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Generalized Star $\omega\alpha$-Closed Sets in Topological Spaces

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Abstract - The aim of this paper is to introduce a new class of closed sets called generalized star $\omega\alpha$-closed sets using $\omega\alpha$-closed sets in topological spaces. This new class of sets lies between the class of closed sets and the class of generalized $\omega\alpha$-closed sets. Further some of their properties are investigated.

Keywords - *Topological spaces; closed sets; $\omega\alpha$-closed sets; generalized $\omega\alpha$-closed sets.*

1 Introduction

The concepts of weaker and stronger forms of closed and open sets play an important role in topological spaces. In 1937, Stone [22] introduced and studied the concept of regular open and regular closed sets which are stronger forms of open and closed sets respectively. After this, Levine [11] and Njastad [13] introduced the concept of new weaker forms of open sets called semi-open and α-open sets in topological spaces respectively.

As a generalization of closed sets, Levine [12] introduced the concept of generalized closed (briefly g-closed) sets which are weaker than closed sets in topological spaces. This idea motivated many topologists for the study of weaker and stronger forms of open and closed sets in topological spaces.

Maki et al [14] [15] introduced and studied generalized α-closed sets and α-generalized closed sets. Sundaram and Sheik John [20] defined and studied ω-closed sets in topological spaces. Maki et al [16], Gnanambal [9], Arya and Nour [3], Bhattacharya and Lahiri [6] introduced and studied the concept of gp-closed, gpr-closed, gs-closed, sg-closed sets which are weaker forms of closed sets and their complements are respective open sets. Recently Benchalli et al [4] [5] studied the concept of weaker forms of closed sets namely $\omega\alpha$-closed sets and generalized $\omega\alpha$-closed (briefly g$\omega\alpha$-closed) sets in topological spaces.

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2 Preliminary

Throughout this paper spaces (X, τ) and (Y, σ) (or simply X and Y) always denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, interior of A and compliment of A respectively.

The α-closure of A is the smallest α-closed set containing A and is denoted by αcl(A). Similarly semi-closure(respectively pre-closure and semipre-closure) of a set A in topological space (X, τ) is the intersection of all semi-closed (respectively pre-closed and semipre-closed) set containing A and is denoted by scl(A) (respectively pcl(A), spcl(A)).

Definition 2.1. *A subset A of a topological space X is called*
(i) semi-open set [11] if A$\subseteq$cl(int(A)) and semi-closed set if int(cl(A))$\subseteq$A.
(ii) pre-open set [17] if A$\subseteq$int(cl(A)) and pre-closed set if cl(int(A))$\subseteq$A.
(iii) α-open set[13] if A$\subseteq$int(cl(int(A))) and α-closed set if cl(int(cl(A)))$\subseteq$A.
(iv) semi-pre-open set [2] (=β-open set [1]) if A$\subseteq$cl(int(cl(A))) and semi-pre-closed (=β-closed set) if cl(int(cl(A)))$\subseteq$A.
(v) regular-open set [22] if A = int(cl(A)) and regular-closed if A = cl(int(A)).

Definition 2.2. *A subset A of a topological space X is called a*
(i) generalized closed (briefly g-closed) set [12] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
(ii) semi-generalized closed (briefly sg-closed) set [6] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in X.
(iii) generalized semi-closed (briefly gs-closed) set [3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
(iv) generalized α-closed (briefly gα-closed) set [14] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in X.
(v) α generalized-closed (briefly αg-closed) set [15] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
(vi) generalized pre-closed (briefly gp-closed) set [16] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
(vii) generalized semipre-closed (briefly gsp-closed) set [7] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
(viii) generalized pre-regular closed (briefly gpr-closed) set [9] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular-open in X.
(ix) ω-closed set [20] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in X.
(x) strongly g-closed (briefly g^*-closed) set [19] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X.
(xi) g^*p-closed [23] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X.
(xii) $\omega\alpha$-closed [4] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ω-open in X.
(xiii) generalized $\omega\alpha$-closed (briefly g$\omega\alpha$-closed) set [5] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\omega\alpha$-open set in X.
(xiv) pre g^*-closed set [10] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\omega\alpha$-open set in X.

