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Cost Varying Interval Transportation Problem under Two Vehicle

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Abstract – In this paper we represent a two-vehicle cost varying interval transportation model (TVCVITM). To determine the cost interval of cost parameters of interval transportation problem (ITP) we use two vehicle cost varying TP. In this model the transportation cost varies due to capacity of vehicles as well as amount of transport quantity. At first we propose an algorithm to determine limits of the interval of unit transportation cost. This is an uncertain multi-level programming model. Then formulate corresponding multi-objective crisp model. To solve this, apply fuzzy programming technique. A numerical example is presented to illustrate the TVCVITM.

Keywords – Interval Transportation Problem, Cost Varying Transportation Problem, Multi-level Programming, Fuzzy Programming.

1 Introduction

Transportation problem of linear programming problem which deals with the distribution of single commodity from various sources of supply to various destination of demand in such a manner that the total transportation cost is minimized. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be fixed at crisp values but in real life applications unit transportation cost may vary.

A interval transportation problem is such a transportation problem in which the supply and demand and cost parameters are lied in some intervals. This problem is

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transformed into a classical bi-objective TP where to minimize the interval objective function, the order relations that represent the decision maker's preference between interval profits is defined by the right limit, left limit, centre, and half-width of an interval.

In transportation problem unit transportation cost is constant from each source to each destination. But in reality, it is not constant; it depends on amount of transport quantity and capacity of vehicles. If amount of quantity is small then small(capacity) vehicle is sufficient for deliver. Where as if amount of quantity is large then big(capacity) vehicle is needed. So, depend on amount of transport quantity and the capacity of vehicles, the unit transportation cost is not constant. The cost varying transportation problem is such a transportation problem where unit transportation cost is varied depending on the selection of vehicles and number of vehicles.

The basic transportation problem was originally developed by Hitchcock [14] and later by Dantzig [6]. Many researchers [13, 15, 18] did work on fixed charge transportation problem. Gupta and Arora [8] presented a capacitated fixed charge bi-criterion indefinite quadratic transportation problem, giving the same priority to cost as well as time is studied. They developed an algorithm which is based on the concept of solving the indefinite quadratic fixed charge transportation problem. Gupta and Arora [11] discussed on a paradox in a capacitated transportation problem where the objective function is a ratio of two linear functions consisting of variable costs and profits respectively. In another paper, Gupta and Arora [9, 10] discussed on restricted flow in a fixed charge capacitated transportation problem with bounds on total source availabilities and total destination requirements. Dahiya and Verma [5] considered a class of the capacitated transportation problems with bounds on total availabilities at sources and total destination requirements. In this paper, unbalanced capacitated transportation problems have been discussed in the present paper as a particular case of original problem. In addition, they have discussed paradoxical situation in a balanced capacitated transportation problem and have obtained the paradoxical solution by solving one of the unbalanced problems. Arora and Ahuja [1] discussed a paradox in fixed charge transportation problem. Then Arora and Khurana [2] introduced three-dimensional fixed charge transportation problem is an extension of the classical three-dimensional transportation problem in which a fixed cost is incurred for every origin. Basu et. al. [3] represented an algorithm for finding the optimum solution of solid fixed charge transportation problem. Then Bit, et. al. developed fuzzy programming technique for multi objective capacitated transportation problem. Singh and Saxena [16] introduced the multiobjective time transportation problem with additional restrictions. Recently, Dutta and Murthy [7] developed fuzzy transportation problem with additional restrictions.

Here we present interval transportation problem. In reality, the interval of the unit cost depending on the interval of sources and demands. In urban region, actually the transportation cost is not depends on the quantities but on the capacity of the transports. So unit cost is vary depended on vehicles. In this paper we determine interval of the parameters of unit cost by our proposed algorithm which develops a multi-level uncertain programming model. Then formulate corresponding multi-objective crisp model. There are various type of methods to solve this type of model, but best one is fuzzy programming technique [20] which is applied here.

2 Preliminary

Definition 1 (Interval) A closed interval is defined by an order pair of brackets as:

$$A = [a_L, a_R] = \{a : a_L \leq a \leq a_R, a \in R\}$$

where a_L and a_R are, respectively, the left and right limits of A .

