ISSN: 1304-7981 Number: 3, Year: 2013, Pages: 38-48 http://jnrs.gop.edu.tr

Gaziosmanpasa University
Graduate School of Natural and Applied Sciences
Journal of New Results in Science

Received: 14.11.2012
Accepted: 19.11.2013

Editors-in-Chief: Naim Çağman
Area Editor: Oktay Muhtaroğlu

Pre m_X Generalized Closed Set and its Application

Sharmistha Bhattacharya (Halder)[1,a] (halder_731@ rediffmail.com)
Gour Pal[b] (gourpal74@gmail.com)

[a]*Department of Mathematics, Tripura University, India*
[b]*Department of Mathematics, Women's College, Agartala, India*

Abstract – In this paper, we Introduce the notions of pre m_X generalized Closed set in the T_{m_X} space generated by m_X structure and investigate some of its properties for such notions. Further we study the new concept of Pre $m_X$generalized * Closed set.

Keywords - m_X open, pre m_X- open, pre m_X- generalized Closed, pre $m_X$generalized *Closed.

1. Introduction

In 1970, Levine [8] introduced the notion of generalized closed (g-Closed) sets in topological spaces. Among many modifications of g closed sets, the notions of α g-closed [10] (resp. gs-closed [3], gp-closed [13], g-closed [6], gsp-closed [5]) sets are investigated by using α -open (resp.semi-open, preopen, b-open, semi-preopen) sets. Further in 2001, V. Popaand T. Noiri [19] introduce some generalized forms of continuity under minimal structure. The concept of pre m_X-open set has been introduced by the authors in 2011[2].

In section 3, we introduce a new class of prem_X generalized closed set in the topological space T_{m_X} generated by m_X structure and investigate some of it's fundamental properties. Major properties of this new concept will be studied as well as relations to the other classes of generalized closed sets will be investigated. Further in section 4 we introduce another new concept of p$m_X$generalized * Closed set using prem_X open set and study of some important property of such notion.

[1]***Corresponding Author***

2. Preliminaries

Let X be a topological space and A $\subseteq$X. The closure of A and the interior of A are denoted by Cl(A) andInt(A) respectively. A subfamily $m_X$of the power set P(X) of a nonempty set X is called a minimalstructure[19] on X if $\phi\in m_X$and X $\in m_X$. By (X,m_X), we denote a nonempty set X with a minimal structure $m_X$on X. Simply we call (X, m_X) a space with a minimal structure $m_X$on X. Elements in $m_X$are called m-open sets. Let (X, m_X) be a space with a minimal structure $m_X$on X. For a subset A of X, the m-closure of A and the m-interior of Aare defined as the following [16]

$$m_X Int(A) = \cup\{U : U \subseteq A; U \in m_X\}$$
$$m_X Cl(A) = \cap\{F : A \subseteq F; X - F \in m_X\}$$

Definition 2.1. ([**12**]) Let (X, m_X) be a space with a minimal structure m_X on X and $A \subseteq X$. Then a set A is called an m_X-preopen set in X if $A \subseteq m_X Int(m_X Cl(A))$.
A set A is called an $m_X - preclosed$ set if the complement of A is m_X-preopen.

Definition 2.2. ([**12**]) Let (X, m_X) be a space with a minimal structurem_X. For A $\subseteq$X, the m-pre-closure and the m-pre-interior of A, denoted by $m_X pCl$(A) and $m_X pInt$(A), respectively, are defined as the following

$$m_X pCl(A) = \cap\{F \subseteq X : A \subseteq F;\ F\ is\ m_X\ preclosed\ in\ X\}$$
$$m_X pInt(A) = \cup\{U \subseteq X : U \subseteq A; U\ is\ m_X\ preopen\ in\ X\}$$

Definition 2.3 ([**2**]) Let (X, T_{m_X}) be a space with a minimal structure $m_X$on X and $A \subseteq X$. Then a subset A of X is said to be a pre-$m_X$open set on an m_X structure if $A \subseteq Int(m_X ClA)$.

Definition 2.4. ([**2**]) Let (X, T_{m_X}) be a space with a minimal structurem_X. For A$\subseteq$X, the pre-m_X-closure and the pre-m_X-interior of A, denoted by p$m_X Int(A)$and$pm_X Cl(A)$**,** respectively, are defined as the following

$$pm_X Int(A) = \cup\{G \subset Pm_X\ O(X) : G \subset A\}$$
$$pm_X Cl(A) = \cap\{F \subset X : F \subset A,\ F\ is\ pre m_X closed\}$$

Lemma 2.5([**2**])
(***a***) $m_X Int(A) \subseteq Int(A)$
(***b***) m_X-$Cl(A) \supseteq Cl(A)$

Lemma 2.6([**12**])
(***a***) $m_X Cl(m_X Int(A)) \subseteq m_X Cl(m_X Int(m_X pCl(A)) \subseteq m_X pCl(A)$
(*b*) $m_X pInt(A) \subseteq m_X Int\left(m_X Cl(m_X pInt(A))\right) \subseteq m_X Int(m_X Cl(A))$.

