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A BRIEF SURVEY ON THE DEFLECTION EFFECTS OF COMPOSITE CHARACTERISTICS

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Abstract

This study presents loading effects of composite characteristics which are generalized by ply orientation, thickness, radius of shell curve etc. The solution methodology is based on Higher Order Shear Deformation Theory (HSDT) for doubly curved, moderately thick, laminated shells which have symmetric and asymmetric ply orientation, under simply supported boundary conditions.

KOMPOZİT ÖZELLİKLERİNİN EĞİLMEYE ETKİLERİNİN İNCELENMESİ

Özetçe

Bu çalışmada, kat dizilişi, kalınlık, eğrilik yarıçapı olarak genellenebilecek kompozit özelliklerinin yükleme koşullarına etkileri incelenmiştir. Çözüm metodu; simetrik ve asimetrik kat dizilimlerine sahip, basit mesnetlenmiş sınır şartları altında, çift eğrilikli, kalın lamine kabuk için Yüksek Mertebeden Kayma deformasyon Teorisine dayanır.

Keywords: Ply-orientation, Thick shells, Laminated composites, HSDT, doubly curved shell
Anahtar Sözcükler : Kat oryantasyonu, Kalın kabuklar, Lamine kompozitler, HSDT, çift eğrilikli kabuk.

1. INTRODUCTION

Modern composites have created a revolution in high performance structures. Their advantages relative to conventional materials such as high strength to weight and stiffness to weight ratios, superior resistance to environmental conditions, design flexibility also known as tailoring the material for desired application, make them attractive for a wide range of applications in marine, chemical, aerospace, automotive industries and for the applications related to medical and sporting goods [1, 2]. Especially, the recent development in the military ships shows that composite structures can be used to increase the operational performance to reduce maintenance and fuel consumption costs [3].

Laminated composite structures are made up of two or more layers of materials bonded together to form a new material. The properties of the laminate can be tailored for a desired application. However, the analysis of composite laminates brings additional difficulties to the analyst such as the inter-laminar or transverse shear stress due to mismatch of material properties among layers, bending-stretching coupling due to asymmetry of lamination, and in-plane orthotropy. Extra complexities arise by the necessity of the satisfaction of the prescribed boundary conditions. Therefore, all these advancements and design requirements place a premium on an in-depth understanding of the response characteristics of such structural components.

The structural analysis of laminated composite plates is performed generally by approximate numerical methods, such as finite element methods (FEM), boundary element methods (BEM), and more recently developed meshless Petrov-Galerkin methods. Derivation of analytical (e.g., Fourier series) solutions for the problems of laminated plates fabricated with such advanced composite materials as graphite/epoxy, Kevlar/epoxy, boron/epoxy, graphite/PEEK, etc., is, however, fraught with many complexities as briefly mentioned above.

Investigations of laminated composite plates usually utilize either the classical lamination theory (CLT) [4-8] or the first order shear deformation theory (FSDT) [9-14]. More accurate theories such as higher order theories (HSDT) assume quadratic, cubic or higher variations of surface-parallel displacements through the entire thickness of the laminates to model the behavior of the structure for thick to thin regions. Analytical solutions utilizing the double Fourier series approach to solve the problems of laminated plates and shells are first considered by Hobson [15] and Green [16] for the solution to the problem of a clamped isotropic plate. Green and Hearmon [17] extended this approach to solve the problems of symmetrically laminated thin isotropic plates with simply supported boundary conditions. Whitney [7] extended this method to cross-ply and angle-ply laminates with clamped boundary conditions.

The use of laminated curved panels is common in many engineering fields. The single most important factor to commercial and military aircraft designers alike is the design flexibility inherent in composite laminates, known as tailoring, which is essentially exploiting the possibility of obtaining optimum design through a combination of structural/material concepts, stacking sequence, ply orientation, choice of component phases, etc., to meet specific design requirements [18].

Tailoring process requires many variations in the material properties of the composite lamina. These variations are more important than the conventional materials as there are large numbers of parameters. In the present study; a few of them will be investigated such as the effect of ply orientation, the effect of geometric form which means radius of the shell, however it should be noted that the material properties are deterministic thus they may be cause to ignore such variations.

