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SOFT SEPARATION AXIOMS AND SOFT PRODUCT OF SOFT TOPOLOGICAL SPACES

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Abstract. In this article, we deal with the soft separation axioms using soft points on soft topological space and discuss the characterizations and properties of them. We extend these separation axioms to the soft product of soft topological spaces. Also we provide correct examples for the wrong examples example:1, example:2 and example:3 given in article [8].

For the vagueness and uncertinity of real life problems, there are several mathematical tools such as fuzzy sets, intuitionistic fuzzy sets, rough sets, vague sets etc. There is one more mathematical tool named soft sets which was introduced by Molodsov[12] in 1999. After that it was developed and used in decision making problems by Maji et. al in [10] and [11]. Aktas and Cagman [1] introduced the applications of soft set theory in algebraic structures in 2007. Kharral and Ahmad [9] introduced and discussed several properties of soft mappings. Shabir and Naz [16] investigated soft seperation axioms defined for crisp points in 2011. Hussain and Ahmad [7] investigate the properties of soft interior, soft closure and soft boundary in 2011. Aygunoglu and Aygun [2] in 2012 generalize Alexander subbase theorem and Tychonoff theorem to the soft topological spaces by defining and using the product of soft topological spaces. Nazmul and Samanta [13] studied the neighbourhood properties of soft topological spaces in 2013. There are several articles related to the properties of soft topological spaces and soft mappings on soft topological spaces. Some of them are [4], [6], [14], [17], [19] [20], [21]. Four different types of sepereation axioms were defined and discussed in [5], [8], [16] and [18]. Singh and Noorie [17] derives the relation among these four types of $T_i,\ i = 1, 2, 3, 4$ spaces in 2017.

In the second section of this article, we give some basic definitions and preliminaries of soft topological spaces.

In the third section of this article, we deal with the soft separation axioms using soft points and discuss about the characterizations and properties of them. In fact

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these separation axioms are stronger than other separation axioms. We extend these separation axioms to the product of soft topological spaces. Also we provide correct examples for the wrong examples Example:1, Example:2 and Example:3 given in article [8]

Throughout this paper, X is the universe set, E is a set of parameters and $\mathcal{P}(X)$ is the set of all subsets of X.

1. Preliminaries

Definition 1.1. [12] A mapping $F : E \to \mathcal{P}(X)$ is called a soft set and is denoted by $(F,\ E)$. The family of all soft sets over X is denoted as $SS(X,\ E)$

Definition 1.2. [12] Let $(F,\ E)$ and $(G,\ E)$ be two soft sets over X. Then $(F,\ E)$ is a soft subset of $(G,\ E)$ written as $(F,\ E)\tilde{\subseteq}(G,\ E)$, if $F(e) \subseteq G(e)$, for all $e \in E$. Also the soft sets $(F,\ E)$ and $(G,\ E)$ are equal written as $(F,\ E)\tilde{=}(G,\ E)$, if $(F,\ E)\tilde{\subseteq}(G,\ E)$ and $(G,\ E)\tilde{\subseteq}(F,\ E)$.

Definition 1.3. [12] Let $\{(F_i,\ E) : i \in I\}\tilde{\subseteq}SS(X,\ E)$, where I is an arbitrary index set. Then

1. the soft union of $\{(F_i,\ E) : i \in I\}$ is the soft set $(F,\ E)$, where F is the mapping defined as $F(e) = \cup\{F_i(e) : i \in I\}$, for every $e \in E$ and is denoted as $(F,\ E) = \tilde{\cup}\{(F_i,\ E) : i \in I\}$.
2. the soft intersection of $\{(F_i,\ E) : i \in I\}$ is the soft set $(F,\ E)$, where F is the mapping defined as $F(e) = \cap\{F_i(e) : i \in I\}$, for every $e \in E$ and is denoted as $(F,\ E) = \tilde{\cap}\{(F_i,\ E) : i \in I\}$.

Definition 1.4. [21] Let $(F,\ E)$ be a soft set over X. Then the soft relative complement F^c of $(F,\ E)$ is the mapping from E to $\mathcal{P}(X)$ defined by $F^c(e) = X - F(e)$ for every $e \in E$ and is denoted as $(F,\ E)^c$ or $(F^c,\ E)$.

Definition 1.5. [12] Let $(F,\ E)$ be a soft set over X. Then

1. $(F,\ E)$ is called as null soft set, if $F(e) = \phi$, for every $e \in E$. We simply write it as $\tilde{\phi}$.
2. $(F,\ E)$ is called as absolute soft set, if $F(e) = X$, for every $e \in E$. We simply write it as $\tilde{X}$.

Definition 1.6. *(*[16], [21]*)* Let $\tau \subseteq SS(X,\ E)$. Then τ is a soft topology on X if it satifies the following three conditions

1. $\tilde{\phi},\ \tilde{X} \in \tau$.
2. The soft union of any number of soft sets in τ is in τ.
3. The soft intersection of finite number of soft sets in τ is in τ.

This soft topological space over X is written as $(X,\ \tau,\ E)$ and the members of τ are called as soft open sets in X. Also the soft complement of soft open sets are called soft closed sets.

Definition 1.7. [21] The soft set $(F,\ E)$ over X is called as a soft point in X, denoted by x_e, if $F(e') = \begin{cases} \{x\} & \text{if } e' = e \\ \phi & \text{if } e' \in E - \{e\} \end{cases}$

Definition 1.8. [2] Let $(X,\ \tau,\ E)$ be a soft topological space. A subcollection $\mathcal{B}$ of τ is said to be a base for τ if every member of τ can be expressed as a union of members of τ.

Definition 1.9. [2] Let $(X,\ \tau,\ E)$ be a soft topological space. A subcollection $\mathcal{S}$ of τ is said to be a subbase for τ if the family of all finite intersetions of members of $\mathcal{S}$ forms a base for τ.

Definition 1.10. [21] A soft set $(G,\ E)$ in a soft topological space $(X,\ \tau,\ E)$ is known as a soft neighbourhood of a soft set $(F,\ E)$ if there exists a soft open set $(H,\ E)$ such that $(F,\ E)\tilde{\subseteq}(H,\ E)\tilde{\subseteq}(G,\ E)$.

Definition 1.11. [16]

Let $(F,\ E)$ be a soft set in a soft topological space $(X,\ \tau,\ E)$. Then the soft closure of $(F,\ E)$ is denoted as $Cl(F,\ E)$ and defined as $Cl(F,\ E) = \tilde{\cap}\{(G,\ E) : (G,\ E)\tilde{\in}\tau^c \textit{ and } (G,\ E)\tilde{\supseteq}(F,\ E)\}$.

Definition 1.12. [16] Let Y be a nonempty soft subset of a soft topological space $(X,\ \tau,\ E)$. Then $\tau_Y = \{(F,\ E)\tilde{\cap}E_Y : (F,\ E)\tilde{\in}\tau\}$ is called a soft relative topology on Y and $(Y,\ \tau_Y, E)$ is called a soft subspace of $(X,\ \tau,\ E)$, where $E_Y : E \to \mathcal{P}(Y)$ such that $E_Y(e) = Y$, for every $e \in E$.

