BŞEÜ Fen Bilimleri Dergisi
7(2), 952-959, 2020

BSEU Journal of Science
DOI: 10.35193/bseufbd.722419

e-ISSN:2458-7575 (http://dergipark.gov.tr/bseufbd)

Araştırma Makalesi-Research Article

Atangana-Baleanu Caputo Anlamında Üçüncü Mertebeden Kesirli Türevli Diferansiyel Denklemler için Implicit Rather Fark Metodu

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Geliş / Received: 17/04/2020 ***Revize / Revised: 07/09/2020*** ***Kabul / Accepted: 30/09/2020***

ÖZ

Atangana-Baleanu Caputo (ABC) türevi ile tanımlı üçüncü mertebeden kesirli kısmi diferansiyel denklemin tam çözümü başlangıç ve sınır değerlerine bağlı olarak hesaplandı. Bu denklem için kararlılık kestirimleri verildi. Bu denklem Implicit Rather fark metodu ile çözüldü. Problem için fark şemalarının kararlılığı gösterildi. Bu teknik ABC üçüncü mertebeden kısmi diferansiyel denklemin $\boldsymbol{\alpha = 0.001, 0.1, 0.5, 0.99, 0.999}$ için kesirli türev değerlerine karşılık uygulanmıştır. Yaklaşık çözüm, tekniğin doğruluğunu ve etkinliğini onaylar**.**

Anahtar Kelimeler- Kesirli Diferansiyel Denklemi, Implicit Rather Fark Şeması, Kararlılık Kestirimi, Yaklaşık Çözüm, Tam Çözüm

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Implicit Rather Difference Method for Third Order Differential Equations in the Sense of Atangana-Baleanu Caputo Fractional Derivative

ABSTRACT

The exact solution of the third order partial differential equation defined by Atangana-Baleanu Caputo (ABC) fractional derivative is calculated for depending on the initial and boundary values. Stability estimates are obtained for this equation. Implicit Rather difference schemes are constructed for this problem. The stability of difference schemes for this problem is presented. This technique has been applied by ABC fractional orders $\alpha = 0.001, 0.1, 0.5, 0.99, 0.999$. Approximation solution confirms the accuracy and effectiveness of the technique.

Keywords- Fractional Differential Equation, Implicit Rather Difference Schemes, Stability Estimates, Approximation Solution, Exact Solution.

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I.INTRODUCTION

Fractional differential equations have several implementations in finance, engineering, physic, and seismology [1-3]. It is developed because of its applications in different areas of geophysics, biology, chemical and petroleum industries [4]. Michele Caputo and Mauro Fabrizio presented a fractional derivative operator based on the exponential function to overcome the problem of the singular kernel in 2015 [5]. But their fractional derivative operator does not have singular kernel [6, 7]. Atangana and Alqahtani applied the concept of Caputo Fabrizio (CF) fractional derivatives to the equation of ground water pollution [8]. CF fractional derivative, as the kernel in integral was non-singular but was still non-local. To eliminate singularity and non-locality, Atangana and Baleanu identified a new derivative with the help of Mittag-Leffler function [9, 10]. However, Atangana et al. provided the numerical approximation to the fractional Advection-Diffusion equation whose fractional derivatives are Atangana- Baleanu derivative of Rieman- Liouville type [11]. In many similar current life problems, the ABC derivative has been used. Modanli has studied two different numerical method for the fractional telegraph differential equation [12]. The Atangana- Baleanu derivative has been applied in modeling of many real problems [13, 14].

In this paper, the impilicit rather difference method is used for the third order partial differential equation defined by ABC fractional derivative. The stability estimates are showed by the Von-Neuman's method.

