Gaziosmanpasa University
Graduate School of
Natural and Applied Sciences
Journal of New Results in Science

Connectedness on Soft Multi Topological Spaces

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Abstract. In this work, first we recall the concepts of soft multiset and soft multi topology. Then we will introduce soft multi connectedness on soft multi topological space and give basic definitions and theorems about it.

1 Introduction

There are uncertainty in most of the engineering, physics, computer sciences, economics, social sciences, and medical sciences problems. The concept of soft set was introduced by Molodtsov [1] as a mathematical tool for dealing with these uncertainties. In [1,2] , Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on.

In addition to Molodtsov's works, soft set theory has applications in other areas and the problems encountered in life [3-7]. Shabir and Naz [8] defined the soft topological space and studied the concepts of soft open set, soft multi interior point, soft neighborhood of a point, soft separation axioms, and subspace of a soft topological space. There are some other studies on the structure of soft topological spaces [9-14].

In classical set theory, there is no repetition of the set members. However, in some cases, repetition of element of set may be helpful. This set is called a multi set which is a collection of objects in which repetition of elements is significant. It has found many

applications in real life in various fields like medicine, banking, engineering, information analysis, data analysis, data mining, etc. Multi set theory was introduced by Cerf et al. [15] in 1971. Peterson [16] and Yager [17] made further contributions to it. Many conclusive results were established by these authors and further study was carried on by Jena et al. [18]. Manjunath and John [19] have done some preliminary work on multi set relations. Following this study , multi set relation and multi set function was introduced by Girish and John [20]. In addition, these authors [21] using the multi sets relations gave multi set topology and some of the definitions of topological structures of this topology.

The concept of soft multisets which is combining soft sets and multisets can be used to solve some real life problems. Also this concept can be used in many areas, such as data storage, computer science, information science, medicine, engineering, etc.

The concept of soft multisets was introduced in [22]. Moreover, in [22] soft multi topology and its some properties was given.

In this work we will introduce soft multi connected on soft multi topological space and will give basic definitions and theorems of soft multi connected.

2   Preliminaries

In this section, we recall concept of soft multiset and soft multi topology which was given in [22].

2.1   Soft Multiset

Definition 2.1. *Let U be an universal multiset , E be set of parameters and $A \subseteq E$. Then an order pair (F, A) is called a soft multiset where F is a mapping given by $F : A \to P^*(U)$. For $\forall e \in A$, $F(e)$ multiset represent by count function $C_{F(e)} : U^* \to N$ where N represents the set of non negative integers and U^* represents the support set of U.*

Let $U = \{2/x, 3/y, 1/z\}$ be a multiset. Then the support set of U is $U^* = \{x, y, z\}$.

Example 2.2. *Let multiset and the parameter set be $U = \{1/x, 5/y, 3/z, 4/w\}$ and $E = \{p, q, r\}$. Define a mapping $F : E \to P^*(U)$ as follows:*

$F(p) = \{1/x, 2/y, 3/z\}, F(q) = \{4/w\}$ and $F(r) = \{3/y, 1/z, 2/w\}$.
Then (F, A) is a soft multiset where for $\forall e \in A$, $F(e)$ multiset represent by count function $C_{F(e)} : U^* \to N$, which are defined as follows:

$$\begin{array}{llll} C_{F(p)}(x) = 1, & C_{F(p)}(y) = 2, & C_{F(p)}(z) = 3, & C_{F(p)}(w) = 0, \\ C_{F(q)}(x) = 0, & C_{F(q)}(y) = 0, & C_{F(q)}(z) = 0, & C_{F(q)}(w) = 4, \\ C_{F(r)}(x) = 0, & C_{F(r)}(y) = 3, & C_{F(r)}(z) = 1, & C_{F(r)}(w) = 2. \end{array}$$

Then $(F, A) = \{F(p), F(q), F(r)\} = \{\{1/x, 2/y, 3/z\}, \{4/w\}, \{3/y, 1/z, 2/w\}\}$.

