ISSN: **1304-7981** Number: **3**, Year: **2013**, Pages: **72-86** http://jnrs.gop.edu.tr

Gaziosmanpasa University
Graduate School of Natural and Applied Sciences
**Journal of New Results in Science**

*Received*: 13.09.2013 *Editors-in-Chief*: *Hikmet Günal*
*Accepted*: 26.11.2013 *Area Editor*: *Oktay Muhtaroğlu*

# On Intuitionistic Fuzzy Soft Groups

**Faruk Karaaslan**[a,1] (f.karaaslan0904@gop.edu.tr)
**Kenan Kaygısız**[a] (kenan.kaygisiz@gop.edu.tr)
**Naim Çağman**[a] (naim.cagman@gop.edu.tr)

[a] *Department of Mathematics, Faculty of Arts and Sciences, Gaziosmanpaşa University, 60250 Tokat, Turkey*

**Abstract** – In this paper, we give some basic properties of intuitionistic fuzzy soft sets and $(\alpha, \beta)$-level sets. Also we define image of an intuitionistic fuzzy soft set under a function, product of two intuitionistic fuzzy soft sets and obtain some results. In addition, we define intuitionistic fuzzy soft group and investigate their properties.

**Keywords** – *Soft set, intuitionistic fuzzy set, intuitionistic fuzzy soft set, intuitionistic fuzzy soft group, $(\alpha, \beta)$-level set.*

## 1 Introduction

Some problems in economy, engineering, environmental science and social science may not be successfully modeled by methods of classical mathematics because of various types of uncertainties. There are some well known mathematical theories for dealing with uncertainties such that; fuzzy set theory [29], soft set theory [25], intuitionistic fuzzy set theory [4], fuzzy soft set theory [21] and so on.

In 1999, Molodtsov [25] firstly introduced the soft set theory as a general mathematical tool for dealing with uncertainty. Since then some authors studied on the operations of soft sets [2, 8, 23]. By using these operations, soft group [3, 5, 6, 11], soft BCK/BCI-algebras [18, 19], soft ordered semigroups [20], soft rings [1, 13, 27] are defined and investigated their properties.

Many interesting results of soft set theory have been obtained by embedding the ideas of fuzzy sets. For example, fuzzy soft sets [9, 10, 14, 21, 24], fuzzy soft groups [7], intuitionistic fuzzy soft sets [16, 17, 22].

Algebraic structures on fuzzy set were firstly studied by the definiton of Rosenfeld [26]. Flep [15] investigated structure and construction of fuzzy subgroup of a group.

[1]Corresponding Author

Intuitionistic fuzzy group is defined by Fathi and Sallah [12] . Zhou et al. [30] applied the concept of intuitionistic fuzzy soft set to semigroup. Yaqoob et al. [28] defined intuitionistic fuzzy soft group induced by $(t, s)$-norm.

In this paper, we give some basic properties of intuitionistic fuzzy soft set and $(\alpha, \beta)$-level set. Then, we define image of an intuitionistic fuzzy soft set under a function and product of two intuitionistic fuzzy soft sets. Moreover, we introduce the notion of intuitionistic fuzzy soft group and investigate its properties analogues to fuzzy groups.

# 2 Preliminaries

In this section, we have presented the basic definitions and results of fuzzy sets, soft sets, fuzzy soft sets, intuitionistic fuzzy set and intuitionistic fuzzy soft sets which are necessary for subsequent discussions.

**Definition 2.1.** *[29] Let $E$ be a crisp set. Then a fuzzy set $\mu$ over $E$ is a function from $E$ into $[0, 1]$.*

**Definition 2.2.** *[26] Let $G$ be an arbitrary group and $\mu$ be a fuzzy set. Then, $\mu$ is called a fuzzy subgroup of $G$ if $\mu(xy) \geq \mu(x) \wedge \mu(y)$ and $\mu(x^{-1}) \geq \mu(x)$ for all $x, y \in G$.*

**Definition 2.3.** *[4] An intuitionistic fuzzy set $(IFS)$ $A$ in $E$ is defined as an object of the following form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in E\},$$

*where the functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$,*

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

*In addition for all $x \in E$, $E = \{\langle x, 1, 0\rangle : x \in E\}$ , $\emptyset = \{\langle x, 0, 1\rangle : x \in E\}$ are intuitionistic fuzzy universal and intuitionistic fuzzy empty set, respectively.*

**Theorem 2.4.** *[4] Let $A$ and $B$ be two intuitionistic fuzzy sets. Then,*

*i.* $A \sqsubseteq B \Leftrightarrow \forall x \in E, \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$

*ii.* $A \sqcap B = \{\langle x, min\{\mu_A(x), \mu_B(x)\}, max\{\nu_A(x), \nu_B(x)\}\rangle : x \in E\}$

*iii.* $A \sqcup B = \{\langle x, max\{\mu_A(x), \mu_B(x)\}, min\{\nu_A(x), \nu_B(x)\}\rangle : x \in E\}$.

**Definition 2.5.** *[8] Let $U$ be an initial universe, $P(U)$ be the power set of $U$, $E$ be a set of all parameters and $A \subseteq E$. Then, a soft set $f_A$ over $U$ is a function from $E$ into $P(U)$ such that $f_A(x) = \emptyset$, if $x \notin A$.*

*Where $f_A$ is called approximate function of the soft set $f_A$ and the value $f_A(x)$ is a set called $x$-element of the soft set for all $x \in E$.*

**Definition 2.6.** *[8] Let $f_A, f_B$ be two soft sets over $U$. Then, $f_A$ is a soft subset of $f_B$, denoted by $f_A \tilde{\subseteq} f_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.*

**Definition 2.7.** *[8] Let $f_A, f_B$ be two soft sets over U. Then, union and intersection of $f_A$ and $f_B$, denoted by $f_A \tilde{\cup} f_B$ and $f_A \tilde{\cap} f_B$, are soft sets defined by the approximate function, for all $x \in E$, $f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$ and $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$, respectively.*

**Definition 2.8.** *[8] Let $f_A$ be a soft set over U. Then, complement of $f_A$, denoted by $f_A^{\tilde{c}}$, is a soft set defined by the approximate function $f_{A^{\tilde{c}}}(x) = f_A^{\tilde{c}}(x)$ for all $x \in E$, where $f_A^{\tilde{c}}$ is complement of the set $f_A(x)$, that is, $f_A^{\tilde{c}} = U \setminus f_A(x)$ for all $x \in E$.*

*It must be noted that to keep clear of confusion, we use two different symbols $\tilde{c}$ and c which represent the complement of a soft set and classical set, respectively. But, in the subscripts, the symbol $\tilde{c}$ indicates that $f_{A^{\tilde{c}}}$ is the approximate function of $f_A^{\tilde{c}}$ but not a set operation.*

**Definition 2.9.** *[11] Let G be an arbitrary group and $f_G$ be a soft set over U. Then $f_G$ is called a soft-int group over U, if $f_G(xy) \supseteq f_G(x) \cap f_G(y)$ and $f_G(x^{-1}) = f_G(x)$ for all $x, y \in G$.*

**Definition 2.10.** *[21] Let U be an initial universe, $\mathcal{F}(U)$ be the set of all fuzzy sets over U, E be a set of parameters and $A \subseteq E$. Then, a fuzzy soft set (f,A) over U is a function from E into $\mathcal{F}(U)$.*

**Definition 2.11.** *[22] Let $U$ be an initial universe, $\mathcal{IF}(U)$ be the set of all intuitionistic fuzzy sets over $U$, $E$ be a set of all parameters and $A \subseteq E$. Then, an intuitionistic fuzzy soft set (IFS-set) $\gamma_A$ over U is a function from E into $\mathcal{IF}(U)$.*

*Where, the value $\gamma_A(x)$ is an intuitionistic fuzzy set over U. That is, $\gamma_A(x) = \{\langle u/\overline{\gamma}_{A(x)}(u)/\underline{\gamma}_{A(x)}(u)\rangle : x \in E, u \in U\}$, where $\overline{\gamma}_{A(x)}(u)$ and $\underline{\gamma}_{A(x)}(u)$ are the membership and non-membership degrees of u for the parameter x, respectively.*

Note that, the set of all intuitionistic fuzzy soft sets over $U$ is denoted by $\mathcal{IFS}(U)$.