Now, we examine the following fractional differential equation defined by ABC derivative

$$\begin{cases} \frac{\partial^3 u(t,x)}{\partial t^3} + m\frac{\partial^2 u(t,x)}{\partial t^2} + {}^{ABC}_{0}D^{\alpha}_{t}u(t,x) - n\frac{\partial^2 u(t,x)}{\partial x^2} - p\frac{\partial u(t,x)}{\partial x} + u(t,x) = f(t,x) \\ 0 < x < L,\ \ 0 < t < T,\ \ 0 < \alpha \leq 1, \\ u(0,x) = h_1(x),\ \ u_t(0,x) = h_2(x),\ \ u_{ttt}(0,x) = h_3(x),\ \ 0 \leq t \leq T, \\ u(t,X_L) = r_1(t),\ \ u(t,X_R) = r_2(t),\ \ X_L < x < X_R, \\ m > 0,\ \ n > 0. \end{cases} \tag{1}$$

Here, h_1, h_2, h_3, r_1, r_2 and f are known functions, $u(t,x)$ is unknown function. For $\alpha = 1$, the problem (1) was called as the third order linear time-varying systems model [13]. For more details see [15-22].

Now, we present some basic definitions and properties of fractional calculus theory for third order differential equation defined by fractional ABC derivative.

Definition 1: Let$f \in H^1(a,b), b > a,\ \alpha \in [0,1]$, then the definition of the new fractional derivative (Atangana-Baleanu derivative in Caputo sense) is given as [13]:

$${}^{ABC}_{\ a}D^{\alpha}_{t}u(t) = \frac{B(\alpha)}{1-\alpha}\int_0^t f'(x)E_{\alpha}[-\alpha\frac{(t-x)^{\alpha}}{1-\alpha}]dx. \tag{2}$$

Where$B(\alpha) = 1 - \alpha + \frac{\alpha}{\Gamma(\alpha)}$ and E_{α} is the Mittag-Leffler function. We defined this function as following:

$$E_{\alpha}\left[-\alpha\frac{(t-x)^{\alpha}}{1-\alpha}\right] = \sum_{i=0}^{\infty}\frac{\left(-\alpha\frac{(t-x)^{\alpha})}{(1-\alpha)}\right)^{k}}{\Gamma(\alpha k + 1)}.$$

Definition 2: Atangana- Baleanu obtained the Laplace transform for the Eq. 1 as [13]:

$$£\{{}^{ABC}_{\ a}D^{\alpha}_{t}(u(t))\} = \frac{B(\alpha)}{1-\alpha}\frac{u(s)s^{\alpha} - s^{\alpha-1}u(0)}{s^{\alpha} + \frac{\alpha}{1-\alpha}}. \tag{3}$$

Next section, we shall calculate the exact solution by the Laplace transform method of the third order partial differential equation defined by ABC fractional derivative operator.

II.EXACT SOLUTION OF THE THIRD ORDER DIFFERENTIAL EQUATION IN SENSE OF ABC FRACTIONAL DERIVATIVE

In this section, we shall find the exact solution of the problem (1) by Laplace transform method. From the problem (1), taking $p=0$, we have the following problem

$$\begin{cases} \frac{\partial^3 u(t,x)}{\partial t^3} + m\frac{\partial^2 u(t,x)}{\partial t^2} + {}^{ABC}_{0}D^{\alpha}_{t} u(t,x) - n\frac{\partial^2 u(t,x)}{\partial x^2} + u(t,x) = f(t,x), \\ 0 < x < L,\ \ 0 < t < T,\ \ 0 < \alpha \le 1, \\ u(0,x) = h_1(x),\ \ u_t(0,x) = h_2(x),\ \ u_{ttt}(0,x) = h_3(x),\ X_L \le x \le X_R, \\ u(t,X_L) = r_1(t),\ \ u(t,X_R) = r_2(t),\ \ 0 \le t \le T, \\ m > 0,\ \ n > 0. \end{cases} \tag{4}$$

The problem (4) was studied in the [14]. In this study, we shall the problem (1). Using the operator method, the exact solution of the problem (1) can be found similar as [14]. For the exact solution of the problem (1), we give one test example problem.