Lemma 2.7 ([**13**])Let X be a nonempty set and m_X a minimal structure on X satisfying property B. For a subset A of X the following properties hold
(i) A $\in m_X iff\ m_X Int(A) = A$
(ii) $A \in m_X iff\ m_X Cl(A) = A$
(iii) $m_X Int(A) \in m_X\ and\ m_X Cl(A) \in m_X$

Definition 2.8. Let (X,τ) be a topological space. A subset A of X is said to be
(1) **g-Closed[8]** if $Cl(A)\subseteq U$ whenever $A \subseteq U$ *and* $U \in \tau$,
(2) **α-g-Closed[10]** if $\alpha Cl(A)\subseteq U$ whenever $A\subseteq U$ *and* $U \in\tau$,
(3) $\boldsymbol{gp-closed}$ [**15**] *if* $pCl(A)\subseteq U$ *whenever* $A \subseteq U$ *and* $U \in \tau$,
(4) $b-\boldsymbol{closed}$ [**6**] *if* $bCl(A)\subseteq U$ *whenever* $A \subseteq U$ *and* $U\in\tau$

Definition 2.9 ([**12**])A subset A of a space (X,m_X) on m_X-structure is said to be m_X-pre closed if $m_X - Cl(m_X Int(A))\subseteq A$.

Definition 2.10 ([**17**]) A subset A of a space (X, m_X) on m_X-structure is said to be m_X-weakly generalized closed (briefly,$m_X wg-closed$) if $m_X Cl(m_X Int(A))\subseteq U$,whenever $A\subseteq U$ and U is m_X-preopen.

***Definition* 2.11** ([**13**]) Let (X,m_X) be a space on m_X-structure.A subset A of X is called m_Xg closedif$m_X Cl(A)\subseteq U$, whenever A⊆U and U is m_X- open.

Definition 2.12. A function $f: X\to Y$ is called
(a) $pre-continuous$[**11**] if $f^{-1}(F)$ is $pre-closed$ in X for every closed set F of Y;
(b) $gp-continuous$ [**7**] if $f^{-1}(F)$ is $gp-closed$ in X for every closed set F of Y;
(c) $g-continuous$[**1**] if $f^{-1}(F)$ is $g-closed$ in X for every closed set F of Y.

***Definition* 2.13** A subset A of a space (X, T_{m_X}) on T_{m_X}generated by$m_X - structure$ is said to be $pre\ m_X - closed$ if $Cl(m_X Int(A))\subseteq A$.

***Definition* 2.14.** Let (X, m_X) be a space on m_X-structure.A subset A of X is called
(a) $m_X -$ ***pre generalized closed***$(briefly, m_X pg-closed)$ if $m_X pCl(A)\subseteq U$, *whenever* $A\subseteq U$ and U is m_X-preopen.
(b) ***Generalized*** $m_X -$ ***preclosed***$(briefly, gm_X p-closed)$if $m_X pCl(A)\subseteq U$ whenever $A\subseteq U$ and U is $m_X - open$.
(c) ***Generalized pre*** m_X***closed***$(briefly, gpm_X closed)$if$pm_X Cl(A)\subseteq U$whenever A⊆U and U is m_X-open.

Definition 2.15. Let (X, m_X) be a space on m_X-structure.A subset A of X is called
(a) **b-m_XClosed** set if $Int(m_X Cl(A))\cup Cl(m_X Int(A))\subseteq A$
(b) ***generalized*** $\boldsymbol{b\ m_X - closed}$ set (simply; $gbm_X - closed$ set) if $bm_X Cl(A) \subseteq U$, whenever A ⊆U and U is$m_X$open.

Definition 2.16 ([**4**]) Let (X,T_{m_X}) be a space with a$m_X - structure$ .For A⊆X, the $pre\ m_X\ closure$ andthe pre- m_X-Interior of A,denoted by Pm_XCl(A) and Pm_XInt(A) respectively are defined as the following

$$Pm_X Cl(A) = \cap\{F\subseteq X: A\subseteq F, F\ is\ Pre\ m_X\ Closed\ in\ X\}$$
$$Pm_X Int(A) = \cup\ \{U \subseteq X: U \subseteq A, U\ is\ Prem_X\ open\ in\ X\}$$

Remark 2.17 ([**2**])Let A be a subset of X on (X,T_{m_X}). Then

$$m_X Int(A)\subseteq Int(A)$$
$$m_X Cl(A)\supseteq Cl(A)$$

Lemma 2.18 ([16])Let (X, m_X)be a space with a minimal structure m_X and $A \subseteq X$.Then $m_X Int(A) \subseteq pm_X Int(A) \subseteq pm_X Cl(A) \subseteq\ m_X Cl(A)$.

Definition 2.19 ([9])A minimal structurem_X on a non-empty set X is said to have property$B$ if the union of any family of subsets belonging to m_X belongs to m_X

Lemma 2.20 ([21]) Let (X, m_X) be an m-space and m_X satisfy property$B$. Then for a subset A of X, the following properties hold

(1) A$\in m_X$ if and only if $m_X - \text{Int} = \text{A}$,
(2) A is m-Closed if and only if m_X −Cl(A)=A,
(3) $m_X - \text{Int(A)} \in m_X$ and $m_X - \text{Cl(A)}$ is m_X-Closed.

