ISSN: **1304-7981** Number: **2**, Year: **2013**, Pages: **60-75**. http://jnrs.gop.edu.tr

Gaziosmanpasa University
Graduate School of Natural and Applied Sciences
Journal of New Results in Science

Received: 20.04.2013
Accepted: 22.08.2013

Editors-in-Chief: *Naim Çağman*
Area Editor: *Oktay Muhtaroğlu*

About Soft Topological Spaces

E. Peyghan[a,1](e-peyghan@araku.ac.ir)
B. Samadi[a] (bsamadi@tafreshu.ac.ir)
A. Tayebi[b] (akbar.tayebi@gmail.com)

[a]*Department of Mathematics, Faculty of Science, Arak University, Arak 38156-8-8349, Iran*
[b]*Department of Mathematics, Faculty of Science, University of Qom, Qom, Iran*

Abstract The soft topological spaces and some their related concepts have studied by Shabir and Naz in [11]. In this paper, we introduce some concepts such as soft connectedness, soft Hausdorff space and exhibit some results related to these concepts.

Keywords - *Soft connected, soft Hausdorff space, soft open set, soft set, soft topological space.*

1 Introduction

Some concepts in mathematics can be considered as mathematical tools for dealing with uncertainties, namely theory of vague sets, theory of rough sets and etc. But all of these theories have their own difficulties. The concept of soft sets was first introduced by Molodtsov as a general mathematical tool for dealing with uncertain objects [8]. He successfully applied the soft theory in several directions, such as smoothness of functions, game theory, probability, Perron integration, Riemann integration, theory of measurement [8][9]. It is remarkable that, Molodtsov used this concept in order to solve complicated problems in other sciences such as, engineering, economics and etc.

The properties and applications of soft set theory have been studied increasingly in [1], after that the operations of soft sets presented by Maji-Biswas-Roy [7]. In [3], Çağman-Enginoglu redefined the operations of the soft sets and constructed a uni-int decision making method by using these new operations, and developed soft set theory. Then to make easy compaction with the operations of soft sets, they presented the soft matrix theory and set up the soft maximin decision making method [4]. These decision making methods can be successfully applied to many problems that contain uncertainties.

[1]Corresponding Author

In [11], Shabir-Naz introduced and studied the concepts soft topological space and some related concepts such as soft interior, soft closed, soft subspace and soft separation axioms. In [2], Aygunoglu-Aygun introduced the soft product topology and defined the version of compactness in soft spaces named soft compactness.

In this paper, we introduce the concept of soft connectedness and we obtain some results that involve the concepts of soft *pu*-continuous functions, soft Hausdorff spaces and soft cartesian procuct.

2 Preliminary

In this section, we recall some definitions and concepts discussed in [6, 10, 11, 12]. Let U be an initial universe and E be a set of parameters. Let $\mathbb{P}(U)$ denote the power set of U and A be a nonempty subset of E. A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \to \mathbb{P}(U)$. For two soft sets (F, A) and (G, B) over common universe U, we say that (F, A) is a *soft subset* (G, B) if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$. In this case, we write $(F, A)\widetilde{\subseteq}(G, B)$ and (G, B) is said to be a *soft super set* of (F, A). Two soft sets (F, A) and (G, B) over a common universe U are said to be *soft equal* if $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$. A soft set (F, A) over U is called a *null soft set*, denoted by Φ_A, if for each $e \in A$, $F(e) = \emptyset$. Similarly, it is called *absolute soft set*, denoted by $\widetilde{U}$, if for each $e \in A$, $F(e) = U$. The *union* of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for each $e \in C$,

$$H(e) = \begin{cases} F(e) & e \in A \\ G(e) & e \in B \\ F(e) \cup G(e) & e \in A \cap B \end{cases}$$

We write $(F, A) \cup (G, B) = (H, C)$. Moreover, the *intersection* (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted by $(F, A)\cap(G, B)$, is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for each $e \in C$.

The *difference* (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E)\backslash(G, E)$, is defined as $H(e) = F(e)\backslash G(e)$, for each $e \in E$.

Let Y be a nonempty subset of X. Then $\widetilde{Y}$ denotes the soft set (Y, E) over X where $Y(e) = Y$, for each $e \in E$. In particular, (X, E) will be denoted by $\widetilde{X}$.

Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$, whenever $x \in F(e)$, for each $e \in E$.

The relative complement of a soft set (F, A) is denoted by $(F, A)'$ and is defined by $(F, A)' = (F', A)$ where $F' : A \to \mathbb{P}(U)$ is a mapping given by $F'(e) = U - F(e)$, for each $e \in A$.

Let τ be the collection of soft sets over X. Then τ is called a soft topology on X if τ satisfies the following axioms:

(i) Φ_E, $\widetilde{X}$ belong to τ.

(ii) The union of any number of soft sets in τ belong to τ.

(iii) The intersection of any two soft sets in τ belong to τ.

The triple (X, τ, E) is called a soft topological space over X. The member of τ are said to be soft open in X, and the soft set (F, E) is called soft closed in X if its relative component $(F, E)'$ belongs to τ.

Proposition 2.1. Let (X, τ, E) be a soft space over X. Then

i) Φ_E, $\widetilde{X}$ are closed soft set over X;

ii) The intersection of any number of soft closed sets is a soft closed set over X;

iii) The union of any two soft closed sets is a soft closed set over X.

Let $SS(X)_E$ be the collection of all soft sets with set of parameter E, over X. The cartesian product of soft sets $(F, A) \in SS(X)_A$ and $(G, B) \in SS(Y)_B$ is a soft set $(F \times G, A \times B)$ in $SS(X \times Y)_{A \times B}$ where $F \times G : A \times B \to \mathbb{P}(X) \times \mathbb{P}(Y)$ is a mapping given by $(F \times G)(a, b) = F(a) \times G(b)$ for each $(a, b) \in A \times B$.

3 Main Results

In this section, we are going to define some new concepts for soft topological spaces and study some properties related to these spaces.

3.1 Soft Connected Spaces

Let (X, τ, E) be a soft topological space over X. A soft separation of $\widetilde{X}$ is a pair (F, E), (G, E) of no-null soft open sets over X such that

$$\widetilde{X} = (F, E) \cup (G, E), \quad (F, E) \cap (G, E) = \Phi_E.$$

A soft topological space (X, τ, E) is said to be soft connected if there does not exist a soft separation of $\widetilde{X}$.

