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BOUNDARY DISCONTINUOUS FOURIER ANALYSIS OF A DOUBLY CURVED CROSS-PLY LAMINATED COMPOSITE SHELL

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Abstract

A new analytical solution to the problem of deformation for a cross-ply, thick laminated composite shell in a rectangular plan form is presented. The solution methodology is based on boundary discontinuous generalized double Fourier series approach and used to solve a system of five highly coupled linear partial differential equations which are generated from Higher Order Shear Deformation Theory (HSDT). Boundary conditions are defined as follows; two opposite edges are SS3 and the others are SS1. The major result of the present study is to declare analytic solution functions of defined boundary discontinuity which is unavailable in the literature.

SÜREKSİZ SINIRLARA SAHİP, ÇİFT EĞRİLİKLİ, ÇAPRAZ KATLI LAMİNE KOMPOZİT KABUĞUN FOURIER ANALİZİ

Özetçe

Bu çalışmada, çapraz katlı, kalın, ortogonal kompozit kabuğun SS3-SS1 sınır şartları altında deformasyonunun belirlenebilmesi için yeni bir analitik çözüm sunulmuştur. Çözüm tekniği süreksiz sınırlar için düzenlenmiş çiftli Fourier serilerine dayanır ve Yüksek Mertebeli Deformasyon Teorileri kullanılarak oluşturulmuş kısmi diferansiyel denklemler kullanılır. Kullanılan sınır şartları; iki karşılıklı kenar için SS3 ve diğer kenarlar için ise SS1 olarak tanımlanmıştır. Çalışmanın başlıca sonucu henüz literatürde bulunmayan bahse konu sınır koşullarının analitik çözümüne ait kısmi diferansiyel denklemlerin belirlenmesidir.

Keywords: Laminated composite plate, HSDT, boundary-discontinuity, Fourier analysis, doubly curved shell
Anahtar Kelimeler : Lamine kompozit levha, sinir süreksizliği, Fourier analizi, HSDT, çift eğrilikli kabuk.

1. INTRODUCTION

Modern composites using fiber reinforced matrices of various types have created a revolution in high performance structures. Advances in the composite material development lead the multi-effectiveness of the material choice. Especially the recent development in the combat scenarios shows that composite structures can be used for relevant advantages not only for the weight strength ratio. The necessity of multifunctional materials in the military equipments such as helicopters, planes, ships, multitasking vehicles etc. has caused the composite material researches to be grown.

The stealth technology, for example, requires radar reflecting / absorbing mechanisms, acoustic absorbents or heat resistive insulators with highest electrical connectivity. Visby Class ships, as an example in the stealth technology, shall have primary focus. The multifunctional material philosophy against the need of multi tasking effectiveness force laminated plate researches to define proper solutions in real world situations. At that point laminated or sandwich composites can be seen the way of development in front of the expensive and restricted nano-science applications. Aircraft, naval or space vehicle designers always choose light weight materials which mean low fuel consumption and prefer easy to find materials not to increase procurement expenses.

These flags can give the idea of industrial necessary materials which became lower weighted, multi layered (with flexibility of re-designing layer properties), suitable for tailoring (with flexibility of re-configuring layers), etc. Kabir et al. has indicated that; the design flexibility inherent in composite laminates, known as tailoring, which is essentially exploiting the possibility of obtaining optimum design through a combination of structural / material concepts, stacking sequence, ply orientation, choice of the component phases, etc. to meet specific design requirements, is the single most important factor to commercial and military developments. [1]

Typically, analysis of laminated plates can be achieved by approximate numerical methods easily, such as finite elements method (FEM) which accuracy has been proven again and again by lots of researchers. However, it is hard to develop an analytical model for a specified problem. Derivation of analytical solutions involves many complexities, such as inplane anisotropy and asymmetry of lamination resulting in stretching-shear, bending-stretching couplings. Additional complexities arise by way of satisfying boundary conditions that can not be handled by traditional analytical approaches, such as Navier's or Levy's [1]. The present study is intended to capture some of these complexities arising from real life boundary conditions.

2. LITERATURE REWIEV

Investigations of laminated composite plate usually utilize either the classical lamination theory (CLT) or the first order shear deformation theory (FSDT). Jones [2], Whitney [3], Kabir et al. [1] and Chaudhuri et al. [4] have presented Double Fourier series based analytical solutions to thin laminated anisotropic plate boundary value problems.