Definition 2.3. *For two soft multisets (F, A) and (G, B) over U, we say that (F, A) is a soft submultiset of (G, B) if*

i. $A \subseteq B$

ii. $C_{F(e)}(x) \leq C_{G(e)}(x), \forall x \in U^*, \forall e \in A \cap B$

We write $(F, A) \tilde{\subset} (G, B)$.

Definition 2.4. *Two soft multisets (F, A) and (G, B) over U are said to be soft multi equal if (F, A) is a soft submultiset of (G, B) and (G, B) is a soft submultiset of (F, A).*

Definition 2.5. *The union of two soft multisets of (F, A) and (G, B) over U is the soft multiset (H, C), where $C = A \cup B$ and $C_{H(e)}(x) = \max\left\{C_{F(e)}(x), C_{G(e)}(x)\right\}, \forall e \in A \cup B, \forall x \in U^*$. We write $(F, A) \tilde{\cup} (G, B)$.*

Definition 2.6. *The intersection of two soft multisets of (F, A) and (G, B) over U is the soft multiset (H, C), where $C = A \cap B$ and $C_{H(e)}(x) = \min\left\{C_{F(e)}(x), C_{G(e)}(x)\right\}, \forall e \in A \cap B, \forall x \in U^*$. We write $(F, A) \tilde{\cap} (G, B)$.*

Definition 2.7. *A soft multiset (F, A) over U is said to be a NULL soft multiset denoted by Φ if for all $e \in A$, $F(e) = \emptyset$.*

Definition 2.8. *A soft multiset (F, A) over U is said to be an absolute soft multiset denoted by $\tilde{A}$ if for all $e \in A$, $F(e) = U$.*

Definition 2.9. *The difference (H, E) of two soft multisets (F, E) and (G, E) over U, denoted by $(F, E) \backslash (G, E)$, is defined as $H(e) = F(e) \backslash G(e)$ for all $e \in E$ where $C_{H(e)}(x) = \max\left\{C_{F(e)}(x) - C_{G(e)}(x), 0\right\}, \forall x \in U^*$.*

Definition 2.10. *Let (F, E) be a soft multiset over U and $a \in U^*$. We say that $a \in (F, E)$ read as a belongs to the soft multiset (F, E) whenever $a \in F(e)$ for all $e \in E$.*

Note that for any $a \in U$, $a \notin (F, E)$, if $a \notin F(e)$ for some $e \in E$.

Note 2.11. *Let (F, E) be soft multiset over U. If for $\forall e \in E$ and $a \in U^*, C_{F(e)}(a) = n$ ($n \geq 1$) then we will write $a \in F(e)$ instead of $a \in^n F(e)$.*

Definition 2.12. *Let V be a non-empty submultiset of U, then $\tilde{V}$ denotes the soft multiset (V, E) over U for which $V(e) = V$, for all $e \in E$.*

In particular, (U, E) will be denoted by $\tilde{U}$.

Definition 2.13. *Let $a \in U^*$, then (a, E) denotes the soft multiset over U for which $a(e) = \{a\}$, for all $e \in E$.*

Definition 2.14. *Let (F, E) be a soft multiset over U and V be a non-empty submultiset of U. Then the sub soft multiset of (F, E) over V denoted by $(^V F, E)$, is defined as follows*

$$^V F(e) = V \cap F(e), \text{ for all } e \in E \text{ where } C_{V\,F(e)}(x) = \min\left\{C_V(x), C_{F(e)}(x)\right\}, \forall x \in U^*.$$

In other words $(^V F, E) = \tilde{V} \tilde{\cap} (F, E)$.

Definition 2.15. *The complement of a soft multiset (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to P^*(U)$ is a mapping given by $F^c(e) = U \backslash F(e)$ for all $e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x), \forall x \in U^*$.*

2.2 Soft Multi Topology

Definition 2.16. *Let X be universal multiset and E be set of parameters. Then the collection of all soft multisets over X with parameters from E is called a soft multi class and is denoted as X_E.*

Definition 2.17. *Let $T \subseteq X_E$, then T is said to be a soft multi topology on X if the following conditions hold.*

i. $\Phi, \tilde{X}$ *belong to T.*

ii. *The union of any number of soft multisets in T belongs to T.*

iii. *The intersection of any two soft multisets in T belongs to T.*

T is called a soft multi topology over X and the binary (X_E, T) is called a soft multi topological space over X.