Proposition 3.1. *Let (F, E) be a soft set in $SS(X)_E$. Then the following hold*

(i) $(F, E) \cup (F, E)' = \widetilde{X}$;

(ii) $(F, E) \cap (F, E)' = \Phi_E$;

(iii) $(F, E) \cap \widetilde{X} = (F, E)$.

Proof. We prove (ii), only. Let $(F, E) \cap (F, E)' = (H, E)$. Then

$$H(e) = F(e) \cap F'(e) = F(e) \cap (X - F(e)) = \emptyset.$$

Therefore $(H, E) = \Phi_E$. □

Using Proposition 3.1, we prove the following.

Theorem 3.2. *A soft topological space (X, τ, E) is soft connected if and only if the only soft sets in $SS(X)_E$ that are both soft open and soft closed over X are Φ_E and $\widetilde{X}$.*

Proof. Let (X, τ, E) be soft connected. Suppose to the contrary that (F, E) is both soft open and soft closed in X different from Φ_E and $\widetilde{X}$. Clearly, $(F, E)'$ is a soft open set in X different from Φ_E and $\widetilde{X}$. By Proposition 3.1, $(F, E), (F, E)'$ is a soft separation of $\widetilde{X}$. This is a contradiction. Thus the only soft closed and open sets in X are Φ_E and $\widetilde{X}$. Conversely, let (F, E), (G, E) be a soft separation of $\widetilde{X}$. Let $(F, E) = \widetilde{X}$. Then Proposition 3.1 implies that $(G, E) = \Phi_E$. This is a contradiction. Hence, $(F, E) \neq \widetilde{X}$. Since $F(e) \cap G(e) = \emptyset$ and $F(e) \cup G(e) = X$, for each $e \in E$, then we have $G'(e) = X - G(e) = F(e)$. Therefore $(F, E) = (G, E)'$. This shows that (F, E) is both soft open and soft closed in X different from Φ_E and $\widetilde{X}$. This is a contradiction. Therefore, (X, τ, E) is soft connected. □

Let $SS(U)_A$ and $SS(V)_B$ be families of soft sets. Suppose that $u : U \rightarrow V$ and $p : A \rightarrow B$ be mappings. Then a mapping $f_{pu} : SS(U)_A \rightarrow SS(V)_B$ is defined as:

(i) Let (F, A) be a soft set in $SS(U)_A$. The image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), B)$ is a soft set in $SS(V)_B$ such that,

$$f_{pu}(F)(y) = \begin{cases} \bigcup_{x \in p^{-1}(y) \cap A} u(F(x)) & p^{-1}(y) \cap A \neq \emptyset \\ \emptyset & p^{-1}(y) \cap A = \emptyset \end{cases}$$

for each $y \in B$.

(ii) Let (G, B) be a soft set in $SS(V)_B$. Then the inverse image of (G, B) under f_{pu}, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), A)$, is a soft set in $SS(U)_A$ such that, $f_{pu}^{-1}(G)(x) = u^{-1}(G(p(x)))$, for each $x \in A$.

Proposition 3.3. *Let $SS(U)_A$ and $SS(V)_B$ be families of soft sets. For a function $f_{pu} : SS(U)_A \rightarrow SS(V)_B$, the following hold*

(i) $f_{pu}^{-1}((F, B) \cup (G, B)) = f_{pu}^{-1}(F, B) \cup f_{pu}^{-1}(G, B)$;

(ii) $f_{pu}^{-1}(\widetilde{V}) = \widetilde{U}$;

(iii) $f_{pu}((F, A) \cap (G, A)) \widetilde{\subseteq} f_{pu}(F, A) \cap f_{pu}(G, A)$;

(iv) $f_{pu}^{-1}((F, B) \cap (G, B)) = f_{pu}^{-1}(F, B) \cap f_{pu}^{-1}(G, B)$;

(v) $f_{pu}^{-1}(\Phi_B) = \Phi_A$.

Proof. (i) Let $(F, B) \cup (G, B) = (H, B)$. Then $f_{pu}^{-1}(H, B) = (f_{pu}^{-1}(H), A)$, where $f_{pu}^{-1}(H)(x) = u^{-1}(H(p(x)))$, for each $x \in A$. On the other hand, let $f_{pu}^{-1}(F, B) \cup f_{pu}^{-1}(G, B) = (O, A)$, where

$$O(x) = f_{pu}^{-1}(F)(x) \cup f_{pu}^{-1}(G)(x) = u^{-1}\Big(F(p(x)) \cup G(p(x))\Big) = u^{-1}(H(p(x))),$$

for each $x \in A$. Therefore $f_{pu}^{-1}(H, B) = (O, A)$.

(ii) $f_{pu}^{-1}(\widetilde{V}) = f_{pu}^{-1}(V, B) = (f_{pu}^{-1}(V), A)$, where $f_{pu}^{-1}(V)(x) = u^{-1}(V(p(x))) = u^{-1}(V) = U = U(x)$.

(iii) Let $(F, A) \cap (G, A) = (H, A)$. Then $f_{pu}(H, A) = (f_{pu}(H), B)$, where

$$f_{pu}(H)(y) = \begin{cases} \bigcup_{x \in p^{-1}(y) \cap A} u(H(x)) & p^{-1}(y) \cap A \neq \emptyset \\ \emptyset & p^{-1}(y) \cap A = \emptyset \end{cases}$$

for each $y \in B$. On the other hand, let $f_{pu}(F, A) \cap f_{pu}(G, A) = (O, B)$, where $O(y) = f_{pu}(F)(y) \cap f_{pu}(G)(y)$, for each $y \in B$. We have

$$O(y) = \begin{cases} (\bigcup_{x \in p^{-1}(y) \cap A} u(F(x))) \cap (\bigcup_{x \in p^{-1}(y) \cap A} u(G(x))) & p^{-1}(y) \cap A \neq \emptyset \\ \emptyset & p^{-1}(y) \cap A = \emptyset \end{cases}$$

for each $y \in B$. Since $H(x) = F(x) \cap G(x)$, for each $x \in A$, then it is easy to see that $f_{pu}(H)(y) \subseteq O(y)$ for each $y \in B$. This implies that $f_{pu}(H, A) \subseteq (O, B)$.