**Definition 2.12.** *[22] Let $A, B \subseteq E$, $\gamma_A$ and $\gamma_B$ be two IFS-sets. Then, $\gamma_A$ is said to be an intuitionistic fuzzy soft subset of $\gamma_B$ if*
*(1) $A \subseteq B$ and*
*(2) $\gamma_A(x)$ is an intuitionistic fuzzy subset of $\gamma_B(x)$, $\forall x \in A$.*

This relationship is denoted by $\gamma_A \tilde{\sqsubseteq} \gamma_B$. Similarly, $\gamma_A$ is said to be an intuitionistic fuzzy soft superset of $\gamma_B$, if $\gamma_B$ is an intuitionistic fuzzy soft subset of $\gamma_A$ and denoted by $\gamma_A \tilde{\sqsupseteq} \gamma_B$.

**Definition 2.13.** *[22] Let $\gamma_A$ and $\gamma_B$ be two intuitionistic fuzzy soft sets over U. Then, $\gamma_A$ and $\gamma_B$ are said to be intuitionistic fuzzy soft equal if and only if $\gamma_A$ is an intuitionistic fuzzy soft subset of $\gamma_B$ and $\gamma_B$ is an intuitionistic fuzzy soft subset of $\gamma_A$, and written by $\gamma_A = \gamma_B$.*

**Definition 2.14.** *[22] Let $\gamma_A$ be an IFS-set over $U$. If $\gamma_A(x) = \emptyset$ for all $x \in E$, then $\gamma_A$ is called empty IFS-set and denoted by $\gamma_\phi$.*

**Definition 2.15.** *[22] Let $\gamma_A$ be an IFS-set over $U$. If $\gamma_A(x) = \{\langle u/1/0\rangle : \forall u \in U\}$ for all $x \in A$, then $\gamma_A$ is called A-universal IFS-set and denoted by $\gamma_{\hat{A}}$.*

*If A=E, then the A-universal IFS-set is called universal IFS-set and denoted by $\gamma_{\hat{E}}$.*

**Definition 2.16.** *[22] Let $\gamma_A$ and $\gamma_B$ be two IFS-sets over $U$. Union of $\gamma_A$ and $\gamma_B$, denoted by $\gamma_A \tilde{\sqcup} \gamma_B$, and is defined by*

$$\gamma_A \tilde{\sqcup} \gamma_B = \{(x, \gamma_{A\tilde{\cup}B}(x)) : x \in E\}$$

*where*

$$\gamma_{A\tilde{\cup}B}(x) = \{\langle u/max\{\overline{\gamma}_{A(x)}(u), \overline{\gamma}_{B(x)}(u)\}/min\{\underline{\gamma}_{A(x)}(u), \underline{\gamma}_{B(x)}(u)\}\rangle : u \in U\}.$$

**Definition 2.17.** *[22] Let $\gamma_A$ and $\gamma_B$ be two IFS-set over $U$. Intersection of $\gamma_A$ and $\gamma_B$, denoted by $\gamma_A \tilde{\sqcap} \gamma_B$, and is defined by*

$$\gamma_A \tilde{\sqcap} \gamma_B = \{(x, \gamma_{A\tilde{\cap}B}(x)) : x \in E\}$$

*where*

$$\gamma_{A\tilde{\cap}B}(x) = \{\langle u/min\{\overline{\gamma}_{A(x)}(u), \overline{\gamma}_{B(x)}(u)\}/max\{\underline{\gamma}_{A(x)}(u), \underline{\gamma}_{B(x)}(u)\}\rangle : u \in U\}.$$

**Definition 2.18.** *[17] Let $\gamma_A \in \mathcal{IFS}(U)$ and $\alpha, \beta \in [0,1]$ such that $\alpha + \beta \leq 1$. Then, $(\alpha, \beta)$-level set of $\gamma_A$ is ${}^{\alpha}_{\beta}\gamma_A(x) = \{u \in U : \overline{\gamma}_{A(x)}(u) \geq \alpha$ and $\underline{\gamma}_{A(x)}(u)) \leq \beta\}$ for all $x \in A$.*

**Proposition 2.19.** *[17] Let $\gamma_A, \gamma_B \in \mathcal{IFS}(U)$. Then, the following assertions hold:*

*i.* $\gamma_A \tilde{\sqsubseteq} \gamma_B \Rightarrow^{\alpha}_{\beta} \gamma_A \tilde{\subseteq} {}^{\alpha}_{\beta}\gamma_B$, *for all* $\alpha, \beta \in [0,1]$

*ii. If* $\alpha_1 \leq \alpha_2$ *and* $\beta_2 \leq \beta_1$, *then* ${}^{\alpha_2}_{\beta_2}\gamma_A \tilde{\subseteq} {}^{\alpha_1}_{\beta_1}\gamma_A$, *for* $\alpha_1, \alpha_2, \beta_1$ *and* $\beta_2 \in [0,1]$

*iii.* $\gamma_A = \gamma_B \Leftrightarrow^{\alpha}_{\beta} \gamma_A =^{\alpha}_{\beta} \gamma_B$, *for all* $\alpha, \beta \in [0,1]$.

# 3 Some results of intuitionistic fuzzy soft sets on a group

In this section, firstly, we give some results of $IFS$-sets on a group $G$ and relations between level subsets of $IFS$-sets.

**Theorem 3.1.** *Let $A$ and $B$ are two subsets of $E$ and $\gamma_A, \gamma_B$ be two $IFS$-sets over $U$. Then*

*i.* ${}^{\alpha}_{\beta}\gamma_A \tilde{\cup}^{\alpha}_{\beta}\gamma_B =^{\alpha}_{\beta} \gamma_{A\tilde{\cup}B}$

*ii.* ${}^{\alpha}_{\beta}\gamma_A \tilde{\cap}^{\alpha}_{\beta}\gamma_B =^{\alpha}_{\beta} \gamma_{A\tilde{\cap}B}$.