The members of T are said to be soft multi open sets in X.

A soft multiset (F, E) in X_E is said to be a soft multi closed set in X, if its complement $(F, E)^c$ belongs to T.

Example 2.18. *Let $X = \{2/x, 3/y, 4/z, 5/w\}$, $E = \{p, q\}$ and $T = \{\Phi, \tilde{X}, (F_1, E),$ $(F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$ where $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$ are soft multisets in X_E, defined as follows*

$$
\begin{aligned}
F_1(p) &= \{1/x, 2/y, 3/z\}, & F_1(q) &= \{4/w\} \\
F_2(p) &= X, & F_2(q) &= \{1/x, 3/y, 4/z, 5/w\} \\
F_3(p) &= \{2/x, 3/y, 3/z, 1/w\}, & F_3(q) &= \{1/x, 4/w\}, \\
F_4(p) &= \{2/y\}, & F_4(q) &= \{2/w\}. \\
F_5(p) &= \{3/y, 3/z, 1/w\}, & F_5(q) &= \{1/x, 4/w\}.
\end{aligned}
$$

Then T defines a soft multi topology on X and hence (X_E, T) is a soft multi topological space over X.

Definition 2.19. *Let X be universal multiset, E be the set of parameters and $T = \{\Phi, \tilde{X}\}$. Then T is called the soft multi indiscrete topology on X and (X_E, T) is said to be a soft multi indiscrete space over X.*

Definition 2.20. *Let X be universal multiset, E be the set of parameters and let $T = X_E$. Then T is called the soft multi discrete topology on X and (X_E, T) is said to be a soft multi discrete space over X.*

Definition 2.21. *Let (X_E, T_1) and (X_E, T_2) be soft multi topological spaces. Then, the following hold.*

If $T_2 \supseteq T_1$, then T_2 is soft multi finer than T_1.
If $T_2 \supset T_1$, then T_2 is soft multi strictly finer than T_1.
If either $T_2 \supseteq T_1$ or $T_2 \subseteq T_1$, then T_1 is comparable with T_2.

Definition 2.22. *Let (X_E, T) be a soft multi topological space over X and Y be a non-empty subset of X. Then*

$$T_Y = \{(^Y F, E) : (F, E) \in T\}$$

is said to be the soft multi topology on Y and (Y_E, T_Y) is called a soft multi subspace of (X_E, T).

We can easily verify that T_Y is, in fact, a soft multi topology on Y.

3   Connectedness on Soft Multi Topological Spaces

In this section we introduced soft multi connectedness on soft multi topological space and gave base definitions and theorems of soft multi connected.

Proposition 3.1. *Let (F, E), (G, E) and (H, E) be soft multisets in X_E. Then the following hold*

1. *$(F, E) \tilde{\cap} \Phi = \Phi$,*

2. *$(F, E) \tilde{\cup} \Phi = (F, E)$,*

3. *$(F, E) \tilde{\cup} \tilde{X} = \tilde{X}$,*

4. *$(F, E) \tilde{\cap} X = (F, E)$,*

5. *$(F, E) \tilde{\subseteq} (G, E)$ iff $(F, E) \tilde{\cup} (G, E) = (G, E)$,*

6. *$(F, E) \tilde{\subseteq} (G, E)$ iff $(F, E) \tilde{\cap} (G, E) = (F, E)$,*

7. *If $(F, E) \tilde{\cap} (G, E) = \Phi$, then $(F, E) \tilde{\subseteq} (G, E)^c$,*

8. *$(F, E) \tilde{\subseteq} (G, E)$ iff $(G, E)^c \tilde{\subseteq} (F, E)^c$,*

9. $(F,E)\tilde{\cap}((G,E)\tilde{\cup}(H,E)) = ((F,E)\tilde{\cap}(G,E))\,\tilde{\cup}\,((F,E)\tilde{\cap}(H,E)).$

Proof. We only prove $(6) - (9)$.