(iv) Let $(F, B) \cap (G, B) = (H, B)$. Then $f_{pu}^{-1}(H, B) = (f_{pu}^{-1}(H), A)$, where $f_{pu}^{-1}(H)(x) = u^{-1}(H(p(x)))$, for each $x \in A$. On the other hand, let $f_{pu}^{-1}(F, B) \cap f_{pu}^{-1}(G, B) = (O, A)$, where

$$\begin{aligned} O(x) &= f_{pu}^{-1}(F)(x) \cap f_{pu}^{-1}(G)(x) = u^{-1}(F(p(x))) \cap u^{-1}(G(p(x))) \\ &= u^{-1}(H(p(x))), \end{aligned}$$

for each $x \in A$. Therefore, $f_{pu}^{-1}(H, B) = (O, A)$. □

Let (U, τ, A) and (V, τ', B) be soft topological spaces. Let $f_{pu} : SS(U)_A \to SS(V)_B$ be a function. Then f_{pu} is said to be soft *pu*-continuous if for each $(F, B) \in \tau'$ we have $f_{pu}^{-1}(F, B) \in \tau$.

Proposition 3.4. *Let (F, E), (G, E) and (H, E) be soft sets in $SS(X)_E$. Then,*

(i) *$(F, E) \cap ((G, E) \cup (H, E)) = ((F, E) \cap (G, E)) \cup ((F, E) \cap (H, E))$;*

(ii) *$(F, E) \widetilde{\subseteq} (G, E)$ if and only if $(F, E) \cap (G, E) = (F, E)$;*

(iii) *$(F, E) \widetilde{\subseteq} (G, E)$ if and only if $(F, E) \cup (G, E) = (G, E)$.*

Proof. We prove (i), only. Let $(G, E) \cup (H, E) = (A, E)$ and $(F, E) \cap (A, E) = (B, E)$. Then,

$$B(e) = F(e) \cap A(e) = F(e) \cap (G(e) \cup H(e)) = (F(e) \cap G(e)) \cup (F(e) \cap H(e)),$$

for each $e \in E$.

On the other hand, if $(F,E) \cap (G,E) = (C,E)$, $(F,E) \cap (H,E) = (D,E)$ and $(C,E) \cup (D,E) = (I,E)$, then $I(e) = C(e) \cup D(e) = (F(e) \cap G(e)) \cup (F(e) \cap H(e))$ for each $e \in E$. Therefore, $(B,E) = (I,E)$. □

Theorem 3.5. *Let f_{pu} be a soft pu-continuous function carrying the soft connected space (U,τ,A) onto the soft space (V,τ',B). Then (V,τ',B) is soft connected.*

Proof. Suppose to the contrary there exists a soft separation (F,B), (G,B) of $\widetilde{V}$. Then Proposition 3.3, implies that

$$\widetilde{U} = f_{pu}^{-1}((F,B) \cup (G,B)) = f_{pu}^{-1}(F,B) \cup f_{pu}^{-1}(G,B),$$
$$f_{pu}^{-1}(F,B) \cap f_{pu}^{-1}(G,B) = f_{pu}^{-1}(\Phi_B) = \Phi_A.$$

Let $f_{pu}^{-1}(F,B) = \Phi_A$. Since f_{pu} is surjective, then by Theorem 3.14 of [12] and Proposition 3.3, we have $(F,B) = \Phi_B$. This is a contradiction. Therefore $f_{pu}^{-1}(F,B)$ and by a similar reason $f_{pu}^{-1}(G,B)$ are different from Φ_A. Now, Proposition 3.4 shows that $f_{pu}^{-1}(F,B), f_{pu}^{-1}(G,B)$ is a soft separation of $\widetilde{U}$. This is a contradiction, and this completes the proof. □

Let (F,E) be a soft set over X and Y be a nonempty subset of X. Then the sub soft set of (F,E) over Y denoted by $({}^YF,E)$ is defined as follows

$$^YF(e) = Y \cap F(e),$$

for each $e \in E$. In other word $({}^YF,E) = \widetilde{Y} \cap (F,E)$.

Now, suppose that (X,τ,E) be a soft topological space over X and Y be a nonempty subset of X. Then

$$\tau_Y = \{({}^YF,E) | (F,E) \in \tau\},$$

is said to be the *soft relative topology* on Y and (Y,τ_Y,E) is called a soft subspace of (X,τ,E).

Proposition 3.6. *If the soft sets (F,E) and (G,E) form a soft separation of $\widetilde{X}$, and (Y,τ_Y,E) is a soft connected subspace of (X,τ,E), then $\widetilde{Y}$ lies entirely within either (F,E) or (G,E).*

Proof. Since $\widetilde{Y} \widetilde{\subseteq} (F,E) \cup (G,E)$, then by Proposition 3.4 we have

$$\widetilde{Y} = (\widetilde{Y} \cap (F,E)) \cup (\widetilde{Y} \cap (G,E)).$$

This means that $\widetilde{Y} \cap (F,E)$ and $\widetilde{Y} \cap (G,E)$ are soft open sets over Y. Suppose to the contrary $\widetilde{Y}$ does not lie entirely within either (F,E) or (G,E). By the hypothesis, Proposition 3.4 of [12] and Proposition 3.4, $\widetilde{Y} \cap (F,E)$ and $\widetilde{Y} \cap (G,E)$ are different from $\widetilde{Y}$ and Φ_E. But $Y(e) \cap F(e) \cap G(e) = \emptyset$, for each $e \in E$. Therefore, $(\widetilde{Y} \cap (F,E)) \cap (\widetilde{Y} \cap (G,E)) = \Phi_E$. Since $(\widetilde{Y} \cap (F,E))$ and $(\widetilde{Y} \cap (G,E))$ are soft open sets over $\widetilde{Y}$, then we have a soft separation of $\widetilde{Y}$. This is a contradiction. This completes the proof. □

It is easy to prove the following.

Proposition 3.7. *Let (F,E), (G,E) and (H,E) be soft sets in $SS(X)_E$. Then the following hold*

(i) $(F,E)\cap((G,E)\cap(H,E))=((F,E)\cap(G,E))\cap(H,E)$;

(ii) $(F,E)\cup((G,E)\cup(H,E))=((F,E)\cup(G,E))\cup(H,E)$.

Also, we can obtain the following easily.

Proposition 3.8. *Let $\{(F_\alpha,E)\}_{\alpha\in J}$ be a family of soft sets in $SS(X)_E$. Then the following hold*

(i) $(F,E)\cap(\cup_{\alpha\in J}(F_\alpha,E))=\cup_{\alpha\in J}((F,E)\cap(F_\alpha,E))$;

(ii) *If* $(F,E)=(G,E)\cup(H,E)$, *then* $(G,E),(H,E)\widetilde{\subset}(F,E)$.