3 Generalized Star $\omega\alpha$-Closed Sets in Topological Spaces

In this section, the notion of new class of sets called generalized star $\omega\alpha$-closed (briefly g*$\omega\alpha$-closed) sets are introduced and obtain some of their properties.

Definition 3.1. *A subset A of a topological space X is called generalized star $\omega\alpha$-closed (briefly $g^*\omega\alpha$-closed) set if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\omega\alpha$-open in X.*

Remark 3.2. *From the definition 3.1, we have the following implification.*
closed $\Rightarrow$ $g^*\omega\alpha$-closed $\Rightarrow$ $g\omega\alpha$-closed.
But the inverse implification need not be true as seen from the following example.

Example 3.3. *Let $X = \{ a, b, c \}$ and $\tau = \{ X, \phi, \{ a \}\}$.*
In this topological space (X, τ), the set $A = \{ b \}$ is $g\omega\alpha$-closed but not $g^*\omega\alpha$-closed and closed in X.

Example 3.4. *Let $X = \{ a, b, c \}$ and $\tau = \{ X, \phi, \{ a, b \}\}$.*
In this topological space (X, τ), the set $A = \{ a , c \}$ is $g^*\omega\alpha$-closed but not closed in X.

Theorem 3.5. *Every regular-closed set in X is $g^*\omega\alpha$-closed in X.*
Proof: *Proof follows from [22] and by the definition 3.1.*
The converse of the above theorem is not true as seen from the following example.

Example 3.6. *In example 3.3, the set $A = \{ b, c \}$ is $g^*\omega\alpha$-closed but not regular-closed in X.*

Thus the class of g*$\omega\alpha$-closed sets contains the class of closed sets and contained in the class of g$\omega\alpha$-closed sets in X.

Theorem 3.7. *Every $g^*\omega\alpha$-closed set in X is αg-closed, gs-closed, gp-closed, gsp-closed, gpr-closed, rg-closed, g^*p-closed and pre g^*-closed set in X.*
Proof: *It follows from the fact that every open set is $\omega\alpha$-open set in X [4].*
The converse of the above theorem is not true as seen from the following example.

Example 3.8. *Let $X = \{ a, b, c \}$ and $\tau = \{ X, \phi, \{ a \}\}$.*
In this topological space (X, τ), the set $A = \{ b \}$ is αg-closed, gs-closed, gp-closed, gsp-closed, gpr-closed, rg-closed, g^*p-closed and pre g^*-closed set in X but not $g^*\omega\alpha$-closed in X.

Theorem 3.9. *The class of $g^*\omega\alpha$-closed set is independent of the class of α-closed, semi-closed, $\omega\alpha$-closed, g-closed and g^*-closed sets in X.*

Example 3.10. *Let $X = \{ a, b, c \}$ and $\tau = \{ X, \phi, \{ a \}\}$.*
In this topological space (X, τ), the set $A = \{ b \}$ is α-closed, semi-closed and $\omega\alpha$-closed but not $g^*\omega\alpha$-closed set in X.

Example 3.11. *Let $X = \{ a, b, c \}$ and $\tau = \{ X, \phi, \{ a \}, \{ b, c \}\}$.*
In this topological space (X, τ), the set $A = \{ a, c \}$ is $g^*\omega\alpha$-closed but not α-closed, semi-closed, $\omega\alpha$-closed set in X.