Proposition 1.1. [16] *Let $(Y,\ \tau_Y, E)$ be a soft subspace of a soft topological space $(X,\ \tau,\ E)$ and (F, A) be a soft set over Y. Then (F, A) is a soft open set in Y if and only if $(F,\ E) = (G,\ E)\tilde{\cap}E_Y$, for some $(G,\ E)\tilde{\in}\tau$.*

Theorem 1.2. [21] *A soft set $(F,\ E)$ is soft open set if and only if $(G,\ E)$ is a soft neighbourhood of a soft set $(F,\ E)$, for each soft set $(F,\ E)$ contained in $(G,\ E)$.*

Proposition 1.3. [16] *Let $(X,\ \tau,\ E)$ be a soft topological space over X. Then the collection $\tau_e = \{F(e) : (F,\ E)\tilde{\in}\tau\}$ defines a topology on X.*

Proposition 1.4. [16] *Let $(X,\ \tau,\ E)$ be a soft topological space over X and $Y \subseteq X$. Then $(Y,\ \tau_{Y_e})$ is a subspace of $(X,\ \tau_e)$.*

Definition 1.13. [3] Let $(F,\ E_1)\tilde{\in}\ SS(X_1,\ E_1)$ and $(G, E_2)\ \tilde{\in}SS(X_2, E_2)$. Then the cartesian product $(F,\ E_1) \times (G,\ E_2)$ is defined by $(F \times G)_{(E_1 \times E_2)}$, where $(F \times G)_{(E_1 \times E_2)}(e_{1_i},\ e_{2_j}) = F(e_{1_i}) \times G(e_{2_j}),\ \forall(e_{1_i}, e_{2_j}) \in E_1 \times E_2$.

Definition 1.14. [2] The soft mappings $(p_q)_i,\ i \in \{1,\ 2\}$ is called soft projection mappings from $X_1 \times X_2$ to X_i defined by $(p_q)_i((F,\ E)_1 \times (F,\ E)_2) = (p_q)_i((F_1 \times F_2)_{(E_1 \times E_2)}) = p_i(F_1 \times F_2)_{q_i(E_1 \times E_2)} = (F, E)_i$, where $(F,\ E)_1 \in SS(X_1,\ E_1)$, $(F,\ E)_2 \in SS(X_2,\ E_2)$ and $p_i : X_1 \times X_2 \to X_i,\ q_i : E_1 \times E_2 \to E_i$ are projection mappings in classical meaning.

Definition 1.15. [2] Let $\{(\phi_\psi)_i : S(X,\ E) \to (Y_i,\ \tau_i)\}_{i \in \Delta}$ be a family of soft mappings where $\{(Y_i,\ \tau_i)\}_{i \in \Delta}$ be a family of soft topological spaces. Then the topology τ generated from the subbase $\{(\phi_\psi)_i^{-1}((F, E)) : (F, E) \in \tau_i,\ i \in \Delta\}$ is called the initial soft topology induced by the family of soft mappings $\{(\phi_\psi)_i\}_{i \in \Delta}$.

Definition 1.16. [2] Let $\{(X_i, \tau_i)\}_{i \in \Delta}$ be a family of soft topological spaces. Then the initial soft topology on $X(= \prod_{i \in \Delta} X_i)$ generated by the family $\{(p_q)_i\}_{i \in \Delta}$ is called soft product topology on X, where $(p_q)_i$ are the soft projection mapping from X to X_i.

Theorem 1.5. [9] *Let X and Y be crisp sets, F_A, $(F_A)_i \in SS(X,\ E)$ and G_B, $(G_B)_i \in SS(Y,\ K)$, where $i \in \Delta$, an index set. Then*

1. *If $(F_A)_1 \tilde{\subseteq} (F_A)_2$, then $\Phi_\psi((F_A)_1) \tilde{\subseteq} \Phi_\psi((F_A)_2)$.*
2. *If $(G_B)_1 \tilde{\subseteq} (G_B)_2$, then $\Phi_\psi^{-1}((G_B)_1) \tilde{\subseteq} \Phi_\psi^{-1}((G_B)_2)$.*
3. *$(F_A) \tilde{\subseteq} \Phi_\psi^{-1}(\Phi_\psi(F_A))$, the equality holds if Φ_ψ is injective.*
4. *$\Phi_\psi(\Phi_\psi^{-1}(F_A)) \tilde{\subseteq} (F_A)$, the equality holds if Φ_ψ is surjective.*
5. *$\Phi_\psi(\tilde{\bigcup}_{i\in\Delta} (F_A)_i) = \tilde{\bigcup}_{i\in\Delta} \Phi_\psi((F_A)_i)$.*
6. *$\Phi_\psi(\bigcap_{i\in\Delta} (F_A)_i) \tilde{\subseteq} \bigcap_{i\in\Delta} \Phi_\psi((F_A)_i)$.*
7. *$\Phi_\psi^{-1}(\bigcap_{i\in\Delta} (G_B)_i) = \bigcap_{i\in\Delta} \Phi_\psi^{-1}((G_B)_i)$.*
8. *$\Phi_\psi^{-1}(\bigcap_{i\in\Delta} (G_B)_i) = \bigcap_{i\in\Delta} \Phi_\psi^{-1}((G_B)_i)$.*
9. *$\Phi_\psi^{-1}(E_Y) = E_X$ and $\Phi_\psi^{-1}(\phi_Y) = \phi_X$.*
10. *$\Phi_\psi(E_X) = E_Y$ if Φ_ψ is surjective.*
11. *$\Phi_\psi(\phi_x) = \phi_Y$.*

2. SOFT SEPARATION AXIOMS AND PRODUCT SOFT TOPOLOGICAL SPACES

Definition 2.1. [8] A soft topological space (X, τ, E) is said to be a soft T_0 -space if for every pair of soft points x_{e_1}, y_{e_2} such that $x_{e_1} \neq y_{e_2}$, there exists $(F, E) \in \tau$ such that $x_{e_1} \tilde{\in} (F, E)$, $y_{e_2} \tilde{\notin} (F, E)$ or there exists $(G, E) \in \tau$ such that $y_{e_2} \tilde{\in} (G, E)$, $x_{e_1} \tilde{\notin} (G, E)$.

Definition 2.2. [8] A soft topological space (X, τ, E) is said to be a soft T_1 -space if every pair of soft points x_{e_1}, y_{e_2}, such that $x_{e_1} \neq y_{e_2}$ there exist $(F, E), (G, E) \in \tau$ such that $x_{e_1} \tilde{\in} (F, E)$, $y_{e_2} \tilde{\notin} (F, E)$ and $x_{e_1} \tilde{\notin} (G, E)$, $y_{e_2} \tilde{\in} (G, E)$.