2. STATEMENT OF THE PROBLEM

The laminated plate, composed of finite number of orthotropic layers of uniform thickness of h is shown in Figure 1,

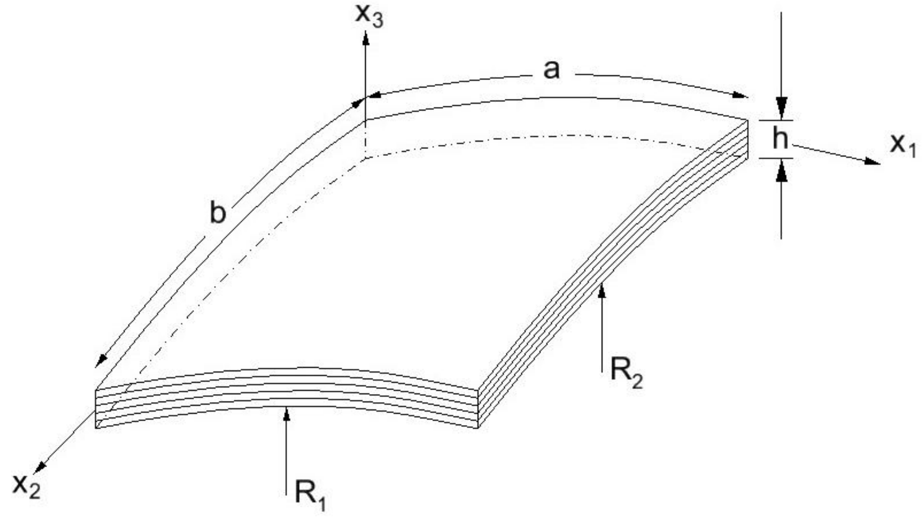

Figure 1. Geometry of a laminated plate.

where; a and b are the dimensions of the shell, R_1 and R_2 are the curve radius at x (represented by x_1) and y (represented by x_2) axes respectively.

A third-order displacement field is considered by expanding the in-plane displacements (u_0, v_0, w_0) as cubic functions of the thickness coordinate, $x_3 = z$.

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) + z\phi_x(x,y,t) - \frac{4}{3h^2} z^3 \left(\phi_x + \frac{\partial w_0}{\partial x} \right) \\
v(x,y,z,t) &= v_0(x,y,t) + z\phi_y(x,y,t) - \frac{4}{3h^2} z^3 \left(\phi_y + \frac{\partial w_0}{\partial y} \right) \\
w(x,y,z,t) &= w_0(x,y,t)
\end{aligned}
\tag{1}
$$

where, u_0, v_0 and w_0 represents displacements of a point at the mid-surface ($z = 0$), while ϕ_x and ϕ_y are rotations about x_2 and x_1 axes, respectively. The details of the strain-displacement relations, and other explanations are given in Reddy [19] and for the sake of brevity, they are not repeated here.

The equilibrium equations derived using the principle of virtual work are given as follows:

$$\frac{\partial N_1}{\partial x}+\frac{\partial N_6}{\partial y}=\bar{I}_1\dot{u}_0+\bar{I}_2\dot{\phi}_1-I_3\frac{\partial \dot{w}_0}{\partial x}$$

$$\frac{\partial N_6}{\partial x}+\frac{\partial N_2}{\partial y}=\bar{I}_1'\dot{v}_0+\bar{I}_2'\dot{\phi}_2-I_3'\frac{\partial \ddot{w}_0}{\partial x}$$

$$\frac{\partial Q_1}{\partial x}+\frac{\partial Q_2}{\partial y}-\frac{4}{h^2}\left(\frac{\partial K_1}{\partial x}+\frac{\partial K_2}{\partial y}\right)+\frac{4}{3h^2}\left(\frac{\partial^2 P_1}{\partial x^2}+\frac{\partial^2 P_2}{\partial y^2}+2\frac{\partial^2 P_6}{\partial x\partial y}\right)$$
$$-\frac{N_1}{R_1}-\frac{N_2}{R_2}=\bar{I}_3\frac{\partial \dot{u}}{\partial x}+\bar{I}_5\frac{\partial \phi_1}{\partial x}+\bar{I}_3'\frac{\partial \dot{v}_0}{\partial y}+\bar{I}_5'\frac{\partial \phi_2}{\partial y}-I_1\dot{w}-I_7\frac{\partial^2 \dot{w}}{\partial x^2}-I_7\frac{\partial^2 \dot{w}}{\partial y^2}-q$$

$$\frac{\partial M_1}{\partial x}+\frac{\partial M_6}{\partial y}-Q_1+\frac{4}{h^2}K_1-\frac{4}{3h^2}\left(\frac{\partial P_1}{\partial x}+\frac{\partial P_6}{\partial y}\right)=\bar{I}_2\dot{u}_0+\bar{I}_4\dot{\phi}_1-\bar{I}_5\frac{\partial \dot{w}}{\partial x}$$