Example 3.12. *Let $X = \{ a, b, c \}$ and $\tau = \{ X, \phi, \{ a \}, \{ a, c \}\}$. In this topological space (X, τ),*
$g^*\omega\alpha$-closed sets are $X, \phi, \{ b, c \}, \{b \}, \{ c \}$
g-closed sets are $X, \phi, \{ a, b \}, \{ b, c \}, \{ b \}$
g^*-closed sets are $X, \phi, \{ a, b \}, \{ b, c \}, \{ b \}$.In this topological space (X, τ), the set $A = \{ c \}$ is $g^*\omega\alpha$-closed but not g-closed and $g^*\omega\alpha$-closed set in X.

Theorem 3.13. *Union of two $g^*\omega\alpha$-closed sets are $g^*\omega\alpha$-closed set in X.*
Proof: *Let A and B be two $g^*\omega\alpha$-closed sets in X. Let U be an $\omega\alpha$-closed set in X, such that $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $g^*\omega\alpha$-closed sets, $cl(A) \subseteq U$ and $cl(B) \subseteq U$. Therefore $cl(A) \cup cl(B) = cl(A \cup B) \subseteq U$. Hence $A \cup B$ is $g^*\omega\alpha$-closed set in X.*

Theorem 3.14. *If a subset A of a space X is $g^*\omega\alpha$-closed in X, then cl(A)-A does not contain any non empty closed set in X.*
Proof: *Let F be a closed set contained in cl(A)-A, such that $cl(A)\text{-}A \subseteq X\text{-}F$ and X-F is open, so $\omega\alpha$-open set with $A \subseteq X\text{-}F$. But A is $g^*\omega\alpha$-closed. Therefore $cl(A) \subseteq X\text{-}F$, consequently $F \subseteq X\text{-}cl(A)$. We have $F \subseteq cl(A)$. Thus $F \subseteq cl(A) \cap (X\text{-}cl(A)) = \phi$. That is $F = \phi$.*

Theorem 3.15. *If a subset A of a space X is $g^*\omega\alpha$-closed in X, then cl(A)-A does not contain any non empty $\omega\alpha$-closed set in X.*
Proof: *It follows from the theorem 3.14 and the fact that every closed set is $g^*\omega\alpha$-closed in X by remark 3.2.*

Example 3.16. *Let $X = \{ a, b, c, d \}$ and $\tau = \{ X, \phi, \{ a, b \}\}$. Then (X, τ) be a topological space. Consider the set $A = \{ b, d \}$, then $cl(A)\text{-}A = X\text{-}\{ b, d \} = \{ a, c \}$ does not contain any non empty $\omega\alpha$-closed set. But A is not a $g^*\omega\alpha$-closed set in X.*

Proposition 3.17. *If A is $\omega\alpha$-open and $g^*\omega\alpha$-closed set of X, then A is closed set of X.*
Proof: *Since A is $\omega\alpha$-open and $g^*\omega\alpha$-closed, then $cl(A) \subseteq A$. Hence A is closed.*

Theorem 3.18. *If A is $g^*\omega\alpha$-closed set in X, then A is closed if and only if cl(A)-A is $\omega\alpha$-closed in X.*
Proof: *Suppose A is closed. Then cl(A)=A and so $cl(A)\text{-}A = \phi$, which is $\omega\alpha$-closed. Conversely, suppose cl(A)-A is $\omega\alpha$-closed. Since A is $g^*\omega\alpha$-closed, by theorem 3.15, cl(A)-A does not contain any non empty $\omega\alpha$-closed set which implies $cl(A)\text{-}A = \phi$. That is $cl(A)\text{-}A = \phi$. That is $cl(A) = A$. Hence A is closed.*

Theorem 3.19. *If A is $g^*\omega\alpha$-closed set in X and $A \subseteq B \subseteq cl(A)$ then B is also $g^*\omega\alpha$-closed set in X.*
Proof: *Let U be $\omega\alpha$-open set in X such that $B \subseteq U$ then $A \subseteq U$. Since $A \subseteq U$ and U is $\omega\alpha$-open set then $cl(A) \subseteq U$. Then $cl(A) \subseteq cl(cl(A)) = cl(A)$. Since $B \subseteq cl(A)$, thus $cl(B) \subseteq cl(A) \subseteq U$. Hence B is $g^*\omega\alpha$-closed set in X.*
However the converse of the above theorem is not true in general.