3. Pre m_X generalized closed

Definition 3.1 Let (X, T_{m_X}) on T_{m_X} generated by m_X-structure. A subset A of X is called ***pre*** m_X***generalized* closed** $(briefly, pm_X g - closed)$ if $pm_X Cl(A) \subseteq U$, whenever $A \subseteq U$ and U is *pre* m_X *open*.

Example 3.2: Let $X = \{a, b, c\}$. Let $m_X = \{\phi, X, \{a\}, \{a, b\}, \{c\}\}$ and the corresponding topology generated by m_X structure is $T_{m_X} = \{\phi,\ X,\ \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$ then pre m_XClosed set ={ϕ, X,{b,c},{c},{a,b},{b}}. Let A ={a ,b} then A⊆{a,b}, X. Here pm_XCl(A)={a,b}⊆ {a,b},X. So A is a pre $m_X$generalized Closed.

Theorem 3.3 ϕ is a prem_X generalizedopen set but X is not.

Proof: (i) Since $m_X Int$(ϕ) = ϕsubset of any pre-$m_X$open set containingϕ. So, ϕ is prem_X generalized open. But X is not contained in any pre$m_X$generalized closed set, so X is not a pre$m_X$generalizedopen set.

Remark 3.4 Finite union of pre$m_X$generalized closed set is not prem_X generalizedclosed set.

Example 3.5 Let X ={a, b,c}. Let m_X = {ϕ, X,{a},{a,b},{c},{b}} and the corresponding topology generated by m_X structure is T_{m_X} ={ϕ, X,{a},{c},{a,c},{a,b},{b,c}{b}}then pre$m_X$closed set = $\{\phi,\ X, \{b, c\}, \{a, b\}, \{b\}, \{c\}, \{a\}, \{a, c\}\}$. If $A = \{a, b\}$then A is a prem_X generalized closed (Example 3.2), B = $\{a, c\} \subseteq \{a, c\}, X$ and $pm_X Cl(B) = \{a, c\} \subseteq \{a, c\}$, then A and B are pre m_X generalized closed sets in T_{m_X}. But $A \cup B = X$ is not pre m_X generalizedclosed set.

Remark 3.6 Intersection of two *pre* m_X*generalized* Closed set is not*pre* m_X *generalized* closedset.

Example 3.7 Let $X = \{a, b, c\}$, $m_X = \{X, \phi, \{b\}, \{a, c\}\{b, c\}\}$, T_{m_X} ={X, ϕ, {b},{c},{a, c},{b,c}}. Let A = {a,c} and B= {b,c}, pre m_XClosed set = {ϕ, X , {a,c},{a,b}, {b}, {a}}then A and B are prem_X generalizedClosed inT_{m_X}but$A \cap B = \{c\}$is notprem_X generalized Closed in T_{m_X}.Since $P\ m_X Cl(\{c\}) = \{a, c\} \not\subseteq \{c\}$.

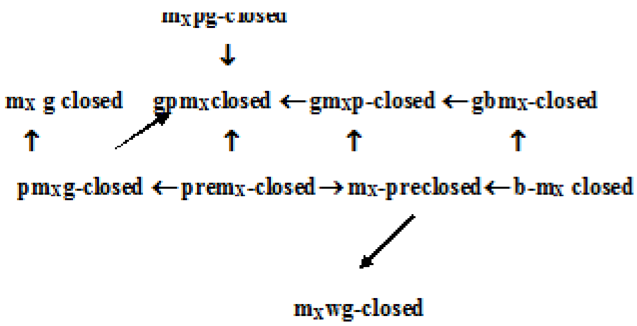
Remark 3.8 The collection of all $pre\ m_X$ $generalized$ open set with X forms a m_X* structure.

Theorem 3.9 If A is prem_X-Closed in X, then it is ap m_Xg-Closed.

Proof: Assume A is prem_X-Closed set in X.
$\Rightarrow Cl(m_X Int(A)) \subseteq A$
Suppose A$\subseteq U$, U being pre m_X open in X.
$\Rightarrow Cl\big(m_X Int(A)\big) \subseteq U.$
$\Rightarrow$p$m_X Cl(A) \subseteq U$ [by **Lemma 2.5**].
$\Rightarrow$ A is p$m_X g$-Closed.

Theorem 3.10 If A is p$m_X g$-Closed set in X then it is a $gp\ m_X$ Closed set.

Proof: Let $A \subseteq U$, U being a $m_X$open subset of X. Now A being a $pm_X g$ $-$closed set $p\ m_X Cl(A) \subseteq U$, Since every m_X open set is prem_X–open set .So $pm_X Cl(A) \subseteq U$ and any $pre\ m_X$opensetUcontaining A. Hence from definition A is a gpm_X closed set.



Example 3.11 Let X $= \{a, b, c\}$. Let$m_X = \{\phi, X, \{a\}, \{a, b\}, \{c\}\}$ and the corresponding topology generated by m_X structure is $T_{m_X} = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$ then prem_XClosed set $= \{\phi, X, \{b, c\}, \{c\}, \{a, b\}, \{b\}$.
Let$A = \{a, b\}$ $then\ A \subseteq \{a, b\}, X.$ $Here\ p\ m_X Cl(A) = \{a, b\} \subseteq \{a , b\}$,X. So A is a pre$m_X$generalized closed. A is also m_X g Closed set.