$$\frac{\partial M_6}{\partial x}+\frac{\partial M_2}{\partial y}-Q_2+\frac{4}{h^2}K_2-\frac{4}{3h^2}\left(\frac{\partial P_6}{\partial x}+\frac{\partial P_2}{\partial y}\right)=\bar{I}_2'\dot{v}_0+\bar{I}_4'\dot{\phi}_2-\bar{I}_5\frac{\partial \dot{w}}{\partial y} \tag{2}$$

where q is the distributed transverse load, and Ni, M_i, P_i, $i = 1, 2, 6,$ denote stress resultants, stress couples, and second stress couples (resultants of the second moment of stress) (see, e.g., Reddy [19]). Q_i, $i = 4, 5$ represents the transverse shear stress resultants. They are given as follows:

$$N_i = A_{ij}\varepsilon_j^0 + B_{ij}\kappa_j^0 + E_{ij}\kappa_j^2, \tag{3a}$$

$$M_i = B_{ij}\varepsilon_j^0 + D_{ij}\kappa_j^0 + F_{ij}\kappa_j^2, \tag{3b}$$

$$P_i = E_{ij}\varepsilon_j^0 + F_{ij}\kappa_j^0 + H_{ij}\kappa_j^2, \ (i,j=1,2,6) \tag{3c}$$

$$Q_1 = A_{5j}\varepsilon_j^0 + D_{5j}\kappa_j^1, \tag{4a}$$

$$Q_2 = A_{4j}\varepsilon_j^0 + D_{4j}\kappa_j^1, \tag{4b}$$

$$K_1 = D_{5j}\varepsilon_j^0 + F_{5j}\kappa_j^1, \tag{4c}$$

$$K_2 = D_{4j}\varepsilon_j^0 + F_{4j}\kappa_j^1. \ (j=4,5) \tag{4d}$$

in which A_{ij}, B_{ij}, etc. are the laminate rigidities (integrated stiffnesses). These are given as follows:

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{N} \int_{\xi_{k-1}}^{\xi_k} Q_{ij}^{(k)}(1, \xi, \xi^2) d\xi, \tag{5a}$$

$$(E_{ij}, F_{ij}, H_{ij}) = \sum_{k=1}^{N} \int_{\xi_{k-1}}^{\xi_k} Q_{ij}^{(k)}(\xi^3, \xi^4, \xi^6) d\xi. \tag{5b}$$

Introduction of Eqs. (3 and 4) into Eqs. (2) gives five highly coupled fourth-order partial differential equations. The set of equations can be expressed in the following form:

$$K_{ij}x_j = f_i \ (i,j=1,...,5) \text{ and } (K_{ij}=K_{ji}) \tag{6a}$$

where;

$$\{x_j\}^T = \{u \quad v \quad w \quad \phi_1 \quad \phi_2\} \tag{6b}$$

$$\{f_j\}^T = \{0 \quad 0 \quad -q \quad 0 \quad 0\} \tag{6c}$$

and $[K_{ij}]$ are given in Appendix A.

3. NUMERICAL RESULTS

Numerical results are presented for [0°/90°/0°/90°/0°/90°] anti-symmetric and [90°/0°/90°/0°/0°/90°/0°/90°] symmetric cross-ply square (a=b) plates which are subjected to uniformly distributed load. The following material properties are assumed:

$E_1 = 25\ Gpa$	$\dfrac{E_1}{E_2} = 25$	$G_{12} = G_{13} = 0,5\ E_2$
$\upsilon_{12} = \upsilon_{13} = 0.25$		$G_{23} = 0,2\ E_2$

Here E_1 and E_2 are the in-plane Young's moduli in x_1 and x_2 coordinate directions, respectively, while G_{12} denotes in-plane shear modulus. G_{13} and G_{23} are transverse shear moduli in the x_1-x_3 and x_2-x_3 planes, respectively, while v_{12} is major Poisson's ratio on the x_1-x_2 plane.

Reddy [20] has defined the solution of a simply supported shell by Third Order Shear Deformation Theory for symmetric and anti-symmetric cross-ply laminates. The following tables indicate the accuracy of the present algorithm by means of dimensionless center deflections $\overline{w} = \left(\dfrac{-wE_2h^3}{q_0a^4} \right) x10^3$ of cross-ply laminated shells under uniformly distributed load (Table 1 and Table 2).