The interval is also denoted by its centre and half width as

$$A = \langle a_c, a_w \rangle = \{a : a_c - a_w \leq a \leq a_c + a_w, a \in R\}$$

where $a_c = \frac{a_R + a_L}{2}$ and $a_w = \frac{a_R - a_L}{2}$ are respectively, the centre and half width of A .

Definition 2 (Operators) If A and B are two closed intervals, and $*$ be a binary operation on the set of real number, then $A * B = \{a * b : a \in A, b \in B\}$ is defined a binary operation.

According to the above definition interval operations are defined as:

$$A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R]$$

$$A + B = \langle a_c, a_w \rangle + \langle b_c, b_w \rangle = \langle a_c + b_c, a_w + b_w \rangle$$

$$kA = k[a_L, a_R] = [ka_L, ka_R] \quad \text{if } k \geq 0$$

$$kA = k[a_L, a_R] = [ka_R, ka_L] \quad \text{if } k \leq 0$$

where k is a real number.

Definition 3 (Order relation \leq_{LR}) The order relation \leq_{LR} between $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is defined as

$$A \leq_{LR} B \text{ iff } a_L \leq b_L \text{ and } a_R \leq b_R$$

$$A <_{LR} B \text{ iff } A \leq_{LR} B \text{ and } A \neq B$$

Definition 4 (Order relation \leq_{cw}) The order relation \leq_{cw} between $A = \langle a_c, a_w \rangle$ and $B = \langle b_c, b_w \rangle$ is defined as

$$A \leq_{cw} B \text{ iff } a_c \leq b_c \text{ and } a_w \leq b_w$$

$$A <_{cw} B \text{ iff } A \leq_{cw} B \text{ and } A \neq B.$$

2.1 Interval Transportation Problem(ITP)

The formulation of ITP is the problem of minimizing interval valued objective function with interval costs, interval sources and interval demands parameters, is given in the following **Model 1**.

Model 1:

$$\begin{aligned}
 & \min \quad \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \\
 \text{subject to} \quad & c_{ij} \in [D_{L_{ij}}, D_{R_{ij}}] \\
 & \sum_{j=1}^n x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \dots, m \\
 & \sum_{i=1}^m x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \dots, n \\
 & \sum_{i=1}^m a_{L_i} = \sum_{j=1}^n b_{L_j} \\
 & \sum_{i=1}^m a_{R_i} = \sum_{j=1}^n b_{R_j} \\
 & x_{ij} \geq 0 \quad \forall i, \quad \forall j
 \end{aligned}$$

where $c_{ij} \in [D_{L_{ij}}, D_{R_{ij}}]$ is an interval representing the uncertain cost for transportation problem. The sources parameter lies in $[a_{L_i}, a_{R_i}]$ and destination parameter lies in $[b_{L_j}, b_{R_j}]$.

Depending on $[a_{L_i}, a_{R_i}]$ and $[b_{L_j}, b_{R_j}]$. We determine $c_{L_{ij}}$ and $c_{R_{ij}}$ which is discussed in the following subsection. Then we define $D_{L_{ij}=\min\{c_{L_{ij}}, c_{R_{ij}}\}}$ and $D_{R_{ij}=\max\{c_{L_{ij}}, c_{R_{ij}}\}}$

2.2 2-vehicle cost varying transportation problem

Suppose there are two types off vehicles V_1, V_2 from each source to each destination. Let C_1 and $C_2 (> C_1)$ are the capacities(in unit) of the vehicles V_1 and V_2 respectively, $C_1 \leq r_{ij}$ and $C_2 \leq r_{ij} \forall i, j$. $R_{ij} = (R_{ij}^1, R_{ij}^2)$ represent transportation cost for each cell (i, j) ; where R_{ij}^1 is the transportation cost from source $O_i, i = 1, \dots, m$ to the destination $D_j, j = 1, \dots, n$ by the vehicle V_1 . And R_{ij}^2 is the transportation cost from source $O_i, i = 1, \dots, m$ to the destination $D_j, j = 1, \dots, n$ by the vehicle V_2 . So, cost varying transportation problem can be represent in the following tabulated form.

	D_1	D_2	..	D_n	stock
O_1	R_{11}^1, R_{11}^2	R_{12}^1, R_{12}^2	R_{1n}^1, R_{1n}^2	a_1
O_2	R_{21}^1, R_{21}^2	R_{22}^1, R_{22}^2	R_{2n}^1, R_{2n}^2	a_2
....
O_m	R_{m1}^1, R_{m1}^2	R_{m2}^1, R_{m2}^2	R_{mn}^1, R_{mn}^2	a_m
Demand	b_1	b_2	b_n	

Table T1: Tabular representation of cost varying transportation problem.