*Proof.* i. For all $x \in E$,

$$\begin{aligned}
({}^{\alpha}_{\beta}\gamma_A \tilde{\cup}^{\alpha}_{\beta}\gamma_B)(x) &= ({}^{\alpha}_{\beta}\gamma_A)(x) \cup ({}^{\alpha}_{\beta}\gamma_B)(x) \\
&= \{u : \overline{\gamma}_{A(x)}(u) \geq \alpha \text{ and } \underline{\gamma}_{A(x)}(u) \leq \beta\} \\
&\quad \cup\{u : \overline{\gamma}_{B(x)}(u) \geq \alpha \text{ and } \underline{\gamma}_{B(x)}(u)) \leq \beta\} \\
&= \{u : (\overline{\gamma}_{A(x)}(u) \geq \alpha \text{ or } \overline{\gamma}_{B(x)}(u)) \geq \alpha) \\
&\quad \text{ and } (\underline{\gamma}_{A(x)}(u) \leq \beta \text{ or } \underline{\gamma}_{B(x)}(u)) \leq \beta)\} \\
&= \{u : \overline{\gamma}_{(A\tilde{\cup}B)(x)}(u) \geq \alpha \text{ and } \underline{\gamma}_{(A\tilde{\cup}B)(x)}(u) \leq \beta\} \\
&= {}^{\alpha}_{\beta}\gamma_{(A\tilde{\cup}B)}(x).
\end{aligned}$$

ii. Similar to the proof of (i).

□

**Theorem 3.2.** *Let $\gamma_A \in \mathcal{IFS}(U)$ and, $\{\alpha_i : i \in I\}$ and $\{\beta_j : j \in I\}$ be two non-empty subsets of $[0,1]$. If $\underline{\alpha} = min\{\alpha_i : i \in I\}, \overline{\alpha} = max\{\alpha_i : i \in I\}$, $\underline{\beta} = min\{\beta_j : j \in I\}$ and $\overline{\beta} = max\{\beta_j : j \in I\}$, then the following assertions hold,*

*i.* $\tilde{\bigcup}_{(i\in I)} {}^{\alpha_i}_{\beta_j}\gamma_A \tilde{\subseteq} {}^{\underline{\alpha}}_{\overline{\beta}}\gamma_A$

*ii.* $\tilde{\bigcap}_{(i\in I)} {}^{\alpha_i}_{\beta_j}\gamma_A = {}^{\overline{\alpha}}_{\underline{\beta}}\gamma_A$.

*Proof.* The proof is clear from Definition 2.18. □

**Definition 3.3.** *Let $\gamma_A$ be an IFS-set over U. Image of A, denoted $Im(\gamma_A)$, is a set of intuitionistic fuzzy sets, consist of image of all $x \in A$ under $\gamma_A$.*

*The image of $x \in A$, $Im(\gamma_{A(x)})$, is the set of all ordered pairs $(\overline{\gamma}_{A(x)}(u), \underline{\gamma}_{A(x)}(u))$ for all $u \in U$. Union of images of all elements in A is denoted by $Im(\gamma_A)(U)$.*

**Proposition 3.4.** *Let $(\alpha_i, \beta_j) \in \gamma_G(U)$, such that $\beta_1 \le \beta_2 \le ...$ , $\alpha_1 \le \alpha_2 \le ...$ and $(\overline{\theta}, \underline{\theta}) \in [0,1] \times [0,1]$.*

*i. If $\alpha_i \le \overline{\theta} \le \alpha_{i+1}$ and $\beta_j \le \underline{\theta} \le \beta_{j+1}$, then, ${}^{\alpha_i}_{\beta_j}\gamma_G = {}^{\overline{\theta}}_{\underline{\theta}}\gamma_G$.*

*ii. $(\overline{\theta}, \underline{\theta})-$level set is equal to one of $(\alpha_i, \beta_j)-$level set, for any ordered pairs $(\overline{\theta}, \underline{\theta})$ such that $\overline{\theta} \le \overline{\gamma}_{G(e)}(u)$ and $\underline{\theta} \ge \overline{\gamma}_{G(e)}(u)$ for all $u \in U$.*

**Definition 3.5.** *Let f be a function from A into B and $\gamma_A, \gamma_B$ be two IFS-sets over U. Then, the intuitionistic fuzzy soft subsets $f(\gamma_A)$ and $f^{-1}(\gamma_B)$ over U are defined, respectively, as follow*

$$f(\gamma_A)(y) = \begin{cases} \sqcup\{\gamma_A(x) : x \in A, f(x) = y\}, & \text{if } f(x) \in f(A) \\ \gamma_\emptyset, & \text{otherwise} \end{cases}$$

*for all $y \in B$ and $f^{-1}(\gamma_B)(x) = \gamma_B(f(x))$ for all $x \in A$. Here, $f(\gamma_A)$ is called the image of $\gamma_A$ under f and $f^{-1}(\gamma_B)$ is called the preimage (or inverse image) of $\gamma_B$ under f.*

**Example 3.6.** *Assume that $U = \{u_1, u_2, u_3\}$ is a universal set. Let $A = \{-1, 0, 1, 2\}$ and $B = \{0, 1, 2, 3, 4\}$ be two subsets of set of parameters, and $f : A \to B$, $f(x) = x^2$. We define an IFS-set over U*

$$\begin{aligned} \gamma_A = \{&(-1, \{\langle u_1/0.5/0.3\rangle, \langle u_2/0.6/0.1\rangle, \langle u_3/0.4/0.5\rangle\}), \\ &(0, \{\langle u_1/0.7/0.2\rangle, \langle u_2/0.4/0.3\rangle, \langle u_3/0.2/0.6\rangle\}), \\ &(1, \{\langle u_1/0.7/0.1\rangle, \langle u_2/0.8/0.2\rangle, \langle u_3/0.5/0.3\rangle\}), \\ &(2, \{\langle u_1/0.3/0.6\rangle, \langle u_2/0.5/0.2\rangle, \langle u_3/0.4/0.5\rangle\})\} \end{aligned}$$

*and*

$$\begin{aligned} \gamma_B = \{&(0, \{\langle u_1/0.4/0.3\rangle, \langle u_2/0,6/0.1\rangle, \langle u_3/0.7/0.2\rangle\}), \\ &(1, \{\langle u_1/0.5/0.3\rangle, \langle u_2/0.6/0.2\rangle, \langle u_3/0.1/0.7\rangle\}), \\ &(2, \{\langle u_1/0.3/0.5\rangle, \langle u_2/0.4/0.2\rangle, \langle u_3/0.4/0.4\rangle\}), \\ &(3, \{\langle u_1/0/1\rangle, \langle u_2/0/1\rangle, \langle u_3/0/1\rangle\}), \\ &(4, \{\langle u_1/0.5/0.5\rangle, \langle u_2/0.4/0.3\rangle, \langle u_3/0.3/0.5\rangle\})\}. \end{aligned}$$

*Then,*

$$\begin{aligned} f(\gamma_A) \;=\; & \{(0, \{\langle u_1/0.7/0.2\rangle, \langle u_2/0.4/0.3\rangle, \langle u_3/0.2/0.6\rangle\}), \\ & (1, \{\langle u_1/0.7/0.1\rangle, \langle u_2/0.8/0.1\rangle, \langle u_3/0.5/0.3\rangle\}), \\ & (2, \{\langle u_1/0/1\rangle, \langle u_2/0/1\rangle, \langle u_3/0/1\rangle\}), \\ & (3, \{\langle u_1/0/1\rangle, \langle u_2/0/1\rangle, \langle u_3/0/1\rangle\}), \\ & (4, \{\langle u_1/0.3/0.6\rangle, \langle u_2/0.5/0.2\rangle, \langle u_3/0.4/0.5\rangle\})\} \end{aligned}$$