(6) If $(F,E)\tilde{\subseteq}(G,E)$, then for $\forall e \in E$ and $\forall x \in X^*$, $C_{F(e)}(x) \le C_{G(e)}(x)$. Let $(F,E)\tilde{\cap}(G,E) = (H,E)$. Then $C_{H(e)}(x) = \min\{C_{F(e)}(x), C_{G(e)}(x)\} = C_{F(e)}(x), \forall e \in E, \forall x \in X^*$. Therefore $(H,E) = (F,E)$ and so $(F,E)\tilde{\cap}(G,E) = (F,E)$.

Let $(F,E)\tilde{\cap}(G,E) = (F,E)$. Then $C_{H(e)}(x) = \min\{C_{F(e)}(x), C_{G(e)}(x)\} = C_{F(e)}(x), \forall e \in E, \forall x \in X^*$. Therefore for $\forall e \in E$ and $\forall x \in X^*$, $C_{F(e)}(x) \le C_{G(e)}(x)$. We have $(F,E)\tilde{\subseteq}(G,E)$.

(7) If $(F,E)\tilde{\cap}(G,E) = \Phi$, then $F(e) \cap G(e) = \emptyset$ for each $e \in E$ where $C_{H(e)}(x) = \min\{C_{F(e)}(x), C_{G(e)}(x)\} = 0, \forall x \in X^*$. Therefore $F(e) \subset U\backslash G(e) = G^c(e)$ for each $e \in E$ where $C_{F(e)}(x) \le C_{G^c(e)}(x), \forall x \in X^*$. Then we have $(F,E)\tilde{\subseteq}(G,E)^c$.

(8) $(F,E)\tilde{\subseteq}(G,E)$ iff $F(e) \subseteq G(e)$ for each $e \in E$ where $C_{F(e)}(x) \le C_{G(e)}(x), \forall x \in X^*$ iff $G^c(e) \subseteq F^c(e)$ foreach $e \in E$ where $C_{G^c(e)}(x) \le C_{F^c(e)}(x), \forall x \in X^*$ iff $(G,E)^c\tilde{\subseteq}(F,E)^c$.

(9) Let $(G,E)\tilde{\cup}(H,E) = (A,E)$ and $(F,E)\tilde{\cap}(A,E) = (B,E)$. Then $B(e) = F(e) \cap A(e) = F(e) \cap (G(e) \cup H(e)) = (F(e) \cap G(e)) \cup (F(e) \cap H(e))$, for each $e \in E$. On the other hand, if $(F,E)\tilde{\cap}(G,E) = (C,E)$, $(F,E)\tilde{\cap}(H,E) = (D,E)$ and $(C,E)\tilde{\cup}(D,E) = (I,E)$. we have $I(e) = C(e) \cup D(e) = (F(e) \cap G(e)) \cup (F(e) \cap H(e))$ for each $e \in E$. Therefore $(B,E) = (I,E)$ where $C_{B(e)}(x) = C_{I(e)}(x), \forall x \in X^*$.

$\square$

Corollary 3.2. *Let $\{(F_\alpha, E)\}_{\alpha \in I}$ be a family of soft multisets in X_E, then*
$(F,E) \cap (\cup_{\alpha \in I}(F_\alpha, E)) = \cup_{\alpha \in I}((F,E) \cap (F_\alpha, E)).$

Definition 3.3. *Let (X_E, T) be a soft multi topological space over X. A soft multi separation of $\tilde{X}$ is a pair (F,E), (G,E) of no-null soft multi open sets in X_E such that*

$$\tilde{X} = (F,E)\tilde{\cup}(G,E),\ (F,E)\tilde{\cap}(G,E) = \Phi.$$

Definition 3.4. *A soft multi topological space (X_E, T) is said to be soft multi connected if there does not exist a soft multi separation of $\tilde{X}$. Otherwise, (X_E, T) is said to be soft multi disconnected.*

Example 3.5. *Let $X = \{2/x, 3/y, 4/z, 5/w\}$, $E = \{p, q\}$ and $T = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft multisets in X_E, defined as follows*

$$\begin{aligned}
F_1(p) &= \{1/x, 2/y, 3/z\}, & F_1(q) &= \{4/w\} \\
F_2(p) &= X, & F_2(q) &= \{1/x, 3/y, 4/z, 5/w\} \\
F_3(p) &= \{2/x, 3/y, 3/z, 1/w\}, & F_3(q) &= \{1/x, 4/w\}.
\end{aligned}$$

Then T defines a soft multi topology on X and hence (X_E, T) is a soft multi topological space over X.