2.2.1 Determination of $c_{L_{ij}}$

To solve this problem, apply our proposed **Algorithms** stated as follows:

Algorithm A1:

Step 1. Since lower limit of unit cost is not determined (because it depends on quantity of transport), so North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S.

Step 2. After the allocate x_{ij} by North-west corner rule, for basic cell we determine $c_{L_{ij}}$ (unit transportation cost from source O_i to destination D_j) as

$$c_{L_{ij}} = \begin{cases} \frac{pL1_{ij}R1_{ij}+pL2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases} \tag{1}$$

where $pL1_{ij}, pL2_{ij}, i = 1, \dots, m; j = 1, \dots, n$ are integer solution of
 min $pL1_{ij}R1_{ij} + pL2_{ij}R2_{ij}$
 s.t. $x_{ij} \leq pL1_{ij}C_1 + pL2_{ij}C_2$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation be x_{ij} , then for non-basic cell $c_{L_{ij}}$ (unit transportation cost from source O_i to destination D_j) as

$$c_{L_{ij}} = \begin{cases} \frac{pL1_{ij}R1_{ij}+pL2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases} \tag{2}$$

where $pL1_{ij}, pL2_{ij}, i = 1, \dots, m; j = 1, \dots, n$ are integer solution of
 min $pL1_{ij}R1_{ij} + pL2_{ij}R2_{ij}$
 s.t. $x_{ij} \leq pL1_{ij}C_1 + pL2_{ij}C_2$

In this manner we convert cost varying transportation problem to a usual transportation problem but $c_{L_{ij}}$ is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix $c_{L_{ij}}$ by **Step 2** and for non-basic we fix $c_{L_{ij}}$ by **Step 3**.

Step 5. Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.

Thus to determine $c_{L_{ij}}$ we solve the following bi-level programming model **Model 2.L** which is as follows:

Model 2.L:

$$\min \sum_{i=1}^m \sum_{j=1}^n c_{Lij} x_{ij}, \tag{3}$$

where, c_{Lij} is determined by following mathematical programming

$$c_{Lij} = \begin{cases} \frac{pL1_{ij}R1_{ij}+pL2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

$$\begin{aligned} \min \quad & pL1_{ij}R1_{ij} + pL2_{ij}R2_{ij} \\ \text{s. t. } \quad & x_{ij} \leq pL1_{ij}C_1 + pL2_{ij}C_2 \end{aligned} \tag{4}$$

$$\sum_{j=1}^n x_{ij} = a_{Li}, \quad i = 1, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_{Lj}, \quad j = 1, \dots, n$$

$$\sum_{i=1}^m a_{Li} = \sum_{j=1}^n b_{Lj}$$

$$0 \leq x_{ij} \quad \forall i, \quad \forall j$$

where $pL1_{ij}, pL2_{ij}, i = 1, \dots, m; j = 1, \dots, n$ are integer

2.2.2 Determination of c_{Rij}

To solve this problem, apply our proposed **Algorithms** stated as follows:

Algorithm A2:

Step 1. Since lower limit of unit cost is not determined (because it depends on quantity of transport), so North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S.

Step 2. After the allocate x_{ij} by North-west corner rule, for basic cell we determine c_{Rij} (unit transportation cost from source O_i to destination D_j) as

$$c_{Rij} = \begin{cases} \frac{pR1_{ij}R1_{ij}+pR2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases} \tag{5}$$

where $pR1_{ij}, pR2_{ij}, i = 1, \dots, m; j = 1, \dots, n$ are integer solution of

$$\begin{aligned} \min \quad & pR1_{ij}R1_{ij} + pR2_{ij}R2_{ij} \\ \text{s.t. } \quad & x_{ij} \leq pR1_{ij}C_1 + pR2_{ij}C_2 \end{aligned}$$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation be x_{ij} , then for

non-basic cell $c_{R_{ij}}$ (unit transportation cost from source O_i to destination D_j) as

$$c_{R_{ij}} = \begin{cases} \frac{pR1_{ij}R1_{ij}+pR2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases} \tag{6}$$

where $pR1_{ij}, pR2_{ij}, i = 1, \dots, m; j = 1, \dots, n$ are integer solution of

$$\begin{aligned} \min \quad & pR1_{ij}R1_{ij} + pR2_{ij}R2_{ij} \\ \text{s.t. } \quad & x_{ij} \leq pR1_{ij}C_1 + pR2_{ij}C_2 \end{aligned}$$

In this manner we convert cost varying transportation problem to a usual transportation problem but $c_{R_{ij}}$ is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix $c_{R_{ij}}$ by **Step 2** and for non-basic we fix $c_{R_{ij}}$ by **Step 3**.

