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New topological approach of generalized closed sets

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Abstract - Closedness is a basic concept for the study and the investigation in the general topological spaces. (Fukutake, Nasef and El- Maghrabi, 2003) introduced a new weakly form of generalized closed sets,$\gamma g-$closed set, which is weaker than both of $gs-$closed sets (Arya and Nour, 1990), $gp-$closed sets (Noiri, Maki and Umehara, 1998) and stronger than $gsp-$closed sets (Dontchev,1995). In this paper, we introduce more study of $\gamma g-$closed sets in a general topological space.

Keywords - *$\gamma-$closed set, $\gamma g-$closed set.*

1 Introduction and preliminaries

The importance of general topological spaces rapidly increases in many fields of applications such as data mining [1]. Information systems are basic tools for producing knowledge from data in any real-life field. Topological structures on the collection of data are suitable mathematical models for mathematizing not only quantitive data but also qualitive ones.

Closedness is a basic concept for the study and the investigation in the general topological spaces. This concept has been generalized and studied by many authors [2, 3, 4, 5] from different points of views.

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Let (X, τ) be a topological space and A be a subset of X. The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$, respectively.

We recall the following definitions:

Definition 1.1. *Let (X, τ) be a topological space. A subset $A \subseteq X$ is called:*

(1) $\alpha-$closed [6] if $cl(int(cl(A))) \subseteq A$,

(2) semi-closed [7] if $int(cl(A)) \subseteq A$,

(3) preclosed [8] if $cl(int(A)) \subseteq A$,

(4) γ-closed [9] or $b-$closed [10] or $sp-$closed [10] if $int(cl(A)) \cap cl(int(A)) \subseteq A$,

(5) semi-preclosed [11] or $\beta-$closed [12] if $int(cl(int(A))) \subseteq A$.

The complement of an $\alpha-$closed (resp. semi-closed, preclosed,$\gamma-$closed, semi-preclosed) set is called $\alpha-$open (resp. semi-open, preopen, $\gamma-$open, semi-preopen). The smallest $\alpha-$closed (resp. semi-closed, preclosed,$\gamma-$closed, semi-preclosed) set containing $A \subseteq X$ is called the $\alpha-$closure (resp. semi-closure, preclosure, $\gamma-$closure, semi-preclosure) of A and shall be denoted by $cl_\alpha(A)$ (resp. $scl(A)$, $pcl(A)$, $cl_\gamma(A), spcl(A)$).

As in Corollary 2.6 of [13], it is easily established that the concept of a $\gamma g-$ closed set [14] yields the only new type of sets that can be gained by utilizing the $\gamma-$closure (resp. the $\gamma-$interior) in the concept of $gr-$closed sets [13, 8]. Thus we give:

Definition 1.2. *[14] Let (X, τ) be a topological space. A subset $A \subseteq X$ is called $\gamma g-$closed if $cl_\gamma(A) \subseteq U$ whenever $A \subseteq U$ and U is open. The complement of a $\gamma g-$closed set is called $\gamma g-$open.*

Definition 1.3. *A subset A of a space (X, τ) is said to be:*

(1) generalized closed [15] (briefly, g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(2) generalized semi-closed [16] (briefly, gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(3) generalized semi-preclosed [17] (briefly, gsp-closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(4) α-generalized closed [18] (briefly, αg-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(5) generalized preclosed [19] (briefly, gp-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(6) semi-generalized closed [20] (briefly, sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

(7) *generalized α-closed [21] (briefly, $g\alpha$-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.*

We also consider the following classes of topological spaces:

Definition 1.4. *A topological space (X, τ) is called:*

(1) *extremally disconnected [22] , if the closure of each open subset of (X, τ) is open,*

(2) *resolvable if (X, τ) is the union of two disjoint dense subsets,*

(3) *$sg-$submaximal [23] if every dense subset of (X, τ) is $sg-$closed,*

(4) *T_{gs} [24] if every $gs-$closed subset of (X, τ) is $sg-$closed.*

Definition 1.5. *[14] A mapping $f : (X, \tau) \to (Y, \sigma)$ is called:*

(1) *γ-generalized continuous (briefly γg-continuous) if the inverse image $f^{-1}(F)$ is γg-closed in (X, τ) for every closed set F of (Y, σ).*

(2) *γ-generalized irresolute (briefly γg- irresolute) if the inverse image $f^{-1}(F)$ is γg-closed in (X, τ) for every γg-closed set F of (Y, σ).*

2 $\gamma g-$closed sets and their relationships

In this section, we shall consider the relationships between $\gamma g-$closed sets and other generalized closed sets.