Example 2.1. *Example for T_0-space.*
Let $X = \{x, y\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E),\ (F_4, E)\}$ where

$$F_1(e) = \begin{cases} \{x\} & \text{if } e = e_1 \\ \{y\} & \text{if } e = e_2 \end{cases},\ F_2(e) = \begin{cases} \{x\} & \text{if } e = e_1 \\ \{x\} & \text{if } e = e_2 \end{cases},$$

$$F_3(e) = \begin{cases} \{x\} & \text{if } e = e_1 \\ X & \text{if } e = e_2 \end{cases},\ F_4(e) = \begin{cases} \{x\} & \text{if } e = e_1 \\ \phi & \text{if } e = e_2 \end{cases},$$

For the soft points x_{e_1}, y_{e_1}, there is a soft open set $(F_1, E) \in \tau$ with $x_{e_1} \tilde{\in} (F_1, E)$ and $y_{e_1} \tilde{\notin} (F_1, E)$. For the soft points x_{e_2}, y_{e_2}, there is a $(F_1, E) \in \tau$ with $x_{e_2} \tilde{\notin} (F_1, E)$ and $y_{e_2} \tilde{\in}\ (F_1, E)$. For the soft points x_{e_1}, y_{e_2}, there is a $(F_2, E) \in \tau$ with $x_{e_1} \tilde{\in}\ (F_2, E)$ and $y_{e_2} \tilde{\notin} (F_2, E)$. For the soft points x_{e_2}, y_{e_1}, there is a $(F_2, E) \in \tau$ with $x_{e_2} \tilde{\in} (F_2, E)$ and $y_{e_1} \tilde{\notin} (F_2, E)$. For the soft points x_{e_1}, x_{e_2}, there is a $(F_1, E) \in \tau$ with $x_{e_1} \tilde{\in} (F_1, E)$ and $x_{e_2} \tilde{\notin} (F_1, E)$. For the soft points y_{e_1}, y_{e_2}, there is a $(F_1, E) \in \tau$ with $y_{e_1} \tilde{\notin}\ (F_1, E)$ and $y_{e_2} \tilde{\in} (F_1, E)$. Thus (X, τ, E) is a soft T_0-space.

Example 2.2. *Let $X = \mathbf{Z}$, the set of all integers and $E = \mathbf{N}$, the set of all natural numbers. Define a soft topology on X as $\tau = \{(F, E)^c : F(e_i)\ is\ finite\ for\ each\ e_i \in E\} \cup \{\tilde{\phi}\}$.*

1. *Clearly $\tilde{\phi} \in \tau$ and $\tilde{X} \in \tau$.*

(2) *If* $(F_\alpha, E) \in \tau$ *for some* $\alpha \in \Delta$, *where* Δ *is some index set, then* $F^c_\alpha(e_i)$ *is finite for each* $e_i \in E$. *Now* $\cap F^c_\alpha(e_i) = (\cup F_\alpha(e_i))^c$ *is finite for each* $e_i \in E$. *So that* $\cup(F_\alpha, E) \in \tau$.

(3) *If* $(F_1, E), (F_2, E) \in \tau$, $F^c_1(e_i)$ *and* $F^c_2(e_i)$ *are finite for each* $e_i \in E$. *Now* $F^c_1(e_i) \cup F^c_2(e_i) = (F_1(e_i) \cap F_2(e_i))^c = ((F_1 \cap F_2)(e_i))^c = (F_1 \cap F_2)^c(e_i)$ *is finite for each* $e_i \in E$. *So that* $(F_1, E) \cap (F_2, E) \in \tau$.

Thus (X, τ, E) *is a soft topological space. For any two distinct soft points* x_{e_i} *and* y_{e_j}, $x^c_{e_i}$ *and* $y^c_{e_j}$ *are soft open sets such that* $x_{e_i} \in y^c_{e_j}$, $y_{e_j} \notin y^c_{e_j}$ *and* $x_{e_i} \notin x^c_{e_i}$, $y_{e_j} \in x^c_{e_i}$. *Thus* (X, τ, E) *is a soft* T_1 *space.*

Theorem 2.1. *Every soft* T_1*-space is a soft* T_0*-space.*

Proof. Proof is straight forward □

Theorem 2.2. *Let* $(X,\ \tau,\ E)$ *be a soft topological space. Then* $(X,\ \tau,\ E)$ *is a soft* T_0 *space if and only if for any two distinct soft points* x_{e_i} *and* y_{e_j}, *there is soft closed set* $(H,\ E)$ *such that* $x_{e_i} \tilde{\in} (H,\ E)$, $y_{e_j} \tilde{\notin} (H,\ E)$ *or there is soft closed set* $(K,\ E)$ *such that* $x_{e_i} \tilde{\notin} (K,\ E)$, $y_{e_j} \tilde{\in} (K,\ E)$.

Proof. Let us consider two distinct soft points x_{e_i} and y_{e_j}. Since $(X,\ \tau,\ E)$ is a soft T_0 space, there is soft open set $(F,\ E)$ such that $x_{e_i} \tilde{\in} (F,\ E)$, $y_{e_j} \tilde{\notin} (F,\ E)$ or there is soft open set $(G,\ E)$ such that $x_{e_i} \tilde{\notin} (G,\ E)$, $y_{e_j} \tilde{\in} (G,\ E)$. Let $(H,\ E) = (G^c,\ E)$ and $(K,\ E) = (F^c,\ E)$. Then $(H,\ E)$ is a soft closed set such that $x_{e_i} \tilde{\in} (H,\ E)$, $y_{e_j} \tilde{\notin} (H,\ E)$ or $(K,\ E)$ is a soft closed set such that $x_{e_i} \tilde{\notin} (K,\ E)$, $y_{e_j} \tilde{\in} (K,\ E)$.

Conversely, for any two distinct soft points x_{e_i} and y_{e_j}, there is a soft closed set $(H,\ E)$ such that $x_{e_i} \tilde{\in} (H,\ E)$, $y_{e_j} \tilde{\notin} (H,\ E)$ or there is soft closed set $(K,\ E)$ such that $x_{e_i} \tilde{\notin} (K,\ E)$, $y_{e_j} \tilde{\in} (K,\ E)$. Then $(H^c,\ E)$ is a soft open set such that $x_{e_i} \tilde{\notin} (H^c,\ E)$, $y_{e_j} \tilde{\in} (H^c,\ E)$ or $(K^c,\ E)$ is a soft open set such that $x_{e_i} \tilde{\in} (K^c,\ E)$, $y_{e_j} \tilde{\notin} (K^c,\ E)$. This proves that $(X,\ \tau,\ E)$ is a soft T_0 space. □

Example:1 given in the artice [8] for soft T_1 space which is not a soft T_0 space is wrong. Because it is not a soft T_0 space too.

Example 2.3. [8] $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$ *and* $\tau = \{\tilde{\phi}, \tilde{X}, (F, A)\}$ *where* $F(e) = \begin{cases} \{x_1\} & \text{if } e = e_1 \\ \{x_2\} & \text{if } e = e_2 \end{cases}$ *This* (X, τ, A) *is verified as soft* T_0 *space in* [8].

consider two soft points $e_F = \begin{cases} \{x_2\} & \text{if } e = e_1 \\ \phi & \text{if } e = e_2 \end{cases}$ *and* $e_G = \begin{cases} \phi & \text{if } e = e_1 \\ \{x_1\} & \text{if } e = e_2 \end{cases}$,

then there is no soft open set (F, A) *in* (X, τ, A) *such that* $e_F \tilde{\in} (F, A)$ *and* $e_G \tilde{\notin} (F, A)$. *Thus* (X, τ, A) *is not a soft* T_0 *space.*

The following example will be a correct example for example:1 of [8]. It also shows that the converse of above theorem 2.1 is not true in general.