*and*

$$\begin{aligned} f^{-1}(\gamma_B) \;=\; & \{(1, \{\langle u_1/0.5/0.3\rangle, \langle u_2/0.6/0.2\rangle, \langle u_3/0.1/0.7\rangle\}), \\ & (0, \{\langle u_1/0.4/0.3\rangle, \langle u_2/0,6/0.1\rangle, \langle u_3/0.7/0.2\rangle\}), \\ & (-1, \{\langle u_1/0.5/0.3\rangle, \langle u_2/0.6/0.2\rangle, \langle u_3/0.1/0.7\rangle\}), \\ & (2, \{\langle u_1/0.5/0.5\rangle, \langle u_2/0.4/0.3\rangle, \langle u_3/0.3/0.5\rangle\})\}. \end{aligned}$$

**Theorem 3.7.** *Let $f$ be a function from $A$ into $B$, $A_i \subseteq A$, $B_i \subseteq B$ and $\gamma_{A_i}, \gamma_{B_i} \in \mathcal{IFS}(U)$ for all $i \in I$. Then*

*i.* $f(\tilde{\sqcup}_{i\in I}\gamma_{A_i}) = \tilde{\sqcup}_{i\in I} f(\gamma_{A_i})$

*ii.* $\gamma_{A_1} \tilde{\sqsubseteq} \gamma_{A_2} \Rightarrow f(\gamma_{A_1}) \tilde{\sqsubseteq} f(\gamma_{A_2})$

*iii.* $\gamma_{B_1} \tilde{\sqsubseteq} \gamma_{B_2} \Rightarrow f^{-1}(\gamma_{B_1}) \tilde{\sqsubseteq} f^{-1}(\gamma_{B_2})$.

*Proof.* i. For all $i \in I$, $IFS$-sets $\gamma_{A_i}$ and $y \in B$

$$\begin{aligned} f(\tilde{\sqcup}_{i\in I}\gamma_{A_i})(y) &= \sqcup\{\gamma_{A_i}(x) : x \in A_i, f(x) = y\} \\ &= \sqcup\{\sqcup\gamma_{A_i(x)}(u) : x \in A_i, f(x) = y, u \in U\} \\ &= \sqcup_{i\in I}\{\gamma_{A_i}(x) : x \in A_i, f(x) = y\} \\ &= \sqcup_{i\in I} f(\gamma_{A_i})(y). \end{aligned}$$

ii. Let $\gamma_{A_1} \tilde{\sqsubseteq} \gamma_{A_2}$, so $A_1 \subseteq A_2$, then

$$\begin{aligned} f(\gamma_{A_1})(y) &= \sqcup\{\gamma_{A_1}(x) : x \in A_1, f(x) = y\} \\ &\sqsubseteq \sqcup\{\gamma_{A_2}(x) : x \in A_2, f(x) = y\} \\ &= f(\gamma_{A_2})(y). \end{aligned}$$

iii. Let $\gamma_{B_1} \tilde{\sqsubseteq} \gamma_{B_2}$ then, for all $x \in A$

$$\begin{aligned} f^{-1}(\gamma_{B_1})(x) &= \gamma_{B_1}(f(x)) \\ &\sqsubseteq \gamma_{B_2}(f(x)) \\ &= f^{-1}(\gamma_{B_2}(x)) \\ &= f^{-1}(\gamma_{B_2})(x). \end{aligned}$$

□

**Theorem 3.8.** *Let $f$ be a function from $A$ into $B$, $I$ be a nonempty index set, $B_i \subseteq B$ and $\gamma_{B_i} \in \mathcal{IFS}(U)$ for all $i \in I$. Then,*

*i.* $f^{-1}(\tilde{\sqcup}_{i\in I}\gamma_{B_i}) = \tilde{\sqcup}_{i\in I} f^{-1}(\gamma_{B_i})$

*ii.* $f^{-1}(\tilde{\sqcap}_{i\in I}\gamma_{B_i}) = \tilde{\sqcap}_{i\in I}f^{-1}(\gamma_{B_i})$.

*Proof.* For all $x \in A$,

i. $f^{-1}(\tilde{\sqcup}_{i\in I}\gamma_{B_i})(x) = \sqcup_{i\in I}\gamma_{B_i}(f(x)) = \sqcup_{i\in I}f^{-1}(\gamma_{B_i})(x)$

ii. $f^{-1}(\tilde{\sqcap}_{i\in I}\gamma_{B_i})(x) = \sqcap_{i\in I}\gamma_{B_i}(f(x)) = \sqcap_{i\in I}f^{-1}(\gamma_{B_i})(x)$.

□

**Theorem 3.9.** *Let $f$ be a function from $A$ into $B$. Then, $f^{-1}(f(\gamma_A))\tilde{\sqsupseteq}\gamma_A$ for all $\gamma_A \in \mathcal{IFS}(U)$. In particular, if $f$ is an injective function, then $f^{-1}(f(\gamma_A)) = \gamma_A$.*

*Proof.* For all $x \in A$,

$$\begin{aligned} f^{-1}(f(\gamma_A))(x) &= f(\gamma_A)(f(x)) \\ &= f(\gamma_A(f(x))) \\ &= \sqcup\{\gamma_A(x') : x' \in A, f(x') = f(x)\} \\ &\sqsupseteq \gamma_A(x) \end{aligned}$$

thus $f^{-1}(f(\gamma_A))\tilde{\sqsupseteq}\gamma_A$.

It is clear that if $f$ is one to one function, then $f(x') = f(x)$ implies $x' = x$ and the last inclusion is reduced to equality. □

**Theorem 3.10.** *Let $f$ be a function from $A$ into $B$. For all $\gamma_B \in \mathcal{IFS}(U)$, $f(f^{-1}(\gamma_B))\tilde{\sqsubseteq}\gamma_B$. In particular, if $f$ is an surjective function, then $f(f^{-1}(\gamma_B)) = \gamma_B$.*

*Proof.* For all $x \in A$,

$$\begin{aligned} f(f^{-1}(\gamma_B))(y) &= \sqcup\{f^{-1}(\gamma_B)(x) : x \in A, f(x) = y\} \\ &= \sqcup\{(\gamma_B)(f(x)) : \forall x \in A, f(x) = y\} \\ &= \begin{cases} \gamma_B(y), & \text{if } y \in f(A) \\ \gamma_\emptyset, & \text{otherwise} \end{cases} \\ &\sqsubseteq \gamma_B(y). \end{aligned}$$

Thus $f(f^{-1}(\gamma_B))\tilde{\sqsubseteq}\gamma_B$. If $f$ is an onto function, then $y \in f(A)$ for all $y \in B$ and so $f(f^{-1}(\gamma_B)) = \gamma_B$. □

**Theorem 3.11.** *Let $f$ be a function from $A$ into $B$. Then, $f(\gamma_A)\tilde{\sqsubseteq}\gamma_B \Leftrightarrow \gamma_A\tilde{\sqsubseteq}f^{-1}(\gamma_B)$ for all $\gamma_A, \gamma_B \in \mathcal{IFS}(U)$.*

*Proof.* We know from Theorem 3.7 that, $f(\gamma_A)\tilde{\sqsubseteq}\gamma_B \Rightarrow f^{-1}(f(\gamma_A))\tilde{\sqsubseteq}f^{-1}(\gamma_B)$ and from Theorem 3.9, $\gamma_A\tilde{\sqsubseteq}f^{-1}(f(\gamma_A))$, so $\gamma_A\tilde{\sqsubseteq}f^{-1}(\gamma_B)$.
Conversely, let $\gamma_A\tilde{\sqsubseteq}f^{-1}(\gamma_B)$. Then from Theorem 3.7 and 3.10, $f(\gamma_A)\tilde{\sqsubseteq}f(f^{-1}(\gamma_B)\tilde{\sqsubseteq}\gamma_B$.
□

**Theorem 3.12.** *Let $f$ be a function from $A$ into $B$ and $g$ be a function from $B$ into $C$. Then,*

*i.* $g(f(\gamma_A)) = (g \circ f)(\gamma_A)$, *for all* $\gamma_A \in \mathcal{IFS}(U)$.