Example 3.12 Let$X = \{a, b, c, d\}$, $m_X = \{\phi, X, \{d\}, \{a, b\}, \{b, c\}\}$ the corresponding $T_{m_X} = \{\phi, X, \{d\}, \{a, b\}, \{b, c\}, \{b\}, \{a, b, d\}, \{b, c, d\}, \{a, b, c\}\}$, m_X $closed\ set$ $=$ $\{\phi, X, \{a, b, c\}, \{c, d\}, \{a, d\}\}$, pre$m_X$Closed set= {ϕ, X,{a,b,c},{c,d},{a,d},{a,c,d},{c}, {a}, {d}}. Let A = {b,d}⊆X then m_XCl(A)= X⊆X. Then A is m_Xg closedbut in T_{m_X}space A=$\{b, d\} \subseteq \{a, b, d\}, \{b, c, d\}$,X and $Pm_X Cl(A) = X \not\subset a, b, d\}, \{b, c, d\}$.Thus A is $m_X\ g$ closedbut $not\ pm_X g - closed$.

Example 3.13 Example (3.11) also shows that every $pm_X g$closed set is gpm_X $closed$ set but example (b) show the converse is not true.

Example 3.14 Let $X = \{a, b, c\}$. Let $m_X = \{\phi, X, \{a\}, \{a, b\}, \{c\}\}$ and the corresponding topology generated by $m_X$structure is $T_{m_X} = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$ then pre$m_X$closed set $= \{\phi, X, \{b, c\}, \{c\}, \{a, b\}, \{b\}$. $Let\ A = \{ b, c\}$, $m_X Int(A) = \{c\}$ implies

$Cl(m_X Int(A)) = Cl(\{c\}) = \{c\} \subseteq A$ i.e. A is pre$m_X$closed set, again A$\subseteq$ X. Here $pm_X Cl(A) = \{b, c\} \subseteq$X. So A is a pre$m_X$ generalized closed.

Example 3.15 X ={a, b, c, d}.Let $m_X = \{\phi, X, \{a\}, \{b, d\}, \{a, c, d\}\}$ then T_{m_X}={ϕ, X, $\{a\}, \{d\}, \{a, b, d\}, \{b, d\}, \{a, c, d\}\}$and pre$m_X$ −Closed set = {ϕ,X,{b,c,d},{a,b,c},{c},{a,c},{b}}.Let $A = \{a, c, d\}$ m_XInt (A) = A implies $Cl(m_X Int (A) = X \not\subset A$ therefore A is not prem_XClosed but $A \subseteq X$ and $pm_X Cl(A) = X \subseteq X$ then A is prem_X generalized Closed.

Lemma 3.16:For subsets A,B of X, the following properties hold
(i) If A is m_X-Closed then A is $pm_X g$-Closed.
(ii) If m_X has property B and A is $pm_X g - Closed$ and m_X-open then A is m_XClosed.
(iii) If A is pm_Xg-Closed and $A \subseteq B \subseteq p\ m_X\ Cl\ (A))$then B is p$m_X$g-Closed.

Proof: To prove (i) Let A be a m_X-Closed set in (X,m_X).To prove A is pm_Xg-Closed set, Let A ⊆U , U being a pre m_X-open set. To prove pm_XCl(A)⊆U. pm_XCl(A))⊆m_X− Cl(A) ⊆U [since A is m_XClosed]
$pm_X Cl(A)) \subseteq U$.
A is $pm_X g$ −Closed.

(ii) To prove A is m_X-Closed i.e. To prove m_X− Cl(A) = A [Reference 17] It is obvious that A ⊆m_X- $Cl(A)$.To prove $m_X - Cl(A) \subseteq A$: $A \subseteq A$ and A is pm_Xg-Closed and m_X-open. ⇒pm_XCl((A)) ⊆A ⇒m_X- Cl(A) ⊆A [since A is m_X-open, we have $m_X int(A)$ = A] m_X-Cl(A) = A and hence A is m_X-Closed.

(iii) To prove $m_X Cl(B) - B \subseteq [pm_X Cl\ (A)] - A$.
since $pm_X Cl(A) - A$ contains no nonempty pm_Xg-closed set neither does pm_XCl (B) − B. Therefore B is pm_Xg -closed.

***Lemma* 3.17:** A subset A of X is pm_Xg -open if and only if F ⊆p$m_X Cl(A)$whenever F ⊆A and F is m_X-Closed.

Proof : Assume that A is $pm_X g - open$ set in X. Then A^c is pm_Xg -Closed set in X. Let F be is pmX$g - Closed$ and F ⊆A. Then F^c is pm_Xg -open and $A^c \subseteq F^c$. So [m_XCl(A^c)] ⊆F^c⇒$[m_X Cl(A)]^c \subseteq F^c$.Thus F ⊆p$m_X Cl$ (A)
Conversely assume that $F \subseteq pm_X Cl\ (A)$whenever F is pm_Xg–Closed
and F ⊆A.
To prove that A^c is pm_Xg -Closed . Let G be a p m_Xg - open set and A^c⊆G.
Now G^c is pm_Xg -Closed. $G^c \subseteq (A^c)^c$= A By hypothesis G ⊆$[Pm_X Int(A)]^c$
Implies G ⊇$[Pm_X Cl(A)]^c$ and hence $A^c$is pm_Xg -closed. Thus A is pm_Xg -open.