Step 5. Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.

Thus to determine $c_{R_{ij}}$ we solve the following bi-level programming model **Model 2.R** which is as follows:

Model 2.R:

$$\min \sum_{i=1}^m \sum_{j=1}^n c_{R_{ij}} x_{ij}, \tag{7}$$

where, $c_{R_{ij}}$ is determined by following mathematical programming

$$c_{R_{ij}} = \begin{cases} \frac{pR1_{ij}R1_{ij}+pR2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

$$\begin{aligned} \min \quad & pR1_{ij}R1_{ij} + pR2_{ij}R2_{ij} \\ \text{s. t. } \quad & x_{ij} \leq pR1_{ij}C_1 + pR2_{ij}C_2 \end{aligned} \tag{8}$$

$$\sum_{j=1}^n x_{ij} = a_{R_i}, \quad i = 1, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_{R_j}, \quad j = 1, \dots, n$$

$$\sum_{i=1}^m a_{R_i} = \sum_{j=1}^n b_{R_j}$$

$$0 \leq x_{ij} \quad \forall i, \quad \forall j$$

where $pR1_{ij}, pR2_{ij}, i = 1, \dots, m; j = 1, \dots, n$ are integer .

2.2.3 Multi-level Mathematical Programming for Cost Varying Interval Transportation Problem under Two Vehicle (CVITPTV)

The Multi-level Mathematical Programming for Cost Varying Interval Transportation Problem under Two Vehicle is formulated in **Model 3** as follows:

Model 3:

$$\begin{aligned}
 & \min \quad \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \\
 \text{subject to} \quad & c_{ij} \in [D_{L_{ij}}, D_{R_{ij}}] \\
 & \sum_{j=1}^n x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \dots, m \\
 & \sum_{i=1}^m x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \dots, n \\
 & \sum_{i=1}^m a_{L_i} = \sum_{j=1}^n b_{L_j} \\
 & \sum_{i=1}^m a_{R_i} = \sum_{j=1}^n b_{R_j} \\
 & x_{ij} \geq 0 \quad \forall i, \quad \forall j
 \end{aligned}$$

Model 2.L (9)

Model 2.R (10)

i.e., **Model 3**

$$\begin{aligned}
 & \min \quad \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \\
 \text{subject to} \quad & c_{ij} \in [D_{L_{ij}}, D_{R_{ij}}] \\
 & \sum_{j=1}^n x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \dots, m \\
 & \sum_{i=1}^m x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \dots, n \\
 & \sum_{i=1}^m a_{L_i} = \sum_{j=1}^n b_{L_j} \\
 & \sum_{i=1}^m a_{R_i} = \sum_{j=1}^n b_{R_j} \\
 & x_{ij} \geq 0 \quad \forall i, \quad \forall j
 \end{aligned}$$

(11)

where, $D_{L_{ij}} = \min\{c_{L_{ij}}, c_{R_{ij}}\}$ and $D_{R_{ij}} = \max\{c_{L_{ij}}, c_{R_{ij}}\}$. And $[c_{L_{ij}}, c_{R_{ij}}]$ is determined by following mathematical programming

$$\min \sum_{i=1}^m \sum_{j=1}^n c_{L_{ij}} x_{ij}, \tag{12}$$

$$\text{where, } c_{L_{ij}} = \begin{cases} \frac{pL1_{ij}R1_{ij} + pL2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

$$\begin{aligned} \min \quad & pL1_{ij}R1_{ij} + pL2_{ij}R2_{ij} \\ \text{s. t. } \quad & x_{ij} \leq pL1_{ij}C_1 + pL2_{ij}C_2 \end{aligned} \tag{13}$$

$$\sum_{j=1}^n x_{ij} = a_{L_i}, \quad i = 1, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_{L_j}, \quad j = 1, \dots, n$$