Chaudhuri and Kabir [5-8], Kabir and Chaudhuri [9] and Kabir [10-11] have presented Double Fourier series based analytical solutions to various FSDT-based laminated plate boundary value problems. Superiority of the FSDT over the CLT in prediction of the transverse deflection of a moderately thick panel notwithstanding, the FSDT requires incorporation of a shear correction factor, due to the fact that the FSDT assumes a uniform transverse shear strain distribution through the thickness, which violates equilibrium conditions at the top and bottom surfaces of the panel [13].

The boundary discontinuous Fourier series theory has been expounded earlier by Hobson and Carslaw, and the method has been applied by other investigators, such as Green, Winslow and Whitney. The boundary discontinuous Fourier method has never been applied to the problem of a plate / shell subjected to asymmetric (with respect to panel centerlines) boundary conditions, which along with the general lack of non-separable Fourier solution has so far remained an enigma in the literature [16]. The proper mathematical explanations of the boundary discontinuous type Fourier series approach to solution of completely coupled system of partial differential equations subjected to admissible general boundary conditions are available in Chaudhuri [16].

A higher order shear deformation theory of elastic shells is developed by Reddy [23] for laminated shells with orthotropic layers. The theory accounts for parabolic distribution of the transverse shear strains through thickness of the shell. Double curved shells with two radius have been examined by Reddy.

Recently, Oktem and Chaudhuri have declared their researches on doubly curved shells with discontinuity in the boundaries [17-22]. The shells have SS2-SS3, SS3-C4, All SS4, All SS1, etc.. boundaries.

Second and higher order shear deformation plate theories use higher order polynomials in the expansion of the displacement components through the thickness of the plate. The higher order theories introduce additional unknowns that are often difficult to interpret in physical terms. Therefore, third order shear deformation theory (TSDT) has been accepted as the most accurate solution methodology between higher order theories, comparing with calculation complexity and accuracy. However, there are a number of third order plate theories in the literature which have been explained by Reddy [15].

3. STATEMENT OF THE PROBLEM

A Third Order displacement field for in-plane displacements is used [13].

$$u_1 = u(x_1, x_2, x_3, t) = u_0(x_1, x_2, t) + z\phi_1(x_1, x_2, t) - \frac{4}{3h^2} z^3 \left(\phi_1 + \frac{\partial w_0}{\partial x_1} \right) \quad (1a)$$

$$u_2 = v(x_1, x_2, x_3, t) = v_0(x_1, x_2, t) + z\phi_2(x_1, x_2, t) - \frac{4}{3h^2} z^3 \left(\phi_2 + \frac{\partial w_0}{\partial x_2} \right) \quad (1b)$$

$$u_3 = w(x_1, x_2, x_3, t) = w_0(x_1, x_2, t) \quad (1c)$$

where u_0,v_0,w_0 represents displacements of a point at the mid-surface (z = 0), while and ϕ_1 and ϕ_2 are rotations about x_2 and x_1 axes respectively.

Three dimensional elasticity strain – displacement relations [15]:

$$\varepsilon_1 = \varepsilon_1^0 + z(K_1^0 + z^2 K_1^2) \quad (2a)$$

$$\varepsilon_2 = \varepsilon_2^0 + z(K_2^0 + z^2 K_2^2) \quad (2b)$$

$$\varepsilon_4 = \varepsilon_4^0 + z^2 K_4^1 \quad (2c)$$

$$\varepsilon_5 = \varepsilon_5^0 + z^2 K_5^1 \quad (2d)$$

$$\varepsilon_6 = \varepsilon_6^0 + z(K_6^0 + z^2 K_6^2) \quad (2e)$$

in which ε represents the components of strain at a point, while ε^0 denotes their mid-surface counterparts. K^0 represents changes of curvature and twist, while K^2 denotes its counterparts due to the higher-order shear deformation effect.

The equilibrium equations derived using the principle of virtual work are given as follows [13] (the derivation process will not be reproduced here in the interest of brevity of the presentation):

$$\frac{\partial N_1}{\partial x_1}+\frac{\partial N_6}{\partial x_2}=0 \tag{3a}$$

$$\frac{\partial N_6}{\partial x_1}+\frac{\partial N_2}{\partial x_2}=0 \tag{3b}$$

