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# On Fuzzy Soft Expert Sets

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**Abstract** – Alkhazaleh and Salleh introduced the fuzzy soft expert sets which allow to know the opinion of more than one expert and the fuzzy set corresponding to it in one model. However, the paper Fuzzy Soft Expert Sets [Applied Mathematics, 2014, 5, 1349-1368] has some mistakes and the decision making algorithm given in this paper has some unnecessary steps. Furthermore the results of this algorithm and Maji et al's algorithm without reduction are equivalent. In this paper, for further study on the fuzzy soft expert sets, we have made it fit into this concept which is important for the development of the concept of soft sets by decontaminating from its own inconsistencies. Besides all these, we propose and apply an algorithm for the new concept by using new definitions and Maji et al's algorithm without reduction. We finally discuss this concept later on works.

***Keywords** - Fuzzy sets, fuzzy soft sets, fuzzy soft expert sets, fuzzy soft operations, fuzzy decision making.*

## 1. Introduction

The concept of soft sets was first introduced by Molodtsov [13] in 1999. Until now, many versions of it have been developed and applied to a lot of areas from algebra to decision making problems. One of these versions is fuzzy soft expert (*fse*) sets propounded by Alkhazaleh and Salleh [6] by using fuzzy sets introduced by Zadeh [14] and soft expert sets introduced by Alkhazaleh and Salleh [5]. Then, Bashir and Salleh [7] introduced the fuzzy parameterized soft expert sets. Afterwards, Hazaymeh et al. [9,10] improved generalized *fse*-sets and fuzzy parameterized *fse*-sets. Then, Alhazaymeh and Hassan [1,2] developed generalized vague soft expert (*gvse*) sets and gave an application of them in decision making. They also studied mapping on *gvse*-sets [3].
Although the concept of *fse*-sets is important for the development of soft sets, it has some own difficulties arising from some definitions. This situation necessitates to arrange some

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parts of it. For example, although the idea based on the principle of time-dependent change of the experts' opinion is impressive, this scenario has not been modelled by using adequate parameterization in [6]. So, we will ignore this idea for the time being. In addition to this case, we should emphasize that the *fse*-sets have become consistent in itself. In other words, some arranges can be necessary when the other types of the *fse*-sets, as fuzzy parameterized soft expert sets and fuzzy parameterized *fse*-sets, are taken into consideration.

## 2. Fuzzy Soft Expert Sets

In this section, we recall some basic notions with some remarks and updates in Fuzzy Soft Expert Sets [6]. Let $U$ be a universe, $E$ be a set of parameters, $X$ be a set of experts (agents), $O = \{0,1\}$ be a set of opinions, $Z = E \times X \times O$ and $A \subseteq Z$.

**Definition 2.1** A pair $(F, A)$ is called a fuzzy soft expert (*fse*) set over $U$, where $F$ is a mapping given by $F: A \to F(U)$ where $F(U)$ denotes the set of all fuzzy subsets of $U$.

**Example 2.2** Suppose that a company produces some new products and wants to obtain the opinion of some experts about these products. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of products, $E = \{e_1, e_2, e_3\}$ a set of decision parameters where, for $i \in \{1, 2,3\}$, $e_i$ denotes the parameters as *easy to use*, *quality* and *cheap*, respectively. Let $X = \{p, q, r\}$ be a set of experts.

Assume that the company has distributed a questionnaire to three experts to make decisions on the products and the results of this questionnaire are as in the following,

$$F(e_1,p,1) = \left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.7},\frac{u_4}{0.1}\right\}, \quad F(e_1,q,1) = \left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.3},\frac{u_4}{1}\right\},$$

$$F(e_1,r,1) = \left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}, \quad F(e_2,p,1) = \left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\},$$

$$F(e_2,q,1) = \left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.7}\right\}, \quad F(e_2,r,1) = \left\{\frac{u_1}{0.1},\frac{u_2}{0.3},\frac{u_3}{0.7},\frac{u_4}{0.4}\right\},$$

$$F(e_3,p,1) = \left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.3}\right\}, \quad F(e_3,q,1) = \left\{\frac{u_1}{0.5},\frac{u_2}{0.3},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\},$$

$$F(e_3,r,1) = \left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}, \quad F(e_1,p,0) = \left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.3},\frac{u_4}{0.9}\right\},$$

$$F(e_1,q,0) = \left\{\frac{u_1}{0.5},\frac{u_2}{0.8},\frac{u_3}{0.7},\frac{u_4}{0}\right\}, \quad F(e_1,r,0) = \left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\},$$

$$F(e_2,p,0) = \left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}, \quad F(e_2,q,0) = \left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.3}\right\},$$

$$F(e_2,r,0) = \left\{\frac{u_1}{0.9},\frac{u_2}{0.7},\frac{u_3}{0.3},\frac{u_4}{0.6}\right\}, \quad F(e_3,p,0) = \left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\},$$

$$F(e_3,q,0) = \left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\}, \quad F(e_3,r,0) = \left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\}$$