$$\sum_{i=1}^m a_{L_i} = \sum_{j=1}^n b_{L_j}$$

$$0 \leq x_{ij} \quad \forall i, \quad \forall j$$

where $pL1_{ij}, pL2_{ij}, i = 1, \dots, m; j = 1, \dots, n$ are integer
AND,

$$\min \sum_{i=1}^m \sum_{j=1}^n c_{R_{ij}} x_{ij},$$

where, c_{ij} is determined by following mathematical programming

$$c_{R_{ij}} = \begin{cases} \frac{pR1_{ij}R1_{ij} + pR2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

$$\min pR1_{ij}R1_{ij} + pR2_{ij}R2_{ij}$$

$$\text{s. t. } x_{ij} \leq pR1_{ij}C_1 + pR2_{ij}C_2$$

$$\sum_{j=1}^n x_{ij} = a_{R_i}, \quad i = 1, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_{R_j}, \quad j = 1, \dots, n$$

$$\sum_{i=1}^m a_{R_i} = \sum_{j=1}^n b_{R_j}$$

$$0 \leq x_{ij} \quad \forall i, \quad \forall j$$

where $pR1_{ij}, pR2_{ij}, i = 1, \dots, m; j = 1, \dots, n$ are integer.

3 Solution Procedure of CVITPTV

3.1 Formulation of the crisp objective function

Let S be the set of all feasible solution of **Model 3**.

Definition 5: Optimal Solution: $x^0 \in S$ is an optimal solution of the **Model 3** iff there is no other solution $x \in S$ which satisfies $Z(x) <_{LR} Z(x^0)$ or $Z(x) <_{cw} Z(x^0)$.

Definition 6 (Order relation \leq_{Rc}) The order relation \leq_{Rc} between A and B is defined as

$$\begin{aligned} A \leq_{Rc} B &\text{ iff } A \leq_{LR} B \text{ and } A \leq_{cw} B \\ A <_{Rc} B &\text{ iff } A <_{LR} B \text{ and } A <_{cw} B \end{aligned}$$

Definition 7: Optimal Solution: $x^0 \in S$ is an optimal solution of the **Model 3** iff there is no other solution $x \in S$ which satisfies $Z(x) <_{Rc} Z(x^0)$.

The right limit $Z_R(x)$ of the interval objective function $Z(x)$ in given problem may be elicited as

$$Z_R(x) = \sum_{i=1}^m \sum_{j=1}^n D_{c_{ij}} x_{ij} + \sum_{i=1}^m \sum_{j=1}^n D_{w_{ij}} x_{ij}$$

The centre of the objective function $Z_c(x)$ can be elicited as

$$Z_c(x) = \sum_{i=1}^m \sum_{j=1}^n D_{c_{ij}} x_{ij}$$

The solution of the **Model 3** by definition 7 can be taken as the optimal solution of following model **Model 4**.

Model 4:

$$\begin{aligned} \min \quad & Z_R(x) = \sum_{i=1}^m \sum_{j=1}^n D_{c_{ij}} x_{ij} + \sum_{i=1}^m \sum_{j=1}^n D_{w_{ij}} x_{ij} \\ \min \quad & Z_c(x) = \sum_{i=1}^m \sum_{j=1}^n D_{c_{ij}} x_{ij} \\ \text{s. t.,} \quad & \sum_{j=1}^n x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \dots, m \\ & \sum_{i=1}^m x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \dots, n \\ & \sum_{i=1}^m a_{L_i} = \sum_{j=1}^n b_{L_j} \\ & \sum_{i=1}^m a_{R_i} = \sum_{j=1}^n b_{R_j} \\ & x_{ij} \geq 0 \quad \forall i, \quad \forall j \end{aligned}$$

Model 2.L

Model 2.R

3.1.1 Fuzzy Programming Technique to solve Model 4

In fuzzy programming technique, we first find the lower bound as LZ_R and the upper bound as UZ_R for the r^{th} objective function $Z_R(x)$. Similarly the lower bound as LZ_c and the upper bound as UZ_c for the r^{th} objective function $Z_c(x)$.