$$\frac{\partial Q_1}{\partial x_1}+\frac{\partial Q_2}{\partial x_2}-\frac{4}{h^2}\left(\frac{\partial K_1}{\partial x_1}+\frac{\partial K_2}{\partial x_2}\right)+\frac{4}{3h^2}\left(\frac{\partial^2 P_1}{\partial x_1{}^2}+\frac{\partial^2 P_2}{\partial x_2{}^2}+2\frac{\partial^2 P_6}{\partial x_1\partial x_2}\right)-\frac{N_1}{R_1}-\frac{N_2}{R_2}=\text{-q} \tag{3c}$$

$$\frac{\partial M_1}{\partial x_1}+\frac{\partial M_6}{\partial x_2}-Q_1+\frac{4}{h^2}K_1-\frac{4}{3h^2}\left(\frac{\partial P_1}{\partial x_1}+\frac{\partial P_6}{\partial x_2}\right)=0 \tag{3d}$$

$$\frac{\partial M_6}{\partial x_1}+\frac{\partial M_2}{\partial x_2}-Q_2+\frac{4}{h^2}K_2-\frac{4}{3h^2}\left(\frac{\partial P_6}{\partial x_1}+\frac{\partial P_2}{\partial x_2}\right)=0 \tag{3e}$$

where q is the distributed transverse load, and N, M, P denote stress resultants, stress couples and second stress couples. Q represents the transverse shear stress resultants.

Generalized stress resultants can be conveniently expressed in terms of strain components. For a cross-ply, orthogonal, doubly curved lamina:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix}=\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}\begin{Bmatrix} u_{1,1}+\dfrac{u_3}{R_1} \\ u_{2,2}+\dfrac{u_3}{R_2} \\ u_{2,1}+u_{1,2} \end{Bmatrix}+\begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}\begin{Bmatrix} \phi_{1,1} \\ \phi_{2,2} \\ \phi_{1,2}+\phi_{2,1} \end{Bmatrix}$$
$$+\left(-\frac{4}{3h^2}\right)\begin{bmatrix} E_{11} & E_{12} & 0 \\ E_{12} & E_{22} & 0 \\ 0 & 0 & E_{66} \end{bmatrix}\begin{Bmatrix} \phi_{1,1}+u_{3,11} \\ \phi_{2,2}+u_{3,22} \\ \phi_{2,1}+\phi_{1,2}+2u_{3,12} \end{Bmatrix} \tag{4a}$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{Bmatrix} u_{1,1} + \dfrac{u_3}{R_1} \\ u_{2,2} + \dfrac{u_3}{R_2} \\ u_{2,1} + u_{1,2} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \phi_{1,1} \\ \phi_{2,2} \\ \phi_{1,2} + \phi_{2,1} \end{Bmatrix} \tag{4b}$$

$$+ \left(-\frac{4}{3h^2} \right) \begin{bmatrix} F_{11} & F_{12} & 0 \\ F_{12} & F_{22} & 0 \\ 0 & 0 & F_{66} \end{bmatrix} \begin{Bmatrix} \phi_{1,1} + u_{3,11} \\ \phi_{2,2} + u_{3,22} \\ \phi_{2,1} + \phi_{1,2} + 2u_{3,12} \end{Bmatrix}$$

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_6 \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & 0 \\ E_{12} & E_{22} & 0 \\ 0 & 0 & E_{66} \end{bmatrix} \begin{Bmatrix} u_{1,1} + \dfrac{u_3}{R_1} \\ u_{2,2} + \dfrac{u_3}{R_2} \\ u_{2,1} + u_{1,2} \end{Bmatrix} + \begin{bmatrix} F_{11} & F_{12} & 0 \\ F_{12} & F_{22} & 0 \\ 0 & 0 & F_{66} \end{bmatrix} \begin{Bmatrix} \phi_{1,1} \\ \phi_{2,2} \\ \phi_{1,2} + \phi_{2,1} \end{Bmatrix} \tag{4c}$$

$$+ \left(-\frac{4}{3h^2} \right) \begin{bmatrix} H_{11} & H_{12} & 0 \\ H_{12} & H_{22} & 0 \\ 0 & 0 & H_{66} \end{bmatrix} \begin{Bmatrix} \phi_{1,1} + u_{3,11} \\ \phi_{2,2} + u_{3,22} \\ \phi_{2,1} + \phi_{1,2} + 2u_{3,12} \end{Bmatrix}$$