Then the *fse*-set $(F, Z)$ as in the following,

$$(F,Z)=\left\{\left((e_1,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.7},\frac{u_4}{0.1}\right\}\right),\left((e_1,q,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.3},\frac{u_4}{1}\right\}\right),\right.$$
$$\left((e_1,r,1),\left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}\right),\left((e_2,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\}\right),$$
$$\left((e_2,q,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.7}\right\}\right),\left((e_2,r,1),\left\{\frac{u_1}{0.1},\frac{u_2}{0.3},\frac{u_3}{0.7},\frac{u_4}{0.4}\right\}\right),$$
$$\left((e_3,p,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.3}\right\}\right),\left((e_3,q,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.3},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\}\right),$$
$$\left((e_3,r,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}\right),\left((e_1,p,0),\left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.3},\frac{u_4}{0.9}\right\}\right),$$
$$\left((e_1,q,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.8},\frac{u_3}{0.7},\frac{u_4}{0}\right\}\right),\left((e_1,r,0),\left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\}\right),$$
$$\left((e_2,p,0),\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),\left((e_2,q,0),\left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.3}\right\}\right),$$
$$\left((e_2,r,0),\left\{\frac{u_1}{0.9},\frac{u_2}{0.7},\frac{u_3}{0.3},\frac{u_4}{0.6}\right\}\right),\left((e_3,p,0),\left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\}\right),$$
$$\left.\left((e_3,q,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\}\right),\left((e_3,r,0),\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\}\right)\right\}$$

In this example, the expert $p$ agrees that the membership grades of $u_1$, $u_2$, $u_3$ and $u_4$ to the set of *easy to use products* are $0.3, 0.5, 0.7$ and $0.1$, respectively.

**Remark 2.3** In a soft set, for the parameter $e_1$, $F(e_1)$ and $G(e_1)$ can be different since the functions $F$ and $G$ may be different. However, in an *fse*-set, for the parameter $(e_1,p,1)$, $F(e_1,p,1)$ and $G(e_1,p,1)$ have to be the same since any variable causing changes, such as time, in the choices of expert $p$ does not exist. In other words, for $t_1 \neq t_2$, $F(e_1,p,1,t_1)$ and $F(e_1,p,1,t_2)$ can be different.

From now on, since an expert $p$ can not claim that a product either provides or does not provide the parameter in the same time, all of the examples given in [6] have been updates.

In the view of such information, the definition of *fse*-inclusion and equality given in [6] are equal to classical inclusion and equality.

**Definition 2.4** For two *fse*-sets $(F,A)$ and $(G,B)$ over $U$, $(F,A)$ is called an *fse*-subset of $(G,B)$, denoted by $(F,A) \widetilde{\subseteq} (G,B)$, if $(F,A) \subseteq (G,B)$ (or briefly $A \subseteq B$).

If $(F,A) \widetilde{\subseteq} (G,B)$, then $(G,B)$ is called an *fse*-superset of $(F,A)$.

**Definition 2.5** Two *fse*-sets $(F,A)$ and $(G,B)$ over $U$ are said to be equal if $(F,A) \widetilde{\subseteq} (G,B)$ and $(G,B) \widetilde{\subseteq} (F,A)$.

**Example 2.6** Let

$$(F,A) = \left\{\left((e_1,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.4},\frac{u_4}{0.2}\right\}\right), \left((e_1,q,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.6}\right\}\right),\right.$$
$$\left((e_2,q,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\}\right), \left((e_2,r,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.4},\frac{u_3}{0.4},\frac{u_4}{0.5}\right\}\right),$$
$$\left((e_3,p,1),\left\{\frac{u_1}{0.7},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}\right), \left((e_3,r,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.3},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\}\right),$$
$$\left.\left((e_1,r,0),\left\{\frac{u_1}{0.4},\frac{u_2}{0.3},\frac{u_3}{0.5},\frac{u_4}{0.2}\right\}\right), \left((e_2,p,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.4},\frac{u_3}{0.6},\frac{u_4}{0.3}\right\}\right)\right\}$$

and

$$(G,B) = \left\{\left((e_1,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.4},\frac{u_4}{0.2}\right\}\right), \left((e_1,q,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.6}\right\}\right),\right.$$
$$\left((e_2,q,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\}\right), \left((e_2,r,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.4},\frac{u_3}{0.4},\frac{u_4}{0.5}\right\}\right),$$
$$\left.\left((e_1,r,0),\left\{\frac{u_1}{0.4},\frac{u_2}{0.3},\frac{u_3}{0.5},\frac{u_4}{0.2}\right\}\right), \left((e_2,p,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.4},\frac{u_3}{0.6},\frac{u_4}{0.3}\right\}\right)\right\}$$

Therefore $(G,B) \tilde{\subseteq} (F,A)$. Clearly $B \subseteq A$.