Lemma 2.1. *[14]*

(1) *If B is a $\gamma g-$closed set, then $cl_\gamma(B) \backslash B$ does not contain nonempty closed set,*

(2) *For each $x \in X$, a singleton $\{x\}$ is closed or its complement $X \backslash \{x\}$ is $\gamma g-$closed in (X, τ).*

Theorem 2.1. (1) *Every $sg-$closed set in a topological space (X, τ) is $\gamma-$closed,*

(2) *If A is open and $\gamma g-$closed in a topological space (X, τ), then A is $\gamma-$closed.*
Proof.

(1) *Let $A \subseteq X$ be $sg-$closed and let $x \in cl_\gamma(A)$. Since singletons are either preopen or nowhere dense (see [25]) we distinguish two cases:*
***Case 1**, If $\{x\}$ is preopen, then it is also $\gamma-$open and hence $\{x\} \cap A \neq \emptyset$, i.e. $x \in A$.*
***Case 2**, If $\{x\}$ is nowwhere dense, then $X \backslash \{x\}$ is semi-open. Suppose that $x \notin A$. Then $A \subseteq X \backslash \{x\}$ and since A is $sg-$closed, we have $cl_\gamma(A) \subseteq scl(A) \subseteq X \backslash \{x\}$. Hence $x \notin cl_\gamma(A)$, and this is a contradiction. Therefore $cl_\gamma(A) \subseteq A$, and so A is $\gamma-$closed.*

(2) *Since A is open and $\gamma g-$closed, then $cl_\gamma(A) \subseteq A$, but always $A \subseteq cl_\gamma(A)$, Then $A = cl_\gamma(A)$. Thus A is $\gamma-$closed.* $\square$

Proposition 2.1. *Every $\alpha g-$closed set is γg-closed and every γg-closed is gsp-closed.*

None of the implications in the proposition above is reversible as the following example shows.

Example 2.1. *(i) Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}, \{a, b, c, d\}, X\}$. Set $A = \{b, c\}$. Note that $cl_\gamma(A) = \{b, c, d, e\}$. Since $A \subseteq \{b, c, d\} \in \tau$, then A is not γg-closed. However it is easily checked that A is gsp-closed*

(ii) Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, \{a, b, c, d\}, X\}$. Set $B = \{c, d\}$. Observe that $\alpha cl(B) = \{b, c, d, e\}$. Clearly, B is not αg-closed, Since $B \subseteq \{b, c, d\} \in \tau$. On the other hand, one can easily verify that $B$is γg-closed.

By Definition 1.2 and Remark 3.2 of [14], we obtain the following relationships between the class of $\gamma g-$closed sets and the classes of generalized closed sets defined above in the following figure.

The following remark enables us to realize that none of the above implications is reversible:

Remark 2.1. *We have the following relationships:*

(1) gs$-$closed set does not imply $\alpha g-$closed set ([14], Example 3.3),

(2) gp$-$closed set does not imply $\alpha g-$closed set ([14], Example 3.4),([26], Example 2.3 (iii)),

(3) gsp$-$closed set does not imply $\gamma g-$closed set ([14], Example 3.5),

(4) gsp$-$closed set does not imply semi-preclosed set ([17], Example 3.4),

(5) gsp$-$closed set does not imply gs-closed set ([17], Example 3.3),

(6) gsp$-$closed set does not imply gp-closed set ([26], Example 2.3(i)),

(7) A semi-closed set need not be sg-closed set ([27], Example 2.5),

(8) A preclosed set need not be gα-closed set ([27], Example 2.5),

(9) $\gamma g-$closed set does not imply gp-closed set ([13], Example 3.3),

(10) $\gamma g-$closed set does not imply gs-closed set ([14], Example 3.4),

We now address the question of when the above implications can be reversed.