$$\begin{Bmatrix} Q_2 \\ Q_1 \end{Bmatrix} = \begin{bmatrix} A_{44} & 0 \\ 0 & A_{55} \end{bmatrix} \begin{Bmatrix} \phi_2 + u_{3,2} \\ \phi_1 + u_{3,1} \end{Bmatrix} + \left(-\frac{4}{h^2} \right) \begin{bmatrix} D_{44} & 0 \\ 0 & D_{55} \end{bmatrix} \begin{Bmatrix} \phi_2 + u_{3,2} \\ \phi_1 + u_{3,1} \end{Bmatrix} \tag{4d}$$

$$\begin{Bmatrix} K_2 \\ K_1 \end{Bmatrix} = \begin{bmatrix} D_{44} & 0 \\ 0 & D_{55} \end{bmatrix} \begin{Bmatrix} \phi_2 + u_{3,2} \\ \phi_1 + u_{3,1} \end{Bmatrix} + \left(-\frac{4}{h^2} \right) \begin{bmatrix} F_{44} & 0 \\ 0 & F_{55} \end{bmatrix} \begin{Bmatrix} \phi_2 + u_{3,2} \\ \phi_1 + u_{3,1} \end{Bmatrix} \tag{4e}$$

Substitution of Eqs.(4a-4e) to Eqs.(3a-3e) yields a system of five highly coupled fourth-order partial differential equations which are given below:

$$A_{11}u_{1,11}+\left(\frac{A_{11}}{R_1}+\frac{A_{12}}{R_2}\right)u_{3,1}+\left(A_{12}+A_{66}\right)u_{2,21}-\frac{4}{3h^2}E_{11}u_{3,111}-\frac{4}{3h^2}\left(E_{12}+2E_{66}\right)u_{3,221}+A_{66}u_{1,22}$$

$$+\left(B_{11}-\frac{4}{3h^2}E_{11}\right)\phi_{1,11}+\left(B_{12}-\frac{4}{3h^2}E_{12}+B_{66}-\frac{4}{3h^2}E_{66}\right)\phi_{2,21}+\left(B_{66}-\frac{4}{3h^2}E_{66}\right)\phi_{1,22}=0 \quad (5a)$$

$$\left(A_{12}+A_{66}\right)u_{1,12}+A_{66}u_{2,11}+\left(\frac{A_{12}}{R_1}+\frac{A_{22}}{R_2}\right)u_{3,2}+A_{22}u_{2,22}-\frac{4}{3h^2}\left(E_{12}+2E_{66}\right)u_{3,112}-\frac{4}{3h^2}E_{22}u_{3,222}$$

$$+\left(B_{66}-\frac{4}{3h^2}E_{66}+B_{12}-\frac{4}{3h^2}E_{12}\right)\phi_{1,21}+\left(B_{66}-\frac{4}{3h^2}E_{66}\right)\phi_{2,11}+\left(B_{22}-\frac{4}{3h^2}E_{22}\right)\phi_{2,22}=0 \quad (5b)$$

$$\begin{aligned}
&\left(A_{55}-\frac{8}{h^2}D_{55}+\frac{16}{h^4}F_{55}+\frac{4}{3h^2}\frac{E_{11}}{R_1}+\frac{4}{3h^2}\frac{E_{12}}{R_2}+\frac{4}{3h^2}\frac{E_{11}}{R_1}+\frac{4}{3h^2}\frac{E_{12}}{R_2}\right)u_{3,11}\\
&+\left(A_{44}-\frac{8}{h^2}D_{44}+\frac{16}{h^4}F_{44}+\frac{4}{3h^2}\frac{E_{12}}{R_1}+\frac{4}{3h^2}\frac{E_{22}}{R_2}+\frac{4}{3h^2}\frac{E_{12}}{R_1}+\frac{4}{3h^2}\frac{E_{22}}{R_2}\right)u_{3,22}\\
&+\left(A_{55}-\frac{8}{h^2}D_{55}+\frac{16}{h^4}F_{55}-\frac{B_{11}}{R_1}+\frac{4}{3h^2}\frac{E_{11}}{R_1}-\frac{B_{12}}{R_2}+\frac{4}{3h^2}\frac{E_{12}}{R_2}\right)\phi_{1,1}\\
&+\left(A_{44}-\frac{8}{h^2}D_{44}+\frac{16}{h^4}F_{44}-\frac{B_{12}}{R_1}+\frac{4}{3h^2}\frac{E_{12}}{R_1}-\frac{B_{22}}{R_2}+\frac{4}{3h^2}\frac{E_{22}}{R_2}\right)\phi_{2,2} \qquad (5c)\\
&+\left(\frac{4}{3h^2}F_{11}-\frac{16}{9h^4}H_{11}\right)\phi_{1,111}+\left(\frac{4}{3h^2}F_{12}-\frac{16}{9h^4}H_{12}+\frac{8}{3h^2}F_{66}-\frac{32}{9h^4}H_{66}\right)\phi_{2,211}\\
&+\left(\frac{8}{3h^2}F_{66}+\frac{4}{3h^2}F_{12}-\frac{16}{9h^4}H_{12}-\frac{32}{9h^4}H_{66}\right)\phi_{1,212}+\left(\frac{4}{3h^2}F_{22}-\frac{16}{9h^4}H_{22}\right)\phi_{2,222}\\
&-\frac{16}{9h^4}H_{11}u_{3,1111}+\frac{4}{h^2}E_{11}u_{1,111}+\frac{4}{3h^2}\left(E_{1,2}+2E_{66}\right)u_{2,211}-\frac{4}{3h^2}\left(\frac{8}{3h^2}H_{12}+\frac{16}{3h^2}H_{66}\right)u_{3,2211}
\end{aligned}$$