R/a	[0/90/90/0] $a/h=100$		[0/90/90/0] $a/h=10$	
	Present	Ref. [20]	Present	Ref. [20]
5	1.533245	**1.5332**	10.476490	**10.476**
10	3.719525	**3.7195**	10.904987	**10.904**
20	5.666076	**5.666**	11.017479	**11.017**
50	6.623414	**6.6234**	11.049382	**11.049**
Plate	6.842756	**6.8427**	11.055479	**11.053**

Table 1: Comparison of results for symmetric lamination.

R/a	[0/90/0] $a/h=100$		[0/90/0] $a/h=10$	
	Present	Ref. [20]	Present	Ref. [20]
5	1.509200	**1.5092**	10.332855	**10.332**
10	3.642694	**3.6426**	10.752603	**10.752**
20	5.550371	**5.5503**	10.862765	**10.862**
50	6.489541	**6.4895**	10.894004	**10.893**
Plate	6.704723	**6.7047**	10.899974	**10.899**

Table 2: Comparison of results for anti-symmetric lamination.

Maximum error percentage is found %0.002. Therefore, the present algorithm shall be suitable to define geometrical effects to the deflection of laminated shell. Same material properties are used at different ply thicknesses to define deflection impression. Hereafter, the theory defined by Reddy [20] shall be used to examine the effects of ply-orientation and curve radius to deformation at the symmetric and anti-symmetric laminated shells.

Ply Orientations	R/a	5	10	20	50	100	Plate
[0/90/0/90/0/90] anti-symmetric	$a/h = 100$	1.5698	3.7798	5.7306	6.6869	6.8497	6.9056
	$a/h = 10$	9.5144	9.4078	9.4908	9.5144	9.5177	9.5188
[90/0/90/0/0/90/0/90 symmetric	$a/h = 100$	1.5695	3.7763	5.7189	6.6688	6.8304	6.8860
	$a/h = 10$	9.0396	9.3542	9.4361	9.4593	9.4626	9.4637

Table 3: Results for different ply orientations.

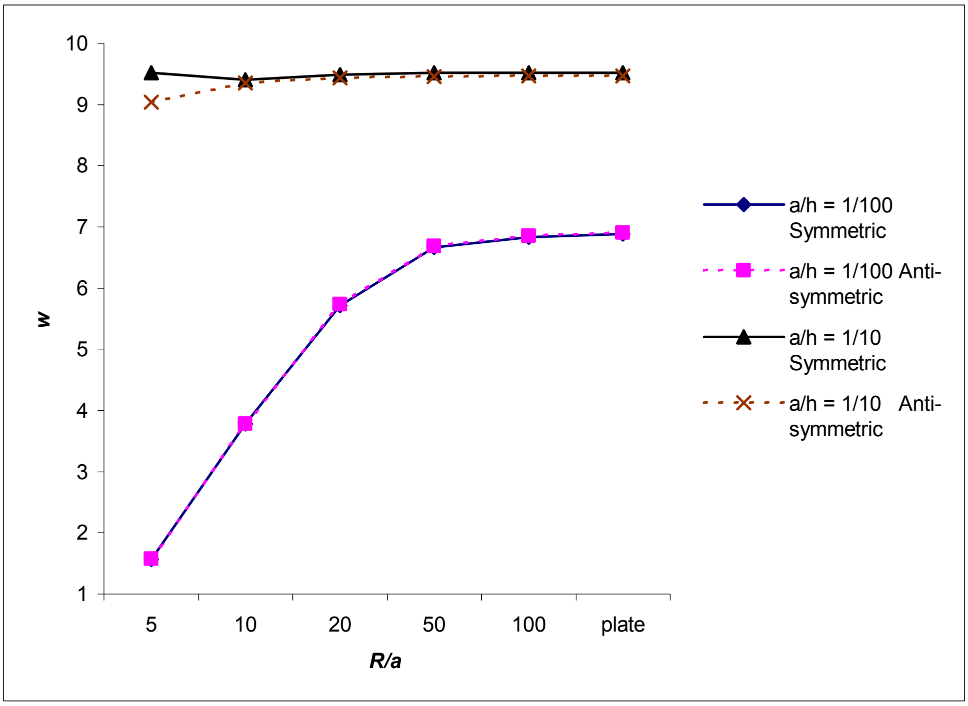

Figure 1: Effect of R/a ratio.