Proposition 2.2. *Let (X, τ) be a topological space. Then*

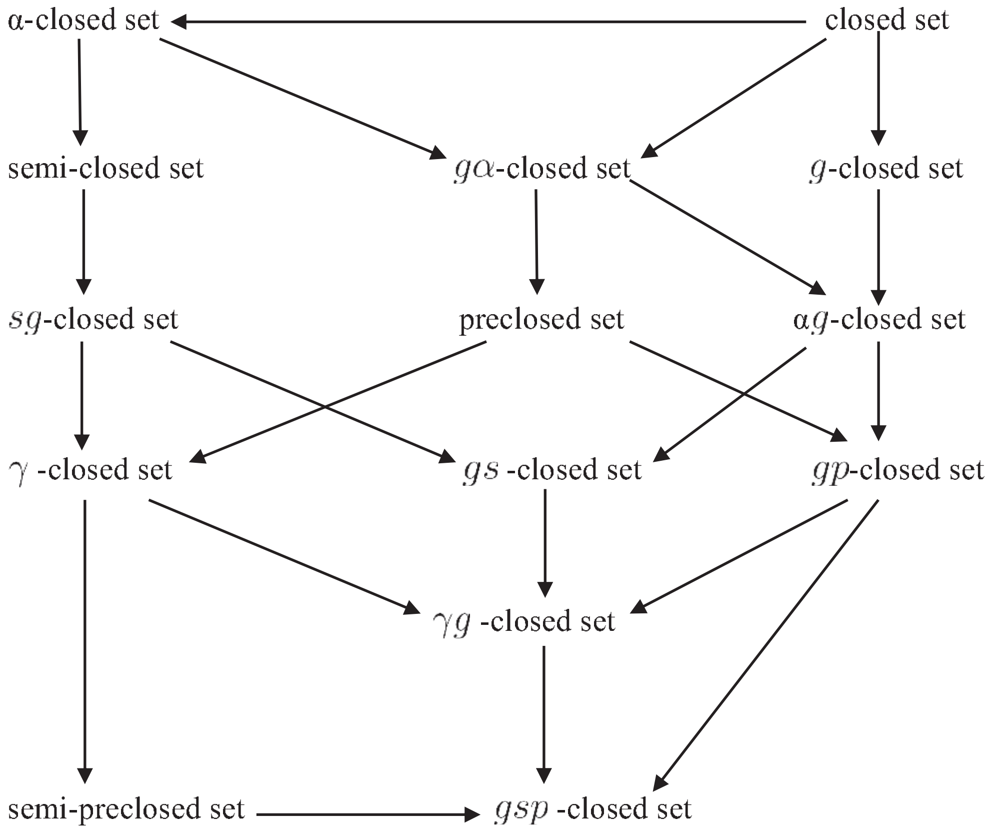

Figure 1:

(1) *Each $\gamma g-$closed set is $gp-$closed if and only if (X, τ) is extremally disconnected.*

(2) *Each semi-preclosed set is $\gamma-$closed if and only if $cl(A)$ is open for every open resolvable subspace A of (X, τ).*

Proof.

(1) *This is an immediate consequence of Definitions 1.2, 1.3 and 1.4.*

(2) *See [28].* □

Theorem 2.2. *Let (X, τ) be a topological space. Then the following statements are equivalent:*

(1) *Each $\gamma-$closed set is $sg-$closed,*

(2) *Each $\gamma-$closed set is $gs-$closed,*

(3) *Each $\gamma g-$closed set is $gs-$closed,*

(4) (X, τ) *is sg−submaximal.*

Proof. *We will show* (1) $\Leftrightarrow$ (4)*: The other equivalences can be proved in a similar manner using the standard methods that can be found in [13]. First recall that a space* (X, τ) *is sg−submaximal if and only if every preclosed set is sg−closed (see [23]). If every* $\gamma-$*closed set is sg−closed, then every preclosed set is sg−closed, i.e.* (X, τ) *is sg−submaximal.*
Conversely, suppose that (X, τ) *is sg−submaximal and let* A *be* $\gamma-$*closed. Then* A *is the intersection of a semi-closed and a preclosed set (see [29]). Since every semi-closed set is sg−closed, by hypothesis,* A *is the intersection of two sg−closed sets. Since the arbitrary intersection of sg−closed sets is always sg−closed (see [30]), we conclude that* A *is sg−closed.* □