Example 3.20. *Let $X = \{ a, b, c \}$ and $\tau = \{ X, \phi, \{ a \}, \{ b, c \}\}$. Then (X, τ) be a topological space. Consider the set $A = \{ a \}$ and $B = \{ a, b \}$ such that A and B are $g^*\omega\alpha$-closed sets but $A \subseteq B \nsubseteq cl(A)$.*

Theorem 3.21. *For each $x \in X$ either $\{ x \}$ is $\omega\alpha$-closed or $\{ x \}^c$ is $g^*\omega\alpha$-closed set in X.*
Proof: *Suppose that $\{ x \}$ is not $\omega\alpha$-closed set in X, then $\{ x \}^c$ is not $\omega\alpha$-open and the only $\omega\alpha$-open set containing $\{ x \}^c$ is the space X itself. Therefore $cl(\{ x \}^c) \subseteq X$ and hence $\{ x \}^c$ is $g^*\omega\alpha$-closed set in X.*

Theorem 3.22. *Let $A \subseteq Y \subseteq X$ and if A is $g^*\omega\alpha$-closed set in X, then A is $g^*\omega\alpha$-closed relative to Y.*
Proof: *Let $A \subseteq Y \cap U$ where U is $\omega\alpha$-open set. Then $A \subseteq U$ and hence $cl(A) \subseteq U$. This implies that $Y \cap cl(A) \subseteq Y \cap U$. Thus A is $g^*\omega\alpha$-closed relative to Y.*

Theorem 3.23. *A regular open, $g^*\omega\alpha$-closed set is pre-closed and hence clopen.*
Proof: *Let A be a regular open, $g^*\omega\alpha$-closed set in X. Since every regular open set is $\omega\alpha$-open[4], $cl(A) \subseteq A$. This implies A is closed. Since every closed (regular) open set is (regular) closed and hence A is clopen.*

Definition 3.24. *[4] A space X is said to be $T_{\omega\alpha}$-space if every $\omega\alpha$-closed set is closed.*

Definition 3.25. *[21] A topological space X is said to be partition space, if every open subset of X is closed.*

Theorem 3.26. *If (X, τ) is $T_{\omega\alpha}$-space, then the following conditions are equivalent.*
(a) X is a partition space.
(b) Every subset of X is $g^*\omega\alpha$-closed set.
Proof: *(a) $\rightarrow$ (b) Let $A \subseteq U$ and U is $\omega\alpha$-open and A is arbitrary subset of X. Since X is $T_{\omega\alpha}$-space, U is open. Since X is partition space, U is clopen. Thus $cl(A) \subseteq cl(U) = U$. Hence every subset of X is $g^*\omega\alpha$-closed.*
(b) $\rightarrow$ (a) If $U \subseteq X$ is open. Then U is $\omega\alpha$-open [4]. Then by (b), $cl(U) \subseteq U$ or U is closed. Hence U is clopen. Therefore X is a partition space.

Theorem 3.27. *Let (X, τ) be a normal space and if Y is $g^*\omega\alpha$-closed subset of X then the subspace Y is normal.*
Proof: *If G_1 and G_2 are disjoint closed sets in X such that $(Y\cap G_1) \cap (Y\cap G_2) = \phi$. Then $Y \subseteq (G_1\cap G_2)^c$ and $(G_1\cap G_2)^c$ is $\omega\alpha$-open and Y is $g^*\omega\alpha$-closed in X. Therefore $cl(Y)\subseteq(G_1\cap G_2)^c$ and hence $(cl(Y)\cap G_1) \cap (cl(Y)\cap G_2) = \phi$. Since X is normal, there exist disjoint open sets A and B such that $cl(Y)\cap G_1 \subseteq A$ and $cl(Y)\cap G_2 \subseteq B$. Thus $Y\cap A$ and $Y\cap B$ are disjoint open sets of Y such that $Y\cap G_1 \subseteq Y\cap A$ and $Y\cap G_2 \subseteq Y\cap B$. Hence Y is normal.*

Now we will introduce the following generalized$^*\omega\alpha$-open sets in topological spaces.