Now, we are going to consider the relation between soft subspaces of (X,τ,E), when one is a subset of the other.

Lemma 3.9. *Let (Y,τ',E) and (Z,τ'',E) be soft subspaces of (X,τ,E) and $(Y,E)\widetilde{\subseteq}(Z,E)$. Then (Y,τ,E) is a soft subspace of (Z,τ'',E).*

Proof. By Proposition 3.4, we have $\widetilde{Y}=\widetilde{Y}\cap\widetilde{Z}$. Moreover each soft open set of (Y,τ',E) is of the form $\widetilde{Y}\cap(F,E)$, where (F,E) is a soft open set of (X,τ,E). Therefore, by Proposition 3.7, we have

$$\widetilde{Y}\cap(F,E)=(\widetilde{Y}\cap\widetilde{Z})\cap(F,E)=\widetilde{Y}\cap[\widetilde{Z}\cap(F,E)].$$

Conversely, it is clear that each soft open set in Y as a soft subspace of (Z,τ'',E) is of the form $\widetilde{Y}\cap(\widetilde{Z}\cap(F,E))=\widetilde{Y}\cap(F,E)$. This completes the proof. □

We are going to answer to this question: *Is the union of a collection of soft connected subspace of (X,τ,E) a soft connected?*

Theorem 3.10. *The union of a collection of soft connected subspace of (X,τ,E) that have non-null intersection is soft connected.*

Proof. Let $\{(Y_\alpha,\tau_{Y_\alpha},E)\}_{\alpha\in J}$ be an arbitrary collection of soft connected soft subspace of (X,τ,E), such that $(\cap_{\alpha\in J}Y_\alpha,E)\neq\Phi_E$. Suppose to the contrary that there exists a soft separation $\widetilde{Y}\cap(F,E)$, $\widetilde{Y}\cap(G,E)$ of $\widetilde{Y}=\cup_{\alpha\in J}\widetilde{Y}_\alpha$. By Proposition 3.8, we have $\widetilde{Y}=(\cup_{\alpha\in J}(F_\alpha,E))\cup(\cup_{\alpha\in J}(G_\alpha,E))$ where $F_\alpha(e)=F(e)\cap Y_\alpha$ and $G_\alpha(e)=G(e)\cap Y_\alpha$, for each $\alpha\in J$ and $e\in E$. Since $\cap_{\alpha\in J}\widetilde{Y}_\alpha\neq\Phi_E$, it is easy to see that $\cap_{\alpha\in J}Y_\alpha\neq\emptyset$, and $x\in\cap_{\alpha\in J}Y_\alpha$. On the other hand, Lemma 3.9 implies that (Y_α,τ_α,E) is a soft subspace of (Y,τ_Y,E), for each $\alpha\in J$. By Proposition 3.6, we can assume that $\widetilde{Y}_\alpha$ lies entirely within $\widetilde{Y}\cap(F,E)$.

Let $\alpha' \in J - \{\alpha\}$. If $\widetilde{Y}_{\alpha'} \widetilde{\subseteq} \widetilde{Y} \cap (G, E)$, then it is easy to see that $x \in Y \cap G(e)$, also $x \in Y \cap F(e)$, for each $\alpha \in J$. This is a contradiction. Therefore $\widetilde{Y}_{\alpha} \widetilde{\subseteq} \widetilde{Y} \cap (F, E)$, for each $\alpha \in J$. Now we can see that $\widetilde{Y} \widetilde{\subseteq} \widetilde{Y} \cap (F, E)$. Proposition 3.4 implies that $\widetilde{Y} \cap (G, E) \widetilde{\subseteq} \widetilde{Y} \cap (F, E)$ and $\Phi_E = \widetilde{Y} \cap (G, E)$. This is a contradiction. This completes the proof. □

3.2 Cartesian Product of Soft Sets

Let (X, τ, E) be a soft topological space and $B \subseteq \tau$. If every element of τ can be written as a union of elements of B, then B is called a soft basis for the soft topology τ. Each element of B is called a soft basis element. Let (F, E_1) and (G, E_2) be soft sets in $SS(X)_{E_1}$ and $SS(Y)_{E_2}$, respectively. Then the cartesian product of (F, E_1) and (G, E_2) denoted by $(F \times G, E_1 \times E_2)$ in $SS(X \times Y)_{E_1 \times E_2}$ is defined as $(F \times G)(e_1, e_2) = F(e_1) \times G(e_2)$.

Proposition 3.11. *Let* $(F_1, E_1), (G_1, E_1) \in SS(X)_{E_1}$ *and* $(F_2, E_2), (G_2, E_2) \in SS(Y)_{E_2}$. *Then*

(i) $\Phi_{E_1} \times (F_2, E_2) = (F_1, E_1) \times \Phi_{E_2} = \Phi_{E_1 \times E_2}$;

(ii) $((F_1, E_1) \times (F_2, E_2)) \cap ((G_1, E_1) \times (G_2, E_2)) = ((F_1, E_1) \cap (G_1, E_1)) \times ((F_2, E_2) \cap (G_2, E_2))$.

Proof. (i) Let $\Phi_{E_1} = (\phi_1, E_1)$ and $\Phi_{E_2} = (\phi_2, E_2)$. Then we have

$$\begin{aligned}(F_1 \times \phi_2)(e_1, e_2) &= F_1(e_1) \times \phi_2(e_2) = F_1(e_1) \times \emptyset = \emptyset \\ &= \emptyset \times F_2(e_2) = \phi_1(e_1) \times F_2(e_2) = (\phi_1 \times F_2)(e_1, e_2).\end{aligned}$$

This implies (i).
(ii) Let $(F_1 \times F_2, E_1 \times E_2) \cap (G_1 \times G_2, E_1 \times E_2) = (H, E_1 \times E_2)$, $(F_1, E_1) \cap (G_1, E_1) = (I, E_1)$ and $(F_2, E_2) \cap (G_2, E_2) = (J, E_2)$. Then

$$\begin{aligned}H(e_1, e_2) &= (F_1 \times F_2)(e_1, e_2) \cap (G_1 \times G_2)(e_1, e_2) = \\ &(F_1(e_1) \times F_2(e_2)) \cap (G_1(e_1) \times G_2(e_2)) = \\ &(F_1(e_1) \cap G_1(e_1)) \times (F_2(e_2) \cap G_2(e_2)) = \\ &I(e_1) \times J(e_2) = (I \times J)(e_1, e_2).\end{aligned}$$

Therefore, $(H, E_1 \times E_2) = (I, E_1) \times (J, E_2)$. □

Proposition 3.12. *Let* (X, τ_1, E_1) *and* (Y, τ_2, E_2) *be soft spaces. Let* $B = \{(F, E_1) \times (G, E_2) | (F, E_1) \in \tau_1,\ (G, E_2) \in \tau_2\}$ *and* τ *be the collection of all arbitrary union of elements of* B. *Then* τ *is a soft topology over* $X \times Y$.