Since $(F_1, E)\tilde{\cap}(F_2, E) \neq \Phi, (F_1, E)\tilde{\cap}(F_3, E) \neq \Phi$ and $(F_2, E)\tilde{\cap}(F_3, E) \neq \Phi$ also $(F_1, E)\tilde{\cup}(F_2, E) \neq \tilde{X}, (F_1, E)\tilde{\cup}(F_3, E) \neq \tilde{X}$ and $(F_2, E)\tilde{\cup}(F_3, E) \neq \tilde{X}$, soft multi topological space (X_E, T) is soft multi connected.

Theorem 3.6. *Soft multi topological space (X_E, T) is soft multi connected if and only if the only soft multisets in X_E that are both soft multi open and soft multi closed in X_E are Φ and $\tilde{X}$.*

Proof. Let (X_E, T) be soft multi connected. Suppose to the contrary that (F, E) is both soft multi open and soft multi closed in X_E different from Φ and $\tilde{X}$. Clearly, $(F, E)^c$ is a soft multi open set in X_E different from Φ and $\tilde{X}$. Also $(F, E)\tilde{\cup}(F, E)^c = \tilde{X}$ and $(F, E)\tilde{\cap}(F, E)^c = \Phi$. Therefore we have $(F, E), (F, E)^c$ is a soft multi separation of $\tilde{X}$. This is a contradiction. Thus the only soft multi closed and open sets in X_E are Φ and $\tilde{X}$.

Conversely, let $(F, E), (G, E)$ be a soft multi separation of $\tilde{X}$. Let $(F, E) = \tilde{X}$. Then Proposition 3.1. implies that $(G, E) = \Phi$. This is a contradiction. Hence, $(F, E) \neq \tilde{X}$. Since $F(e) \cap G(e) = \emptyset$ where $C_{H(e)}(x) = \min\left\{C_{F(e)}(x), C_{G(e)}(x)\right\} = 0, \forall e \in e, x \in X^*$ and $F(e) \cup G(e) = X$ for each $e \in E$, we have $G^c(e) = X \backslash G(e) = F(e)$ where $C_{G^c(e)}(x) = \max\left\{C_X(x) - C_{G(e)}(x), 0\right\} = C_{F(e)}(x), \forall x \in X^*$. Therefore $(F, E) = (G, E)^c$. This shows that (F, E) is both soft multi open and soft multi closed in X_E different from Φ and $\tilde{X}$. This is a contradiction. Therefore, (X_E, T) is soft connected. $\square$

Example 3.7. *Since the only soft multisets in X_E that are both soft multi open and soft multi closed in X_E are $\tilde{X}$ and Φ, soft multi indiscrete topological space (X_E, T) is soft multi connected.*

Example 3.8. *Soft multi discrete topological space (X_E, T) is soft multi disconnected. Because for at least one soft multiset (F, E) in X_E, soft multiset (F, E) is both soft multi open set and soft multi closed.*

Corollary 3.9. *Let (X_E, T) be a soft multi topological space over X. Then following statements are equivalent.*

1. *(X_E, T) is soft multi connected.*

2. *No-null soft multi open sets $(F, E), (G, E)$ over X_E and $\tilde{X} = (F, E)\tilde{\cup}(G, E)$ but $(F, E)\tilde{\cap}(G, E) \neq \Phi$.*

3. *The only soft multisets in X_E that are both soft multi open and soft multi closed in X_E are Φ and $\tilde{X}$.*

4. *If $\tilde{X} = (F, E)\tilde{\cup}(G, E)$ and $(F, E)\tilde{\cap}(G, E) = \Phi$ then $(F, E) = \Phi$ or $(G, E) = \Phi$.*