Figure 1 defines the variation of deflection according to ply-orientations, which are given at Table 3. It shows that the change in the ply orientation has a minimal effect in the deflection. However the change in the geometric form which changes the R/a ratio, has an average effect on the dimensionless center deflection (w). In the thick regime ($a/h = 10$), deflection has minimal variations.

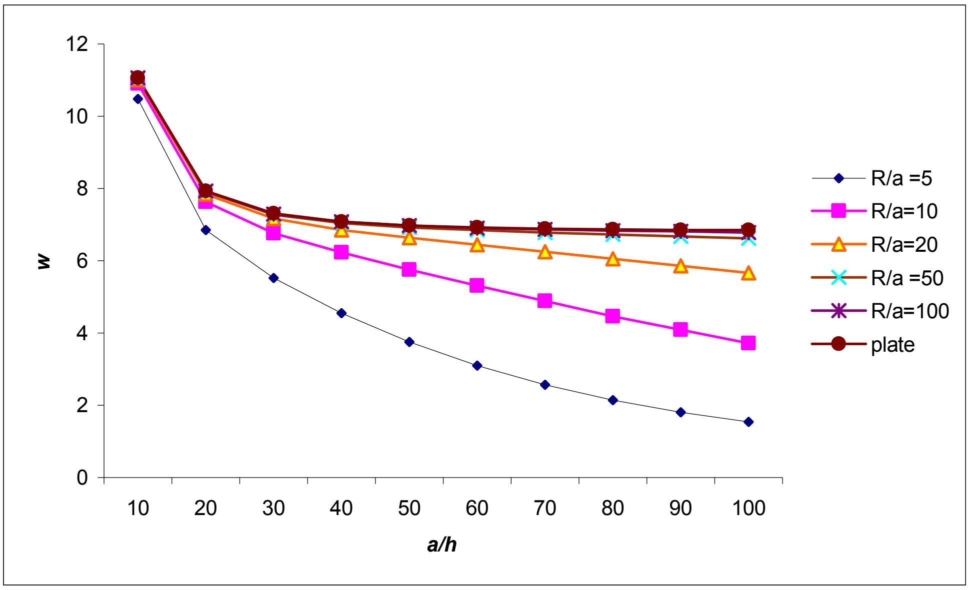

Figure 2: Effect of thickness according to curvature.

Figure 2 defines the variation of deflection according to thickness. It shows that the change in the curvature has a remarkable effect in the deflection in the thin regime (*a/h>20*).

4. CONCLUSION

A higher-order theory based analytical solution to the problem of symmetric and antisymmetric cross-ply shells with the simply supported boundary condition prescribed at all four edges which completely defines Navier solution, is presented.

A system of five highly coupled linear partial differential equations, which are generated from Third Order Shear Deformation Theory, is solved for a variation of geometric form parameter in both thick and thin regime. Finally, following results are gained;

1. The accurate changes are observed at the change of *R/a* in thin regime without any changes in material properties such as Young's modules or Poisson's ratios.

2. It can be clearly seen that the effect of the ply thickness (h) is more pronounced in the thin laminate regime. In the moderately thick laminate regimes, this effect is compensated by shear deformation effect.

3. It should be noted that this study considers the geometric effects, however material properties are deterministic for deflection, thus they may be cause to ignore geometric variations.

Appendix A. Constant Definitions of $[K_{ij}]$

$$K_{11} = A_{11}\frac{\partial^2}{\partial x^2} + A_{66}\frac{\partial^2}{\partial y^2} \tag{A.1}$$

$$K_{12} = \left(A_{12} + A_{66}\right)\frac{\partial^2}{\partial x \partial y} \tag{A.2}$$

$$K_{13} = \left(\frac{A_{11}}{R_1} + \frac{A_{12}}{R_2}\right)\frac{\partial}{\partial x} - c_1 E_{11}\frac{\partial^3}{\partial x^3} - (2c_1 E_{66} + c_1 E_{12})\frac{\partial^3}{\partial x \partial y^2} \tag{A.3}$$

$$K_{14} = \left(B_{11} - c_1 E_{11}\right)\frac{\partial^2}{\partial x^2} + \left(B_{66} - c_1 E_{66}\right)\frac{\partial^2}{\partial y^2} \tag{A.4}$$

$$K_{15} = \left(B_{12} - c_1 E_{12} + B_{66} - c_1 E_{66}\right)\frac{\partial^2}{\partial x \partial y} \tag{A.5}$$