Corollary 3.18 For subsets A, B of X the following properties hold:
(i) If A is m_X-open then A is pm_Xg -open.
(ii) If m_X has property B and A is pm_Xg -open and m_X-Closed then A is $m_X$open.
(iii) If A is pm_Xg -open and pm_XCl (A) ⊆B ⊆A then B is pm_Xg –open.

Theorem 3.19 For a subset A of X the following properties are equivalent.
(1) A is $pm_X\ g$ -Closed;

(2) $m_X Cl(A)- A$ does not contain any non empty m_X-Closed set;
(3) $m_X - Cl(A)- A$ is $pm_X g$ -open.

Proof: **(1)⇒(2)**
Suppose that F is non-empty pm_Xg -Closed subset of $p\ m_X\ Cl(A) - A$.Now F$\subseteq pm_X Cl\ (A) - A$.Then $F \subseteq pm_X Cl\ (A) \cap A^c$ [since$pm_X Cl$ (A)−A = $pm_X Cl$ (A)$\cap A^c$], F $\subseteq$p$m_X Cl$ (A) and F$\subseteq A^c$ .Since F^c is $pm_X g$ -open set.
A is pm_Xg–closedpm_XCl (A) $\subseteq$F^c. F $\subseteq$[pm_XCl (A)]^c
Hence F $\subseteq$[p$m_X Cl$ (A)] $\cap$[pm_XCl (A)]^c= ϕ.
F = ϕ.Thus $pm_X Cl\ (A) - A$ contains no nonempty $pm_X g$ -closed set.
Conversely assume that pm_XCl (A) −A contains no nonempty m_X−closed set.
Let A $\subseteq$G, G is pm_Xg -open. Suppose that pm_XCl(A) is not contained in G. Then Then[pm$_X$Cl(A)]$\cap G^c$is a non-empty pm_XgClosed set of pm_XCl(A) -A which is contradiction. A is pm_Xg -Closed.

(2) ⇒(3) Assume that pm_XCl(A) -A contains no nonempty pm_Xg -Closed set.
Let A $\subseteq$G is pm_Xg -open. Suppose that $pm_X Cl(A)$is contained in G.Then $pm_X Cl(A) \cap G^c$ is empty.It is m_X Closed set i.e. p$m_X Cl(A)$- A is pre m_X open.

Theorem 3.20 A subset A of X is $pm_X g$-Closed iff [p$m_X Cl(A)$]$\cap$F = ϕwhenever $A \cap F = \phi$and F ism_X-Closed.

Proof :Assume that A is $pm_X g$ -Closed. By definition $pm_X Cl\ (A)) \subseteq U$ and also $F \subseteq U$ i.e.$[pm_X Cl\ (A)] - F = \phi$isp$m_X$g -Closed.
∴ F is m_X-Closed.
Conversely assume that pm_XCl (A) - A is m_X closed. By the abovetheorem pm_XCl (A) − A contains no nonempty Closed set i.e. p$m_X Cl(A)$- $A = \phi$.
∴ A is $pm_X g$ -Closed.

Definition 3.21 Let (X, T_{m_X}) on T_{m_X} space generated by m_X-structure.A subset A of X is called pre$m_X$generalized Closed set relative to B if there exist a $m_X$open set U where A$\subseteq$U such that B$\cap$ Pm_XCl(A) $\subseteq$ B $\cap$ U.

Theorem 3.22 Suppose that B $\subseteq$ A, B is a pm_Xg-Closed set relative to A and that A is a pm_Xg-Closed subset of X. Then B is pm_Xg-Closed relative to X.

Proof. Let B $\subseteq$U and suppose that U is $m_X$open set in X. Then B$\subseteq$A$\cap$U and hence $Pm_X Cl\ (B) \subseteq A \cap U$. It follows that$A \cap Pm_X Cl(B) \subseteq A \cap U$ and $A \subseteq U \cup (X - Pm_X Cl(B)$.Since A is pre$m_X$g-Closed in X, we $have\ Pm_X Cl(A) \subseteq U \cup (X - Pm_X ClB)$.
$\therefore Pm_X Cl(B) \subseteq Pm_X Cl(A) \subseteq U \cup (X - Pm_X Cl\ (B)$)

Corollary 3.23 Let A be a $pm_X g$-Closed set and suppose that F is a pm_XClosed set. Then $A \cap F$ is a $pm_X g$- Closed set.

Proof: A$\cap$F is p m_X Closed in A and hence pm_Xg-Closed in A applying theorem 3.20.

Theorem 3.24 Let A $\subseteq$Y$\subseteq$ X and suppose that A is pm_Xg-Closed in X. Then A is pm_Xg-Closed relative to Y.

Proof: Let A$\subseteq$Y$\cap$U and suppose that U is $m_X$open in X. Then A$\subseteq$U and hence Pm_XCl(A) $\subseteq$U. It follows that Y $\cap Pm_X Cl(A) \subseteq$ Y $\cap$U.