Example 1: We consider the following problem for the special values $m = n = p = 1$

$$\begin{cases} \frac{\partial^3 u(t,x)}{\partial t^3} + m\frac{\partial^2 u(t,x)}{\partial t^2} + {}^{ABC}_{0}D^{\alpha}_{t} u(t,x) - n\frac{\partial^2 u(t,x)}{\partial x^2} + u(t,x) = f(t,x), \\ f(t,x) = (\frac{6(1-\alpha)}{\beta(\alpha)} + \frac{6t^{\alpha}}{\beta(\alpha)\Gamma(\alpha)} + \frac{6t(1-\alpha)}{\beta(\alpha)} + \frac{6t^{\alpha+1}}{(\alpha+1)\Gamma(\alpha)} + t^3 \\ + \frac{1-\alpha}{\beta(\alpha)}t^3 + \frac{6\alpha}{\Gamma(\alpha+4)\beta(\alpha)}t^{\alpha+3})(x - x^2) + 2\left(\frac{1-\alpha}{\beta(\alpha)}t^3 + \frac{6\alpha}{\Gamma(\alpha+4)\beta(\alpha)}\right) \\ + \left(\frac{1-\alpha}{\beta(\alpha)}t^3 + \frac{6\alpha}{\Gamma(\alpha+4)\beta(\alpha)}t^{\alpha+3}\right)(1-2x), \\ u(0,x) = u_t(0,x) = u_{ttt}(0,x) = 0,\ \ 0 \le x \le 1, \\ u(t,0) = \ u(t,1) = 0,\ \ 0 \le t \le 1. \end{cases} \tag{5}$$

Eq. (2) using the initial conditions in the Eq. (5) and taking the Laplace transform to both of the Eq. (5), we obtain

$$(s^3 + s^2\frac{\beta(\alpha)}{1-\alpha}\frac{s^{\alpha}}{s^{\alpha}+\frac{\alpha}{1-\alpha}}) - \frac{\partial^2 u(s,x)}{\partial x^2} - \frac{\partial u(s,x)}{\partial x}$$

$$= \frac{6}{\beta(\alpha)}\left(\frac{6(1-\alpha)}{s^4} + \frac{\alpha+1}{s^{\alpha+1}} + \frac{1-\alpha}{s^2} + \frac{\alpha+1}{s^{\alpha+2}} + \frac{\beta(\alpha)}{s^4} + \frac{1-\alpha}{s^4} + \frac{\alpha}{s^{\alpha+4}}\right)(x - x^2)$$

$$+ \frac{6}{\beta(\alpha)}\left[\frac{2(1-\alpha)}{s^4} + \frac{2\alpha}{s^{\alpha+4}} + \left(\frac{(1-\alpha)}{s^4} + \frac{\alpha}{s^{\alpha+4}}\right)(1-2x)\right].$$

Solving this equation for homogenous and non-homogenous part as to x, using boundary value conditions, we have

$$u(s,x) = \frac{6}{s^4}\left(\frac{1-\alpha}{\beta(\alpha)} + \frac{\alpha}{s^{\alpha}\beta(\alpha)}\right)(x - x^2). \tag{6}$$

Taking the inverse Laplace transform for the Eq. (6) we get the exact solution of this example following as:

$$u(t,x) = \left(\frac{1-\alpha}{\beta(\alpha)}t^3 + \frac{6\alpha}{\Gamma(\alpha+4)\beta(\alpha)}t^{\alpha+3}\right)(x - x^2).$$

Next section, we shall construct difference scheme for the implicit rather difference scheme method. Then we prove the stability estimates for this difference scheme method.

