\mathcal{I}$-sets to obtain the further decompositions of

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πg-continuity. Let (X, τ) be a topological space. An ideal is defined as a nonempty collection $\mathcal{I}$ of subsets of X satisfying the following two conditions:

(i) If $A \in \mathcal{I}$ and $B \subseteq A$, then $B \in \mathcal{I}$

(ii) If $A \in \mathcal{I}$ and $B \in \mathcal{I}$, then $A \cup B \in \mathcal{I}$.

For a subset $A \subseteq X$, $A^*(\mathcal{I}) = \{x \in X / U \cap A \notin \mathcal{I}$ for each neighborhood U of $x\}$ is called the local function of A with respect to $\mathcal{I}$ and τ [10]. We simply write A^* instead of $A^*(\mathcal{I})$ in case there is no chance for confusion. X^* is often a proper subset of X. For every ideal topological space $(X, \tau, \mathcal{I})$ there exists a topology $\tau^*(\mathcal{I})$, finer than τ, generated by $\beta(\mathcal{I}, \tau) = \{U \setminus I : U \in \tau$ and $I \in \mathcal{I}\}$, but in general $\beta(\mathcal{I}, \tau)$ is not always a topology [24]. Also, $\text{cl}^*(A) = A \cup A^*$ defines a Kuratowski closure operator for $\tau^*(\mathcal{I})$ [24]. Additionally, $\text{cl}^*(A) \subseteq \text{cl}(A)$ for any subset A of X [8]. Throughout this paper, X denotes the ideal topological space $(X, \tau, \mathcal{I})$ and also $\text{cl}(A)$ and $\text{int}(A)$ denote the closure of A and the interior of A in (X, τ), respectively.

Definition 1.1. A subset A of (X, τ) is said to be

1. *α-open [15] if $A \subseteq int(cl(int(A)))$,*
2. *preopen [14] if $A \subseteq int(cl(A))$,*
3. *regular open [21] if $A = int(cl(A))$,*
4. *π-open [25] if the finite union of regular open sets,*
5. *πg-open [5] iff $F \subseteq int(A)$ whenever $F \subseteq A$ and F is π-closed in (X, τ),*
6. *πgp-open [19] iff $F \subseteq pint(A)$ whenever $F \subseteq A$ and F is π-closed in (X, τ),*
7. *$\pi g\alpha$-open [2] iff $F \subseteq \alpha int(A)$ whenever $F \subseteq A$ and F is π-closed in (X, τ),*
8. *a t-set [23] if $int(A) = int(cl(A))$,*
9. *an α^*-set [7] if $int(A) = int(cl(int(A)))$,*
10. *a E_r-set [20] if $A = U \cap V$, where U is πg-open and V is a t-set in (X, τ),*
11. *a E_r^*-set [20] if $A = U \cap V$, where U is πg-open and V is an α^*-set in (X, τ).*

The complements of the above mentioned open sets are called their respective closed sets. The preinterior $pint(A)$ (resp. α-interior, $\alpha int(A)$) of A is the union of all preopen sets (resp. α-open sets) contained in A. The α-closure $\alpha cl(A)$ of A is the intersection of all α-closed sets containing A.

Lemma 1.2. *[1] If A is a subset of X, then*

1. *$pint(A) = A \cap int(cl(A))$,*
2. *$\alpha int(A) = A \cap int(cl(int(A)))$ and $\alpha cl(A) = A \cup cl(int(cl(A)))$.*

Definition 1.3. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be

1. *pre-$\mathcal{I}$-open [4] if $A \subseteq int(cl^*(A))$,*

2. *α-$\mathcal{I}$-open [6] if $A \subseteq int(cl^*(int(A)))$,*

3. *a t-$\mathcal{I}$-set [6] if $int(cl^*(A)) = int(A)$,*

4. *an α^*-$\mathcal{I}$-set [6] if $int(cl^*(int(A))) = int(A)$.*

Also, we have α-$\mathcal{I}$-$int(A) = A \cap int(cl^*(int(A)))$ [16] and p-$\mathcal{I}$-$int(A) = A \cap int(cl^*(A))$ [16], where α-$\mathcal{I}$-int(A) denotes the α-$\mathcal{I}$ interior of A in (X, τ, $\mathcal{I}$) which is the union of all α-$\mathcal{I}$-open sets of (X, τ, $\mathcal{I}$) contained in A. p-$\mathcal{I}$-int(A) has similar meaning.