Theorem 2.3. *Let* (X, τ) *be a topological space. Then the following statements are equivalent:*

(1) Each gs−closed set is $\gamma-$*closed,*

(2) Each $\gamma g-$*closed set is* $\gamma-$*closed,*

(3) Each $\gamma g-$*closed set is semi-preclosed,*

(4) (X, τ) *is a* $T_{gs}-$*space.*

Proof. *The proof is similar to that of Theorem 2.9. and of Theorem 3.6 of [31].* □

Theorem 2.4. *Let* (X, τ) *be a topological space. Then the following statements are equivalent:*

(1) Each gsp-closed set is γg*-closed,*

(2) Each semi-preclosed set is γg*-closed.*

Proof. *The necessity is clear, so we only have to show the sufficiency. Let* A *be a gsp−closed set and* U *be an open subset of* X *such that* $A \subseteq U$*. Since* A *is gsp-closed, we have* $spcl(A) \subseteq U$*. Now,* $spcl(A)$ *is semi-preclosed and hence* $\gamma g-$*closed by hypothesis. Therefore,* $cl_\gamma(A) \subseteq cl_\gamma(spcl(A)) \subseteq U$ *and thus our claim is proved.* □

Remark 2.2. *If* $A \subseteq X$*, then the largest* $\gamma-$*open subset of* A *is called the* $\gamma-$*interior of* A *and is denoted by* $int_\gamma(A)$*. It is well known that* $int_\gamma(A) = (cl(int(A))) \cup int(cl(A)) \cap A$ *(see[9]). Consequently, a subset* A *is* $\gamma g-$*closed if and only if for every closed subset* F *satisfying* $F \subseteq A$ *we have* $F \subseteq cl(int(A)) \cup int(cl(A))$*.*

Now, we present one of our major results.

Theorem 2.5. *Let* (X, τ) *be a topological space. Then the following statements are equivalent:*

(1) Each semi-preclosed set is $\gamma-$closed,

(2) Each semi-preclosed set is $\gamma g-$closed,

(3) $cl(G)$ is open for every open resolvable subspace G of (X, τ).

Proof. *It is obvious that* $(1) \Rightarrow (2)$. *Furthermore, it has been shown in [28] that* $(3) \Leftrightarrow (1)$, *so we only have to prove that* $(2) \Rightarrow (3)$. *Let G be a nonempty open resolvable subspace and let D_1 and D_2 be disjoint dense subsets of $(G, \tau|G)$. Suppose that there exists a point $x \in cl(G)\backslash int(cl(G))$. Let $S = D_1 \cup cl(\{x\})$. It is easily checked that $int(S) = \emptyset$, $cl(S) = cl(G)$ and that S is semi-preopen. By hypothesis, S is $\gamma g-$open and so, since $cl(\{x\}) \subseteq S$, we conclude that $\{x\} \subseteq cl(\{x\}) \subseteq int(cl(S)) = int(cl(G))$. This is a contradiction to our assumption and so $cl(G)$ has to be open.* □

Theorem 2.6. *Let (X, τ) be a topological space. Then the following statements are equivalent:*

(1) Each $\gamma g-$closed set of (X, τ) is $\gamma-$closed,

(2) For each $x \in X$, the singleton $\{x\}$ is closed or $\gamma-$open in (X, τ).