Theorem 3.25 Let (X, T_{m_X}) be a compact topological space and suppose that A is a pm_Xg-Closed subset of X. Then A is compact.

Proof: Let S be an open cover of A. Then $m_X Cl(A) \subseteq \cup$S since A is pm_Xg-Closed.
But $m_X Cl(A)$is compact and it follows that A $\subseteq m_X Cl(A) \subseteq U_1 \cup \ ... \cup U_n$ for some $U_i \in S$.

***Theorem* 3.26** Let (X, T_{m_X}) be a normal space and suppose that Y is a Pm_Xg-closed subset of X. Then (Y,Y$\cap$ T_{m_X}) is normal.

Proof: Let E and F be m_XClosed in X and suppose that $(Y \cap E) \cap (Y \cap F) = \phi$. Then $Y \subseteq X - (E \cap F) \in T_{m_X}$ and hence $m_X Cl(Y) \subseteq X - (E \cap F)$.
Thus $(m_X Cl(Y) \cap E) \cap (m_X Cl(Y) \cap F) = \phi$. Since (X, T_{m_X}) is normal, there exist m_X open disjoint sets $U_1$and U_2 such that m_XCl(Y) $\cap$E $\subseteq U_1$and m_XCl(Y) $\cap$F$\subseteq U_2$. It follows that $Y \cap E \subseteq U_1 \cap Y$ *and* $Y \cap F \subseteq U_2 \cap Y$.

Theorem 3.27. If (X, T_{m_X}) is pre$m_X$regular and if A is compact, then A is pm_Xg-Closed.

Proof: Suppose that A $\subseteq$U$\in$ T_{m_X}. Then there exists an $V \in T_{m_X}$ such that A $\subseteq$V $\subseteq m_X Cl(V) \subseteq U$ and it follows that $Pm_X Cl(A) \subseteq U$.

Theorem 3.28 If (X, T_{m_X}) is pre$m_X$regular and locally compact and if A is a pm_Xg-Closed subset of X, then A is locally $m_X$compact in the relative topology.

Proof: Let $x \in A$. Then x$\in$V$\subseteq$X where V is a $m_X$compact neighborhood of x.Since (X, T_{m_X}) is pre $m_X$regular, there exists an U$\in$ T_{m_X} such that $x \in U \subseteq m_X Cl(U) \subseteq V$.Now $A \cap m_X Cl(U)$ is a neighborhood of x in A and is pm_Xg-Closed in X by corollary 3.23.By theorem 3.24, A $\cap m_X Cl(U)$ is pm_Xg-Closed in V and therefore $m_X$compact by theorem 3.25.

Theorem 3.29 If (X, T_{m_X}) is normal and F $\cap$ A = ϕ where F is pm_XClosed and A is pm_Xg-Closed, then there exist disjoint$m_X$open sets U_1 and U_2 such that F $\subseteq U_1$and $A \subseteq U_2$.

Proof: $A \subseteq X - F \in T_{m_X}$and hence $Pm_X Cl(A) \subseteq X - F$.Thus $Pm_X Cl(A) \cap F = \phi$ .Now apply theorem 3.26.

4. Pre m_X generalized $*$ closed set

Definition 4.1 Let (X, T_{m_X}) on T_{m_X} space generated by m_X-structure. A subset A of X is called Pre $m_X$generalized * Closed set (briefly, pm_Xg*-Closed) if $pm_X Cl(A) \supseteq U$, whenever A$\subseteq$U and U is pre $m_X$open set.

Example4.2 Let $X = \{a, b, c, d\}$. Let $m_X = \{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ and the corresponding topology generated by m_X structure is $T_{m_X}= \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$. Let $A = \{a\}$ be any sub set of X and U={a,b} pre $m_X$open set i.e. A⊆U then Clearly P$m_X Cl(A) = \{a, b, d\} \supseteq U$. So A is a Pre m_X generalized* Closed set.

***Theorem* 4.3** A Subset A of X is a Pre m_X generalized * Closed set $iff pm_X Cl(A) = X$

Proof: Let U∈m_XP be a pre $m_X$open set **.**Since A is pre m_X generalized* Closed set then A⊆U whenever p$m_X Cl(A) \supseteq U$.Again $X \in m_X P$ then $pm_X Cl(A) \supseteq X$ but $pm_X Cl(A) \subseteq X$ which implies $pm_X Cl(A) = X$.
By definition converse part is obvious.

Theorem 4.4. Every m_X closed and m_X dense set is pre $m_X$generalized * closed set.

Proof: Let A is m_X dense in X then $m_X Cl(A) = X$ **.**Also A is m_XClosed then A= $m_X Cl(A)$implies $pm_X Cl(A)$ = pm_XCl($m_X Cl(A)$) =p$m_X Cl(X)$ = X.By **Theorem 4.3** A is Pre$m_X$generalized * Closed set.

***Theorem* 4.5.**
(i) Xis a pre $m_X$generalized * Closed set but ϕ is not
(ii) Arbitrary union of pre $m_X$generalized * Closed set is a pre $m_X$generalized * Closed set if it is contained in any pre m_X open set.
(iii) Finite union of pre$m_X$generalized * Closed set is a pre $m_X$generalized * Closed set if it is contained in any pre m_X open set.
(iv) Arbitrary Intersection of pre $m_X$generalized * Closed set is a pre $m_X$generalized * Closed set .