Proof. We have $\Phi_{E_1} = (\phi_1, E_1) \in \tau_1$ and $\Phi_{E_2} = (\phi_2, E_2) \in \tau_2$. Then, by Proposition 3.11, $\Phi_{E_1} \times \Phi_{E_2} = \Phi_{E_1 \times E_2} \in \tau$. Moreover $\widetilde{X} = (X, E_1) \in \tau_1$ and $\widetilde{Y} = (Y, E_2) \in \tau_2$. Then $\widetilde{X} \times \widetilde{Y} = (X \times Y, E_1 \times E_2)$ such that the following holds

$$(X \times Y)(e_1, e_2) = X(e_1) \times Y(e_2) = X \times Y,$$

for each $(e_1, e_2) \in E_1 \times E_2$. Therefore, $\widetilde{X} \times \widetilde{Y} = \widetilde{X \times Y} \in \tau$. Let $(F, E_1 \times E_2)$, $(G, E_1 \times E_2) \in \tau$. There exist the elements $(F_\alpha, E_1) \times (G_\alpha, E_2)$, $(F_\beta, E_1) \times (G_\beta, E_2)$, $\alpha \in I$, $\beta \in J$, of B such that

$$\begin{aligned}(F, E_1 \times E_2) &= \bigcup_{\alpha \in I}((F_\alpha \times G_\alpha, E_1 \times E_2)),\\ (G, E_1 \times E_2) &= \bigcup_{\beta \in J}((F_\beta \times G_\beta, E_1 \times E_2)).\end{aligned}$$

Let $(H, E_1 \times E_2) = (F, E_1 \times E_2) \cap (G, E_1 \times E_2)$. Then, we have

$$\begin{aligned}H(e_1, e_2) &= F(e_1, e_2) \cap G(e_1, e_2)\\ &= (\bigcup_{\alpha \in I}(F_\alpha(e_1) \times G_\alpha(e_2))) \cap (\bigcup_{\beta \in J}(F_\beta(e_1) \times G_\beta(e_2)))\\ &= \bigcup_{\beta \in J}\Big[\bigcup_{\alpha \in J}((F_\alpha(e_1) \times G_\alpha(e_2))) \cap (F_\beta(e_1) \times G_\beta(e_2))\Big]\\ &= \bigcup_{\beta \in J}\bigcup_{\alpha \in I}((F_\alpha(e_1) \times G_\alpha(e_2)) \cap (F_\beta(e_1) \times G_\beta(e_2)))\\ &= \bigcup_{\beta \in J}\bigcup_{\alpha \in I}((F_\alpha(e_1) \cap F_\beta(e_1)) \times (G_\alpha(e_2) \cap G_\beta(e_2)))\\ &= \bigcup_{\alpha \in I, \beta \in J}((F_\alpha \cap F_\beta)(e_1) \times (G_\alpha \cap G_\beta)(e_2))\\ &= \bigcup_{\alpha \in I, \beta \in J}(F_\alpha \cap F_\beta \times G_\alpha \cap G_\beta)(e_1, e_2).\end{aligned}$$

This shows that

$$\begin{aligned}(H, E_1 \times E_2) = &\bigcup_{\alpha \in I, \beta \in J}((F_\alpha \cap F_\beta) \times (G_\alpha \cap G_\beta), E_1 \times E_2) =\\ &\bigcup_{\alpha \in I, \beta \in J}((F_\alpha \cap F_\beta, E_1) \times (G_\alpha \cap G_\beta, E_2)).\end{aligned}$$

This implies that $(H, E_1 \times E_2) \in \tau$. Finally, It is obvious that an arbitrary union of elements of τ is an elements in τ. This completes the proof. □

Let (X, τ_1, E_1) and (Y, τ_2, E_2) be soft spaces. Then the soft space $(X \times Y, \tau, E_1 \times E_2)$ as defined in previous proposition is called soft product topological space over $X \times Y$.

Proposition 3.13. *Let (F, E_1) and (G, E_2) be soft sets in $SS(X)_{E_1}$ and $SS(Y)_{E_2}$, respectively. Then,*

$$((F, E_1) \times (G, E_2))' = ((F, E_1)' \times \widetilde{Y}) \cup (\widetilde{X} \times (G, E_2)').$$

Proof. Let $(F \times G, E_1 \times E_2)' = ((F \times G)', E_1 \times E_2)$. Then,

$$(F \times G)'(e_1, e_2) = (X \times Y) - (F(e_1) \times G(e_2)) = [(X - F(e_1)) \times Y] \cup [X \times (Y - G(e_2))].$$

On the other hand,

$$((F, E_1)' \times \widetilde{Y}) \cup (\widetilde{X} \times (G, E_2)') = (F' \times Y, E_1 \times E_2) \cup (X \times G', E_1 \times E_2).$$

Let us denote this soft set by $(H, E_1 \times E_2)$. Then we have

$$\begin{aligned} H(e_1, e_2) &= (F' \times Y)(e_1, e_2) \cup (X \times G')(e_1, e_2) = (F'(e_1) \times Y) \cup (X \times G'(e_2)) \\ &= ((X - F(e_1)) \times Y) \cup (X \times (Y - G(e_2))). \end{aligned}$$

This completes the proof. □

Corollary 3.14. *Let (F, E_1) and (G, E_2) be soft closed set in soft topological spaces (X, τ_1, E_1) and (Y, τ_2, E_2), respectively. Then $(F, E_1) \times (G, E_2)$ is soft closed set in soft product space $(X \times Y, \tau, E_1 \times E_2)$.*

Proof. It is obvious that $(F, E_1)'$, $\widetilde{X}$ are soft open sets in (X, τ_1, E_1) and $(G, E_2)'$, $\widetilde{Y}$ are soft open sets in (Y, τ_2, E_2). Now, Proposition 3.13 implies that $((F, E_1) \times (G, E_2))'$ is soft open in $(X \times Y, \tau, E_1 \times E_2)$. This completes the proof. □

3.3 Soft Hausdorff Topological Spcase

We are going to define soft Hausdorff topological spaces [5, 10, 11] and study some properties of these spaces.

Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$. If there exist soft open sets (F, E) and (G, E) such that $x \in (F, E), y \in (G, E)$ and $(F, E) \cap (G, E) = \Phi_E$, then (X, τ, E) is called a soft T_2-space or soft Hausdorff.

Proposition 3.15. *Let (F, E_1) and (G, E_2) be soft sets in $SS(X)_{E_1}$ and $SS(Y)_{E_2}$, respectively. Suppose that $x \in (F, E_1)$ and $y \in (G, E_2)$. Then $(x, y) \in (F, E_1) \times (G, E_2)$, and vice versa.*

Proof. By the hypothesis, we have $x \in \bigcap_{e_1 \in E_1} F(e_1)$ and $y \in \bigcap_{e_2 \in E_2} G(e_2)$. Therefore,

$$\begin{aligned} (x, y) \in (\bigcap_{e_1 \in E_1} F(e_1)) \times (\bigcap_{e_2 \in E_2} G(e_2)) &= \bigcap_{(e_1, e_2) \in E_1 \times E_2} (F(e_1) \times G(e_2)) \\ &= \bigcap_{(e_1, e_2) \in E_1 \times E_2} (F \times G)(e_1, e_2). \end{aligned}$$

This shows that $(x, y) \in (F, E_1) \times (G, E_2)$. Conversely is similar. □

Proposition 3.16. *The product of two soft Hausdorff spaces is soft Hausdorff.*

Proof. Let (X, τ_1, E_1) and (Y, τ_2, E_2) be soft Hausdorff spaces. we consider distinct points (x_1, y_1) and (x_2, y_2) of $X \times Y$. Without loss of generality let $x_1 \neq x_2$. Then there exist soft open sets (F, E_1) and (G, E_1) in (X, τ, E_1) such that $x_1 \in (F, E_1), x_2 \in (G, E_1)$ and $(F, E_1) \cap (G, E_1) = \Phi_{E_1}$. By Proposition 3.15 we have,

$$(x_1, y_1) \in (F, E_1) \times \widetilde{Y}, \quad (x_2, y_2) \in (G, E_1) \times \widetilde{Y}.$$

These soft sets are soft open in $(X \times Y, \tau, E_1 \times E_2)$. Finally Proposition 3.11 shows that

$$((F, E_1) \times \widetilde{Y}) \cap ((G, E_1) \times \widetilde{Y}) = \Phi_{E_1 \times E_2}.$$

This completes the proof. □

Proposition 3.17. *Let $\{(F_\alpha, B)\}_{\alpha \in J}$ be an arbitrary family of soft sets in $SS(V)_B$. Then, $f_{pu}^{-1}(\cup_{\alpha \in J}(F_\alpha, B)) = \cup_{\alpha \in J} f_{pu}^{-1}(F_\alpha, B)$*

Proof. Let $\cup_{\alpha \in J}(F_\alpha, B) = (F, B)$, where $F(b) = \cup_{\alpha \in J} F_\alpha(b)$, for each $b \in B$. Then $f_{pu}^{-1}(F, B) = (f_{pu}^{-1}(F), A)$, where

$$f_{pu}^{-1}(F)(a) = u^{-1}(F(p(a))) = u^{-1}(\cup_{\alpha \in J} F_\alpha(p(a))) = \cup_{\alpha \in J} u^{-1}(F_\alpha(p(a))),$$

for each $a \in A$. On the other hand if

$$\cup_{\alpha \in J} f_{pu}^{-1}(F_\alpha, B) = \cup_{\alpha \in J}(f_{pu}^{-1}(F_\alpha), A) = (G, A),$$

then,

$$G(a) = \cup_{\alpha \in J} f_{pu}^{-1}(F_\alpha)(a) = \cup_{\alpha \in J} u^{-1}(F_\alpha(p(a))),$$

for each $a \in A$. This completes the proof. □

Lemma 3.18. *Let the soft topological space (V, τ', B) is given by soft basis $\mathcal{B}$. Then f_{pu} is soft pu-continuous if the inverse image of every soft basis element is soft open.*

Proof. We consider $f_{pu} : SS(U)_A \to SS(V)_B$. Let (F, B) be a soft open set in soft space (V, τ', B). We can write

$$(F, B) = \cup_{\alpha \in J}(F_\alpha, B),$$

where $\mathcal{B} = \{(F_\beta, B)\}_{\beta \in I}$ is a soft basis of (V, τ, B) and $J \subseteq I$. By Proposition 3.17, we have

$$f_{pu}^{-1}(F, B) = \cup_{\alpha \in J} f_{pu}^{-1}(F_\alpha, B),$$

that is a soft open set in (U, τ, A). □

Let (X, τ_1, E) and (X, τ_2, E) be soft topological spaces. Then the following hold:

(i) if $\tau_1 \subseteq \tau_2$, then τ_2 is soft finer than τ_1;

(ii) if $\tau_1 \subset \tau_2$, then τ_2 is soft strictly finer than τ_1;

(iii) if $\tau_1 \subseteq \tau_2$ or $\tau_2 \subseteq \tau_1$, then τ_1 is soft comparable with τ_2.

Then, we have the following.

Proposition 3.19. *Let (X, τ_2, E) be a soft connected space and $\tau_1 \subseteq \tau_2$. Then (X, τ_1, E) is soft connected.*

Proof. Suppose to the contrary that (F, E), (G, E) is a soft separation of $\widetilde{X}$ with soft topology τ_1. Since $\tau_1 \subseteq \tau_2$, then (F, E) and (G, E) is a soft separation of $\widetilde{X}$ with soft topology τ_2. This is a contradiction. Therefore (X, τ_1, E) is soft connected. □

Definition 3.20. Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then the soft closure of (F, E) denoted by $\overline{(F, E)}$ is the intersection of all soft closed super sets of $\overline{(F, E)}$.