$$+\frac{4}{3h^2}(E_{12}+2E_{66})u_{1,122}+\frac{4}{3h^2}E_{22}u_{2,222}-\frac{16}{9h^4}H_{66}u_{3,2222}-\left(\frac{A_{11}}{R_1}+\frac{A_{12}}{R_2}\right)u_{1,1}-\left(\frac{A_{12}}{R_1}+\frac{A_{22}}{R_2}\right)u_{2,2}$$

$$-\frac{1}{R_1}\left(\frac{A_{11}}{R_1}+\frac{A_{12}}{R_2}\right)u_3-\frac{1}{R_1}\left(\frac{A_{12}}{R_1}+\frac{A_{22}}{R_2}\right)u_3=-q$$

$$\begin{aligned}&\left(B_{11}-\frac{4}{3h^2}E_{11}\right)u_{1,11}+\left(\frac{B_{11}}{R_1}+\frac{B_{12}}{R_2}-A_{55}+\frac{8}{h^2}D_{55}-\frac{16}{h^4}F_{55}-\frac{4}{3h^2}\frac{E_{11}}{R_1}-\frac{4}{3h^2}\frac{E_{12}}{R_2}\right)u_{3,1}+\left(B_{12}+B_{66}-\frac{4}{3h^2}E_{12}-\frac{4}{3h^2}E_{66}\right)u_{2,21}\\&+\left(B_{66}-\frac{4}{3h^2}E_{66}\right)u_{1,22}+\left(D_{11}-\frac{8}{3h^2}F_{11}+\frac{16}{9h^4}H_{11}\right)\phi_{1,11}+\left(D_{12}-\frac{8}{3h^2}F_{66}+D_{66}-\frac{4}{3h^2}F_{12}+\frac{16}{9h^4}H_{12}+\frac{16}{9h^4}H_{66}\right)\phi_{2,12}\\&-\left(\frac{4}{3h^2}F_{11}-\frac{16}{9h^4}H_{11}\right)u_{3,111}-\left(\frac{4}{3h^2}F_{12}+\frac{8}{3h^2}F_{66}-\frac{16}{9h^4}H_{12}-\frac{32}{9h^4}H_{66}\right)u_{3,122}+\left(D_{66}-\frac{8}{3h^2}F_{66}+\frac{16}{9h^4}H_{66}\right)\phi_{1,22}\\&-\left(A_{55}-\frac{8}{3h^2}D_{55}+\frac{16}{h^4}F_{55}\right)\phi_1=0\end{aligned}\tag{5d}$$