4 Generalized Star $\omega\alpha$-Open Sets in Topological Spaces

Definition 4.1. *A subset A of a topological space X is said to be generalized star $\omega\alpha$-open (briefly $g^*\omega\alpha$-open) set if its complement A^c is $g^*\omega\alpha$-closed set in X.*

Theorem 4.2. *A subset A of a topological space X is $g^*\omega\alpha$-open if and only if U $\subseteq$int(A) whenever U is $\omega\alpha$-closed and U $\subseteq$ A.*

Proof: *Assume that A is $g^*\omega\alpha$-open set in X and U is $\omega\alpha$-closed such that U $\subseteq$ A. Then X-A is $g^*\omega\alpha$-closed set in X. Also X-A $\subseteq$ X-U and X-U is $\omega\alpha$-open set of X. This implies cl(X-A) $\subseteq$ X-U. But cl(X-A) = X-int(A).*
Thus X-int(A) $\subseteq$ X-U. So U $\subseteq$ int(A).
Conversely, suppose U $\subseteq$ int(A) whenever U is $\omega\alpha$-closed and U $\subseteq$ A.
To prove that A is $g^*\omega\alpha$-open. Let G be $\omega\alpha$-open set of X such that X-A $\subseteq$ G, then X-G $\subseteq$ A. Now X-G is $g^*\omega\alpha$-closed set containing A, so that X-G $\subseteq$ int(A), X-int(A) $\subseteq$ G but cl(X-A) = X-int(A). Thus cl(X-A) $\subseteq$ G, that is X-A is $g^*\omega\alpha$-closed set and hence A is $g^*\omega\alpha$-open set in X.

Theorem 4.3. *If int(A) $\subseteq$ B $\subseteq$ A and A is $g^*\omega\alpha$-open set in X, then B is $g^*\omega\alpha$-open set in X.*

Proof: *If int(A) $\subseteq$ B $\subseteq$ A, then X-A $\subseteq$ X-B $\subseteq$ X-int(A) = cl(X-A). Since X-A is $g^*\omega\alpha$-closed set, then by theorem 3.19, X-B is also $g^*\omega\alpha$-closed set in X. Therefore B is $g^*\omega\alpha$-open set in X.*

Theorem 4.4. *If A is $g^*\omega\alpha$-closed set in X, then cl(A)-A is $g^*\omega\alpha$-open set in X.*

Proof: *Let A be $g^*\omega\alpha$-closed set in X. Let F be an $\omega\alpha$-open set such that F $\subseteq$ cl(A)-A. Since A is $g^*\omega\alpha$-closed, then by theorem 3.14, cl(A)-A does not contain any non empty closed set in X. Thus F = ϕ. Then F $\subseteq$ int(cl(A)-A). Therefore by theorem 4.2, cl(A)-A is $g^*\omega\alpha$-open set in X.*

Theorem 4.5. *A subset A is $g^*\omega\alpha$-open in X if and only if G = X whenever G is $\omega\alpha$-open and int(A)$\cup$(X-G) $\subseteq$ G.*

Proof: *Let A be $g^*\omega\alpha$-open set, U be $\omega\alpha$-open set and int(A)$\cup$(X-U) $\subseteq$ U. This gives X-U $\subseteq$ (X-int(A)) $\cap$ (X-(X-A)) = X-int(A)-(X-A) = cl(X-A)-(X-A). Since X-A is $g^*\omega\alpha$-closed and X-U is $\omega\alpha$-closed. Then by theorem 3.14, it follows that X-U = ϕ. Therefore X = U.*
Conversely, suppose F is $\omega\alpha$-closed and F $\subseteq$ A. Then int(A)$\cup$(X-A) $\subseteq$ int(A)$\cup$(X-F). It follows that int(A)$\cup$(X-F) = X and hence F $\subseteq$ int(A). Therefore A is $g^*\omega\alpha$-open in X.