Remark 1.4. *The following hold in a topological space.*

1. *Every πg-open set is πgp-open but not conversely.[19]*

2. *Every πg-open set is $\pi g\alpha$-open but not conversely.[2]*

2 $\pi g\alpha$-$\mathcal{I}$-Open Sets and πgp-$\mathcal{I}$-Open Sets

Definition 2.1. A subset A of an ideal topological space *(X, τ, $\mathcal{I}$)* is called

1. *$\pi g\alpha$-$\mathcal{I}$-open if $F \subseteq \alpha$-$\mathcal{I}$-int(A) whenever $F \subseteq A$ and F is π-closed in X.*

2. *πgp-$\mathcal{I}$-open if $F \subseteq$ p-$\mathcal{I}$-int(A) whenever $F \subseteq A$ and F is π-closed in X.*

Proposition 2.2. *For a subset of an ideal topological space, the following hold:*

1. *Every $\pi g\alpha$-$\mathcal{I}$-open set is $\pi g\alpha$-open.*

2. *Every πgp-$\mathcal{I}$-open set is πgp-open.*

3. *Every $\pi g\alpha$-open set is πgp-open.*

Proof. (1) Let A be an $\pi g\alpha$-$\mathcal{I}$-open set. Let $F \subseteq A$ and F is π-closed in X. Then, $F \subseteq \alpha$-$\mathcal{I}$-$\text{int}(A) = A \cap (\text{int}(\text{cl}^*(\text{int}(A)))) \subseteq A \cap \text{int}(\text{cl}(\text{int}(A))) = \alpha\text{int}(A)$. This shows that A is $\pi g\alpha$-open.

(2) Let A be πgp-$\mathcal{I}$-open set. Let $F \subseteq A$ and F is π-closed in X. Then, $F \subseteq p$-$\mathcal{I}$-$\text{int}(A) = A \cap \text{int}(\text{cl}^*(A)) \subseteq A \cap \text{int}(\text{cl}(A)) = p\text{int}(A)$. This shows that A is πgp-open.

(3) It follows from the definitions.

Remark 2.3. *The converses of Proposition 2.2 are not true, in general.*

Example 2.4. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then $\{a, b, d\}$ is $\pi g\alpha$-open but not an $\pi g\alpha$-$\mathcal{I}$-open set.*

Example 2.5. *In Example 2.4, $\{a, b, d\}$ is πgp-open but not a πgp-$\mathcal{I}$-open set.*

Example 2.6. *Let X= $\{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$. Then $\{b, c, e\}$ is πgp-open set but not an $\pi g\alpha$-open.*

Proposition 2.7. *For a subset of an ideal topological space, the following hold:*

1. *Every $\pi g\alpha$-$\mathcal{I}$-open set is πgp-$\mathcal{I}$-open.*

2. *Every πg-open set is πgp-$\mathcal{I}$-open.*

3. *Every πg-open set is $\pi g\alpha$-$\mathcal{I}$-open.*

Proof. 1. Let A be $\pi g\alpha$-$\mathcal{I}$-open. Then, for any π-closed set F with F $\subseteq A$, we have F $\subseteq \alpha$-$\mathcal{I}$-int$(A) = A \cap \text{int}(\text{cl}^*(\text{int}(A))) \subseteq A \cap \text{int}(\text{cl}^*(A)) = p$-$\mathcal{I}$-int$(A)$ which implies that A is πgp-$\mathcal{I}$-open.

2. Let A be an πg-open set. Then, for any π-closed set F with $F \subseteq A$, we have $F \subseteq \text{int}(A) \subseteq \text{int}((\text{int}(A))^*) \cup \text{int}(A) = \text{int}((\text{int}(A))^*) \cup \text{int}(\text{int}(A)) \subseteq \text{int}((\text{int}(A))^* \cup \text{int}(A)) = \text{int}(\text{cl}^*(\text{int}(A)))$. That is, $F \subseteq A \cap \text{int}(\text{cl}^*(\text{int}(A))) = \alpha$-$\mathcal{I}$-int$(A) =$ A$\cap$int(cl*(int(A))) $\subseteq$ A$\cap$int(cl*(A)) = p-$\mathcal{I}$-int(A) which implies that A is πgp-$\mathcal{I}$-open.