**Definition 2.7** An agree-*fse*-set $(F,A)_1$ which is also an *fse*-subset of $(F,A)$ over $U$ is defined as in the following,

$$(F,A)_1 = \{(\alpha, F(\alpha)) : \alpha \in A_1\}$$

where $A_1 \subseteq Z_1$ such that $Z_1 := E \times X \times \{1\}$.

**Definition 2.8** A disagree-*fse*-set $(F,A)_0$ which is also an *fse*-subset of $(F,A)$ over $U$ is defined as in the following,

$$(F,A)_0 = \{(\alpha, F(\alpha)) : \alpha \in A_0\}$$

where $A_0 \subseteq Z_0$ such that $Z_0 := E \times X \times \{0\}$.

**Example 2.9** Let's consider Example 2.1. Then

$$(F,Z)_1 = \left\{\left((e_1,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.7},\frac{u_4}{0.1}\right\}\right), \left((e_1,q,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.3},\frac{u_4}{1}\right\}\right),\right.$$
$$\left((e_1,r,1),\left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}\right), \left((e_2,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\}\right),$$
$$\left((e_2,q,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.7}\right\}\right), \left((e_2,r,1),\left\{\frac{u_1}{0.1},\frac{u_2}{0.3},\frac{u_3}{0.7},\frac{u_4}{0.4}\right\}\right),$$
$$\left((e_3,p,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.3}\right\}\right), \left((e_3,q,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.3},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\}\right),$$
$$\left.\left((e_3,r,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}\right)\right\}$$

and

$$(F,Z)_0 = \left\{\left((e_1,p,0),\left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.3},\frac{u_4}{0.9}\right\}\right),\left((e_1,q,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.8},\frac{u_3}{0.7},\frac{u_4}{0}\right\}\right),\right.$$
$$\left((e_1,r,0),\left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\}\right),\left((e_2,p,0),\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),$$
$$\left((e_2,q,0),\left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.3}\right\}\right),\left((e_2,r,0),\left\{\frac{u_1}{0.9},\frac{u_2}{0.7},\frac{u_3}{0.3},\frac{u_4}{0.6}\right\}\right),$$
$$\left((e_3,p,0),\left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\}\right),\left((e_3,q,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\}\right),$$
$$\left.\left((e_3,r,0),\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\}\right)\right\}$$

**Remark 2.10** According to the definition of *fse*-sets given in [6], "not α" and "not Z" are defined by $\neg\alpha = (\neg e_i, x_j, o_k) \in Z$ and $\neg Z = \{\neg\alpha : \alpha \in Z\}$, respectively. Since $\neg Z_1 = \neg E \times X \times \{1\}$ and $Z_0 = E \times X \times \{0\}$, $\neg Z_1 \neq Z_0$ and $\neg Z \not\subseteq Z$. So, some expected propositons such as

i. $(F,Z)_1^{\tilde{c}} = (F,Z)_0$

ii. $(F,Z)_0^{\tilde{c}} = (F,Z)_1$

are not held. It can be overcome this kind of difficulties by accepting as $(\neg e_1, p, 1) = (e_1, p, 0)$. Therefore, $\neg Z_1 = Z_0$ and the propositions mentioned above are held.

In the view of such information, the definition of "not Z" and *fse*-complement can be rewritten as in the following,

**Definition 2.11** Let $\alpha = (e_i, x_j, o_k) \in Z$. Then "not α" and "not Z" are defined by $\neg\alpha = (e_i, x_j, 1-o_k) \in Z$ and $\neg Z = \{\neg\alpha : \alpha \in Z\}$, respectively. It can easily be seen that $\neg Z = Z$ but $\neg A \neq A$, for some $A \subseteq Z$.

**Definition 2.12** The complement of an *fse*-set $(F,A)$ is denoted by $(F,A)^{\tilde{c}}$ and is defined by $(F,A)^{\tilde{c}} = (F^{\tilde{c}}, \neg A)$ where $F^{\tilde{c}}: \neg A \to F(U)$ is mapping given by $F^{\tilde{c}}(\neg\alpha) = F^c(\alpha)$, for all $\neg\alpha \in \neg A$, where $F^c(\alpha)$ is a fuzzy complement of $F(\alpha)$.