*ii.* $f^{-1}(g^{-1}(\gamma_C)) = (g \circ f)^{-1}(\gamma_C)$, *for all* $\gamma_C \in \mathcal{IFS}(U)$.

*Proof.* Consider any $\gamma_A \in \mathcal{IFS}(U)$ and any $z \in C$, then

i.
$$\begin{aligned} g(f(\gamma_A))(z) &= \sqcup\{f(\gamma_A)(y) : y \in B, g(y) = z\} \\ &= \sqcup\{\sqcup\{(\gamma_A)(x) : x \in A, f(x) = y\} : y \in B, g(y) = z\} \\ &= \sqcup\{(\gamma_A)(x) : x \in A, (g \circ f)(x) = z\} \\ &= (gof)\gamma_A(z). \end{aligned}$$

ii. For any $\gamma_C \in \mathcal{IFS}(U)$ and for all $x \in A$,
$$\begin{aligned} (g \circ f)^{-1}(\gamma_C)(x) &= \gamma_C(g(f(x))) \\ &= g^{-1}(\gamma_C)(f(x)) \\ &= f^{-1}(g^{-1}(\gamma_C))(x). \end{aligned}$$

□

**Definition 3.13.** *Let $G$ be an arbitrary group and $\gamma_G$, $\beta_G$ be two $IFS$-sets over $U$. Then, product of $\gamma_G$ and $\beta_G$ is defined as follow, for all $x \in G$,*

$$(\gamma_G * \beta_G)(x) = \sqcup\{\gamma_G(y) \sqcap \beta_G(z) : y, z \in G \text{ and } yz = x\}$$

*and inverse of $\gamma_G$ is*
$$\gamma_G^{-1}(x) = \gamma_G(x^{-1}).$$

**Theorem 3.14.** *The product, defined in Definition 3.13, is associative.*

*Proof.* Let $G$ be a group and $\gamma_G, \beta_G, \theta_G \in \mathcal{IFS}(U)$. Then,

$$\begin{aligned} & [(\gamma_G * \beta_G) * \theta_G](x) \\ = \; & \sqcup\{(\gamma_G * \beta_G)(y) \sqcap \theta_G(z) : yz = x, \quad y, z \in G\} \\ = \; & \sqcup\{\sqcup\{(\gamma_G(u) \sqcap \beta_G(v)) : uv = y\} \sqcap \theta_G(z) : yz = x, \quad y, z \in G\} \\ = \; & \sqcup\{(\gamma_G(u) \sqcap \beta_G(v)) \sqcap \theta_G(z) : uvz = x, \quad u, v, z \in G\} \\ = \; & \sqcup\{\gamma_G(u) \sqcap (\beta_G(v) \sqcap \theta_G(z)) : uvz = x, \quad u, v, z \in G\} \\ = \; & \sqcup\{\gamma_G(u) \sqcap \{\beta_G(v) \sqcap \theta_G(z) : vz = t, \; v, z \in G\} : ut = x, \; u, t \in G\} \\ = \; & \sqcup\{\gamma_G(u) \sqcap (\beta_G * \theta_G)(t) : ut = x, \quad u, t \in G\} \\ = \; & [\gamma_G * (\beta_G * \theta_G)](x) \end{aligned}$$

so it is associative. □

**Theorem 3.15.** *Let $\gamma_G, \beta_G, \gamma_{i_G} \in \mathcal{IFS}(U)$ for all $i \in I$. Then the following assertions hold,*

*i.* $(\gamma_G * \beta_G)(x) = \sqcup_{y \in G}\{\gamma_G(y) \sqcap \beta_G(y^{-1}x)\} = \sqcup_{y \in G}\{\gamma_G(xy^{-1}) \sqcap \beta_G(y)\}$

*ii.* $[(\gamma_G^{-1})]^{-1} = \gamma_G$

*iii.* $\gamma_G \tilde{\sqsubseteq} \gamma_G^{-1} \Leftrightarrow \gamma_G^{-1} \tilde{\sqsubseteq} \gamma_G \Leftrightarrow \gamma_G^{-1} = \gamma_G$

*iv.* $\gamma_G \tilde{\sqsubseteq} \beta_G \Leftrightarrow \gamma_G^{-1} \tilde{\sqsubseteq} \beta_G^{-1}$

*v.* $(\tilde{\sqcup}_{i \in I}\gamma_{i_G})^{-1} = \tilde{\sqcup}_{i \in I}\gamma_{i_G}^{-1}$

*vi.* $(\tilde{\sqcap}_{i \in I}\gamma_{i_G})^{-1} = \tilde{\sqcap}_{i \in I}\gamma_{i_G}^{-1}$

*vii.* $(\gamma_G * \beta_G)^{-1} = \beta_G^{-1} * \gamma_G^{-1}$.

*Proof.* i. For all $y \in G$, $y(y^{-1}x) = x$ or $(xy^{-1})y = x$ includes all alternatives of components of $x$, so equality holds.

ii. For all $x \in G$,
$(\gamma_G^{-1})^{-1}(x) = \gamma_G^{-1}(x^{-1}) = \gamma_G((x^{-1})^{-1}) = \gamma_G(x)$.

iii. Let $\gamma_G \tilde{\sqsubseteq} \gamma_G^{-1}$. Then, for all $x \in G$,

$$\gamma_G(x) \sqsubseteq \gamma_G^{-1}(x) = \gamma_G(x^{-1})$$

In last expression substituting $x^{-1}$ instead of $x$ we get

$$\begin{aligned} \gamma_G(x^{-1}) \sqsubseteq \gamma_G((x^{-1})^{-1}) &\Leftrightarrow \gamma_G^{-1}(x) \sqsubseteq \gamma_G(x) \\ &\Leftrightarrow \gamma_G^{-1} \tilde{\sqsubseteq} \gamma_G \end{aligned}$$

therefore $\gamma_G^{-1} = \gamma_G$.

iv. The proof is direct from (3).

v. For all $x \in G$,

$$\begin{aligned} (\tilde{\sqcup}_{i\in I}\gamma_{i_G})^{-1}(x) &= (\tilde{\sqcup}_{i\in I}\gamma_{i_G})(x^{-1}) \\ &= \sqcup_{i\in I}\gamma_{i_G}(x^{-1}) \\ &= \sqcup_{i\in I}\gamma_{i_G}^{-1}(x). \end{aligned}$$

vi. The proof is similar to 3.

vii. For all $x \in G$,

$$\begin{aligned} (\gamma_G * \beta_G)^{-1}(x) &= (\gamma_G * \beta_G)(x^{-1}) \\ &= \sqcup\{\gamma_G(y) \sqcap \beta_G(z) : y, z \in G \, and \, yz = x^{-1}\} \\ &= \sqcup\{\gamma_G(y) \sqcap \beta_G(z) : y, z \in G \, and \, (yz)^{-1} = (x^{-1})^{-1}\} \\ &= \sqcup\{\beta_G(z^{-1})^{-1} \sqcap \gamma_G(y^{-1})^{-1} : y^{-1}, z^{-1} \in G \, and \, z^{-1}y^{-1} = x\} \\ &= \sqcup\{\beta_G^{-1}(z^{-1}) \sqcap \gamma_G^{-1}(y^{-1}) : y^{-1}, z^{-1} \in G \, and \, z^{-1}y^{-1} = x\} \\ &= (\beta_G^{-1} * \gamma_G^{-1})(x). \end{aligned}$$

□

# 4 Intuitionistic fuzzy soft groups

In this section, we introduce notion of intuitionistic fuzzy soft group ($IFS$-group) by inspiring from the fuzzy group of Rosenfefld [26].