3. Let A be an πg-open set. Then, for any π-closed set F with $F \subseteq A$, we have $F \subseteq \text{int}(A) \subseteq \text{int}((\text{int}(A))^*) \cup \text{int}(A) = \text{int}((\text{int}(A))^*) \cup \text{int}(\text{int}(A)) \subseteq \text{int}((\text{int}(A))^* \cup \text{int}(A)) = \text{int}(\text{cl}^*(\text{int}(A)))$. That is, $F \subseteq A \cap \text{int}(\text{cl}^*(\text{int}(A))) = \alpha$-$\mathcal{I}$-int$(A)$ which implies that A is $\pi g\alpha$-$\mathcal{I}$-open.

Remark 2.8. *The converses of Proposition 2.7 are not true, in general.*

Example 2.9. *Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{b\}, \{e\}, \{b, e\}, \{c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{e\}, \{b, e\}\}$. Then $\{a, d, e\}$ is πgp-$\mathcal{I}$-open but not an $\pi g\alpha$-$\mathcal{I}$-open set.*

Example 2.10. *In Example 2.9, $\{a, d, e\}$ is πgp-$\mathcal{I}$-open but not a πg-open set.*

Example 2.11. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{a, b, d\}$ is $\pi g\alpha$-$\mathcal{I}$-open but not a πg-open set.*

Remark 2.12. *By Remark 1.4, Propositions 2.2 and 2.7, we have the following diagram. In this diagram, there is no implication which is reversible as shown by examples above.*

$$\begin{array}{ccccc} & & \pi g\alpha\text{-}\mathcal{I}\text{-open} & \longrightarrow & \pi g\alpha\text{-open} \\ & \nearrow & & & \\ \pi g\text{-open} & & \downarrow & & \downarrow \\ & \searrow & & & \\ & & \pi gp\text{-}\mathcal{I}\text{-open} & \longrightarrow & \pi gp\text{-open} \end{array}$$

3 E_r-$\mathcal{I}$-Sets and E_r^*-$\mathcal{I}$-Sets

Definition 3.1. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called*

1. *a E_r-$\mathcal{I}$-set if $A = U \cap V$, where U is πg-open and V is a t-$\mathcal{I}$-set,*

2. *a E_r^*-$\mathcal{I}$-set if $A = U \cap V$, where U is πg-open and V is an α^*-$\mathcal{I}$-set.*

We have the following proposition:

Proposition 3.2. *For a subset of an ideal topological space, the following hold:*

1. *Every t-$\mathcal{I}$-set is an α^*-$\mathcal{I}$-set [6] and a E_r-$\mathcal{I}$-set.*

2. *Every α^*-$\mathcal{I}$-set is a E_r^*-$\mathcal{I}$-set.*
3. *Every E_r-$\mathcal{I}$-set is a E_r^*-$\mathcal{I}$-set.*
4. *Every πg-open set is a E_r-set.*
5. *Every E_r-set is a E_r-$\mathcal{I}$-set and a E_r^*-set.*
6. *Every E_r^*-set is a E_r^*-$\mathcal{I}$-set.*

From Proposition 3.2, We have the following diagram.

$$\begin{array}{ccccccc} \pi g\text{-open set} & \longrightarrow & E_r\text{-set} & \longrightarrow & E_r\text{-}\mathcal{I}\text{-set} & \longleftarrow & t\text{-}\mathcal{I}\text{-set} \\ & & \downarrow & & \downarrow & & \downarrow \\ & & E_r^*\text{-set} & \longrightarrow & E_r^*\text{-}\mathcal{I}\text{-set} & \longleftarrow & \alpha^*\text{-}\mathcal{I}\text{-set} \end{array}$$

Remark 3.3. *The converses of implications in Diagram II need not be true as the following examples show.*