Proof. $(1) \Rightarrow (2)$: *Suppose that, for $x \in X$, $\{x\}$ is not closed. By Lemma 1.1, $X\backslash\{x\}$ is $\gamma g-$closed set. Therefore, $X\backslash\{x\}$ is $\gamma-$closed by using assumption and hence $\{x\}$ is $\gamma-$open.*
$(2) \Rightarrow (1)$: *Let B be a $\gamma g-$closed set and $x \in cl_\gamma(B)$. Then the singleton $\{x\}$ is closed or $\gamma-$open by assumption.*
Case 1. *Suppose that $\{x\}$ is closed. It follows from Lemma 1.1 that $cl_\gamma(B)\backslash B$ does not contain $\{x\}$. Since $x \in cl_\gamma(B)$, we obtain $x \in B$ and hence B is $\gamma-$closed.*
Case 2. *Suppose that $\{x\}$ is $\gamma-$open, we have $\{x\} \cap B \neq \emptyset$ and hence $x \in B$. Therefore, this shows that $cl_\gamma(B) \subseteq B$, but $B \subseteq cl_\gamma(B)$ and so $cl_\gamma(B) = B$ and hence B is $\gamma-$closed.* □

3 γg-compactness and γg-connectedness

Definition 3.1. *A collection $\{A_\alpha : \alpha \in \nabla\}$ of γg-open sets in a topological space X is called a γg-open cover of a subset B of X if $B \subset \bigcup\{A_\alpha : \alpha \in \nabla\}$ holds.*

Definition 3.2. *A topological space X is γ-generalized-compact (or γg- compact) if every γg-open cover of X has a finite subcover.*

Definition 3.3. *A subset B of a topological space X is said to be γg-compact relative to X if, for every collection $\{A_\alpha : \alpha \in \nabla\}$ of γg-open subsets of X such that $B \subset \bigcup\{A_\alpha : \alpha \in \nabla\}$, there exists a finite subset ∇_0 of ∇ such that $B \subset \bigcup\{A_\alpha : \alpha \in \nabla_0\}$.*

Definition 3.4. *A subset B of a topological space X is said to be γg-compact if B is γg-compact as a subspace of X.*

Theorem 3.1. *Every sg-closed subset of a γg-compact space X is γg-compact relative to X.* ***Proof.*** *Let A be a γg-closed subset of X. Then A^c is γg-open in X. Let $M = \{G_\alpha : \alpha \in \nabla\}$ be a cover of A by γg-open subsets in X. Then $M^* = M \cup A^c$ is a γg-open cover of X, i.e., $X = (\bigcup\{G_\alpha : \alpha \in \nabla\}) \bigcup A^c$. By hypothesis, X is γg-compact, hence M^* is reducible to a finite cover of X, say $X = G_{\alpha_1} \cup G_{\alpha_2} \cup ... \cup G_{\alpha_m} \cup A^c$, $G_{\alpha k} \in M$. But A and A^c are disjoint, hence $A \subset G_{\alpha_1} \cup G_{\alpha_2} ... \cup G_{\alpha_m}, G_{\alpha k} \in M$. We have just shown that any γg-open cover M of A contains a finite subcover, i.e., A is γg-compact relative to X.* □

Theorem 3.2. *Let (X, τ) be a topological space.*

(i) A γg-continuous image of a γg-compact space is compact.

(ii) If a map $f : X \to Y$ is γg-irresolute and a subset B of X is γg-compact relative to X, then the image $f(B)$ is γg-compact relative to Y.

Proof.

(i) Let $f : X \to Y$ be a γg-continuous map from a γg-compact space X onto a topological space Y. Let $\{A_\alpha : \alpha \in \nabla\}$ be an open cover of Y. Then $\{f^{-1}(A_\alpha) : \alpha \in \nabla\}$ is a γg-open cover of X. Since X is γg-compact, it has a finite subcover, say $\{f^{-1}(A_1), ..., f^{-1}(A_n)\}$. Since f is onto $\{A_1, ..., A_n\}$ is a cover of Y and so Y is compact.