Proposition 3.21. *Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. If $x \in \overline{(F, E)}$, then every soft open set (U, E) containing x intersects (F, E).*

Proof. Let $x \in \overline{(F, E)}$. Let there is a soft open set (U, E) containing x such that $(F, E) \cap (U, E) = \Phi_E$. Proposition 3.6 of [12] shows that $(F, E)\widetilde{\subseteq}(U, E)'$. Therefore $\overline{(F, E)}\widetilde{\subseteq}(U, E)'$. Hence $x \in (U, E) \cap (U, E)'$. This is a contradiction. Therefore $(F, E) \cap (U, E) \neq \Phi_E$. □

The following example shows that the converse of Proposition 3.21 is not true.

Example 3.22. Let

$$X = \{h_1, h_2, h_3\},\ E = \{e_1, e_2\},\quad \tau = \{\Phi_E, \widetilde{X}, (F_1, E), (F_2, E), \ldots, (F_{30}, E)\},$$

where $F_1, F_2, \ldots, F_{30}$ are given in Example 9 of [11]. Then (X, τ) is a soft topological space over X. We consider the soft set (F_{25}, E), where

$$F_{25}(e_1) = \{h_2\}, \quad F_{25}(e_2) = X.$$

It is easy to see that the following hold

$$\overline{(F_{25}, E)} = (F_{25}, E), \quad h_1 \notin \overline{(F_{25}, E)}.$$

But for every soft open set (F, E) containing h_1 we have $(F, E) \cap (F_{25}, E) \neq \Phi_E$.

Theorem 3.23. *Let* (Y, τ_Y, E) *be a soft connected subspace of* (X, τ, E). *If* $(Y, E)\widetilde{\subseteq}(Z, E)\widetilde{\subseteq}\overline{(Y, E)}$, *then* (Z, τ_Z, E) *is also soft connected.*

Proof. Suppose to the contrary that $(Z, E) \cap (U, E)$, $(Z, E) \cap (V, E)$ is a soft separation of $\widetilde{Z}$. By Proposition 3.4, we have

$$\widetilde{Y} = [(Y, E) \cap (U, E)] \cup [(Y, E) \cap (V, E)].$$

It is easy to see that

$$((Y, E) \cap (U, E)) \cap ((Y, E) \cap (V, E)) = \Phi_E.$$

If $(Y, E) \cap (U, E) = \Phi_E$, then

$$(Y, E)\widetilde{\subseteq}(U, E)'.$$

Therefore $(Z, E)\widetilde{\subseteq}\overline{(Y, E)}\widetilde{\subseteq}(U, E)'$. This shows that $(Z, E) \cap (U, E) = \Phi_E$. This is a contradiction. Hence

$$(Y, E) \cap (U, E) \neq \Phi_E.$$

By a similar reason, we have $(Y, E) \cap (V, E) \neq \Phi_E$. Therefore $(Y, E) \cap (U, E)$, $(Y, E) \cap (V, E)$ is a soft separation of $\widetilde{Y}$. This is a contradiction. Therefore (Z, τ_Z, E) is soft connected. □

Remark: There are some differences between topological space and soft topological spaces. The following examples exhibit some of them.

Example 3.24. Let X be a nonempty set, $E = \{e_1, e_2\}$ and $\tau = \{\Phi_E, \widetilde{X}, (F_1, E), (F_2, E)\}$ where

$$F_1(e_1) = \emptyset, \quad F_1(e_2) = X, \quad F_2(e_1) = X, \quad F_2(e_2) = \emptyset.$$

Then (X, τ, E) is a soft topological space and it is easy to see that (F_1, E), (F_2, E) is a soft separation of (X, E). Therefore a soft space (X, τ, E) with $|X| = 1$ can be soft disconnected.

Example 3.25. Let X be a nonempty set, $Y = \{h\}$, $E = \{e_1, e_2\}$, $\tau_1 = \{(F_1, E), (F_2, E)\}$ and $\tau_2 = \{(G_1, E), (G_2, E), (G_3, E), (G_4, E)\}$ where,

$$F_1(e_1) = F_1(e_2) = X, \quad F_2(e_1) = F_2(e_2) = \emptyset,$$

and

$$G_1(e_1) = G_2(e_2) = G_3(e_1) = G_3(e_2) = Y, G_1(e_2) = G_2(e_1) = G_4(e_1) = G_4(e_2) = \emptyset.$$

Then (X, τ_1, E) and (Y, τ_2, E) are soft spaces. Now, we consider soft open sets $(F_1 \times G_1, E \times E)$ and $(F_1 \times G_2, E \times E)$ of the soft product space. Then

$$(F_1 \times G_1)(e_1, e_1) = (F_1 \times G_1)(e_2, e_1) = (F_1 \times G_2)(e_1, e_2) = (F_1 \times G_2)(e_2, e_2) = X \times Y,$$

and

$$(F_1 \times G_1)(e_1, e_2) = (F_1 \times G_1)(e_2, e_2) = (F_1 \times G_2)(e_1, e_1) = (F_1 \times G_2)(e_2, e_1) = \emptyset.$$

Therefore $(F_1 \times G_1, E \times E)$, $(F_1 \times G_2, E \times E)$ is a soft separation of $X \times Y$. Hence, the soft space $(X \times Y, E \times E)$ is not soft connected.

Theorem 3.26. *Let (X, τ_1, E) and (Y, τ_2, E) be two soft connected topological spaces. Let each soft subset $(\{x\}, E)$ be soft connected as a soft subspace of (X, τ_1, E). Then the soft cartesian product of these two soft spaces is soft connected.*

Proof. We exhibit the proof of this theorem in three steps.

Step1. We show that there exists a soft subspace $(\{y\}, E)$ of (Y, E) that is soft connected. Suppose to the contrary that for every $y \in Y$, $(\{y\}, E)$ has a soft separation (F_{yy}, E), (G_{yy}, E), such that $F_{yy}(e) = F_y(e) \cap \{y\}$ and $G_{yy}(e) = G_y(e) \cap \{y\}$, where (F_y, E) and (G_y, E) are soft open sets over X. Then

$$(Y, E) = \bigcup_{y \in Y}(\{y\}, E) = \bigcup_{y \in Y}((F_{yy}, E) \cup (G_{yy}, E)) \widetilde{\subseteq} (\bigcup_{y \in Y} F_y, E) \cup (\bigcup_{y \in Y} G_y, E).$$

Obviously, $(\cup_{y \in Y} F_y, E)$ and $(\cup_{y \in Y} G_y, E)$ are different from Φ_E. Let $(\cup_{y \in Y} F_y, E) \cap (\cup_{y \in Y} G_y, E) \neq \Phi_E$. Therefore $F_y(e) \cap G_{y'}(e) \neq \emptyset$, for some $e \in E$ and $y, y' \in Y$. This implies that $y = y'$. This is a contradiction. Therefore $(\cup_{y \in Y} F_{yy}, E)$, $(\cup_{y \in Y} G_{yy}, E)$ is a soft separation of (Y, E) that is impossible.