Example 2.4. *Example for a soft* T_0*-space which is not a soft* T_1*-space.*
Let $X = \{x, y\}$, $E = \{e_1, e_2\}$ *and* $\tau = \{\tilde{\phi},\ \tilde{X},\ (F_1, E),\ (F_2, E),\ (F_3, E),\ (F_4, E),\ (F_5, E)\}$ *where*

$$F_1(e) = \begin{cases} \{x\} & \text{if } e = e_1 \\ \{y\} & \text{if } e = e_2 \end{cases},\ F_2(e) = \begin{cases} \{x\} & \text{if } e = e_1 \\ \phi & \text{if } e = e_2 \end{cases},\ F_3(e) = \begin{cases} \phi & \text{if } e = e_1 \\ \{x\} & \text{if } e = e_2 \end{cases},$$

$$F_4(e) = \begin{cases} \{x\} & \textit{if } e = e_1 \\ \{x\} & \textit{if } e = e_2 \end{cases}, \; F_5(e) = \begin{cases} \{x\} & \textit{if } e = e_1 \\ X & \textit{if } e = e_2 \end{cases}$$

For the soft points x_{e_1}, y_{e_1}, *there is a* $(F_2, E) \in \tau$ *with* $x_{e_1} \tilde{\in} (F_2, E)$ *and* $y_{e_1} \tilde{\notin} (F_2, E)$. *For the soft points* x_{e_2}, y_{e_2}, *there is a* $(F_3, E) \in \tau$ *with* $x_{e_2} \tilde{\in} (F_3, E)$ *and* $y_{e_2} \tilde{\notin} (F_3, E)$. *For the soft points* x_{e_1}, y_{e_2}, *there is a* $(F_2, E) \in \tau$ *with* $x_{e_1} \tilde{\in} (F_2, E)$ *and* $y_{e_2} \tilde{\notin} (F_2, E)$. *For the soft points* x_{e_2}, y_{e_1}, *there is a* $(F_3, E) \in \tau$ *with* $x_{e_2} \tilde{\in} (F_3, E)$ *and* $y_{e_1} \tilde{\notin} (F_3, E)$. *For the soft points* x_{e_1}, x_{e_2}, *there is a* $(F_2, E) \in \tau$ *with* $x_{e_1} \tilde{\in} (F_2, E)$ *and* $x_{e_2} \tilde{\notin} (F_2, E)$. *For the soft points* y_{e_1}, y_{e_2}, *there is a* $(F_1, E) \in \tau$ *with* $y_{e_2} \tilde{\in} (F_1, E)$ *and* $y_{e_1} \tilde{\notin} (F_1, E)$. *Thus* (X, τ, E) *is a soft* T_0*-space. But for the pair of soft points* y_{e_1}, y_{e_2}, *we dont have* $(K, E) \in \tau$ *such that* $y_{e_1} \tilde{\in} (K, E)$ *and* $y_{e_2} \tilde{\notin} (K, E)$. *Thus* $(X, \tau,\ E)$ *is not a soft* T_1*-space.*

Theorem 2.3.
1. *A subspace of a soft* T_0*-space is a soft* T_0*-space.*
2. *A subspace of a soft* T_1*-space is a soft* T_1*-space*

Proof.
1. Let $(X,\ \tau,\ E)$ be a soft T_0-space and $(Y,\ \tau_Y,\ E)$ be a soft subspace. Let x_{e_i}, y_{e_j} be two soft points in $SS(Y,\ E)$. Then x_{e_i}, $y_{e_j} \tilde{\in} SS(X,\ E)$. Since (X, τ, E) is a soft T_0 space, there is a soft open set $(F,\ E)$ in (X, τ, E) such that $x_{e_i} \tilde{\in} (F,\ E)$, $y_{e_j} \tilde{\notin} (F,\ E)$ or there is a soft open set $(G,\ E)$ in $(X,\ \tau,\ E)$ such that $y_{e_j} \tilde{\in} (G,\ E)$, $x_{e_i} \tilde{\notin} (G,\ E)$. Then $(F,\ E)\ \tilde{\cap} E_Y$ is a soft open set in $(Y,\ \tau_Y,\ E)$ such that $x_{e_i} \tilde{\in} (F,\ E) \tilde{\cap} E_Y$ $y_{e_j} \tilde{\notin} (F,\ E) \tilde{\cap} E_Y$ or $(G,\ E) \tilde{\cap} E_Y$ is a soft open set in $(Y,\ \tau_Y,\ E)$ such that $y_{e_j} \tilde{\in} (G,\ E) \tilde{\cap} E_Y$, $x_{e_i} \tilde{\notin} (G,\ E) \tilde{\cap} E_Y$. Thus $(Y,\ \tau,\ E)$ is a soft T_0-space
2. Proof is similar to (1)

□

Theorem 2.4. *Let* $(X,\ \tau,\ E)$ *be a soft topological space. Then* $(X,\ \tau,\ E)$ *is a soft* T_1 *space if and only if for any soft points* x_{e_i} *and* y_{e_j}, *there exist two soft closed sets* $(H,\ E)$ *and* $(K,\ E)$ *such that* $x_{e_i} \tilde{\in} (H,\ E)$, $y_{e_j} \tilde{\notin} (H,\ E)$, $y_{e_j} \tilde{\in} (K,\ E)$ *and* $x_{e_i} \tilde{\notin} (K,\ E)$.

Proof. Proof is similar to the theorem 2.2 □

The following example shows that the product of soft T_0-spaces need not be a soft T_0-space

Definition 2.3. Let $\{(X_i,\ \tau_i,\ E_i) : i \in I\}$ be a family of soft topological spaces and $(\prod X_i, \prod \tau_i,\ \prod E_i)$ be their product soft toplogical space. Then a soft point in $(\prod X_i, \prod \tau_i,\ \prod E_i)$ is denoted as $\mathbf{x_e}$, where $\mathbf{x} =< x_i >_{i \in I},\ x_i \in X_i$ and $\mathbf{e} =< e_i >_{i \in I},\ e_i \in E_i$.

Example 2.5. *Let* $X_1 = \{x_1, y_1\}$, $E_1 = \{e_{11}, e_{12}\}$ *and* $\tau_1 = \{\tilde{\phi}, \tilde{X}_1,\ (F_1, E_1),\ (F_2, E_1),\ (F_3, E_1),\ (F_4, E_1),\ (F_5, E_1),\ (F_6, E_1),\ (F_7, E_1)\}$. $X_2 = \{x_2, y_2\}$, $E_2 = \{e_{21}, e_{22}\}$ *and* $\tau_2 = \{\tilde{\phi}, \tilde{X}_2, (G_1, E_2), (G_2, E_2), (G_3, E_2), (G_4, E_2), (G_5, E_2), (G_6, E_2), (G_7, E_2)\}$ *where*

$$F_1(e) = \begin{cases} \{x_1\} & \textit{if } e = e_{11} \\ \phi & \textit{if } e = e_{12} \end{cases}, \; G_1(e) = \begin{cases} \{x_2\} & \textit{if } e = e_{21} \\ \phi & \textit{if } e = e_{22} \end{cases}, \; F_2(e) = \begin{cases} \phi & \textit{if } e = e_{11} \\ \{x_1\} & \textit{if } e = e_{12} \end{cases},$$

$$G_2(e) = \begin{cases} \phi & \textit{if } e = e_{21} \\ \{x_2\} & \textit{if } e = e_{22} \end{cases}, \; F_3(e) = \begin{cases} \{x_1\} & \textit{if } e = e_{11} \\ \{x_1\} & \textit{if } e = e_{12} \end{cases}, \; G_3(e) = \begin{cases} \{x_2\} & \textit{if } e = e_{21} \\ \{x_2\} & \textit{if } e = e_{22} \end{cases},$$

$$F_4(e)=\begin{cases}\{y_1\} & if\ e=e_{11}\\ \{x_1\} & if\ e=e_{12}\end{cases},\ G_4(e)=\begin{cases}\{x_2\} & if\ e=e_{21}\\ \{y_2\} & if\ e=e_{22}\end{cases},\ F_5(e)=\begin{cases}\{y_1\} & if\ e=e_{11}\\ \phi & if\ e=e_{12}\end{cases},$$

$$G_5(e)=\begin{cases}\phi & if\ e=e_{21}\\ \{y_2\} & if\ e=e_{22}\end{cases},\ F_6(e)=\begin{cases}X_1 & if\ e=e_{11}\\ \phi & if\ e=e_{12}\end{cases},\ G_6(e)=\begin{cases}\phi & if\ e=e_{21}\\ X_2 & if\ e=e_{22}\end{cases},$$

$$F_7(e)=\begin{cases}X_1 & if\ e=e_{11}\\ \{x_1\} & if\ e=e_{12}\end{cases},\ G_7(e)=\begin{cases}\{x_2\} & if\ e=e_{21}\\ X_2 & if\ e=e_{22}\end{cases}.$$