$$\begin{aligned}&\left(B_{66}-\frac{4}{3h^2}E_{66}\right)u_{2,11}+\left(B_{66}+B_{12}-\frac{4}{3h^2}E_{66}-\frac{4}{3h^2}E_{12}\right)u_{1,12}-\left(\frac{4}{3h^2}F_{22}-\frac{16}{9h^4}H_{22}\right)u_{3,222}\\&+\left(D_{66}-\frac{8}{3h^2}F_{66}+D_{12}-\frac{8}{3h^2}F_{12}+\frac{16}{9h^4}H_{66}+\frac{16}{9h^4}H_{12}\right)\phi_{1,21}+\left(B_{22}-\frac{4}{3h^2}E_{22}\right)u_{2,22}\\&+\left(D_{66}-\frac{8}{3h^2}F_{66}+\frac{16}{9h^4}H_{66}\right)\phi_{2,11}-\left(\frac{8}{3h^2}F_{66}+\frac{4}{3h^2}F_{12}-\frac{32}{9h^4}H_{66}-\frac{16}{9h^4}H_{12}\right)u_{3,112}\\&+\left(\frac{B_{12}}{R_1}+\frac{B_{22}}{R_2}-A_{44}+\frac{8}{h^2}D_{44}-\frac{16}{h^4}F_{44}-\frac{4}{3h^2}\frac{E_{12}}{R_1}-\frac{4}{3h^2}\frac{E_{22}}{R_2}\right)u_{3,2}+\left(D_{22}-\frac{8}{3h^2}F_{22}+\frac{16}{9h^4}H_{22}\right)\phi_{2,22}\\&-\left(A_{44}-\frac{8}{h^2}D_{44}+\frac{16}{h^4}F_{44}\right)\phi_2=0\end{aligned}\tag{5e}$$

4. BOUNDARY CONDITIONS

SS1 type simply supported boundary conditions are prescribed at the edges x_1=0,a:

$$u_3(0,x_2) = u_3(a,x_2) = 0 \tag{6a}$$

$$\phi_2(0,x_2) = \phi_2(a,x_2) = 0 \tag{6b}$$

$$M_1(0,x_2) = M_1(a,x_2) = 0 \tag{6c}$$

$$N_1(0,x_2) = N_1(a,x_2) = 0 \tag{6d}$$

$$N_6(0,x_2) = N_6(a,x_2) = 0 \tag{6e}$$

SS3 type simply supported boundary conditions are prescribed at the edges x_2=0,b:

$$u_3(x_1,0) = u_3(x_1,b) = 0 \tag{7a}$$

$$N_2(x_1,0) = N_2(x_1,b) = 0 \tag{7b}$$

$$u_1(x_1,0) = u_1(x_1,b) = 0 \tag{7c}$$

$$M_2(x_1,0) = M_2(x_1,b) = 0 \tag{7d}$$

$$\phi_1(x_1,0) = \phi_1(x_1,b) = 0 \tag{7e}$$

5. SOLUTION METHODOLOGY

The particular solution to the boundary-value problem of a HSDT-based cross-ply plate is assumed as follows:

$$u_1 = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} U_{mn} \cos \alpha x_1 \sin \beta x_2 \qquad 0 \le x_1 \le a\,; \quad 0 \le x_2 \le b \qquad \text{(8a)}$$

$$u_2 = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} V_{mn} \sin \alpha x_1 \cos \beta x_2 \qquad 0 < x_1 < a\,; \quad 0 \le x_2 \le b \qquad \text{(8b)}$$

$$u_3 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} \sin \alpha x_1 \sin \beta x_2 \qquad 0 \le x_1 \le a\,; \quad 0 \le x_2 \le b \qquad \text{(8c)}$$

$$\phi_1 = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} X_{mn} \cos \alpha x_1 \sin \beta x_2 \qquad 0 \le x_1 \le a\,; \quad 0 \le x_2 \le b \qquad \text{(8d)}$$

$$\phi_2 = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} Y_{mn} \sin \alpha x_1 \cos \beta x_2 \qquad 0 \le x_1 \le a\,; \quad 0 \le x_2 \le b \qquad \text{(8e)}$$

where

$$\alpha = \frac{m\pi}{a}, \qquad \beta = \frac{n\pi}{b} \qquad \text{(9)}$$

It may be noted that the assumed solution functions, given by Eqs. (8a – 8e), satisfy the SS3 type simply supported conditions at the edges, x_2 =0,b. The total number of unknown Fourier Coefficients introduced in Eqs. (8a – 8e) numbers 5mn+2m+2n.

The next operation is substituting Eqs (8a – 8e) into Eqs. (5a – 5e). The procedure for differentiation of these functions is based on Lebesque integration theory that introduces boundary Fourier coefficients arising from discontinuities of the particular solutions at the edges x_1 =0,a. As has been noted by Chaudhuri [16], the boundary Fourier coefficients serve as complementary solution to the problem under investigation [17-22]. The partial derivatives, which can not be obtained by termwise differentiation, are given as follows;