Step2. We choose a base point $(x, y) \in X \times Y$ and by Step 1, we can assume that $(\{y\}, E)$ is soft connected. Suppose to the contrary that $(X \times \{y\}, E \times E)$ is not soft connected. Therefore there exists a soft separation $\cup_{\alpha \in A} \cup_{\beta \in B} ((F_\alpha, E) \times (G_\beta, E))$, $\cup_{\gamma \in C} \cup_{\eta \in D} ((F_\gamma, E) \times (G_\eta, E))$ of $X \times \{y\}$. We can deduce that

$$(X \times \{y\}, E \times E) = (\bigcup_{\alpha \in A} \bigcup_{\beta \in B} (F_\alpha \times G_\beta), E \times E) \cup (\bigcup_{\gamma \in C} \bigcup_{\eta \in D} (F_\gamma \times G_\eta), E \times E),$$

and

$$\begin{aligned} &(\bigcup_{\alpha \in A} \bigcup_{\beta \in B} (F_\alpha \times G_\beta), E \times E) \cap (\bigcup_{\gamma \in C} \bigcup_{\eta \in D} (F_\gamma \times G_\eta), E \times E) \\ &= (\bigcup_{\alpha \in A} \bigcup_{\beta \in B} \bigcup_{\gamma \in C} \bigcup_{\eta \in D} ((F_\alpha \cap F_\gamma) \times (G_\beta \cap G_\eta)), E \times E) = \Phi_{E \times E} \end{aligned}$$

Moreover,

$$(\{y\}, E) = (\bigcup_{\beta \in B} (G_\beta, E)) \cup (\bigcup_{\eta \in D} (G_\eta, E)) = (\bigcup_{\beta \in B} G_\beta, E) \cup (\bigcup_{\eta \in D} G_\eta, E).$$

It is easy to see that $(\cup_{\beta \in B} G_\beta, E)$ and $(\cup_{\eta \in D} G_\eta, E)$ are different from Φ_E. Since $(\{y\}, E)$ is soft connected, we have $(\cup_{\beta \in B} G_\beta, E) \cap (\cup_{\eta \in D} G_\eta, E) \neq \Phi_E$. Therefore

$$(\bigcup_{\beta \in B} \bigcup_{\eta \in D} (G_\beta \cap G_\eta), E) \neq \Phi_E.$$

This implies that $G_\beta(e) \cap G_\eta(e) \neq \emptyset$ for some $e \in E$, $\beta \in B$ and $\eta \in D$. On the other hand $(X, E) = (\cup_{\alpha \in A} F_\alpha, E) \cup (\cup_{\gamma \in C} F_\gamma, E)$. Since (X, E) is soft connected then $(F_\alpha \cap F_\gamma)(e) \neq \emptyset$, for some $e' \in E$, $\alpha \in A$ and $\gamma \in C$. Therefore $((F_\alpha \cap F_\gamma) \times (G_\beta \cap G_\gamma))(e', e) \neq \emptyset$. Hence

$$(\bigcup_{\alpha \in A} \bigcup_{\beta \in B} \bigcup_{\gamma \in C} \bigcup_{\eta \in D} ((F_\alpha \cap F_\gamma) \times (G_\beta \cap G_\eta)), E \times E) \neq \Phi_{E \times E}.$$

This is a contradiction. Therefore $(X \times \{y\}, E \times E)$ is soft connected. By hypothesis and a similar way $(\{x\}' \times Y, E \times E)$ is soft connected, for each $x' \in X$.

Step 3. Now, we complete the proof. As a result $(T_{x'}, E \times E) = ((\{x'\} \times Y) \cup (X \times \{y\}), E \times E)$ is soft connected, for each $x' \in X$, being the union of two soft connected subspace that have non-null intersection. It is easy to see that $(X \times Y, E \times E)$ is soft connected, where $(X \times Y, E \times E) = \cup_{x' \in X}((\{x'\} \times Y) \cup (X \times \{y\}), E \times E)$, because it is the union of a collection of soft connected subspace that have non-null intersection containing $(\{x\} \times \{y\}, E \times E)$. This completes the proof. □

Question. At the end we pose a natural question here: Is the soft cartesian product of two soft connected spaces soft connected?

References

[1] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, *On some new operations in soft set theory*, Comput. Math. Appl. **57**(2009), 1547-1553.

[2] A. Aygunoglu and H. Aygun, *Some notes on soft topological spaces*, Neural. Comput. Appl. (2011), 1-7.

[3] N. Çağman and S. Enginoglu, *Soft set theory and uniint decision making*, Europ. J. Operat. Res, **207**(2010), 848-855.

[4] N. Çağman and S. Enginoglu, *Soft matrix theory and its decision making*, Comput. Math. Appl. **59**(2010), 3308-3314.

[5] N. Çağman, S. Karatas and S. Enginoglu, *Soft topology*, Comput. Math. Appl. **62**(2011), 351-358.

[6] S. Hussain and B. Ahmad, *Some properties of soft topological spaces*, Comput. Math. Appl. **62**(2011), 4058-4067.

[7] P. K. Maji, R. Biswas and A. R. Roy, *Soft set theory*, Comput. Math. Appl. **45**(2003), 555-562.

[8] D. A. Molodtsov, *Soft set theory-first results*, Comput. Math. Appl. **37**(1999), 19-31.

[9] D. A. Molodtsov, V. Y. Leonov and D. V. Kovkov, *Soft sets technique and its application*, Nechetkie Sistemy i Myagkie Vychisleniya, 1 (1)(2006), 8-39.

[10] W. K. Min, *A note on soft topological spaces*, Comput. Math. Appl. **62**(2011), 3524-3528.

[11] M. Shabir and M. Naz, *On soft topological spaces*, Comput. Math. Appl. **61**(2011), 1786-1799.

[12] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, *Remarks on soft topological spaces*, Annal. Fuzzy. Math. Inform., 3 (2012), 171- 185.