For the soft points $x_{1_{e_{11}}}$, $y_{1_{e_{11}}}$, there is a soft open set $(F_1,E_1)\in\tau_1$ with $x_{1_{e_{11}}}\tilde{\in}(F_1,E_1)$ and $y_{1_{e_{11}}}\tilde{\notin}(F_1,E_1)$. For the soft points $x_{1_{e_{11}}}$, $y_{1_{e_{12}}}$, there is a soft open set $(F_1,E_1)\in\tau_1$ with $x_{1_{e_{11}}}\tilde{\in}(F_1,E_1)$ and $y_{1_{e_{12}}}\tilde{\notin}(F_1,E_1)$. For the soft points $x_{1_{e_{12}}}$, $y_{1_{e_{11}}}$, there is $(F_2,E_1)\in\tau_1$ with $x_{1_{e_{12}}}\tilde{\in}(F_2,E_1)$ and $y_{1_{e_{11}}}\tilde{\notin}(F_2,E_1)$. For the soft points $x_{1_{e_{12}}}$, $y_{1_{e_{12}}}$, there is $(F_2,E_1)\in\tau_1$ with $x_{1_{e_{12}}}\tilde{\in}(F_2,E_1)$ and $y_{1_{e_{12}}}\tilde{\notin}(F_2,E_1)$. For the soft points $x_{1_{e_{11}}}$, $x_{1_{e_{12}}}$, there is a soft open set $(F_1,E_1)\in\tau_1$ with $x_{1_{e_{11}}}\tilde{\in}(F_1,E_1)$ and $x_{1_{e_{12}}}\tilde{\notin}(F_1,E_1)$. For the soft points $y_{1_{e_{11}}}$, $y_{1_{e_{12}}}$, there is a soft open set $(F_5,E_1)\in\tau_1$ with $y_{1_{e_{11}}}\tilde{\in}(F_5,E_1)$ and $y_{1_{e_{12}}}\tilde{\notin}(F_5,E_1)$. Thus (X_1,τ_1,E_1) is a soft T_0-space.

For the soft points $x_{2_{e_{21}}}$, $y_{2_{e_{21}}}$, there is a soft open set $(G_1,E_2)\in\tau_2$ with $x_{2_{e_{21}}}\tilde{\in}(G_1,E_2)$ and $y_{2_{e_{21}}}\tilde{\notin}(G_1,E_2)$. For the soft points $x_{2_{e_{21}}}$, $y_{2_{e_{22}}}$, there is a soft open set $(G_1,E_2)\in\tau_2$ with $x_{2_{e_{21}}}\tilde{\in}(G_1,E_2)$ and $y_{2_{e_{22}}}\tilde{\notin}(G_1,E_2)$. For the soft points $x_{2_{e_{22}}}$, $y_{2_{e_{21}}}$, there is a soft open set $(G_2,E_2)\in\tau_2$ with $x_{2_{e_{22}}}\tilde{\in}(G_2,E_2)$ and $y_{2_{e_{21}}}\tilde{\notin}(G_2,E_2)$. For the soft points $x_{2_{e_{22}}}$, $y_{2_{e_{22}}}$, there is a soft open set $(G_2,E_2)\in\tau_2$ with $x_{2_{e_{22}}}\tilde{\in}(G_2,E_2)$ and $y_{2_{e_{22}}}\tilde{\notin}(G_2,E_2)$. For the soft points $x_{2_{e_{21}}}$, $x_{2_{e_{22}}}$, there is a soft open set $(G_1,E_2)\in\tau_2$ with $x_{2_{e_{21}}}\tilde{\in}(G_1,E_2)$ and $x_{2_{e_{22}}}\tilde{\notin}(G_1,E_2)$. For the soft points $y_{2_{e_{21}}}$, $y_{2_{e_{22}}}$, there is a soft open set $(G_4,E_2)\in\tau_2$ with $y_{2_{e_{21}}}\tilde{\notin}(G_4,E_2)$ and $y_{2_{e_{22}}}\tilde{\in}(G_4,E_2)$. Thus (X_2,τ_2,E_2) is a soft T_0-space.

Now $E_1\times E_2=\{(e_{11},\ e_{21}),(e_{11},\ e_{22}),(e_{12},\ e_{21}),(e_{12}\ e_{22})\}$ *and* $\tau_1\times\tau_2=\{\tilde{\phi}$, $X_1\tilde{\times}X_2$, $(F_1\times G_1,E_1\times E_2)$, $(F_1\times G_2,E_1\times E_2)$, $(F_1\times G_3,E_1\times E_2)$, $(F_1\times G_4,E_1\times E_2)$, $(F_1\times G_5,E_1\times E_2)$, $(F_1\times G_6,E_1\times E_2)$, $(F_1\times G_7,E_1\times E_2)$, $(F_2\times G_1,E_1\times E_2)$, $(F_2\times G_2,E_1\times E_2)$, $(F_2\times G_3,E_1\times E_2)$, $(F_2\times G_4,E_1\times E_2)$, $(F_2\times G_5,E_1\times E_2)$, $(F_2\times G_6,E_1\times E_2)$, $(F_2\times G_7,E_1\times E_2)$, $(F_3\times G_1,E_1\times E_2)$, $(F_3\times G_2,E_1\times E_2)$, $(F_3\times G_3,E_1\times E_2)$, $(F_3\times G_4,E_1\times E_2)$, $(F_3\times G_5,E_1\times E_2)$, $(F_3\times G_6,E_1\times E_2)$, $(F_3\times G_7,E_1\times E_2)$, $(F_4\times G_1,E_1\times E_2)$, $(F_4\times G_2,E_1\times E_2)$, $(F_4\times G_3,E_1\times E_2)$, $(F_4\times G_4,E_1\times E_2)$, $(F_4\times G_5,E_1\times E_2)$, $(F_4\times G_6,E_1\times E_2)$, $(F_4\times G_7,E_1\times E_2)$, $(F_5\times G_1,E_1\times E_2)$, $(F_5\times G_2,E_1\times E_2)$, $(F_5\times G_3,E_1\times E_2)$, $(F_5\times G_4,E_1\times E_2)$, $(F_5\times G_5,E_1\times E_2)$, $(F_5\times G_6,E_1\times E_2)$, $(F_5\times G_7,E_1\times E_2)$, $(F_6\times G_1,E_1\times E_2)$, $(F_6\times G_2,E_1\times E_2)$, $(F_6\times G_3,E_1\times E_2)$, $(F_6\times G_4,E_1\times E_2)$, $(F_6\times G_5,E_1\times E_2)$, $(F_6\times G_6,E_1\times E_2)$, $(F_6\times G_7,E_1\times E_2)$, $(F_7\times G_1,E_1\times E_2)$, $(F_7\times G_2,E_1\times E_2)$, $(F_7\times G_3,E_1\times E_2)$, $(F_7\times G_4,E_1\times E_2)$, $(F_7\times G_5,E_1\times E_2)$, $(F_7\times G_6,E_1\times E_2)$, $(F_7\times G_7,E_1\times E_2)\}$.