$$K_{22} = A_{66}\frac{\partial^2}{\partial x^2} + A_{22}\frac{\partial^2}{\partial y^2} \tag{A.6}$$

$$K_{23} = \left(\frac{A_{12}}{R_1} + \frac{A_{22}}{R_2}\right)\frac{\partial}{\partial y} - c_1 E_{22}\frac{\partial^3}{\partial y^3} - (2c_1 E_{66} + c_1 E_{12})\frac{\partial^3}{\partial x^2 \partial y} \tag{A.7}$$

$$K_{24} = \left(B_{12} - c_1 E_{12} + B_{66} - c_1 E_{66}\right)\frac{\partial^2}{\partial x \partial y} \tag{A.8}$$

$$K_{25} = \left(B_{66} - c_1 E_{66}\right)\frac{\partial^2}{\partial x^2} + \left(B_{22} - c_1 E_{22}\right)\frac{\partial^2}{\partial y^2} \tag{A.9}$$

$$K_{33}=\left[A_{55}-6c_1D_{55}+9c_1^2F_{55}+c_1(\frac{E_{12}}{R_1}+\frac{E_{22}}{R_2})+c_1(\frac{E_{11}}{R_1}+\frac{E_{12}}{R_2})\right]\frac{\partial^2}{\partial x^2}$$
$$+\left[A_{44}-6c_1D_{44}+9c_1^2F_{44}+2c_1(\frac{E_{12}}{R_1}+\frac{E_{22}}{R_2})\right]\frac{\partial^2}{\partial y^2}$$
$$-9c_1^2H_{11}\frac{\partial^4}{\partial x^4}-2c_1^2\left(H_{12}+2H_{66}\right)\frac{\partial^4}{\partial x^2\partial y^2} \tag{A.10}$$
$$-c_1^2H_{22}\frac{\partial^4}{\partial y^4}-\left[\left(\frac{A_{11}}{R_1^2}+\frac{A_{12}}{R_1R_2}\right)+\left(\frac{A_{12}}{R_1R_2}+\frac{A_{22}}{R_2^2}\right)\right]$$

$$K_{34}=\left[A_{55}-6c_1D_{55}+9c_1^2F_{55}-\frac{1}{R_1}\left(B_{11}-c_1E_{11}\right)-\frac{1}{R_2}\left(B_{12}-c_1E_{12}\right)\right]\frac{\partial}{\partial x}$$
$$+c_1\left(F_{11}-c_1H_{11}\right)\frac{\partial^3}{\partial x^3}+\left[c_1\left(F_{12}-c_1H_{12}\right)+2c_1\left(F_{66}-c_1H_{66}\right)\right]\frac{\partial^3}{\partial x\partial y^2} \tag{A.11}$$

$$K_{35}=\left[A_{44}-6c_1D_{44}+9c_1^2F_{44}-\frac{1}{R_1}\left(B_{12}-c_1E_{12}\right)-\frac{1}{R_2}\left(B_{22}-c_1E_{22}\right)\right]\frac{\partial}{\partial y}$$
$$+c_1\left(F_{22}-c_1H_{22}\right)\frac{\partial^3}{\partial y^3}+\left[c_1\left(F_{12}-c_1H_{12}\right)+2c_1\left(F_{66}-c_1H_{66}\right)\right]\frac{\partial^3}{\partial x^2\partial y} \tag{A.12}$$

$$K_{44}=\left[D_{11}-2c_1F_{11}+c_1^2H_{11}\right]\frac{\partial^2}{\partial x^2}+\left[D_{66}-2c_1F_{66}+c_1^2H_{66}\right]\frac{\partial^2}{\partial y^2}$$
$$-(A_{55}+6c_1D_{55}+9c_1^2F_{55}) \tag{A.13}$$

$$K_{45}=\left[D_{12}-c_1F_{12}+D_{66}-c_1F_{66}-c_1\left(F_{12}-c_1H_{12}\right)-c_1\left(F_{66}-c_1H_{66}\right)\right]\frac{\partial^2}{\partial x\partial y} \tag{A.14}$$

$$K_{55} = \left[D_{66} - 2c_1 F_{66} + c_1^2 H_{66} \right] \frac{\partial^2}{\partial x^2} + \left[D_{22} - 2c_1 F_{22} + c_1^2 H_{22} \right] \frac{\partial^2}{\partial y^2} \\ - A_{44} + 3c_1 D_{44} + 3c_1 \left(D_{44} - 3c_1 F_{44} \right) \tag{A.15}$$

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