Definition 4.6. *[8] A topological space X is said to be submaximal if each dense subset is open.*

Theorem 4.7. *For a topological space X the following are equivalent.*
(a) X is submaximal.
(b) Every $g^*\omega\alpha$-closed set in X is closed.

Proof: *(a) $\Rightarrow$ (b) Let A be $g^*\omega\alpha$-closed set in X. Since all submaximal spaces are $T_{1/2}$ spaces, then every singleton of X is open and pre-open or closed. Hence A is closed.*
(b) $\Rightarrow$ (a) Let A be a dense subset of X. Clearly A is $g^*\omega\alpha$-open, from (b) A is open and thus X is submaximal.

Theorem 4.8. *In a topological space X, if $\omega\alpha O(X) = \{X, \phi\}$ then every subset of X is $g^*\omega\alpha$-closed set in X.*

Proof: *Let X be a topological space and $\omega\alpha O(X) = \{X, \phi\}$. Let A be any subset of X. Suppose A = ϕ, then ϕ is $g^*\omega\alpha$-closed set in X. Suppose A $\neq \phi$, then X is the only $\omega\alpha$-open set containing A and so cl(A) $\subseteq$ X. Hence A is $g^*\omega\alpha$-closed set in X.*

The converse of the above theorem need not be true in general as seen from the following example.

Example 4.9. *Let X = { a, b, c, d } and* τ *= { X,* ϕ*, { a, b }, { c, d }}. Let (X,* τ*) be a topological space. Then every subset of X is* $g^*\omega\alpha$*-closed in X, but* $\omega\alpha O(X)$ *= { X,* ϕ*, { a, b }, { c, d }}.*

Theorem 4.10. *If a subset A of a topological space X is both semi-open and* ω*-closed then it is* $g^*\omega\alpha$*-closed.*
Proof: *Let A be semi-open and* ω*-closed set in X. Let* $A \subseteq U$ *and U is* $\omega\alpha$*-open in X. Now* $A \subseteq A$*, by hypothesis* $cl(A) \subseteq A$ *then* $cl(A) \subseteq A \subseteq U$*. Thus A is* $g^*\omega\alpha$*-closed set in X.*
If A is both semi-open and $g^*\omega\alpha$*-closed then A need not be* ω*-closed as seen from the following example.*

Example 4.11. *X = { a, b, c, d } and* τ *= { X,* ϕ*, { a }, { a, b }}. In this topological space (X,* τ*), the set A = { a, c, d } which is both semi-open and* $g^*\omega\alpha$*-closed but not* ω*-closed.*

Now we will introduce kernel in a $g^*\omega\alpha$-open sets in topological space.

Definition 4.12. *[10] For any subset A of X,* $\omega\alpha$*-ker(A) is defined as the intersection of all* $\omega\alpha$*-open sets containing A.*
That is $\omega\alpha\text{-}ker(A) = \cap\{ U: A \subset U, U \in \omega\alpha O(\tau) \}$

Definition 4.13. *For any subset A of X,* $g^*\omega\alpha$*-ker(A) is defined as the intersection of all* $g^*\omega\alpha$*-open sets containing A.*
That is $g^*\omega\alpha\text{-}ker(A) = \cap\{ U: A \subset U, U \in g^*\omega\alpha O(\tau) \}$