Suppose if the soft product of (X_1, τ_1, E_1) *and* (X_2, τ_2, E_2) *is a soft* T_0 *space, then for any two distinct soft points* $(x_1,\ y_2)_{(e_{11},\ e_{21})} = \begin{cases} \{(x_1,\ y_2)\} & \text{if } e = (e_{11},\ e_{21}) \\ \phi & \text{if } e = (e_{11},\ e_{22}) \\ \phi & \text{if } e = (e_{12},\ e_{21}) \\ \phi & \text{if } e = (e_{12},\ e_{22}) \end{cases}$ *and* $(y_1,\ y_2)_{(e_{11},\ e_{21})} = \begin{cases} \{(y_1,\ y_2)\} & \text{if } e = (e_{11},\ e_{21}) \\ \phi & \text{if } e = (e_{11},\ e_{22}) \\ \phi & \text{if } e = (e_{12},\ e_{21}) \\ \phi & \text{if } e = (e_{12},\ e_{22}) \end{cases}$, *there is a soft open set* $(F_m \times G_n, E_1 \times E_2)$ *in* $\tau_1 \times \tau_2$ *such that* $(x_1,\ y_2)_{(e_{11},\ e_{21})} \tilde{\in} (F_m \times G_n, E_1 \times E_2)$ *and* $(y_1,\ y_2)_{(e_{11},\ e_{21})} \tilde{\notin} (F_m \times G_n, E_1 \times E_2)$, *for some* $m, n \in \{1, 2, 3 \ldots, 7\}$. *Now* $(p_q)_2((x_1,\ y_2)_{(e_{11},\ e_{21})}) \tilde{\in} (p_q)_2((F_m \times G_n, E_1 \times E_2))$. *That is* $p_2(x_1,\ y_2)_{q2(e_{11},\ e_{21})} \tilde{\in} p_2(F_m \times G_n)_{q2(E_1 \times E_2)}$. *This implies* $y_{2_{e_{21}}} \tilde{\in} (G_n, E_2)$, *for some* $m, n \in \{1, 2, 3 \ldots, 7\}$. *Since* $(p_q)_2$ *is a soft projection mapping and* $(F_m \times G_n, E_1 \times E_2)$ *is a soft open set in* $X_1 \times X_2$, (G_n, E_2) *is a soft open set in* (X_2, τ_2, E_2) *containing* $y_{2_{e_{21}}}$. *But there is no soft open set* (G_n, E_2) *in* (X_2, τ_2, E_2) *containing* $y_{2_{e_{21}}}$, *for any* $n \in \{1, 2, 3 \ldots, 7\}$ *and hence* $(X_1 \times X_2, \tau_1 \times \tau_2, E_1 \times E_2)$ *is not a soft* T_0 *space.*

Definition 2.4. Let $(X,\ \tau,\ E)$ be a soft topological space and $A = \{x_{e_i} : x_{e_i}$ is a soft points of $(X,\ \tau,\ E)\}$.

(1) If the number of elements of the set A is finite, then $(X,\ \tau,\ E)$ is called a finite soft topological space.
(2) If the number of elements of the set A is countable, then $(X,\ \tau,\ E)$ is called a countable soft topological space.

Theorem 2.5. *If* $(X,\ \tau,\ E)$ *is a finite soft* T_1 *space, then* $(X,\ \tau,\ E)$ *is a soft discrete space.*

Proof. Let x_{e_i} be a soft point, $x \in X$ and $e_i \in E$. $(X,\ \tau,\ E)$ is a soft T_1 space, for any soft point $y_{e_j} \neq x_{e_i}$, there is a soft open set $(F_{x_j},\ E)$ such that $x_{e_i} \tilde{\in} (F_{x_j},\ E)$ and $y_{e_j} \tilde{\notin} (F_{x_j},\ E)$. Since $(X,\ \tau,\ E)$ is a finite soft topological space, $\tilde{\bigcap}_{y_{e_j} \neq x_{e_i}} (F_{x_j},\ E)$ is a soft open set such that $\tilde{\bigcap}_{y_{e_j} \neq x_{e_i}} (F_{x_j},\ E) = \begin{cases} \{x\} & \text{if } e = e_i \\ \phi & \text{if } e \neq e_i \end{cases}$. Thus x_{e_i} is soft open and hence $(X,\ \tau,\ E)$ is a soft discrete space. □

Definition 2.5. Let $(X,\ \tau,\ E)$ be a soft topological space. Then the soft set $(F,\ E)$ is called a soft G_δ set if it is a countable intersection of soft open sets.

Theorem 2.6. *If* $(X,\ \tau,\ E)$ *is a countable soft* T_1 *space and if every soft* G_δ *set is soft open in* $(X,\ \tau,\ E)$, *then* $(X,\ \tau,\ E)$ *is a soft discrete space.*

Proof. Let x_{e_i} be a soft point. Since $(X,\ \tau,\ E)$ is a soft T_1 space, for any soft point $y_{e_j} \neq x_{e_i}$, there is a soft open set $(F_{x_j},\ E)$ such that $x_{e_i} \tilde{\in} (F_{x_j},\ E)$ and $y_{e_j} \tilde{\notin} (F_{x_j},\ E)$. Since every soft G_δ set is soft open and $(X,\ \tau,\ E)$ is a countable soft topological space, $\tilde{\bigcap}_{y_{e_j} \neq x_{e_i}} (F_{x_j},\ E)$ is a soft open set such that $\tilde{\bigcap}_{y_{e_j} \neq x_{e_i}} (F_{x_j},\ E) = \begin{cases} \{x\} & \text{if } e = e_i \\ \phi & \text{if } e \neq e_i \end{cases}$. Thus x_{e_i} is soft open and hence $(X,\ \tau,\ E)$ is a soft discrete space. □

Theorem 2.7. *Product of soft T_1-spaces is a soft T_1-space*

Proof. Let $\{(X_i,\ \tau_i,\ E_i) : i \in I\}$ be a family of soft topological spaces and $(\prod X_i, \prod \tau_i, \prod E_i)$ be their product soft toplogical space. Suppose $\mathbf{x_e}$ and $\mathbf{y_f}$ be two distinct soft points, where $\mathbf{x} =< x_i >_{i\in I},\ \mathbf{y} =< y_i >_{i\in I}\ \ x_i, y_i \in X_i$ and $\mathbf{e} =< e_i >_{i\in I},\ \mathbf{f} =< f_i >_{i\in I}, e_i, f_i \in E_i$. Then there exists atleast one $\beta \in I$ such that $x_\beta \neq y_\beta$ or there exist $e_{i_k},\ e_{i_m} \in E_i$ such that $e_{i_k} \neq e_{i_m}$.