**Example 2.13** Let's consider Example 2.1. Then

$$(F,Z)^{\tilde{c}} = \left\{\left((e_1,p,0),\left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.3},\frac{u_4}{0.9}\right\}\right),\left((e_1,q,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.8},\frac{u_3}{0.7},\frac{u_4}{0}\right\}\right),\right.$$
$$\left((e_1,r,0),\left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\}\right),\left((e_2,p,0),\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),$$
$$\left((e_2,q,0),\left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.3}\right\}\right),\left((e_2,r,0),\left\{\frac{u_1}{0.9},\frac{u_2}{0.7},\frac{u_3}{0.3},\frac{u_4}{0.6}\right\}\right),$$
$$\left((e_3,p,0),\left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\}\right),\left((e_3,q,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\}\right),$$
$$\left((e_3,r,0),\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\}\right),\left((e_1,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.7},\frac{u_4}{0.1}\right\}\right),$$
$$\left((e_1,q,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.3},\frac{u_4}{1}\right\}\right),\left((e_1,r,1),\left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}\right),$$

$$\left((e_2,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\}\right),\ \left((e_2,q,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.7}\right\}\right),$$
$$\left((e_2,r,1),\left\{\frac{u_1}{0.1},\frac{u_2}{0.3},\frac{u_3}{0.7},\frac{u_4}{0.4}\right\}\right),\ \left((e_3,p,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.3}\right\}\right),$$
$$\left.\left((e_3,q,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.3},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\}\right),\ \left((e_3,r,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}\right)\right\}= (F,Z)$$

**Proposition 2.14** Let $(F,A)$ be an *fse*-set over $U$. Then $\left((F,A)^{\tilde{c}}\right)^{\tilde{c}} = (F,A)$

**Definition 2.15** The union of two *fse*-sets $(F,A)$ and $(G,B)$ over $U$, denoted by $(F,A)\ \tilde{\cup}\ (G,B)$, is the *fse*-set $(H,C)$ where $C = A \cup B$ and for all $\alpha \in C$,

$$H(\alpha) = \begin{cases} F(\alpha), & \alpha \in A - B \\ G(\alpha), & \alpha \in B - A \\ F(\alpha) = G(\alpha), & \alpha \in A \cap B \end{cases}$$

**Example 2.16** Let

$$(F,A) = \left\{\left((e_1,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\}\right),\ \left((e_1,q,1),\left\{\frac{u_1}{0.4},\frac{u_2}{0.2},\frac{u_3}{0.6},\frac{u_4}{0.3}\right\}\right),\right.$$
$$\left((e_2,q,1),\left\{\frac{u_1}{0.1},\frac{u_2}{0.2},\frac{u_3}{0.7},\frac{u_4}{0.4}\right\}\right),\ \left((e_2,r,1),\left\{\frac{u_1}{0.4},\frac{u_2}{0.4},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\}\right),$$
$$\left((e_3,p,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.3},\frac{u_3}{0.4},\frac{u_4}{0.2}\right\}\right),\ \left((e_3,r,1),\left\{\frac{u_1}{0.1},\frac{u_2}{0.5},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}\right),$$
$$\left((e_1,r,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.8},\frac{u_4}{0.3}\right\}\right),\ \left((e_2,p,0),\left\{\frac{u_1}{0.1},\frac{u_2}{0.3},\frac{u_3}{0.7},\frac{u_4}{0.3}\right\}\right),$$
$$\left.\left((e_3,q,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.5}\right\}\right)\right\}$$

and

$$(G,B) = \left\{\left((e_1,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\}\right),\ \left((e_1,q,1),\left\{\frac{u_1}{0.4},\frac{u_2}{0.2},\frac{u_3}{0.6},\frac{u_4}{0.3}\right\}\right),\right.$$
$$\left((e_2,q,1),\left\{\frac{u_1}{0.1},\frac{u_2}{0.2},\frac{u_3}{0.7},\frac{u_4}{0.4}\right\}\right),\ \left((e_2,r,1),\left\{\frac{u_1}{0.4},\frac{u_2}{0.4},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\}\right),$$
$$\left((e_1,r,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.8},\frac{u_4}{0.3}\right\}\right),\ \left((e_3,p,0),\left\{\frac{u_1}{0.4},\frac{u_2}{0.7},\frac{u_3}{0.6},\frac{u_4}{0.8}\right\}\right),$$
$$\left.\left((e_3,q,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.5}\right\}\right),\ \left((e_2,p,0),\left\{\frac{u_1}{0.1},\frac{u_2}{0.3},\frac{u_3}{0.7},\frac{u_4}{0.3}\right\}\right)\right\}$$

Thus,

$$(F,A)\ \tilde{\cup}\ (G,B) = \left\{\left((e_1,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\}\right),\ \left((e_1,q,1),\left\{\frac{u_1}{0.4},\frac{u_2}{0.2},\frac{u_3}{0.6},\frac{u_4}{0.3}\right\}\right),\right.$$
$$\left((e_2,q,1),\left\{\frac{u_1}{0.1},\frac{u_2}{0.2},\frac{u_3}{0.7},\frac{u_4}{0.4}\right\}\right),\ \left((e_2,r,1),\left\{\frac{u_1}{0.4},\frac{u_2}{0.4},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\}\right),$$

$$\left((e_3,p,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.3},\frac{u_3}{0.4},\frac{u_4}{0.2}\right\}\right),\quad \left((e_3,r,1),\left\{\frac{u_1}{0.1},\frac{u_2}{0.5},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\}\right),$$
$$\left((e_1,r,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.8},\frac{u_4}{0.3}\right\}\right),\quad \left((e_2,p,0),\left\{\frac{u_1}{0.1},\frac{u_2}{0.3},\frac{u_3}{0.7},\frac{u_4}{0.3}\right\}\right),$$
$$\left((e_3,q,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.5}\right\}\right),\quad \left((e_3,p,0),\left\{\frac{u_1}{0.4},\frac{u_2}{0.7},\frac{u_3}{0.6},\frac{u_4}{0.8}\right\}\right)\Bigg\}.$$