Example 3.4. *In Example 2.4, $\{a, b, d\}$ is E_r-$\mathcal{I}$-set but not a E_r-set.*

Example 3.5. *In Example 2.4, $\{a, b, c\}$ is E_r-$\mathcal{I}$-set but not a t-$\mathcal{I}$-set.*

Example 3.6. *Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, X, \{b\}, \{e\}, \{b, e\}, \{c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$. Then $\{a\}$ is E_r-set but not a πg-open set.*

Example 3.7. *In Example 3.6, $\{a, b, d, e\}$ is E_r^*-set but not a E_r-set.*

Example 3.8. *In Example 2.10, $\{a, b, d, e\}$ is E_r^*-$\mathcal{I}$-set but not a E_r-$\mathcal{I}$-set.*

Example 3.9. *In Example 2.4, $\{a, b, d\}$ is E_r^*-$\mathcal{I}$-set but not a E_r^*-set.*

Example 3.10. *In Example 2.4, $\{a, b, c\}$ is E_r^*-$\mathcal{I}$-set but not an α^*-$\mathcal{I}$-set.*

Example 3.11. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{b\}$ is α^*-$\mathcal{I}$-set but not a t-$\mathcal{I}$-set.*

Remark 3.12. *Examples 3.13 and 3.14 show that E_r-$\mathcal{I}$-sets and E_r^*-sets are independent of each other.*

Example 3.13. *Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{b\}, \{e\}, \{b, e\}, \{c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{e\}, \{b, e\}\}$. Then $\{a, b, d, e\}$ is E_r^*-set but not a E_r-$\mathcal{I}$-set.*

Example 3.14. *In Example 2.4, $\{a, b, d\}$ is E_r-$\mathcal{I}$-set but not a E_r^*-set.*

Proposition 3.15. *A subset A of X is πg-open if and only if it is both πgp-$\mathcal{I}$-open and a E_r-$\mathcal{I}$-set in X.*

Proof. Necessity is trivial. We prove the sufficiency. Assume that A is πgp-$\mathcal{I}$-open and a E_r-$\mathcal{I}$-set in X. Let $F \subseteq A$ and F is π-closed in X. Since A is a E_r-$\mathcal{I}$-set in X, $A = U \cap V$, where U is πg-open and V is a t-$\mathcal{I}$-set. Since A is πgp-$\mathcal{I}$-open, $F \subseteq p\text{-}\mathcal{I}\text{-int}(A) = A \cap \text{int}(\text{cl}^*(A)) = (U \cap V) \cap \text{int}(\text{cl}^*(U \cap V)) \subseteq (U \cap V) \cap \text{int}(\text{cl}^*(U) \cap \text{cl}^*(V)) = (U \cap V) \cap \text{int}(\text{cl}^*(U)) \cap \text{int}(\text{cl}^*(V))$. This implies $F \subseteq \text{int}(\text{cl}^*(V)) = \text{int}(V)$ since V is a t-$\mathcal{I}$-set. Since F is π-closed, U is πg-open and $F \subseteq U$, we have $F \subseteq \text{int}(U)$. Therefore, $F \subseteq \text{int}(U) \cap \text{int}(V) = \text{int}(U \cap V) = \text{int}(A)$. Hence A is πg-open in X.

Corollary 3.16. *A subset A of X is πg-open if and only if it is both $\pi g\alpha$-$\mathcal{I}$-open and a E_r-$\mathcal{I}$-set in X.*

Proof. This is an immediate consequence of Proposition 3.15.

Proposition 3.17. *A subset A of X is πg-open if and only if it is both $\pi g\alpha$-$\mathcal{I}$-open and a E_r^*-$\mathcal{I}$-set in X.*