(ii) Let $\{A_\alpha : \alpha \in \nabla\}$ be any collection of γg-open subsets of Y such that $f(B) \subset \bigcup\{A_\alpha : \alpha \in \nabla\}$. Then $B \subset \bigcup\{f^{-1}(A_\alpha) : \alpha \in \nabla_0\}$ holds. By hypothesis there exists a finite subset ∇_0 of ∇ such that $B \subset \bigcup\{f^{-1}(A_\alpha) : \alpha \in \nabla_0\}$. Therefore we have $f(B) \subset \bigcup\{A_\alpha : \alpha \in \nabla_0\}$ which shows that $f(B)$ is γg-compact relative to Y. □

Definition 3.5. *A topological space X is said to be γg-connected if X can not be written as a disjoint union of two non-empty γg-open sets. A subset of X is γg-connected if it is γg-connected as a subspace.*

In view of the Definition 4.5, we can give a characterization of γg-connected spaces.

Theorem 3.3. *For a topological space X, the following are equivalent.*

(i) X is γg-connected.

(ii) X and $\emptyset$ are the only subsets of X which are both γg-open and γg-closed.

(iii) Each γg-continuous map of X into a discrete space Y with at least two points is a constant map.

Proof. ***(i) $\Rightarrow$ (ii)***: *Let Q be a γg-open and γg-closed subset of X. Then Q^c is both γg-open and γg-closed. Since X is the disjoint union of the γg-open sets Q and Q^c, one of these must be empty, that is $Q = \emptyset$ or $Q = \emptyset$.*
(ii) $\Rightarrow$ (i): *Suppose that $X = A \cup B$ where A and B are disjoint non-empty γg-open subsets of X. Then A is both γg-open and γg-closed. By assumption, $A = \emptyset$ or X. Therefore X is γg-connected.*
(ii) $\Rightarrow$ (iii) *Let $f : X \to Y$ be a γg-continuous map then X is covered by γg-open and γg-closed covering $\{f^{-1}(y) : y \in Y\}$. By assumption $f^{-1}(y) = \emptyset$ or X for each $y \in Y$. If $f^{-1}(y) = \emptyset$ for all $y \in Y$, then f fails to be map. Then, there exists only one point $y \in Y$ such that $f^{-1}(y) \neq \emptyset$ and hence $f^{-1}(y) = X$. This shows that f is a constant map.*
(iii) $\Rightarrow$ (ii): *Let Q be both γg-open and γg-closed in X. Suppose $Q \neq \emptyset$. Let $f : X \to Y$ be a γg-continuous map defined by $f(Q) = \{y\}$ and $f(Q^c) = \{w\}$ for some distinct points y and w in Y. By assumption f is constant. Therefore we have $Q = X$.* □

It is obvious that every γg-connected space is connected. The following example shows that the converse is not true.

Example 3.1. *Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then the topological space (X, τ) is connected. However, since $\{a\}$ is both γg-open and γg-closed, X is not γg-connected by Theorem 3.3.*

As a direct consequence of Theorem 3.3, we have:

Corollary 3.1. *In a topological space (X, τ) with at least two points, if $\gamma O(X, \tau) = \gamma C(X, \tau)$, X is not γg-connected.*
Proof. *Using the hypothesis and Theorem 5 due to in [20] there is a proper non-empty subset of X which is both γg-open and γg-closed in X. By Theorem 3.3, X is not γg-connected.* □

Finally, we proved γg-connectedness is preserved under γg-irresolute surjections.

Theorem 3.4. *(i) $f : X \to Y$ is a γg-continuous surjection and X is γg-connected, then Y is connected.*

(ii) $f : X \to Y$ is a γg-irresolute surjection and X is γg-connected, then Y is γg-connected.

Proof.

(i) Suppose that Y is not connected. Let $Y = A \cup B$ where A and B are disjoint non-empty open set in Y . Since f is γg-continuous and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty and γg-open in X. This contradicts the fact that X is γg-connected. Hence Y is connected.

(ii) The argument is a minor modification of the proof (i). □

4 Conclusion

Generalizations of closed sets in point-set topology will give some new topological properties (for example,separation axioms, compactness, connectedness and continuity) which has been found to be very useful in the study of certain objects of digital topology. Thus we may stress once more the importance of γg-closed sets as a branch of them and the possible application in computer graphics [32, 33] and physics [34].

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