**Proposition 2.17** Let $(F,A),(G,B)$ and $(H,C)$ be three *fse*-sets over $U$. Then

i. $(F,A)\,\tilde{\cup}\,(F,A)=(F,A)$
ii. $(F,A)\,\tilde{\cup}\,(G,B)=(G,B)\,\tilde{\cup}\,(F,A)$
iii. $(F,A)\,\tilde{\cup}\,\big((G,B)\,\tilde{\cup}\,(H,C)\big)=\big((F,A)\,\tilde{\cup}\,(G,B)\big)\,\tilde{\cup}\,(H,C)$

**Definition 2.18** The intersection of two *fse*-sets $(F,A)$ and $(G,B)$ over $U$, denoted by $(F,A)\,\tilde{\cap}\,(G,B)$, is the *fse*-set $(H,C)$ where $C=A\cap B$, for all $\alpha\in C$,

$$H(\alpha)=\begin{cases}F(\alpha)=G(\alpha), & C\neq\emptyset\\ \emptyset, & otherwise\end{cases}$$

**Example 2.19** Let's consider Example 2.5. Then

$$(F,A)\,\tilde{\cap}\,(G,B)=\Bigg\{\left((e_1,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\}\right),\quad \left((e_1,q,1),\left\{\frac{u_1}{0.4},\frac{u_2}{0.2},\frac{u_3}{0.6},\frac{u_4}{0.3}\right\}\right),$$
$$\left((e_2,q,1),\left\{\frac{u_1}{0.1},\frac{u_2}{0.2},\frac{u_3}{0.7},\frac{u_4}{0.4}\right\}\right),\quad \left((e_2,r,1),\left\{\frac{u_1}{0.4},\frac{u_2}{0.4},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\}\right),$$
$$\left((e_1,r,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.8},\frac{u_4}{0.3}\right\}\right),\quad \left((e_2,p,0),\left\{\frac{u_1}{0.1},\frac{u_2}{0.3},\frac{u_3}{0.7},\frac{u_4}{0.3}\right\}\right),$$
$$\left((e_3,q,0),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.5}\right\}\right)\Bigg\}$$

**Remark 2.20** For all $\alpha\in A\cap B$, $F(\alpha)=G(\alpha)$. That is, $(F,A)\,\tilde{\cup}\,(G,B)=(F,A)\,\tilde{\cap}\,(G,B)$ is as in [6]. Therefore, the set $C$ may consider as $A\cap B$ for the intersection *fse*-set $(H,C)$.

**Proposition 2.21** Let $(F,A),(G,B)$ and $(H,C)$ be three *fse*-sets over $U$. Then
i. $(F,A)\,\tilde{\cap}\,(F,A)=(F,A)$
ii. $(F,A)\,\tilde{\cap}\,(G,B)=(G,B)\,\tilde{\cap}\,(F,A)$
iii. $(F,A)\,\tilde{\cap}\,\big((G,B)\,\tilde{\cap}\,(H,C)\big)=\big((F,A)\,\tilde{\cap}\,(G,B)\big)\,\tilde{\cap}\,(H,C)$

**Proposition 2.22** Let $(F,A),(G,B)$ and $(H,C)$ be three *fse*-sets over $U$. Then

i. $(F,A)\,\tilde{\cup}\,\big((G,B)\,\tilde{\cap}\,(H,C)\big)=\big((F,A)\,\tilde{\cup}\,(G,B)\big)\,\tilde{\cap}\,\big((F,A)\,\tilde{\cup}\,(H,C)\big)$
ii. $(F,A)\,\tilde{\cap}\,\big((G,B)\,\tilde{\cup}\,(H,C)\big)=\big((F,A)\,\tilde{\cap}\,(G,B)\big)\,\tilde{\cup}\,\big((F,A)\,\tilde{\cap}\,(H,C)\big)$

**Definition 2.23** Let $(F,A)$ and $(G,B)$ be two *fse*-sets over $U$. Then $(F,A)$ AND $(G,B)$, denoted by $(F,A)\wedge(G,B)$, is defined by

$$(F,A)\wedge(G,B)=(H,A\times B)$$

where $H(\alpha,\beta)=F(\alpha)\cap G(\beta)$, for all $(\alpha,\beta)\in A\times B$.