$$u_{2,1} = \frac{1}{4}\overline{e_0} + \sum_{m=1}^{\infty} C\,\mathrm{os}(\alpha x_1)\left\{\alpha V_{m0} + \frac{1}{2}(\overline{e_0}\gamma_m + \overline{f_0\delta_m})\right\} + \frac{1}{2}\sum_{n=1}^{\infty}\overline{e_n}Cos(\beta x_2)$$
$$+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Cos(\beta x_2)Cos(\alpha x_1)\left\{\alpha V_{mn} + \overline{e_n}\gamma_m + \overline{f_n\delta_m}\right\} \quad (10)$$

$$u_{2,12} = -\frac{1}{2}\sum_{n=1}^{\infty}\overline{e_n}\beta Sin(\beta x_2) - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\beta Sin(\beta x_2)Cos(\alpha x_1)\left\{\alpha V_{mn} + \overline{e_n}\gamma_m + \overline{f_n}\delta_m\right\} \quad (11)$$

$$u_{2,11} = -\sum_{m=1}^{\infty}\alpha Sin(\alpha x_1)\left\{\alpha V_{m0} + \frac{1}{2}(\overline{e_0}\gamma_m + \overline{f_0}\delta_m)\right\}$$
$$-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\alpha Sin(\alpha x_1)Cos(\beta x_2)\left\{\alpha V_{mn} + \overline{e_n}\gamma_m + \overline{f_n}\delta_m\right\} \quad (12)$$

where

$$(\overline{e_n};\overline{f_n}) = \frac{4}{ab}\int_0^b \left\{\pm u_2(a,x_2) - u_2(0,x_2)\right\} Cos(\beta x_2)dx_2 \quad (13)$$

$$(\gamma_m;\delta_m) = \left\{\begin{matrix}(0,1), m = odd \\ (1,0), m = even\end{matrix}\right\} \quad (14)$$

Introduction of the displacement functions and their appropriate partial derivatives into the governing partial differential equations will supply the following 5mn+2m+2n linear algebraic equations:

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Cos(\alpha x_1)Sin(\beta x_2)\begin{Bmatrix} -(A_{11}\alpha^2 + A_{66}\beta^2)U_{mn} - f_1\alpha\beta V_{mn} \\ +(a_4\alpha^3 - f_3\alpha\beta^2 + a_1\alpha)W_{mn} \\ -(a_2\alpha^2 + a_9\beta^2)X_{mn} \\ -f_2\alpha\beta Y_{mn} - f_1\beta(\overline{e_n}\gamma_m + \overline{f_n}\delta_m) \end{Bmatrix} = 0 \tag{15a}$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Sin(\alpha x_1)Cos(\beta x_2)\begin{Bmatrix} -f_1\alpha\beta U_{mn} - (A_{66}\alpha^2 + A_{22}\beta^2)V_{mn} \\ +(a_6\beta^3 + a_8\beta^3 - f_3\alpha^2\beta)W_{mn} \\ -f_2\alpha\beta X_{mn} - (a_9\alpha^2 + a_7\beta^2)Y_{mn} \\ -A_{66}\alpha(\overline{e_n}\gamma_m + \overline{f_n}\delta_m) \end{Bmatrix} = 0 \tag{15b}$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Sin(\alpha x_1)Sin(\beta x_2)\begin{Bmatrix} (a_4\alpha^3 + f_8\alpha\beta^2 + a_1\alpha)U_{mn} \\ +(f_8\alpha^2\beta + a_8\beta^3 + a_6\beta)V_{mn} \\ +(f_{15} - f_{14}\beta^4 + f_{12}\alpha^2\beta^2 - f_{11}\alpha^4 \\ -f_7\beta^2 - f_6\alpha^2)W_{mn} \\ +(-f_4\alpha + f_9\alpha^3 + f_{10}\alpha\beta^2)X_{mn} \\ -(f_5\beta - f_{10}\alpha^2\beta - f_{13}\beta^3)Y_{mn} \\ +f_8\alpha\beta(\overline{e_n}\gamma_m + \overline{f_n}\delta_m) \end{Bmatrix} = -q \tag{15c}$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Cos(\alpha x_1)Sin(\beta x_2)\begin{Bmatrix} -(a_2\alpha^2 + a_9\beta^2)U_{mn} \\ -e_1\alpha\beta V_{mn} \\ -(e_2\alpha - e_6\alpha^3 - e_7\alpha\beta^2)W_{mn} \\ -(e_3\alpha^2 + e_5\beta^2 - e_8)X_{mn} \\ -e_4\alpha\beta Y_{mn} \\ -e_1\beta(\overline{e_n}\gamma_m + \overline{f_n}\delta_m) \end{Bmatrix} = 0 \tag{15d}$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Sin(\alpha x_1)Cos(\beta x_2)\left\{\begin{array}{l} -e_1\alpha\beta U_{mn} \\ -(a_9\alpha^2+a_7\beta^2)V_{mn} \\ +(e_{12}\beta-e_7\alpha^2\beta-e_{10}\beta^3)W_{mn} \\ -e_4\alpha\beta X_{mn} \\ -(e_5\alpha^2+e_9\beta^2-e_{11})Y_{mn} \\ -a_9\alpha(\overline{e_n}\gamma_m+\overline{f_n}\delta_m) \end{array}\right\}=0 \tag{15e}$$