Case: 1

If $x_\beta \neq y_\beta$, $(p_q)_\beta(\mathbf{x_e}) = (p_{\beta_{q_\beta}})(\mathbf{x_e}) = p_\beta(\mathbf{x})_{q_\beta(\mathbf{e})} = x_{\beta_{e_\beta}}$ and $(p_q)_\beta\ (\mathbf{y_f}) = (p_{\beta_{q_\beta}})(\mathbf{y_f}) = p_\beta(\mathbf{y})_{q_\beta(\mathbf{f})} = y_{\beta_{f_\beta}}$. Since X_β is a soft T_1 space, there exist soft open sets (F_β, E_β) and (G_β, E_β) such that $x_{\beta_{e_\beta}} \tilde{\in} (F_\beta, E_\beta),\ y_{\beta_{f_\beta}} \tilde{\notin} (F_\beta, E_\beta)$ and $y_{\beta_{f_\beta}} \tilde{\in} (G_\beta, E_\beta),\ x_{\beta_{e_\beta}} \tilde{\notin} (G_\beta, E_\beta)$. Then the soft subbasic members $(p_q)_\beta^{-1}(F_\beta, E_\beta)$ and $(p_q)_\beta^{-1}(G_\beta, E_\beta)$ are the soft open sets containing $\mathbf{x_e}$ and $\mathbf{y_f}$ respectively. Suppose if $\mathbf{y_f} \tilde{\in}\ (p_q)_\beta^{-1}(F_\beta, E_\beta)$, then $p_\beta(\mathbf{y})_{q_\beta(f)} = (p_q)_\beta(\mathbf{y_f}) \tilde{\in}\ (p_q)_\beta((p_q)_\beta^{-1}(F_\beta, E_\beta))$. That is $y_{\beta_{f_\beta}} \tilde{\in}\ (F_\beta,\ E_\beta)$ which is a contradiction. Similarly we can prove $\mathbf{x_e} \tilde{\notin} (p_q)_\beta^{-1}\ (G_\beta, E_\beta)$. Thus $(p_q)_\beta^{-1}(F_\beta, E_\beta)$ and $(p_q)_\beta^{-1}(G_\beta, E_\beta)$ are the soft open sets such that $\mathbf{x_e}\ \tilde{\in} (p_q)_\beta^{-1}(F_\beta, E_\beta)$, $\mathbf{y_f}\ \tilde{\notin} (p_q)_\beta^{-1}\ (F_\beta, E_\beta)$ and $\mathbf{y_f}\ \tilde{\in} (p_q)_\beta^{-1}\ (G_\beta, E_\beta)$, $\mathbf{x_e}\ \tilde{\notin} (p_q)_\beta^{-1}(G_\beta, E_\beta)$.

Case: 2

If $e_{i_k} \neq e_{i_m}$, there are soft open sets (F_{i_k}, E_i) and (F_{i_m}, E_i) in $(X_i,\ \tau_i,\ E_i)$ such that $x_{e_{i_k}} \tilde{\in} (F_{i_k}, E_i),\ x_{e_{i_m}}\ (= y_{e_{i_m}}) \tilde{\notin}\ (F_{i_k}, E_i)$ and $x_{e_{i_m}} \tilde{\in} (F_{i_m}, E_i)$, $x_{e_{i_k}}\ \tilde{\notin} (F_{i_m}, E_i)$. Then $(p_q)_i^{-1}\ (F_{i_k}, E_i)$ and $(p_q)_i^{-1}\ (F_{i_m}, E_i)$ are soft open sets such that $\mathbf{x_e}\ \tilde{\in}\ (p_q)_i^{-1}(F_{i_k}, E_i)$ and $\mathbf{y_f} \tilde{\in}\ (p_q)_i^{-1}(F_{i_m}, E_i)$. We can prove $\mathbf{y_f}\ \tilde{\notin} (p_q)_i^{-1}(F_{i_k},\ E_i)$ and $\mathbf{x_e}\ \tilde{\notin} (p_q)_i^{-1}\ (F_{i_m}, E_i)$ as we proved in case:1. This completes the proof. □

Theorem 2.8. *Let (X, τ, E) be a soft topological space. Then the following are equivalent.*

(1) (X, τ, E) *is a soft* τ_1*-space*
(2) $x_{e_i} = \tilde{\cap}\{(G, E) : (G, E) \in \tau$ *and* $x_{e_i} \tilde{\in} (G, E)\}$
(3) $x_{e_i} = \tilde{\cap}\{(F, E) : (F, E) \in \tau^c$ *and* $x_{e_i} \tilde{\in} (F, E)\}$

Proof. $(i) \Rightarrow (ii)$. Clearly $x_{e_i} \tilde{\subseteq} \tilde{\cap}\{(G, E) : (G, E) \in \tau$ and $x_{e_i} \tilde{\in} (G, E)\}$. Suppose if $y_{e_j} \tilde{\in} \tilde{\cap}\{(G, E) : (G, E) \in \tau$ *and* $x_{e_i} \tilde{\in} (G, E)\}$ such that $x_{e_i} \neq y_{e_j}$. Then $x \neq y$ or $e_i \neq e_j$. In either cases, by our assumption, there is a soft open set (G, E) such that $x_{e_i} \tilde{\in} (G, E)$ and $y_{e_j} \tilde{\notin} (G, E)$.So $y_{e_j} \tilde{\notin} \tilde{\cap}\{(G, E)\ :\ (G, E) \tilde{\in} \tau$ *and* $x_{e_i} \tilde{\in} (G, E)\}$. Thus $x_{e_i} = \tilde{\cap}\{(G, E) : (G, E) \in \tau$ *and* $x_{e_i} \tilde{\in} (G, E)\}$.
$(ii) \Rightarrow (iii)$. Clearly $x_{e_i} \tilde{\subseteq} \tilde{\cap}\{(F, E) : (F, E) \in \tau^c$ *and* $x_{e_i} \tilde{\in} (F, E)\}$. Let $y_{e_j} \tilde{\in} \tilde{\cap}\{(F, E)$ $: (F, E) \in \tau^c$ *and* $x_{e_i} \tilde{\in} (F, E)\}$ such that $x_{e_i} \neq y_{e_j}$. By (ii), there exists $(G, E) \in \tau$ such that $y_{e_j} \tilde{\in} (G, E)$ and $x_{e_i} \tilde{\notin} (G, E)$. Now $(G, E)^c \in \tau^c$ and $y_{e_j} \tilde{\notin} (G, E)^c$ and $x_{e_i} \tilde{\in} (G, E)^c$ and hence $y_{e_j} \tilde{\notin} \tilde{\cap}\{(F, E) : (F, E) \in \tau^c$ *and* $x_{e_i} \tilde{\in} (F, E)\}$. Thus $x_{e_i} = \tilde{\cap}$ $\{(F, E) : (F, E) \in \tau^c$ *and* $x_{e_i} \tilde{\in}\ (F, E)\}$.
$(iii) \Rightarrow (i)$. Let x_{e_i} and y_{e_j} be two distinct soft points. Then by (iii), $x_{e_i} \neq y_{e_j} = \tilde{\cap}\{(F, E) : (F, E) \in \tau^c$ *and* $y_{e_j} \tilde{\in} (F, E)\}$. There is some soft closed set (F_1, E) such that $y_{e_j} \tilde{\in} (F_1, E)$ and $x_{e_i} \tilde{\notin} (F_1, E)$. Then $(F_1, E)^c$ is a soft open set such that $x_{e_i} \tilde{\in} (F_1, E)^c$ and $y_{e_j} \tilde{\notin} (F_1, E)^c$. Similarly, from $y_{e_j} \neq x_{e_i} = \tilde{\cap}\{(F, E) :$ $(F, E) \in \tau^c$ *and* $y_{e_j} \tilde{\in} (F, E)\}$, we can find another soft open set $(F_2, E)^c$ such that $x_{e_i} \tilde{\notin} (F_2, E)^c$ and $y_{e_j} \tilde{\in} (F_2, E)^c$. This proves that (X, τ, E) is a soft τ_1-space. □