Troughout this section $G$ denotes an arbitrary group with identity element e.

**Definition 4.1.** *Let $G$ be an arbitrary group and $\gamma_G \in \mathcal{IFS}(U)$. $\gamma_G$ is called intuitionistic fuzzy soft groupoid if $\gamma_G(xy) \sqsupseteq \gamma_G(x) \sqcap \gamma_G(y)$ for all $x, y \in G$ and is called intuitionistic fuzzy soft group ($IFS$-group) if $\gamma_G(x^{-1}) = \gamma_G(x)$ for all $x \in G$.*

From now on, set of all intuitionistic fuzzy soft groups on $G$ over $U$ is denoted by $\mathcal{IFS}(G_U)$.

**Definition 4.2.** *Let $\gamma_G \in \mathcal{IFS}(G_U)$ and $e$ be the identity of $G$. Then $e$-set of $\gamma_G$, denoted by $\gamma_G^e$, is defined by*

$$\gamma_G^e = \{x \in G : \gamma_G(x) = \gamma_G(e)\}.$$

**Example 4.3.** *Assume that $U = \{u_1, u_2, u_3\}$ is a universal set and $G = Z_4$ is a set of parameters. We define an $IFS$-set $\gamma_G$ by*

| $\gamma_G$ | $\gamma_G(0)$ | $\gamma_G(1)$ | $\gamma_G(2)$ | $\gamma_G(3)$ |
|---|---|---|---|---|
| $u_1$ | $(0.6, 0.2)$ | $(0.4, 0.5)$ | $(0.5, 0.3)$ | $(0.4, 0.5)$ |
| $u_2$ | $(0.5, 0.4)$ | $(0.3, 0.6)$ | $(0.4, 0.5)$ | $(0.3, 0.6)$ |
| $u_3$ | $(0.7, 0.1)$ | $(0.4, 0.3)$ | $(0.6, 0.2)$ | $(0.4, 0.3)$ |

Table 1. *An IFS-group*

*which satisfies all conditions of $IFS$-group.*

**Proposition 4.4.** *Let $\gamma_G \in \mathcal{IFS}(G_U)$. Then,*

*i.* $\gamma_G(x^n) \sqsupseteq \gamma_G(x)$ *for all* $x \in G$

*ii.* $\gamma_G(e) \sqsupseteq \gamma_G(x)$ *for all* $x \in G$

*iii.* $\gamma_G(xy) \sqsupseteq \gamma_G(y)$ *for all* $x, y \in G$ *if and only if* $\gamma_G(x) = \gamma_G(e)$.

*Proof.* From Definition 4.1, the results are trivial. □

**Theorem 4.5.** *An $IFS$-set $\gamma_G$ over $U$ is an $IFS$-group over $U$ if and only if $\gamma_G(xy^{-1}) \sqsupseteq \gamma_G(x) \sqcap \gamma_G(y)$ for all $x, y \in G$.*

*Proof.* Let $\gamma_G \in \mathcal{IFS}(G_U)$. Then for all $x, y \in G$, $\gamma_G(xy^{-1}) \sqsupseteq \gamma_G(x) \sqcap \gamma_G(y^{-1}) = \gamma_G(x) \sqcap \gamma_G(y)$.

Suppose that $\gamma_G(xy^{-1}) \sqsupseteq \gamma_G(x) \sqcap \gamma_G(y)$ for all $x, y \in G$. Substituting $x = e$ we get $\gamma_G(y^{-1}) \sqsupseteq \gamma_G(y)$. Thus, $\gamma_G(y) = \gamma_G((y^{-1})^{-1}) \sqsupseteq \gamma_G(y^{-1})$ and so $\gamma_G(y) = \gamma_G(y^{-1})$. In addition $\gamma_G(xy) = \gamma_G(x(y^{-1})^{-1}) \sqsupseteq \gamma_G(x) \sqcap \gamma_G(y^{-1}) = \gamma_G(x) \sqcap \gamma_G(y)$. Therefore $\gamma_G$ is a $IFS$-group over $U$. □

**Definition 4.6.** *Let $G$ be a group, $H$ be a subgroup of $G$, $\gamma_G$ be an $IFS$-group over $U$. Then, $\gamma_H$ is said to be an intuitionistic fuzzy soft subgroup of $\gamma_G$ over $U$ if $\gamma_H$ is an $IFS$-group over $U$. It is denoted by $\gamma_H \leq_{ifs} \gamma_G$.*

**Example 4.7.** *Let $\gamma_G$ be as in Example 4.3 and $H = \{0, 2\}$. Then, $\gamma_H \leq_{ifs} \gamma_G$.*

**Theorem 4.8.** *Let $\gamma_G$ be an $IFS$-group over $U$ and, $\gamma_H$ and $\gamma_N$ be two intuitionistic fuzzy soft subgroups of $\gamma_G$ over $U$. Then $\gamma_H \tilde{\sqcap} \gamma_N$ is an intuitionistic fuzzy soft subgroup of $\gamma_G$ over $U$.*

*Proof.* From Definition 2.17, $\gamma_A \tilde{\sqcap} \gamma_B = \{(x, \gamma_{A \tilde{\cap} B}(x)) : x \in G\}$, so for $x, y \in G$,

$$\begin{aligned}
\gamma_{H \tilde{\cap} N}(xy^{-1}) &= \gamma_H(xy^{-1}) \sqcap \gamma_N(xy^{-1}) \\
&\sqsupseteq (\gamma_H(x) \sqcap \gamma_H(y)) \sqcap (\gamma_N(x) \sqcap \gamma_N(y)) \\
&= (\gamma_H(x) \sqcap \gamma_N(x)) \sqcap (\gamma_H(y) \sqcap \gamma_N(y)) \\
&= \gamma_{H \tilde{\cap} N}(x) \sqcap \gamma_{H \tilde{\cap} N}(y).
\end{aligned}$$

Thus $\gamma_H \tilde{\sqcap} \gamma_N$ is a intuitionistic fuzzy soft subgroup of $\gamma_G$ over $U$. □

It is also possible to prove that the union of two intuitionistic fuzzy soft subgroup need not be an $IFS$-*group.*