Proof. Necessity is trivial. We prove the sufficiency. Assume that A is $\pi g\alpha$-$\mathcal{I}$-open and a E_r^*-$\mathcal{I}$-set in X. Let $F \subseteq A$ and F is π-closed in X. Since A is a E_r^*-$\mathcal{I}$-set in X, $A = U \cap V$, where U is πg-open and V is an α^*-$\mathcal{I}$-set. Now since F is π-closed, $F \subseteq U$ and U is πg-open, $F \subseteq$ int(U). Since A is $\pi g\alpha$-$\mathcal{I}$-open, $F \subseteq \alpha$-$\mathcal{I}$-int$(A) = A \cap \text{int}(\text{cl}^*(\text{int}(A)))$ $= (U \cap V) \cap \text{int}(\text{cl}^*(\text{int}(U \cap V))) = (U \cap V) \cap \text{int}(\text{cl}^*(\text{int}(U) \cap \text{int}(V))) \subseteq (U \cap V) \cap \text{int}(\text{cl}^*(\text{int}(U)) \cap \text{cl}^*(\text{int}(V))) = (U \cap V) \cap \text{int}(\text{cl}^*(\text{int}(U))) \cap \text{int}(\text{cl}^*(\text{int}(V))) = (U \cap V) \cap \text{int}(\text{cl}^*(\text{int}(U))) \cap \text{int}(V)$, since V is an α^*-$\mathcal{I}$-set. This implies $F \subseteq \text{int}(V)$. Therefore, $F \subseteq \text{int}(U) \cap \text{int}(V) = \text{int}(U \cap V) = \text{int}(A)$. Hence A is πg-open in X.

Remark 3.18.
1. *The concepts of πgp-$\mathcal{I}$-open sets and E_r-$\mathcal{I}$-sets are independent of each other.*
2. *The concepts of $\pi g\alpha$-$\mathcal{I}$-open sets and E_r-$\mathcal{I}$-sets are independent of each other.*
3. *The concepts of $\pi g\alpha$-$\mathcal{I}$-open sets and E_r^*-$\mathcal{I}$-sets are independent of each other.*

Example 3.19. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then*
(1) $\{b, c, d\}$ is E_r-$\mathcal{I}$-set but not a πgp-$\mathcal{I}$-open.
(2) In Example 3.13, $\{a, b, d, e\}$ is πgp-$\mathcal{I}$-open but not a E_r-$\mathcal{I}$-set.

Example 3.20. *Let $X =\{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then*
(1) $\{b, c, d\}$ is E_r-$\mathcal{I}$-set but not an $\pi g\alpha$-$\mathcal{I}$-open set.
(2) $\{a, b, d\}$ is $\pi g\alpha$-$\mathcal{I}$-open set but not a E_r-$\mathcal{I}$-set.

Example 3.21. *Let $X =\{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then*
(1) $\{b, c, d\}$ is E_r^*-$\mathcal{I}$-set but not an $\pi g\alpha$-$\mathcal{I}$-open set.
(2) $\{a, b, d\}$ is $\pi g\alpha$-$\mathcal{I}$-open set but not a E_r^*-$\mathcal{I}$-set.

4 Decompositions of πg-Continuity

Definition 4.1. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be πg-continuous [5] (resp. πgp-continuous [19], $\pi g\alpha$-continuous [20], E_r-continuous [20] and E_r^*-continuous [20]) if $f^{-1}(V)$ is πg-open (resp. πgp-open, $\pi g\alpha$-open, E_r-set and E_r^*-set) in (X, τ) for every open set V in (Y, σ).*

Definition 4.2. *A mapping $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be $\pi g\alpha$-$\mathcal{I}$-continuous (resp. πgp-$\mathcal{I}$-continuous, E_r-$\mathcal{I}$-continuous and E_r^*-$\mathcal{I}$-continuous) if for every $V \in \sigma$, $f^{-1}(V)$ is $\pi g\alpha$-$\mathcal{I}$-open (resp. πgp-$\mathcal{I}$-open, a E_r-$\mathcal{I}$-set and a E_r^*-$\mathcal{I}$-set) in $(X, \tau, \mathcal{I})$.*

From Propositions 3.15 and 3.17 and Corollary 3.16 we have the following decompositions of πg-continuity.