Lemma 4.14. *For a subsets A and B of a topological space X, the following are equivalent.*
(a) $A \subset g^*\omega\alpha ker(A)$
(b) If $A \subset B$ *then* $g^*\omega\alpha\text{-}ker(A) \subset g^*\omega\alpha\text{-}ker(B)$*.*
(c) $g^*\omega\alpha\text{-}ker(g^*\omega\alpha\text{-}ker(A)) = g^*\omega\alpha\text{-}ker(A)$
(d) If A is $g^*\omega\alpha$*-open then* $A = g^*\omega\alpha\text{-}ker(A)$*.*
(e) $g^*\omega\alpha\text{-}ker(\cap\{ A_\alpha : \alpha \in \Lambda\}) \subset \cap\{ g^*\omega\alpha\text{-}ker(A_\alpha) : \alpha \in \Lambda\}$
Proof: *(a) Clearly follows from the definition of* $g^*\omega\alpha$*-kernel.*
(b) Suppose that $x \notin g^*\omega\alpha\text{-}ker(B)$*, then there exist a subset* $U \in g^*\omega\alpha O(\tau)$ *such that* $U \supset B$ *with* $x \notin g^*\omega\alpha\text{-}ker(A)$*. Thus* $g^*\omega\alpha\text{-}ker(A) \subset g^*\omega\alpha\text{-}ker(B)$*.*
(c) Follows from (a) and definition.
(d) By the definition of kernel and $A \in g^*\omega\alpha O(\tau)$*, we have* $g^*\omega\alpha\text{-}ker(A) \subset A$*. By(a), we get* $A = g^*\omega\alpha\text{-}ker(A)$*.*
(e) Suppose that $x \notin \cap g^*\omega\alpha ker \{\cap A_\alpha : \alpha \in \Lambda\}$ *then there exist an* $\alpha_0 \in \Lambda$ *such that* $x \notin g^*\omega\alpha ker(A\alpha_0)$ *and there exist an* $g^*\omega\alpha$*-open set U such that* $x \notin U$ *and* $A\alpha_0 \subset U$*. We have* $\cap_{\alpha\in\Lambda} A\alpha_\alpha \subset A\alpha_0 \subset U$ *and* $x \notin U$*. Therefore* $x \notin g^*\omega\alpha ker\{\cap A_\alpha : \alpha \in \Lambda\}$*. Hence* $\cap\{g^*\omega\alpha ker A_\alpha : \alpha \in \Lambda\} \supset g^*\omega\alpha ker(\cap\{A_\alpha : \alpha \in \Lambda\}$

Remark 4.15. *In (e) of Lemma 4.14, the equality does not necessarily hold as given in the following example.*

Example 4.16. *$X = \{ a, b, c \}$ and $\tau = \{ X, \phi, \{ a, b \}\}$. Then (X, τ) be a topological space. Let $A = \{ a, b \}$ and $B = \{ a, c \}$. Then $g^*\omega\alpha ker(A) = \{ a, b \}$ and $g^*\omega\alpha ker(B) = X$. Consider $g^*\omega\alpha ker(A \cap B) = g^*\omega\alpha ker(\{ a \}) = \{ a \}$ and $g^*\omega\alpha ker(A) \cap g^*\omega\alpha ker(B) = \{ a, b \} \cap \{ X \} = \{ a, b \}$.*

Theorem 4.17. *A subset A of a topological space X is $g^*\omega\alpha$-closed if and only if $cl(A) \subseteq \omega\alpha ker(A)$.*

Proof: *Suppose A is $g^*\omega\alpha$-closed. Then $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\omega\alpha$-open. Let $x \in cl(A)$. If $x \notin \omega\alpha ker(A)$, then there is an $\omega\alpha$-open set containing A such that $x \notin U$. Since U is $\omega\alpha$-open set containing A, we have $x \notin cl(A)$. This is contradiction. Hence $cl(A) \subseteq \omega\alpha ker(A)$.*

Conversely, let $cl(A) \subseteq \omega\alpha ker(A)$. If U is an $\omega\alpha$-open set containing A, then $cl(A) \subseteq \omega\alpha ker(A) \subseteq U$. Hence A is $g^*\omega\alpha$-closed.

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