BSEU Journal of Science
DOI: 10.35193/bseufbd.722419

e-ISSN:2458-7575 (http://dergipark.gov.tr/bseufbd)

III.STABILITY ESTIMATES FOR THE IMPLICIT RATHER DIFFERENCE SCHEME METHOD

Now, we will obtain difference schemes of implicit rather difference method for the partial differential equation 1. Let's assume that $h = \frac{x_R - x_L}{M}$ for $x -$ axis, $\tau = \frac{T}{N}$ for $t -$axisandthenwe can write

$$x_n = x_L + nh; n = 1,2,\dots M, t_k = k\tau, k = 1,2,\dots N.$$

We construct implicit rather difference method for third order partial differential equation defined by ABC derivative of the formula (1). Using Taylor series formula, we obtain the following formulas:

$$\frac{\partial^3 u(t_k,x_n)}{\partial t^3} \cong \frac{u_n^{k+2} - 3u_n^{k+1} + 3u_n^k - u_n^{k-1}}{\tau^3}, \tag{7}$$

$$\frac{\partial^2 u(t_k,x_n)}{\partial t^2} \cong \frac{u_n^{k+1} - 2u_n^k + u_n^{k-1}}{\tau^2}, \tag{8}$$

$$\frac{\partial^2 u(t_k,x_n))}{\partial x^2} \cong \frac{u_{n+1}^k - 2u_n^k + u_{n-1}^k}{h^2}, \tag{9}$$

$$\frac{\partial u(t_k,x_n)}{\partial x} \cong \frac{u_{n+1}^k - u_n^k}{h}, \tag{10}$$

and difference scheme for ${}^{ABC}_{0}D_t^{\alpha} u(t_k, x_n)$ fractional derivative was obtained in [8] as:

$${}^{ABC}_{0}D_t^{\alpha} u(t_k, x_n) \cong \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{k} \frac{u_{n+1}^k - u_n^k}{h} d_{j,k}, \tag{11}$$

where $d_{j,k} = (t_j - t_{k+1})^{1-\alpha}$.

Using the formulas (7-11), we obtain the implicit rather difference method for third order partial differential equation defined by ABC derivative of the formula (1) as:

$$\begin{cases} \frac{u_n^{k+2} - 3u_n^{k+1} + 3u_n^k - u_n^{k-1}}{\tau^3} + m\frac{u_n^{k+1} - 2u_n^k + u_n^{k-1}}{\tau^2} + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{k} \frac{u_{n+1}^k - u_n^k}{h} d_{j,k} \\ -n\frac{u_{n+1}^k - 2u_n^k + u_{n-1}^k}{h^2} - p\frac{u_{n+1}^k - u_n^k}{h} + u_n^k = f_n^k = f(t_k, x_n), \\ u_n^0 = h_1(x_n), \ \frac{u_n^1 - u_n^0}{\tau} = h_2(x_n), \ \frac{u_n^2 - 2u_n^1 + u_n^0}{\tau^2} = h_3, \ 0 \le n \le M, \\ u_0^k = u_M^k = 0, \ 0 \le k \le N, \\ m > 0, \ n > 0. \end{cases} \tag{12}$$

Using the general Von-Neumann's method as the formula $n_n^k = r^k e^{in\theta}$ for stability estimates of the formula (12), we obtain

$$\begin{cases} r^3 + \left(-3 + m\tau + \frac{\tau^2 (2\tau)^{1-\alpha}}{\Gamma(\alpha)}\right) r^2 + \left(3 - 2m\tau - \frac{\tau^2(-2\tau)^{1-\alpha}}{\Gamma(\alpha)} + \frac{2n\tau^3}{h^2} cos\theta - \frac{2\lambda\tau^3}{h^2} - \frac{p\tau^3 e^{i\theta}}{h} + 1\right) r \\ -1 + k\tau + \frac{p\tau^3}{h} - f_0^1 = 0 \\ f_0^1 = f(\tau, 0), \ \text{for } n = 0, \ k = 1. \end{cases} \tag{13}$$