**Definition 2.24** Let $(F,A)$ and $(G,B)$ be two *fse*-sets over $U$. Then $(F,A)$ OR $(G,B)$, denoted by $(F,A)\vee(G,B)$, is defined by

$$(F,A)\vee(G,B)=(O,A\times B)$$

where $O(\alpha,\beta)=F(\alpha)\cup G(\beta)$, for all $(\alpha,\beta)\in A\times B$.

**Example 2.25** Let

$$(F,A)=\left\{\left((e_1,p,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.2}\right\}\right),\ \left((e_1,r,0),\left\{\frac{u_1}{0.3},\frac{u_2}{0.1},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),\right.$$
$$\left.\left((e_3,q,0),\left\{\frac{u_1}{0.2},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.8}\right\}\right),\ \left((e_2,r,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\}\right)\right\}$$

and

$$(G,B)=\left\{\left((e_1,p,1),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.2}\right\}\right),\ \left((e_1,r,0),\left\{\frac{u_1}{0.3},\frac{u_2}{0.1},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),\right.$$
$$\left.\left((e_2,r,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\}\right)\right\}$$

Then $(F,A)\wedge(G,B)$ and $(F,A)\vee(G,B)$ as in the following, respectively,

$$\left\{\left(((e_1,p,1),(e_1,p,1)),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.2}\right\}\right),\ \left(((e_1,p,1),(e_1,r,0)),\left\{\frac{u_1}{0.3},\frac{u_2}{0.1},\frac{u_3}{0.2},\frac{u_4}{0.2}\right\}\right),\right.$$
$$\left(((e_1,p,1),(e_2,r,1)),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.2}\right\}\right),\ \left(((e_1,r,0),(e_1,p,1)),\left\{\frac{u_1}{0.3},\frac{u_2}{0.1},\frac{u_3}{0.2},\frac{u_4}{0.2}\right\}\right),$$
$$\left(((e_1,r,0),(e_1,r,0)),\left\{\frac{u_1}{0.3},\frac{u_2}{0.1},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),\ \left(((e_1,r,0),(e_2,r,1)),\left\{\frac{u_1}{0.3},\frac{u_2}{0.1},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\}\right),$$
$$\left(((e_3,q,0),(e_1,p,1)),\left\{\frac{u_1}{0.2},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.2}\right\}\right),\ \left(((e_3,q,0),(e_1,r,0)),\left\{\frac{u_1}{0.2},\frac{u_2}{0.1},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),$$
$$\left(((e_3,q,0),(e_2,r,1)),\left\{\frac{u_1}{0.2},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\}\right),\ \left(((e_2,r,1),(e_1,p,1)),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.2}\right\}\right),$$
$$\left.\left(((e_2,r,1),(e_1,r,0)),\left\{\frac{u_1}{0.3},\frac{u_2}{0.1},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\}\right),\ \left(((e_2,r,1),(e_2,r,1)),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\}\right)\right\}$$

$$\left\{\left(((e_1,p,1),(e_1,p,1)),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.2}\right\}\right),\left(((e_1,p,1),(e_1,r,0)),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),\right.$$
$$\left(((e_1,p,1),(e_2,r,1)),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\}\right),\left(((e_1,r,0),(e_1,p,1)),\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),$$
$$\left(((e_1,r,0),(e_1,r,0)),\left\{\frac{u_1}{0.3},\frac{u_2}{0.1},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),\left(((e_1,r,0),(e_2,r,1)),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),$$
$$\left(((e_3,q,0),(e_1,p,1)),\left\{\frac{u_1}{0.5},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.8}\right\}\right),\left(((e_3,q,0),(e_1,r,0)),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.8}\right\}\right),$$
$$\left(((e_3,q,0),(e_2,r,1)),\left\{\frac{u_1}{0.6},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.8}\right\}\right),\left(((e_2,r,1),(e_1,p,1)),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\}\right),$$
$$\left.\left(((e_2,r,1),(e_1,r,0)),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),\left(((e_2,r,1),(e_2,r,1)),\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\}\right)\right\}$$

**Proposition 2.26** Let $(F, A)$ and $(G, B)$ be two *fse*-sets over $U$. Then

i. $\big((F, A) \wedge (G, B)\big)^{\tilde{c}} = (F, A)^{\tilde{c}} \vee (G, B)^{\tilde{c}}$
ii. $\big((F, A) \vee (G, B)\big)^{\tilde{c}} = (F, A)^{\tilde{c}} \wedge (G, B)^{\tilde{c}}$

**Proposition 2.27** Let $(F, A)$, $(G, B)$ and $(H, C)$ be three *fse*-sets over $U$. Then

i. $(F, A) \wedge \big((G, B) \wedge (H, C)\big) = \big((F, A) \wedge (G, B)\big) \wedge (H, C)$
ii. $(F, A) \vee \big((G, B) \vee (H, C)\big) = \big((F, A) \vee (G, B)\big) \vee (H, C)$

**Remark 2.28** Since the domains of functions which lay on the right side of the equalities are different from the other side of them, the propositions given in [6]

iii. $(F, A) \vee \big((G, B) \wedge (H, C)\big) = \big((F, A) \vee (G, B)\big) \wedge \big((F, A) \vee (H, C)\big)$
iv. $(F, A) \wedge \big((G, B) \vee (H, C)\big) = \big((F, A) \wedge (G, B)\big) \vee \big((F, A) \wedge (H, C)\big)$

are not held as it is also shown in [4] for the soft sets.