$$\sum_{n=1}^{\infty} Sin(\beta x_2)\left\{-A_{66}\beta^2 U_{0n}-a_9\beta^2 X_{0n}-\frac{1}{2}f_1\overline{e_n}\beta\right\}=0 \tag{15f}$$

$$\sum_{m=1}^{\infty} Sin(\alpha x_1)\left\{-A_{66}\alpha^2 V_{m0}-a_9\alpha^2 Y_{m0}-\frac{1}{2}A_{66}(\overline{e_0}\gamma_m+\overline{f_0}\delta_m)\right\}=0 \tag{15g}$$

$$\sum_{n=1}^{\infty} Sin(\beta x_2)\left\{-a_9\beta^2 U_{0n}+(e_8-e_5\beta^2)X_{0n}-\frac{1}{2}e_1\overline{e_n}\beta\right\}=0 \tag{15h}$$

$$\sum_{m=1}^{\infty} Sin(\alpha x_1)\left\{-a_9\alpha^2 V_{m0}+(e_{11}-e_5\alpha^2)Y_{m0}-\frac{1}{2}a_9\alpha(\overline{e_0}\gamma_m+\overline{f_0}\delta_m)\right\}=0 \tag{15i}$$

The remaining equations are supplied by the geometric and natural boundary conditions. Satisfying the geometric boundary conditions given by Eqs. (7c) such that u_1 should vanish at the edges, x_2=0,b and equating the the coefficients yield the following algebraic equations:

$$\sum_{m=1}^{\infty}\delta_m U_{mn}=0 \tag{16a}$$

$$U_{0n}+\sum_{m=1}^{\infty}\gamma_m U_{mn}=0 \tag{16b}$$

The rest of the equations are obtained from satisfying the natural boundary conditions. Satisfying $N_6=0$ at these edges ($x_1=0,a$) yield the following equations:

$$\sum_{n=1}^{\infty} Cos(\beta x_2) \begin{Bmatrix} \frac{1}{2} A_{66}\overline{e_n} + \sum_{m=1}^{\infty} A_{66}\alpha V_{mn}\overline{H} + \sum_{m=1}^{\infty} A_{66}(\overline{e_n}\gamma_m + \overline{f_n}\delta_m)\overline{H} \\ + A_{66}\beta U_{0n} + \sum_{m=1}^{\infty} A_{66}\beta U_{mn}\overline{H} + a_{11}\beta X_{0n} \\ + \sum_{m=1}^{\infty} a_{11}\beta X_{mn}\overline{H} + \sum_{m=1}^{\infty} a_{11}\alpha Y_{mn}\overline{H} - \sum_{m=1}^{\infty} a_{12}\alpha\beta W_{mn}\overline{H} \end{Bmatrix} = 0 \quad (17a)$$

$$\sum_{m=1}^{\infty} \left\{ A_{66}\alpha V_{m0}\overline{H} + \frac{A_{66}}{2}(\overline{e_0}\gamma_m + \overline{f_0}\delta_m) + a_{11}\alpha Y_{m0}\overline{H} \right\} + \frac{A_{66}}{4}\overline{e_0} = 0 \quad (17b)$$

In which $\overline{H}=1$ and $\overline{H}=(-1)^m$, represents the condition of $N_6=0$, at the edges $x_1=0,a$ respectively. Other constants have been defined at [17-22].

6. CONCLUSION

A heretofore unavailable boundary discontinuous Fourier solution to the problem of deformation of a finite dimensional general cross-ply thick rectangular shell is presented. Unlike the conventional Navier and Levy type approaches which can provide only particular solutions, the present method is general enough to provide the complete (particular as well as complementary) solution for any arbitrary combination of admissible boundary conditions with relative ease [22]. SS3-SS1 discontinuity has been solved analytically, therefore researchers shall complete analytic modeling and computer aided numeric solution.

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