$dZ_R = UZ_R - LZ_R$ the degradation allowance for objective $Z_R(x)$. $dZ_c = UZ_c - LZ_{Rc}$ the degradation allowance for objective $Z_c(x)$

When the aspiration levels for each of the objective have been specified, a fuzzy model is formed and then the fuzzy model is converted into a crisp model. The solution of

Model 4 can be obtained by the following steps:

step 1. Solve the **Model 4** as a single-objective transportation problem 2 times by taking one of the objective at a time.

step 2. From the above results, determine the corresponding values for objective at each solution derived. According to each solution and value for every objective, we can find a pay-off matrix as follows:

	Z_R	Z_c
x^1	Z_{1R}	Z_{1c}
x^2	Z_{2R}	Z_{2c}

Where x^1, x^2 are the isolated optimal solutions of the k different transportation prob-

lems for 2 different objective functions. $Z_{1R}, Z_{1c}, Z_{2R}, Z_{2c}$ are the values of objective functions.

step 3. From **Step 2**, find for each objective the U_r and the L_r corresponding to the set of solutions, where,

$$UZ_R = \max\{Z_{1R}, Z_{2R}\} \quad , \quad LZ_R = \min\{Z_{1R}, Z_{2R}\}$$

$$UZ_c = \max\{Z_{1c}, Z_{2c}\} \quad , \quad LZ_c = \min\{Z_{1c}, Z_{2c}\}$$

An initial fuzzy model of the problem can be

Find x_{ij} , $i = 1, \dots, m$; $j = 1, \dots, n$

$$\begin{aligned} Z_R &\preceq LZ_R, \\ Z_c &\preceq LZ_c \\ \text{s. t., } \sum_{j=1}^n x_{ij} &\in [a_{L_i}, a_{R_i}], \quad i = 1, \dots, m \\ \sum_{i=1}^m x_{ij} &\in [b_{L_j}, b_{R_j}], \quad j = 1, \dots, n \\ \sum_{i=1}^m a_{L_i} &= \sum_{j=1}^n b_{L_j} \\ \sum_{i=1}^m a_{R_i} &= \sum_{j=1}^n b_{R_j} \\ x_{ij} &\geq 0 \quad \forall i, \quad \forall j \end{aligned}$$

Model 2.L

Model 2.R

step 4. Define a membership function $\mu(Z_R), \mu(Z_c)$, for the Z_R, Z_c respectively.

step 5. Convert the fuzzy model of the problem, obtained in **Step 3**, into the following crisp model, namely, **Model 5**.

Model 5:

$$\begin{aligned}
 & \max \quad \lambda \\
 \text{subject to} \quad & \lambda \leq \mu(Z_R) \\
 & \lambda \leq \mu(Z_c) \\
 & \sum_{j=1}^n x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \dots, m \\
 & \sum_{i=1}^m x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \dots, n \\
 & \sum_{i=1}^m a_{L_i} = \sum_{j=1}^n b_{L_j} \\
 & \sum_{i=1}^m a_{R_i} = \sum_{j=1}^n b_{R_j} \\
 & x_{ij} \geq 0 \quad \forall i, \quad \forall j
 \end{aligned}$$

Model 2.L

Model 2.R

$$\lambda \geq 0$$

step 6. Solve the crisp model by an appropriate mathematical programming algorithm.

step 7. The solution obtained in **step 6** will be optimal compromise solution of the **Model 4**.

3.1.2 Fuzzy Programming Technique with Linear Membership Function

A linear membership function is defined as

$$\mu_1(Z_R) = \begin{cases} 1 & \text{if } Z_R \leq LZ_R \\ 1 - \frac{Z_R - LZ_R}{UZ_R - LZ_R} & \text{if } LZ_R \leq Z_R \leq UZ_R, \\ 0 & \text{if } UZ_R \leq Z_R \end{cases}$$

and

$$\mu_2(Z_c) = \begin{cases} 1 & \text{if } Z_c \leq LZ_c \\ 1 - \frac{Z_c - LZ_c}{UZ_c - LZ_c} & \text{if } LZ_c \leq Z_c \leq UZ_c, \\ 0 & \text{if } UZ_c \leq Z_c \end{cases}$$

If we use a linear membership function, the crisp model can be simplified in **Model 6** as follows:

Model 6:

$$\begin{aligned}
 & \max \quad \lambda \\
 \text{subject to} \quad & Z_R + \lambda(UZ_R - LZ_R) \leq UZ_R, \\
 & Z_c + \lambda(UZ_c - LZ_c) \leq UZ_c, \\
 & \sum_{j=1}^n x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \dots, m \\
 & \sum_{i=1}^m x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \dots, n \\
 & \sum_{i=1}^m a_{L_i} = \sum_{j=1}^n b_{L_j} \\
 & \sum_{i=1}^m a_{R_i} = \sum_{j=1}^n b_{R_j} \\
 & x_{ij} \geq 0 \quad \forall i, \quad \forall j
 \end{aligned}$$

Model 2.L

Model 2.R

$$\lambda \geq 0$$

3.1.3 Fuzzy Programming Technique with Exponential Membership Function

A exponential membership function is defined as

Considering $Z_1 = Z_R, Z_2 = Z_c, U_1 = UZ_R, U_2 = UZ_c, L_1 = LZ_R, L_2 = LZ_c$

$$\mu_l(Z_r) = \begin{cases} 1 & \text{if } Z_r \leq L_r \\ 1 - \frac{e^{-s}Z_r - L_r}{1 - e^{-s}} & \text{if } L_r \leq Z_r \leq U_r, \quad r = 1, 2 \\ 0 & \text{if } U_r \leq Z_r \end{cases}$$

If we use a exponential membership function, the crisp model can be simplified in **Model 7** as follows

Model 7:

$$\begin{aligned}
 & \max \quad \lambda \\
 & \text{subject to} \\
 & e^{-s} \frac{Z_r - L_r}{U_r - L_r} - \lambda(1 - e^{-s}) \geq e^{-s}, \quad r = 1, \dots, k \\
 & \sum_{j=1}^n x_{ij} \in [a_{L_i}, a_{R_i}], \quad i = 1, \dots, m \\
 & \sum_{i=1}^m x_{ij} \in [b_{L_j}, b_{R_j}], \quad j = 1, \dots, n \\
 & \sum_{i=1}^m a_{L_i} = \sum_{j=1}^n b_{L_j} \\
 & \sum_{i=1}^m a_{R_i} = \sum_{j=1}^n b_{R_j} \\
 & x_{ij} \geq 0 \quad \forall i, \quad \forall j
 \end{aligned}$$

Model 2.L

Model 2.R

$$\lambda \geq 0$$

4 Numerical Example

Let us consider the following interval transportation problem

	D_1	D_2	D_3	<i>stock</i>
O_1	c_{11}	c_{12}	c_{13}	a_1
O_2	c_{21}	c_{22}	c_{23}	a_2
O_3	c_{31}	c_{32}	c_{33}	a_3
<i>Demand</i>	b_1	b_2	b_n	

where the intervals of c_{ij} 's are unknown but $=a_1 \in [50, 70]$, $a_2 \in [40, 50]$, $a_3 \in [30, 80]$, $b_1 \in [60, 90]$, $b_2 \in [40, 80]$, $b_3 \in [20, 30]$

It is also given that there are two types of vehicle V_1 and V_2 . The cost of V_1 from source 'i' destination 'j' is R_{ij}^1 for a single trip. The cost of V_2 from source 'i' destination 'j' is R_{ij}^2 for a single trip. It is also given that the capacity of V_1 is $C_1 = 10$ and that of V_2 is $C_2 = 20$

So the cost varying interval transportation problem can be considered as

	D_1	D_2	D_3	<i>stock</i>
O_1	5, 7	4, 6	8, 10	[50, 70]
O_2	2, 3	6, 8	7, 9	[40, 50]
O_3	3, 4	10, 12	4, 6	[30, 80]
<i>Demand</i>	[60, 90]	[40, 80]	[20, 30]	

To determine c_{Lij} we consider the cost varying TP as

	D_1	D_2	D_3	<i>stock</i>
O_1	5, 7	4, 6	8, 10	50
O_2	2, 3	6, 8	7, 9	40
O_3	3, 4	10, 12	4, 6	30
<i>Demand</i>	60	40	20	

Then we have by **Model 2.L**

$$c_{L11} = \frac{5}{10}, c_{L12} = \frac{12}{40}, c_{L13} = \frac{8}{10}, c_{L21} = \frac{6}{40}, c_{L22} = \frac{16}{40},$$

$$c_{L23} = \frac{9}{20}, c_{L31} = \frac{3}{10}, c_{L32} = \frac{10}{10}, c_{L33} = \frac{6}{20}$$