## 3. An Application of Fuzzy Soft Expert Sets in Decision Making

In this section, we show that the algorithm given in [6] has some unnecessary steps and that the results of this algorithm and Maji et al's algorithm [11] without reduction are equivalent. Afterwards, we suggest a new algorithm and give an application on decision making by using updated definitions and propositions as a result of remarks above.

Let's consider the algorithm in [6] as in the following,

**Algorithm 1.**

i. Input the *fse*-set $(F, Z)$,
ii. Find an agree-*fse*-set and a disagree-*fse*-set,
iii. Find $c_j = \sum_i R_X(\alpha_i, u_j)$ for agree-*fse*-set,
iv. Find $k_j = \sum_i R_X(\alpha_i, u_j)$ for disagree-*fse*-set,
v. Find $s_j = c_j - k_j$,
vi. Find m, for which $s_m = \max_j s_j$

Here, $R_X$ is a fuzzy relation on $Z \times U$, defined by $R_X(\alpha_i, u_j) = \mu_{F(\alpha_i)}(u_j)$ such that $R_X(\alpha_i, u_j)$ is the entries corresponding the $i$th row and $j$th column in table representation of $R_X$ and $\mu_{F(\alpha_i)}(u_j)$ is the membership grade of $u_j$ to the fuzzy set $F(\alpha_i)$ in $(F, Z)$.

It is easy to show that, from the Definition 2.8,

$$k_j = |E \times X| - c_j$$

then

$$s_j = c_j - \{|E \times X| - c_j\} = 2c_j - |E \times X|$$

and

$$c_i \leq c_j \Leftrightarrow 2c_i \leq 2c_j \Leftrightarrow \big(2c_i - |E \times X| \leq 2c_j - |E \times X|\big) \Leftrightarrow s_i \leq s_j$$

where, the symbol $|E \times X|$ is the cardinality of $E \times X$. That is, $s_j$ and $\max_j\{s_j\}$ are redundant. So, step 5, step 4 and the last part of step 2 are unnecessary. Hence, the algorithm has become Maji et al's algorithm, i.e.,

i. Input the *fse*-set $(F, Z)$,
ii. Find the agree-*fse*-set,
iii. Find $c_j = \sum_i R_X(\alpha_i, u_j)$ for the agree-*fse*-set,
iv. Find $m$, for which $c_m = \max_j c_j$

To illustrate, let's consider the application given in [6]. Assume that a company wants to fill a position. There are four candidates who form the universe $U = \{u_1, u_2, u_3, u_4\}$, the hiring committee considers a set of parameters, $E = \{e_1, e_2, e_3\}$ where the parameters $e_i$, for $i \in \{1, 2, 3\}$, stand for *experience*, *computer knowledge* and *elocution*, respectively. Let $X = \{p, q, r\}$ be a set of experts (committee members). Suppose that, after a serious discussion, the committee constructs the *fse*-set $(F, Z)$ given in Example 2.1. Then the table representation of $(F, Z)_1$ as in the following,

**Table 1.** The table of agree-*fse*-sets

| $R_X$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ |
|---|---|---|---|---|
| $(e_1, p, 1)$ | 0.3 | 0.5 | 0.7 | 0.1 |
| $(e_2, p, 1)$ | 0.3 | 0.2 | 0.5 | 0.6 |
| $(e_3, p, 1)$ | 0.6 | 0.2 | 0.4 | 0.3 |
| $(e_1, q, 1)$ | 0.5 | 0.2 | 0.3 | 1.0 |
| $(e_2, q, 1)$ | 0.6 | 0.4 | 0.3 | 0.7 |
| $(e_3, q, 1)$ | 0.5 | 0.3 | 0.5 | 0.7 |
| $(e_1, r, 1)$ | 0.4 | 0.8 | 0.3 | 0.4 |
| $(e_2, r, 1)$ | 0.1 | 0.3 | 0.7 | 0.4 |
| $(e_3, r, 1)$ | 0.3 | 0.4 | 0.3 | 0.4 |
| $c_j = \sum_i R_X(\alpha_i, u_j)$ | $c_1 = 3.6$ | $c_2 = 3.3$ | $c_3 = 4$ | $c_4 = 4.6$ |

Hence, the committee can choose candidate 4 for the job since $\max_j c_j = c_4$.