Proof: (i) Since $pm_X Cl(X) = X$subset of any pre m_X open set containing X. So,X is a pre $m_X$generalized * Closed set. But ϕ is not contained in any pre m_X open set, so ϕ is not a pre $m_X$generalized * Closed set.

(ii) Let if possible {A_i : i∈I} be an arbitrary collection of p $m_X$generalized* Closed sets. Let$A_i \subseteq \cup\{A_i : i \in I\} \subseteq U$, a pre m_X open subset of X. Since $\{A_i$: i ∈I}is an arbitrary collection of p$m_X$generalized* Closed set , so, $\{Pm_X Cl(A_i): i \in I\} \subseteq U$. i.e.$\{Pm_X Cl(\cup A_i): i \in I\} \supseteq \cup\{Pm_X Cl(A_i): i \in I\} \supseteq U$ [1] i.e. arbitrary union of p$m_X$generalized* Closed set is a p$m_X$generalized* Closed set if it is contained in any pre m_X open set.

(iii) Let if possible $\{A_i$: i = 1, 2, ., n }be a finite collection of pre$m_X$generalized * Closed sets. Let $A_i \subseteq \cup\{A_i$: i = 1, 2, ., n } ⊆U, a pre m_X open subset of X. Since $\{A_i$: i = 1, 2, ., n }is a finite collection of pre $m_X$generalized * Closed set , $so, \{Pm_X Cl(A_i)$: i = 1, 2, ., n } ⊇U . i.e. $\cup\{Pm_X Cl(A_i): i = 1,2,.,n\}$= $\{Pm_X Cl(\cup A_i : i = 1,2,.,n)\} \supseteq U$ i.e. finite union of pre $m_X$generalized * Closed set is a pre $m_X$generalized * Closed set if it is contained in any Pre m_X open set.

(iv) Let if possible $\{A_i$: i∈I} be an arbitrary collection of pre $m_X$generalized * Closed sets. Let A = $\cap\{A_i$: i∈I} ⊆U a pre m_X open set. If all {A_i: i∈I}⊆U_i, some independent pre m_X

open set then $\cap\{A_i$: i∈I}=ϕ, a pre $m_X$generalized * Closed set. Now let if possible $\cap\{A_i$: i∈I} = ϕ, then $\cap\{A_i$: i∈I} ⊆U implies $\{A_i$: i∈I} ⊆U. Since $\{A_i$: i∈I} is an arbitrary collection of pre $m_X$generalized * Closed sets, $Pm_XCl\{A_i : i\in I\}\supseteq U$. Now $\cap\{A_i$: i∈I} ⊆ $\{A_i$: i∈I}. So, $Pm_XCl\{\cap\{A_i : i\in I\}\} = \cap Pm_XCl\{A_i : i\in I\} \supseteq U$. So, arbitrary intersection of pre$m_X$generalized * Closed set is a pre $m_X$generalized * Closed set.

Remark **4.6.** The collection ofpre $m_X$generalized * Closed set with ϕ forms a Alekxendroff space.

Example 4.7. Let $X = \{a, b, c, d\}$. Let $m_X = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$ and the corresponding topology generated by$m_X$structure is $T_{m_X} = \{\phi, X, \{a\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$ and pre m_XClosed set ={X, ϕ, $\{b, c, d\}, \{b, d\}, \{c, d\}, \{d\}$} . Let $A = \{a, b\}$and B ={a,c} be any sub set of X and $U = \{a, b, c\}$ pre m_X open set i.e. A⊆U and B⊆U then Clearly $Pm_XCl(A)$ = X⊇U and $Pm_XCl(B) = X\supseteq U$ .Then $A\cup B = \{a, b, c\}\subseteq U$ then $pm_XCl(A\cup B) = X\supseteq U$ .So A∪B is a Pre m_X generalized* Closed set.Again A∩B={a}⊂U and $Pm_XCl(A\cap B) = X = Pm_XCl(A)\cap Pm_XCl(B) = X \cap X = X\supseteq U$. Hence A∩B is Pre m_X generalized* Closed.

Theorem 4.8 The intersection of two Pm_Xg*-Closed set is Pm_Xg*-Closed set in (X, T_{m_X}).

Proof: Let A and B be any two Pm_Xg*--Closed sets in (X, T_{m_X}). To prove A∩B is Pm_Xg*-Closed set. Let G be Pre m_X-open set such that A ∩B ⊆G ⇒A ⊆G and B ⊆G. Since A and B are Pm_Xg*-Closed sets, $Pm_XCl(A)\supseteq$G and $m_XCl(B) \supseteq G$, $Pm_XCl(A)\cap Pm_XCl(B)\supseteq G$. Hence $Pm_XCl(A \cap B)\supseteq G$.

Remark 4.9 Union of two nonPm_Xg*-Closed set may be Pm_Xg*-Closed.