5. *If $\tilde{X} = (F,E)\tilde{\cup}(G,E)$ and $(F,E)\tilde{\cap}(G,E) = \Phi$ then $(F,E) = \tilde{X}$ or $(G,E) = \tilde{X}$.*

Proposition 3.10. *If the soft multisets (F,E) and (G,E) form a soft multi separation of $\tilde{X}$, and (Y_E, T_Y) is a soft multi connected subspace of (X_E, T), then $\tilde{Y}$ lies entirely within either (F,E) or (G,E).*

Proof. Since $\tilde{Y} \tilde{\subseteq} (F,E)\tilde{\cup}(G,E)$ by Proposition 3.1. we have, $\tilde{Y} = (\tilde{Y}\tilde{\cap}(F,E)\tilde{\cup}(\tilde{Y}\tilde{\cap}(G,E))$ that $\tilde{Y}\tilde{\cap}(F,E)$ and $\tilde{Y}\tilde{\cap}(G,E)$ are soft multi open sets over Y . Suppose to the contrary $\tilde{Y}$ does not lie entirely within either (F,E) or (G,E). By the hypothesis and Proposition 3.1., $(\tilde{Y}\tilde{\cap}(F,E))$ and $(\tilde{Y}\tilde{\cap}(G,E))$ are different from $\tilde{Y}$ and Φ . But $Y(e)\cap F(e)\cap G(e) = \emptyset$, for each $e \in E$. Therefore, $(\tilde{Y}\tilde{\cap}(F,E))\tilde{\cap}(\tilde{Y}\tilde{\cap}(G,E)) = \Phi$. Since $(\tilde{Y}\tilde{\cap}(F,E))$ and $(\tilde{Y}\tilde{\cap}(G,E))$ are soft multi open sets over $\tilde{Y}$, then we have a soft multi separation of $\tilde{Y}$. This is a contradiction. This completes the proof. $\square$

Example 3.11. *Let $X = \{1/x, 2/y, 4/z, 3/w\}$, $E = \{p,q,r\}$ and $T = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ where $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ are soft multisets in X_E, defined as follows*

$$
\begin{array}{lll}
F_1(p) = \{1/x, 2/y\}, & F_1(q) = \{1/x, 2/y, 4/z\}, & F_1(r) = X, \\
F_2(p) = \{4/z, 3/w\}, & F_2(q) = \{3/w\}, & F_2(r) = \emptyset, \\
F_3(p) = \{1/x\}, & F_3(q) = \{1/x, 1/y\}, & F_3(r) = \{1/x, 1/y, 1/z\}, \\
F_4(p) = \{1/x, 4/z, 3/w\}, & F_4(q) = \{1/x, 1/y, 3/w\}, & F_4(r) = \{1/x, 1/y, 1/z\}.
\end{array}
$$

Then T defines a soft multi topology on X and hence (X_E, T) is a soft multi topological space over X. Also (F_1, E) and (F_2, E) is a soft multi separation of $\tilde{X}$. Let $Y = \{1/x, 1/y\}$. Then $T_Y = \{\Phi, \tilde{Y}, ({}^Y F_1, E), ({}^Y F_2, E), ({}^Y F_3, E), ({}^Y F_4, E)\}$ where $({}^Y F_1, E), ({}^Y F_2, E), ({}^Y F_3, E)$ and $({}^Y F_4, E)$ are soft multisets in X_E, defined as follows

$$
\begin{array}{lll}
{}^Y F_1(p) = \{1/x, 1/y\}, & {}^Y F_1(q) = \{1/x, 1/y\}, & {}^Y F_1(r) = \{1/x, 1/y\}, \\
{}^Y F_2(p) = \emptyset, & {}^Y F_2(q) = \emptyset, & {}^Y F_2(r) = \emptyset, \\
{}^Y F_3(p) = \{1/x\}, & {}^Y F_3(q) = \{1/x, 1/y\}, & {}^Y F_3(r) = \{1/x, 1/y\}, \\
{}^Y F_4(p) = \{1/x\}, & {}^Y F_4(q) = \{1/x, 1/y\}, & {}^Y F_4(r) = \{1/x, 1/y\}.
\end{array}
$$

If we arrange T_Y, then we get $T_Y = \{\Phi, \tilde{Y}, ({}^Y F_3, E)\}$. If we consider Proposition 3.10., then we have $\tilde{Y} \tilde{\subseteq} (F_1, E)$. This is obviously seen above.