Note that the order of $c_j$,

$$c_4 > c_3 > c_1 > c_2$$

obtained by Maji et al's algorithm without reduction, is the same as the order obtained by Alkhazaleh and Salleh's algoritm.

Let's give a new definition and an algorithm which is different from the others.

**Definition 3.1** The *fse*-set $(F, A)$ is called $p$-part of $(F, Z)$, denoted by $p(F, Z)$, such that $A = E \times \{p\} \times O$ for $p \in X$.

For example;

$$p(F,Z) = \left\{\left((e_1,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.7},\frac{u_4}{0.1}\right\}\right),\left((e_2,p,1),\left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\}\right),\right.$$
$$\left((e_3,p,1),\left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.3}\right\}\right),\left((e_1,p,0),\left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.3},\frac{u_4}{0.9}\right\}\right),$$
$$\left.\left((e_2,p,0),\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\}\right),\left((e_3,p,0),\left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\}\right)\right\}$$

is the p-part of $(F,Z)$ given in Example 2.1.

Note that $p(F,Z)_1$ can be seen as a fuzzy soft set over $U$ and written simply as in the following,

$$p(F,Z)_1 = \left\{\left(e_1,\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.7},\frac{u_4}{0.1}\right\}\right),\left(e_2,\left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\}\right),\left(e_3,\left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.3}\right\}\right)\right\}$$

**Algorithm 2.**

i. Construct an *fse*-set,
ii. Find the parts of agree-*fse*-set,
iii. Find the consensus fuzzy soft set by using s-intersection to all parts of agree-*fse*-set,
iv. Find $c_j = \frac{\sum_i R_C(e_i,u_j)}{|E|}$ for consensus fuzzy soft set by using Cesàro means,
v. Find $\{u_k : c_k = \max_j c_j\}$.

To illustrate, let's consider the application above. Then the table representation of all parts of agree-*fse*-set as in the following,

**Table 1.** The table of $p(F,Z)_1$

| $R_p$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ |
|---|---|---|---|---|
| $e_1$ | 0.3 | 0.5 | 0.7 | 0.1 |
| $e_2$ | 0.3 | 0.2 | 0.5 | 0.6 |
| $e_3$ | 0.6 | 0.2 | 0.4 | 0.3 |

**Table 2.** The table of $q(F,Z)_1$

| $R_q$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ |
|---|---|---|---|---|
| $e_1$ | 0.5 | 0.2 | 0.3 | 0.1 |
| $e_2$ | 0.6 | 0.4 | 0.3 | 0.7 |
| $e_3$ | 0.5 | 0.3 | 0.5 | 0.7 |

**Table 3.** The table of $r(F,Z)_1$

| $R_r$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ |
|---|---|---|---|---|
| $e_1$ | 0.4 | 0.8 | 0.3 | 0.4 |
| $e_2$ | 0.1 | 0.3 | 0.7 | 0.4 |
| $e_3$ | 0.3 | 0.4 | 0.3 | 0.4 |

Here, $R_p$ is a fuzzy relation on $E \times U$, defined by $R_p(e_i,u_j) = \mu_{F(e_i)}(u_j)$ such that $R_p(e_i,u_j)$ is the entries corresponding the $i$th row and $j$th column in table representation of $R_p$ and $\mu_{F(e_i)}(u_j)$ is the membership grade of $u_j$ to the fuzzy set $F(e_i)$ in $p(F,Z)_1$.

Let's obtain the consensus fuzzy soft set by using s-intersection of all parts of the agree-*fse*-set and show as in the following,

**Table 4.** The table of the consensus fuzzy soft set

| $R_C$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ |
|---|---|---|---|---|
| $e_1$ | 0.3 | 0.2 | 0.3 | 0.1 |
| $e_2$ | 0.1 | 0.2 | 0.3 | 0.4 |
| $e_3$ | 0.3 | 0.2 | 0.3 | 0.3 |
| $c_j = \frac{\sum_i R_C(e_i,u_j)}{\lvert E \rvert}$ | $c_1 = 0.23$ | $c_2 = 0.20$ | $c_3 = 0.30$ | $c_4 = 0.27$ |

By Table 4, we have the following results;

$$c_3 > c_4 > c_1 > c_2$$

Since $\max_j c_j = c_3$, the committee can choose the candidate with number 3 for the job.

## 4. Conclusion

The concept of soft sets has idiosyncratic serious problems because of some of their definitions as the soft complement. Enginoğlu [8] overcame such problems by characteristic sets in 2012. Similarly, the concept of *fse*-sets can provide dealing with the difficulty arising from the definition of soft complement in [12] by assuming $(\neg e_i, p_j, 1) = (e_i, p_j, 0)$. This is important for the development of soft sets, and it is worth doing the study on it when viewed from this aspect. People who want to study on this concept should not ignore this detail.

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