To determine c_{Rij} we consider the cost varying TP as

	D_1	D_2	D_3	<i>stock</i>
O_1	5, 7	4, 6	8, 10	70
O_2	2, 3	6, 8	7, 9	50
O_3	3, 4	10, 12	4, 6	80
<i>Demand</i>	90	80	30	

Then we have by **Model 2.R**

$$c_{R11} = \frac{11}{40}, c_{R12} = \frac{22}{70}, c_{R13} = \frac{18}{30}, c_{R21} = \frac{6}{40}, c_{L22} = \frac{6}{10},$$

$$c_{L23} = \frac{16}{30}, c_{L31} = \frac{11}{50}, c_{L32} = \frac{10}{10}, c_{L33} = \frac{10}{30},$$

So, interval c_{ij} of interval TP determined as $c_{ij} \in [\min\{c_{Lij}, c_{Rij}\}, \max\{c_{Lij}, c_{Rij}\}]$, i.e., we have

$$c_{11} \in \left[\frac{11}{40}, \frac{5}{10}\right], c_{12} \in \left[\frac{12}{40}, \frac{22}{70}\right], c_{13} \in \left[\frac{18}{30}, \frac{8}{10}\right], c_{21} \in \left[\frac{6}{40}, \frac{6}{40}\right], c_{22} \in \left[\frac{16}{40}, \frac{6}{10}\right],$$

$$c_{23} \in \left[\frac{9}{20}, \frac{16}{30}\right], c_{31} \in \left[\frac{11}{50}, \frac{3}{10}\right], c_{32} \in \left[\frac{10}{10}, \frac{10}{10}\right], c_{33} \in \left[\frac{6}{20}, \frac{10}{30}\right]$$

Then we formulate cost varying interval TP by **Model 4** which is **Model 8**

Model 8:

$$\begin{aligned}
\min Z_R(x) = zr &= (0.388 + 0.1125) * x_{11} + (0.307 + 0.007) * x_{12} + (0.7 + 0.1) * x_{13} \\
&+ (0.15 + 0.0) * x_{21} + (0.5 + 0.1) * x_{22} + (0.49 + 0.042) * x_{23} + \\
&= (0.26 + 0.04) * x_{31} + (1 + 0.0) * x_{32} + (0.3166 + 0.017) * x_{33}; \\
\min Z_C(x) &= (0.0 + 0.1125) * x_{11} + (0.0 + 0.007) * x_{12} + \\
&(0.0 + 0.1) * x_{13} + (0.0 + 0.0) * x_{21} + (0.0 + 0.10) * x_{22} + \\
&(0.0 + 0.042) * x_{23} + (0.0 + 0.04) * x_{31} + (0.0 + 0.2) * x_{32} + (0.0 + 0.017) * x_{33}; \\
&x_{11} + x_{12} + x_{13} \geq 50; x_{11} + x_{12} + x_{13} \leq 70; \\
&x_{21} + x_{22} + x_{23} \geq 40; x_{21} + x_{22} + x_{23} \leq 50; \\
&x_{31} + x_{32} + x_{33} \geq 30; x_{31} + x_{32} + x_{33} \leq 80; \\
&x_{11} + x_{21} + x_{31} \geq 60; x_{11} + x_{21} + x_{31} \leq 90; \\
&x_{12} + x_{22} + x_{32} \geq 40; x_{12} + x_{22} + x_{32} \leq 80; \\
&x_{13} + x_{23} + x_{33} \geq 20; x_{13} + x_{23} + x_{33} \leq 30; \\
&x_{ij} \geq 0, i = 1, 2, 3; j = 1, 2, 3
\end{aligned}$$

Then solved by **Model 6** by Lingo package we have the following result $\lambda = .4093522$, $Z_R = 40.40619$, $Z_C = 4.824721$, $x_{11} = 6$, $x_{12} = 44$, $x_{21} = 44$, $x_{31} = 10$, $x_{33} = 20$.

5 Conclusion

In this paper we have presented a solution procedure of cost varying interval transportation problem under two vehicles. Here the source and destination parameters are considered as intervals. Initially, depending on cost of vehicles we determine interval of the parameter of the objective function, and the problem is converted into classical single objective interval transportation problem. Then this model converted to a bi-objective transportation problem, one is the right limit and other is center of the objective which are minimized.

To obtain the solution of this bi-objective model, the fuzzy programming technique is used. Here different types of membership functions may be used (like, linear, hyperbolic, exponential). But we use only linear membership function.

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