Lemma 3.12. *Let (Y_E, T_Y) and (Z_E, T_Z) be soft multi subspaces of (X_E, T) and $(Y,E)\tilde{\subseteq}(Z,E)$. Then (Y_E, T_Y) is a soft multi subspace of (Z_E, T_Z).*

Theorem 3.13. *The union of a collection of soft multi connected subspace of (X_E, T) that have non-null intersection is soft multi connected.*

Proof. Let $\{(Y_E, T_{Y_\alpha})\}_{\alpha \in I}$ be an arbitrary collection of soft multi connected soft multi subspace of (X_E, T). Suppose to the contrary that there exists a soft multi separation of $\tilde{Y} = \cup_{\alpha \in I}\tilde{Y}_\alpha$, $\tilde{Y}\tilde{\cap}(F,E)$, $\tilde{Y}\tilde{\cap}(G,E)$. By the Proposition 3.1., we have

$\tilde{Y} = (\tilde{U}_{\alpha \in I}(F,E) \tilde{U}(\tilde{U}_{\alpha \in I}(G,E))$ where $F_\alpha(e) = F(e) \cap Y_\alpha$ and $G_\alpha(e) = G(e) \cap Y_\alpha$, for each $\alpha \in I$ and $e \in E$. Since $\tilde{\cap}_{\alpha \in I}\tilde{Y} \neq \Phi$, it is easy to see that $\cap_{\alpha \in I}Y_\alpha \neq \emptyset$, and $x \in \cap_{\alpha \in I}Y_\alpha$. On the other hand Lemma 3.12. implies that (Y_E, T_α) is a soft multi subspace of (Y_E, T_Y), for each $\alpha \in I$. By Proposition 3.1., we can assume that $\tilde{Y}$ lies entirely within $\tilde{Y}\tilde{\cap}(F,E)$. Let $\alpha' \in I \setminus \{\alpha\}$. If $Y_{\alpha'}\tilde{\subseteq}\tilde{Y}\tilde{\cap}(G,E)$, it is easy to see that $x \in Y \cap G(e)$, also $x \in Y \cap F(e)$, for each $\alpha \in I$. This is a contradiction. Therefore $\tilde{Y}_\alpha\tilde{\subseteq}\tilde{Y}\tilde{\cap}(F,E)$, for each $\alpha \in I$. Now we can see that $\tilde{Y}\tilde{\subseteq}\tilde{Y}\tilde{\cap}(F,E)$. Proposition of [12] implies that $\tilde{Y}\tilde{\cap}(G,E)\tilde{\subseteq}\tilde{Y}\tilde{\cap}(F,E)$ and $\Phi = \tilde{Y}\tilde{\cap}(G,E)$. This is a contradiction. This completes the proof. $\square$

Definition 3.14. *A soft multi topological space (X_E, T) is said to be soft multi locally connected at $x \in X$ if for every soft multi open set (F,E) containing x, there is a soft multi connected soft multi open (G,E) containing x contained in (F,E). If (X_E, T) is soft multi locally connected at each of its points, it is said simply to be soft multi locally connected.*

Definition 3.15. *A soft multi topological space (X_E, T) is said to be soft multi weakly locally connected at $x \in X^*$ if for every soft multi open set (Y,E) containing x, there is a soft multi connected subspace of (X_E, T) contained in (Y,E) that contains a soft multi open set (F,E) containing x and similar to the soft multi locally connectedness, if (X_E, T) is soft multi weakly locally connected at each of its points, it is said simply to be soft multi weakly locally connected.*

Theorem 3.16. *If (X_E, T) is soft multi weakly locally connected, then it is soft multi locally connected.*

4   Conclusion

There are many problems encountered in real life. Various mathematical methods have been developed to solve these problems. One of them is concept of soft multiset. In this work, we introduced soft multi connectedness on soft multi topological spaces and gave basic definitions and theorems of soft multi connectedness.

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