Theorem 4.3. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. For a mapping $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$, the following properties are equivalent:*

1. *f is πg-continuous;*
2. *f is πgp-$\mathcal{I}$-continuous and E_r-$\mathcal{I}$-continuous;*
3. *f is $\pi g\alpha$-$\mathcal{I}$-continuous and E_r-$\mathcal{I}$-continuous;*
4. *f is $\pi g\alpha$-$\mathcal{I}$-continuous and E_r^*-$\mathcal{I}$-continuous.*

Remark 4.4.
1. *The concepts of πgp-$\mathcal{I}$-continuity and E_r-$\mathcal{I}$-continuity are independent of each other.*
2. *The concepts of $\pi g\alpha$-$\mathcal{I}$-continuity and E_r-$\mathcal{I}$-continuity are independent of each other.*
3. *The concepts of $\pi g\alpha$-$\mathcal{I}$-continuity and E_r^*-$\mathcal{I}$-continuity are independent of each other.*

Example 4.5. *(1) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$, $\mathcal{I} = \{\emptyset, \{c\}\}$ and $\sigma = \{\emptyset, Y, \{b, c, d\}\}$. Let $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ be the identity function. Then f is E_r-$\mathcal{I}$-continuous but not πgp-$\mathcal{I}$-continuous.*
(2) Let $X = Y = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{b\}, \{e\}, \{b, e\}, \{c, d\}, \{b, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$, $\mathcal{I} = \{\emptyset, \{b\}, \{e\}, \{b, e\}\}$ and $\sigma = \{\emptyset, Y, \{a, b, d, e\}\}$. Let $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ be the identity function. Then f is πgp-$\mathcal{I}$-continuous but not E_r-$\mathcal{I}$-continuous.

Example 4.6. *(1) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$, $\mathcal{I} = \{\emptyset, \{c\}\}$ and $\sigma = \{\emptyset, Y, \{b, c, d\}\}$. Let $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ be the identity function. Then f is E_r-$\mathcal{I}$-continuous but not $\pi g\alpha$-$\mathcal{I}$-continuous.*
(2) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$, $\mathcal{I} = \{\emptyset, \{c\}\}$ and $\sigma = \{\emptyset, Y, \{a, b, d\}\}$. Let $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ be the identity function. Then f is $\pi g\alpha$-$\mathcal{I}$-continuous but not E_r-$\mathcal{I}$-continuous.

Example 4.7. *In Example 4.6 (1) f is E_r^*-$\mathcal{I}$-continuous but not $\pi g\alpha$-$\mathcal{I}$-continuous. In Example 4.6 (2) f is $\pi g\alpha$-$\mathcal{I}$-continuous but not E_r^*-$\mathcal{I}$-continuous.*

Corollary 4.8. *[20] Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $\mathcal{I} = \{\emptyset\}$. For a mapping $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$, the following properties are equivalent:*

1. *f is πg-continuous;*
2. *f is πgp-continuous and E_r-continuous;*
3. *f is $\pi g\alpha$-continuous and E_r-continuous;*
4. *f is $\pi g\alpha$-continuous and E_r^*-continuous.*

Proof. Since $\mathcal{I} = \{\emptyset\}$, we have $A^* = \text{cl}(A)$ and $\text{cl}^*(A) = A^* \cup A = \text{cl}(A)$ for any subset A of X [[6], Proposition 2.4(a)]. Therefore, we obtain (1) A is $\pi g\alpha$-$\mathcal{I}$-open (resp. πgp-$\mathcal{I}$-open) if and only if it is $\pi g\alpha$-open (resp. πgp-open) and (2) A is a E_r-$\mathcal{I}$-set (resp. a E_r^*-$\mathcal{I}$-set) if and only if it is a E_r-set (resp. a E_r^*-set). The proof follows from Theorem 4.3 immediately.

5 Conclusion

Topology as a field of mathematics is concerned with all questions directly or indirectly related to open/closed sets. Therefore, generalization of open/closed sets is one of the most important subjects in topology. Topology plays a significant role in quantum physics, high energy physics and superstring theory. In this paper, we introduce the notions of $\pi g\alpha$-$\mathcal{I}$-open sets, πgp-$\mathcal{I}$-open sets, E_r-$\mathcal{I}$-sets and E_r^*-$\mathcal{I}$-sets in ideal topological spaces and investigate some of their properties and using these notions we obtain three decompositions of πg-continuity. Moreover, some notions of the sets and functions in topological spaces and ideal topolgical spaces are highly developed and used extensively in many practical and engineering problems.

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