**Theorem 4.9.** *Let $\gamma_G$ be an $IFS$-group over $U$. Then $\gamma_G^e$ is a subgroup of $G$.*

*Proof.* Since $e \in \gamma_G^e$ then $\gamma_G^e \neq \emptyset$. Let $x, y \in \gamma_G^e$, then $\gamma_G(x) = \gamma_G(e) = \gamma_G(y)$. Firstly,

$$\begin{aligned} \gamma_G(xy^{-1}) &\sqsupseteq \gamma_G(x) \sqcap \gamma_G(y) \\ &= \gamma_G(e) \sqcap \gamma_G(e) \\ &= \gamma_G(e). \end{aligned}$$

Since $\gamma_G(e) \sqsupseteq \gamma_G(xy^{-1})$, then $\gamma_G(xy^{-1}) = \gamma_G(e)$. Thus $xy^{-1} \in \gamma_G^e$.
Therefore $\gamma_G^e$ is an intuitionistic fuzzy soft subgroup of $G$. □

**Theorem 4.10.** *Let $T = Im(\gamma_G)(U) \cup \{(\overline{\theta}, \underline{\theta}) : 0 \leq \overline{\theta} \leq \overline{\gamma}_{G(e)}(u), \underline{\gamma}_{G(e)}(u) \leq \underline{\theta} \leq 1, \overline{\theta}, \underline{\theta} \in R\}$. If $\gamma_G \in \mathcal{IFS}(G_U)$, then ${}^{\alpha}_{\beta}\gamma_G$ is a soft subgroup of $G$ for all $(\alpha, \beta) \in T$.*

*Proof.* Suppose $\gamma_G \in \mathcal{IFS}(G_U)$ and let $(\alpha, \beta) \in T$. Since $\gamma_G(e) \sqsupseteq \gamma_G(x)$, for all $x \in G$ then $e \in^{\alpha}_{\beta} \gamma_G$. Thus ${}^{\alpha}_{\beta}\gamma_G \neq \emptyset$. Let $x, y \in^{\alpha}_{\beta} \gamma_G$. Then, for all $u \in Im(\gamma_{G(x)}), \overline{\gamma}_{G(x)}(u) \geq \alpha$ and $\underline{\gamma}_{G(x)}(u) \leq \beta$, and for all $u \in Im(\gamma_{G(y)}), \overline{\gamma}_{G(y)}(u) \geq \alpha$ and $\underline{\gamma}_{G(y)}(u) \leq \beta$. Since $\gamma_G$ is an intuitionistic fuzzy soft subgroup,

$$\begin{aligned} \gamma_G(xy^{-1}) &\sqsupseteq \gamma_G(x) \sqcap \gamma_G(y) \\ &= \{\langle u/min\{\overline{\gamma}_{G(x)}(u), \overline{\gamma}_{G(y)}(u)\}/max\{\underline{\gamma}_{G(x)}(u), \underline{\gamma}_{G(y)}(u)\}\rangle : u \in U\} \\ &\sqsupseteq \{\langle u/\alpha/\beta\rangle : u \in U\}. \end{aligned}$$

So for all $u \in U$, $\overline{\gamma}_{G(xy^{-1})}(u) \geq \alpha$ and $\underline{\gamma}_{G(xy^{-1})}(u) \leq \beta$, thus $xy^{-1} \in^{\alpha}_{\beta} \gamma_G$. This follows that ${}^{\alpha}_{\beta}\gamma_G$ is soft subgroup of $G$. □

**Theorem 4.11.** *Let $\gamma_G \in \mathcal{IFS}(U)$. Then, $\gamma_G \in \mathcal{IFS}(G_U)$ if and only if $\gamma_G$ satisfies the following conditions;*

*i.* $\gamma_G * \gamma_G \tilde{\sqsubseteq} \gamma_G$

*ii.* $\gamma_G^{-1} \tilde{\sqsubseteq} \gamma_G$ *(or* $\gamma_G^{-1} \tilde{\sqsupseteq} \gamma_G$ *or* $\gamma_G^{-1} = \gamma_G$*).*

*Proof.* Let $\gamma_G \in \mathcal{IFS}(G_U)$. Then,

i.

$$\begin{aligned} (\gamma_G * \gamma_G)(x) &= \sqcup\{\gamma_G(y) \sqcap \gamma_G(z) : yz = x, y, z \in G\} \\ &\sqsubseteq \sqcup\{\gamma_G(yz) : yz = x, y, z \in G\} \\ &= \gamma_G(x). \end{aligned}$$

ii. By the definition of $IFS$-group and Theorem 3.15-(iii) the necessary condition is obvious. Conversely; suppose $(\gamma_G * \gamma_G) \tilde{\sqsubseteq} \gamma_G$ then $(\gamma_G * \gamma_G)(x) \sqsubseteq \gamma_G(x)$, for all $x \in G$ so

$$\begin{aligned} \gamma_G(x) &\sqsupseteq (\gamma_G * \gamma_G)(x) \\ &= \sqcup\{\gamma_G(y) \sqcap \gamma_G(z) : yz = x, \quad y, z \in G\} \\ &\sqsupseteq \{\gamma_G(y) \sqcap \gamma_G(z) : yz = x, \quad y, z \in G\}, \end{aligned}$$

as a result for any $x = yz$, $\gamma_G(yz) \sqsupseteq \gamma_G(y) \sqcap \gamma_G(z)$ and by Definition 4.1, $\gamma_G \in \mathcal{IFS}(G_U)$.

□

**Theorem 4.12.** *Let $\gamma_G$, $\beta_G \in \mathcal{IFS}(G_U)$. Then, $\gamma_G * \beta_G \in \mathcal{IFS}(G_U)$ if and only if $\gamma_G * \beta_G = \beta_G * \gamma_G$.*

*Proof.* Suppose that $\gamma_G * \beta_G \in \mathcal{IFS}(G_U)$. Then,

$$\gamma_G * \beta_G = \gamma_G^{-1} * \beta_G^{-1} = (\beta_G * \gamma_G)^{-1} = \beta_G * \gamma_G.$$

Conversely, suppose that $\gamma_G * \beta_G = \beta_G * \gamma_G$. Then,

$$\begin{array}{rcl}(\gamma_G \ * \beta_G) * (\gamma_G \ * \beta_G) & = & \gamma_G * (\beta_G * \gamma_G) * \beta_G \\ & = & \gamma_G * (\gamma_G * \beta_G) * \beta_G \\ & = & (\gamma_G * \gamma_G) * (\beta_G * \beta_G) \\ & \tilde{\sqsubseteq} & \gamma_G * \beta_G\end{array}$$

and

$$(\gamma_G \ * \beta_G)^{-1} = (\beta_G * \gamma_G)^{-1} = \gamma_G^{-1} * \beta_G^{-1} = \gamma_G * \beta_G.$$

Consequently by Theorem 4.11, $\gamma_G * \beta_G \in \mathcal{IFS}(G_U)$. □

**Theorem 4.13.** *Let $\gamma_G \in \mathcal{IFS}(G_U)$ and $f$ be a homomorphism of $G$ into $H$. Then, $f(\gamma_G) \in \mathcal{IFS}(H_U)$.*