Example 4.10 Let $X = \{a, b, c\}$.Let $m_X = \{\phi, X, \{a\}, \{b\}, \{a, c\}\}$ then $T_{m_X} = \{\phi, X, \{a\}, \{b\}, \{a, c\}, \{a, b\}\}$, Pre m_XClosed set = $\{\phi, X, \{b, c\}, \{a, c\}, \{b\}, \{c\}\}$.Let $A = \{a\}\subseteq\{a\}, \{a, c\}, \{a, b\}$, X. $Pm_XCl(A) = \{a, c\}\not\supseteq\{a, b\}, X$ and B={b}$\subseteq\{b\}, \{a, b\}, X$ then $Pm_XCl(B)$ = {b} $\not\supseteq$ {a,b},X. But $A\cup B = \{a, b\}\subseteq\{a, b\}, X$, p m_XCl(A∪B) = X $\supseteq\{a, b\}, X$.Here A and B are not Pm_Xg*-Closed set but their union is Pm_Xg*-Closed set.

Theorem 4.11 Every Pre$m_X$generalized * Closed set is $m_X$generalized * Closed set.

Proof: Follows from definition 4.1 and Lemma 2.18[15].

References

[1] K. Balachandran, P. Sundaram, H. Maki, On generalized continuous functions in topological spaces, Mem. Fac. Sci. Kochi Univ. Ser. A Math.,12(1991),5-13

[2] SharmisthaBhattacharya(Halder),Gour pal, Study on Prem$_X$-open Set, International Mathematical forum, Vol. 6, 2011, no. 45, 2231 -2237.

[3] P. Bhattacharyya, B. K .Lahiri, Semi-generalized closed sets in topology, Indian Journal of Mathematics, 29(1987) pp. 375–382.

[4] SharmisthaBhattacharya(Halder),Gour pal, a study on pre- m$_X$ Continuous function, Communicated .

[5] J. Dontchev , On generalizing semi-preopen sets, Memoirs of Faculty of Science Kochi

University, Series A, Mathematics, 16(1995), pp. 35–48,

[6] T. Fukutake, A. Nasef, A.I. El-Maghrabi. Some topological concepts via γ-generalized closed sets, Bulletin of Fukuoka University, Ed. III , 52(2003) pp. 1–9.

[7] Y. Gnanambal, On generalized pre-regular closed sets in topological spaces, Indian J. pure. Appl.Math., 28(3)(1997), 351-360.

[8] N. Levine , Generalized closed sets in topology, Rendiconti del CircoloMatematico di Palermo (2), 19(1970),p.p. 89–96.

[9] H. Maki, K.C. Rao, A. NagoorGani, On generalizing semi-open and preopen sets, Pure Appl. Math. Sci. 49(1999), 17–29.

[10] H. Maki, R Devi, K. Balachandran, Associated topologies of generalized α-closed sets and α- generalized closed sets, Memoirs of Faculty of Science Kochi University, Series A, Mathematics,15(1994) pp. 51–63.

[11] A. S. Mashhour, M. E. Abd El-Monsef, S. N. El-Deeb, Onpre continuous and weak pre continuous mappings, Proc. Math. Phys. Soc. Egypt, 53(1982),47-53.

[12] W. K. Min and Y. K. Kim, m-Preopen Sets and M-Precontinuity On Spaces With Minimal Structures, Advances in Fuzzy Sets and Systems 4 (2009), no.3, 237-245.

[13] T. Noiri, 11th meetings on topological spaces and its applications, Fukuoka University Seminar House, 2006, 1-9.

[14] T. Noiri, A unified theory for certain modifications of generalized closed sets, Rendiconti del Circolomatematico di palermo, serie II, tomo LVI(2007), pp. 171-184.

[15] T. Noiri, H. Maki ,J. Umehara, Generalized preclosed functions, Memoirs of Faculty of Science Kochi University, Series A, Mathematics, 19(1998), pp. 13–20,

[16]T. Noiri, V. Popa, A generalization of some forms of g-irresolute functions,European journal of pure and applied mathematics,Vol.2,No. 4(2009),473-493.

[17] R. Parimelazhagan, K. Balachandran, N. Nagaveni, Weakly generalized closed sets in minimal structure,Int. J. Contemp. Math. Sciences, Vol. 4, 2009, no. 27, 1335 – 1343.

[18] V. Popa. And T. Noiri, On M-continuous functions, Anal. Univ. ”Dunarea de Jos” Galati. Ser. Mat. Fiz. Mec. Teor. (2), 18(23) (2000), 31 - 41.

[19] V.Popa,Noiri T, On the definitions of some generalized forms of continuity under minimal conditions, memoirs of Faculty of Science Kochi University, Series A, Mathematics, 22(2001), pp.31–41.

[20] V. Popa and T. Noiri.On m-continuous functions.Anal.Uni'Dunarea De, jos'Galati, Ser. Mat.,Fiz., 18(23)(2000)31-41.

[21] V. Popa, T. Noiri, A unified theory of weak continuity for functions, Rend.Circ. Mat. Palermo51(2002), 439–464.

[22] A. Pushpalatha and E. Subha, Strongly generalized closed sets in minimal structures. Int. J. of Math. Anal., 3(26)(2009),1259-1263.

[23] O. Ravi , S. Jeyashri, K. Vijayalakshmi and J. Antony Rex Rodrigo, On Strongly Minimal g-continuous functions in minimal structures, Journal of Advanced Studies in Topology ISSN: 2090-388X online Vol. 1, No. 2, 2010, 29-36.