Taking $f_0^1 \to 0$, we can rewrite the formula (13) the following polynomial function form

$$\begin{cases} P(r) = r^3 + \left(-3 + m\tau + \frac{\tau^2(2\tau)^{1-\alpha}}{\Gamma(\alpha)}\right) r^2 + (3 - 2m\tau - \frac{\tau^2(-2\tau)^{1-\alpha}}{\Gamma(\alpha)} + \frac{2n\tau^3}{h^2} cos\theta \\ -\frac{2\lambda\tau^3}{h^2} - \frac{p\tau^3 e^{i\theta}}{h} + 1)r - 1 + k\tau + \frac{p\tau^3}{h} = 0, \end{cases} \tag{14}$$

BSEU Journal of Science
DOI: 10.35193/bseufbd.722419

e-ISSN:2458-7575 (http://dergipark.gov.tr/bseufbd)

where

$$a_0 = -1 + k\tau + \frac{p\tau^3}{h},$$

$$a_1 = 3 - 2m\tau - \frac{\tau^2(-2\tau)^{1-\alpha}}{\Gamma(\alpha)} + \frac{2n\tau^3}{h^2}cos\theta - \frac{2\lambda\tau^3}{h^2} - \frac{p\tau^3 e^{i\theta}}{h} + 1,$$

$$a_2 = -3 + m\tau + \frac{\tau^2(2\tau)^{1-\alpha}}{\Gamma(\alpha)},$$

$$a_3 = 1.$$

From the article [14], we write the following lemma.

Lemma 3: Taking$p = \frac{3a_1 - a_2^2}{9}$ and $q = \frac{9a_1a_2 - 27a_0 - 2a_2^2}{54}$, then this formula have that the following roots

$$r_1 = \sqrt[3]{q + \sqrt{p^3 + q^2}},$$

$$r_2 = \sqrt[3]{q - \sqrt{p^3 + q^2}},$$

$$r_3 = -r_1 - r_2 - a_2.$$

When the found values a_0, a_1 and $a_2$are written instead in this lemma, we obtained $r_1 < 1$, $r_2 < 1$ and $r_3 < 1$. r_1 and r_2 are stable but r_3 is unstable.

Next section, we shall show one example by the implicit rather difference method for the approximation solution.

IV. NUMERICAL IMPLEMENTATION

In section 2, we obtained the exact solution for one test problem. In this section, we shall investigate the approximation of this example by the implicit rather difference method. We utilize a procedure of modified Gauss elimination method for difference equation (12). We obtain the maximum norm of the error of the numerical solution by:

$$\varepsilon = \max\|u(t,x) - u(t\_k, x\_n)\|, n = 0, 1, \dots M, k = 0, 1, 2, \dots N$$

where $u_n^k = u(t_k, x_n)$ is the approximate solution and $u(t,x)$ is exact solution. The error analysis for this method.

BSEU Journal of Science
DOI: 10.35193/bseufbd.722419

e-ISSN:2458-7575 (http://dergipark.gov.tr/bseufbd)

Table 1. Error Analysis

	$\tau = \frac{1}{N}, h = \frac{1}{M}$					
	The difference scheme (12)					
α	0.1	0.50	0.01	0.99	0.999	1
$N = M = 20$	0.2155	0.1860	0.2008	0.0403	0.0380	0.0378
$N = M = 80$	1.8796	1.0237	1.6434	0.0680	0.0462	0.0418
$N = 400, M = 20$	0.1493	0.1277	0.1381	0.0230	0.0216	0.0214

From the error analysis table, obtained results are satisfied stability estimates for $N = M = 20$ and $N = 400$, $M = 20$ and all α. But it is not satisfied for $N = M = 80$, $0.01 < \alpha < 0.5$. This result is due to the stability of the roots in lemma.

V. RESULTS

In this study, exact solution is obtained for by Laplace transform method for third order partial differential equation with ABC fractional derivative. First order difference schemes are presented. Stability inequalities are given for first order difference. We have utilized the implicit rather difference method to get algorithms for investigating third order partial differential equation with ABC fractional derivative. Approximate solutions are obtained by this method. MATLAB software program has been utilized for all results. Finally, the exact and the approximation solutions are compared. Obtained results showed that this approximation method is effective and good.

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