*Proof.* Let $u, v \in H$. If either $u \notin f(G)$ or $v \notin f(G)$, then $f(\gamma_G)(u) \sqcap f(\gamma_G)(v) = \hat{\emptyset} \sqsubseteq f(\gamma_G)(uv)$. Assume $u \notin f(G)$ then $u^{-1} \notin f(G)$, thus $f(\gamma_G)(u) = \hat{\emptyset} = f(\gamma_G)(u^{-1})$ so subgroup conditions are satisfied. Now suppose that $u = f(x)$ and $v = f(y)$ for some $x, y \in G$. Then

$$\begin{array}{rcl}(f(\gamma_G))(uv) & = & \sqcup\{\gamma_G(z) : z \in G, f(z) = uv\} \\ & = & \sqcup\{\gamma_G(xy) : x, y \in G, f(xy) = uv\} \\ & \sqsupseteq & \sqcup\{\gamma_G(x) \sqcap \gamma_G(y) : x, y \in G, f(x) = u, f(y) = v\} \\ & = & (\sqcup\{\gamma_G(x) : x \in G, f(x) = u\}) \sqcap (\sqcup\{\gamma_G(y) : y \in G, f(y) = v\}) \\ & = & (f(\gamma_G))(u) \sqcap (f(\gamma_G))(v).\end{array}$$

In addition,

$$\begin{array}{rcl}(f(\gamma_G))(u^{-1}) & = & \sqcup\{\gamma_G(z) : z \in G, f(z) = u^{-1}\} \\ & = & \sqcup\{\gamma_G(z^{-1}) : z \in G, f(z^{-1}) = u\} \\ & = & (f(\gamma_G))(u).\end{array}$$

Hence $f(\gamma_G)$, is an intuitionistic fuzzy soft group over $U$. □

**Theorem 4.14.** *Let $H$ be a group, $\gamma_H \in \mathcal{IFS}(H_U)$ and $f$ be a homomorphism of $G$ into $H$. Then, $f^{-1}(\gamma_H) \in \mathcal{IFS}(G_U)$.*

*Proof.* Let $x, y \in G$. Then,

$$\begin{array}{rcl}f^{-1}(\gamma_H)(xy) & = & \gamma_H(f(xy)) \\ & = & \gamma_H(f(x)f(y)) \\ & \sqsupseteq & \gamma_H(f(x)) \sqcap \gamma_H(f(y)) \\ & = & f^{-1}(\gamma_H)(x) \sqcap f^{-1}(\gamma_H)(y).\end{array}$$

In addition,

$$\begin{array}{rcl} f^{-1}(\gamma_H)(x^{-1}) & = & \gamma_H(f(x^{-1})) \\ & = & \gamma_H((f(x))^{-1}) \\ & = & \gamma_H(f(x)) \\ & = & f^{-1}(\gamma_H)(x). \end{array}$$

Hence $f^{-1}(\gamma_H) \in \mathcal{IFS}(G_U)$. □

# 5 Conclusion

In this paper, we present some results of intuitionistic fuzzy soft sets on a group. In addition, we give some properties of them. Then, we define intuitionistic fuzzy soft groups and investigate their properties. This study affords us an opportunity to go further on intuitionistic fuzzy soft group, that is, normal intuitionistic fuzzy soft group, quotient group, isomorphism theorems etc.

# References

[1] U. Acar, F. Koyuncu, B. Tanay, Soft sets and soft rings, Computer and Mathemetics with Application, 59, 3458-3463, 2010.

[2] M.I. Ali, F.Feng, X.Liu, W.K. Min, M. Shabir, On some new operations in soft set theory. Computer Mathematics with Application 57, 1547-1553, 2009.

[3] H. Aktaş, N. Çağman, Soft sets and soft groups, Information Sciences 177, 2726-2735, 2007.

[4] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Set and Systems, 20, 87-96, 1986.

[5] K. Kaygısız, On soft int groups, Annals of Fuzzy Mathematics and Informatics. 4(2), 363-375, 2012.

[6] K. Kaygısız, Normal soft int groups, arXiv.1209.3157v1 [math, GR], 14 Sep 2012.

[7] A. Aygünoglu, H. Aygün, Introduction to fuzzy soft groups, Computer and Mathematics with Application, 58, 1279-1286, 2009.

[8] N. Çağman, S. Enginoğlu, Soft set theory and uni-int decision making, European Journal of Operational Research, 207, 848-855, 2010.

[9] N. Çağman, F. Çıtak, S. Enginoğlu, Fuzzy parameterized fuzzy soft set theory and its applications, Turkish Journal of Fuzzy Systems, 1, 21-35, 2010.

[10] N. Çağman, S. Enginoğlu, F. Çıtak Fuzzy soft set theory and its applications, Iranian Journal of Fuzzy Systems 8(3), 137-147, 2011.

[11] N. Çağman, F. Çıtak, H. Aktaş Soft int-group, Neural Computing and Application, 21(1), 151-158, 2012.

[12] M. Fatih, A.R. Salleh, Intuitionistic fuzzy group, Asian Journal of Algebra, 2(1), 1-10, 2009.

[13] F. Feng, Y.B. Jun, X. Zhao, Soft semirings, Computer and Mathematics with Applications, 56, 2621-2628, 2008.

[14] F. Feng, C. Li, B. Davvaz, M.I. Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Computing 14(9), 899-911, 2010.

[15] L. Flep Structure and construction of fuzzy subgroup of a group, Fuzzy Set and Systems 51, 105-109, 1992.

[16] Y. Jiang, Y. Tang, Q. Chen, H. Liu, J. Tang, Interval-valued intuitionistic fuzzy soft sets and their properties, Computer Mathematics with Applications, 60(3), 906-918, 2010.

[17] Y. Jiang, Y. Tang, Q. Chen, An adjustable approach to intuitionistic fuzzy soft sets based decision making, Applied Mathematical Modelling, 35, 824-836, 2011.

[18] Y.B. Jun, Soft BCK/BCI-algebras, Computer Mathematics with Applications, 56, 1408-1413, 2008.

[19] Y.B Jun, C.H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebra Information Sciences, 178, 2466-2475, 2008.

[20] Y.B. Jun, K.J Lee, A. Khan, (2010) Soft ordered semigroups. Mathematical Logic Quarterly, 56(1), 42-50, 2010.

[21] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, Journal of Fuzzy Mathematics 9(3), 589-602, 2001.

[22] P.K. Maji, R. Biswas, A.R. Roy, Intuitionistic fuzzy soft sets, Journal of Fuzzy Mathematics 9(3), 677-692, 2001.

[23] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Computer Mathematics with Applications, 45, 555-562, 2003.

[24] P. Majumdar, S.K. Samanta, Generalised fuzzy soft sets, Computer Mathematics with Application 59, 1425-1432, 2010.

[25] D.A. Molodtsov, Soft set theory-first results, Computer and Mathematics with Applications 37, 19-31, 1999.

[26] A. Rosenfeld, Fuzzy groups, Journal of Mathematics Analysis and Applications, 35, 512-517, 1971.

[27] A. Sezgin, A.O. Atagün, N. Çağman, Union soft substructures of near-rings and N-groups, Neural Computing and Applications, 21(1), 133-143, 2012.

[28] N. Yaqoob, M. Akram, M. Aslam, Intuitionistic fuzzy soft groups induced by $(t, s)$-norm, Indian Journal of Science and Technology, 6(4), 4282-4289, 2013.

[29] L.A. Zadeh, Fuzzy sets, Information Control, 8, 338-353, 1965.

[30] J. Zhou, Y. Li, Y. Yin, Intuitionistic fuzzy soft semigroups, Mathematica Aeterna, 1 03, 173-183, 2011.