Remark.
(1) *From (iii) of theorem 2.8, it is clear that each soft point* x_{e_i} *is a soft closed set in a soft* T_1 *space.*
(2) *Let* T_i= *Number of elements in* $F(e_i), i \in I$ *an indexed set of* E. *If* $T = \sum_{i \in I} T_i$ *is finite, then the soft set* (F, E) *can be written as a finite union of soft points. Each soft point is a soft closed set, we have* (F, E) *is a soft closed set.*
(3) *If* $T = \sum_{i \in I} T_i$ *is infinite,* (F, E) *need not be a closed set. Following example shows this.*

Example 2.6. *Let* X *be an infinite set and* $E = \mathbf{N}$. *Let* $\tau = \{(F, E)^c : \{e_i : F(e_i) \neq \phi\} \ is \ finite\} \cup \{\tilde{\phi}\}$.

(1) *Clearly* $\tilde{\phi} \in \tau$ *and* $\tilde{X} \in \tau$.
(2) *If* $(F_{\alpha_i}, E) \in \tau, \alpha_i \in I$, *for some index set* I, *then* $\{e_j : F^c_{\alpha_i}(e_j) \neq \phi\}$ *is a finite set. Now* $\{e_j : (\cup F_{\alpha_i})^c(e_j) \neq \phi\} = \{e_j : \tilde{\cap} F^c_{\alpha_i}(e_j) \neq \phi\} \subseteq \{e_j : F^c_{\alpha_k}(e_j) \neq \phi\}$, *for all* $\alpha_k \in I$. *Since* $\{e_j : F^c_{\alpha_k}(e_j) \neq \phi\}$ *is a finite set,* $\{e_j : (\cup F_{\alpha_i})^c(e_j) \neq \phi\}$ *is a finite set and hence* $\tilde{\cup}(F_{\alpha_i}, E) \in \tau$.
(3) *If* (F_{α_1}, E) *and* $(F_{\alpha_2}, E) \in \tau$, *then* $\{e_j : F^c_{\alpha_1}(e_j) \neq \phi\}$ *and* $\{e_j : F^c_{\alpha_2}(e_j) \neq \phi\}$ *are finite sets. Now* $\{e_j : (F^c_{\alpha_1} \tilde{\cup} F^c_{\alpha_2})(e_j) \neq \phi\} = \{e_j : (F_{\alpha_1} \tilde{\cap} F_{\alpha_2})^c(e_j) \neq \phi\}$ *is a finite set. Thus* $(F_{\alpha_1}, E) \tilde{\cap} (F_{\alpha_2}, E) \in \tau$

Thus (X, τ, E) *is a soft topological space. Let us take two distinct soft points* x_{e_i} *and* y_{e_j}. *Then either* $x \neq y$ *or* $e_i \neq e_j$. *In either cases* $x^c_{e_i}$ *and* $y^c_{e_j}$ *are two soft open sets such that* $x_{e_i} \tilde{\in} y^c_{e_j}, y_{e_j} \tilde{\notin} y^c_{e_j}$ *and* $x_{e_i} \tilde{\notin} x^c_{e_i}$, $y_{e_j} \tilde{\in} x^c_{e_i}$. *This proves that* (X, τ, E) *is a soft* T_1 *space.*

Let us consider a soft set (G, E) *such that* $G(e_i) = \begin{cases} \{x\} & if \ e_i \ is \ even \\ \phi & if \ e_i \ is \ odd \end{cases}$. *Define* $T(e_i) = \begin{cases} 1 & if \ e_i \ is \ even \\ 0 & if \ e_i \ is \ odd \end{cases}$. $T = \sum T(e_i) = \infty$, *because* $2\mathbf{N}$ *is an infinite set. Since* $\{e_j : G(e_j) \neq \phi\}$ *is not a finite set,* (G, E) *is not a soft closed set.*

Definition 2.6. [8] A soft topological space (X, τ, E) is said to be a soft T_2 -space if for every pair of soft points x_{e_i} and y_{e_j} such that $x_{e_i} \neq y_{e_j}$ there exist soft open sets (F, E) and (G, E) such that $x_{e_i} \tilde{\in} (F, E)$, $y_{e_j} \tilde{\in} (G, E)$ and $(F, E) \tilde{\cap} (G, E) = \tilde{\phi}$.

Example:2 given in the artice [8] for soft T_1 and soft T_2 space is wrong. Because it is neither soft T_1 nor soft T_2 space.

Example 2.7. [8] $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$ *and* $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$ *where*

$F_1(e) = \begin{cases} \{x_2\} & if \ e = e_1 \\ \{x_1\} & if \ e = e_2 \end{cases}$, $F_2(e) = \begin{cases} \{x_1\} & if \ e = e_1 \\ \{x_2\} & if \ e = e_2 \end{cases}$ $F_3(e) = \begin{cases} \{x_1\} & if \ e = e_1 \\ \phi & if \ e = e_2 \end{cases}$

$F_4(e) = \begin{cases} X & if \ e = e_1 \\ \{x_1\} & if \ e = e_2 \end{cases}$

This (X, τ, A) *is verified as soft* T_1 *and soft* T_2 *spaces in* [8].

consider two soft points $e_F = \begin{cases} \{x_1\} & if \ e = e_1 \\ \phi & if \ e = e_2 \end{cases}$ *and* $e_G = \begin{cases} \phi & if \ e = e_1 \\ \{x_2\} & if \ e = e_2 \end{cases}$, *then there is no soft open set* (F_i, A), $i \in \{1, 2, 3, 4\}$ *in* (X, τ, A) *such that* $e_G \tilde{\in} (F_i, A)$ *and* $e_F \tilde{\notin} (F_i, A)$. *Thus* (X, τ, A) *is not a soft* T_1 *space.*

Similarly, there is no two soft open sets (F_i, A) (F_j, A), $i, j \in \{1, 2, 3, 4\}$, $i \neq j$ in (X, τ, A) such that $e_F \tilde{\in} (F_i, A)$, $e_G \tilde{\in} (F_j, A)$ and $(F_i, A) \tilde{\cap} (F_j, A) = \tilde{\phi}$ Thus (X, τ, A) is not a soft T_2 space too.

Next the example:3 given in article [8] is wrong.

Example 2.8. [8] $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$ *and* $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, A), (F_2, A), (F_3, A)\}$ *where*

$$F_1(e) = \begin{cases} \{x_1\} & \text{if } e = e_1 \\ \phi & \text{if } e = e_2 \end{cases}, \quad F_2(e) = \begin{cases} \phi & \text{if } e = e_1 \\ \{x_2\} & \text{if } e = e_2 \end{cases} \quad F_3(e) = \begin{cases} \{x_1\} & \text{if } e = e_1 \\ \{x_2\} & \text{if } e = e_2 \end{cases}$$

This (X, τ, A) is verified as soft T_1 and soft T_0 spaces in [8].

consider two soft points $$e_F = \begin{cases} \{x_2\} & \text{if } e = e_1 \\ \phi & \text{if } e = e_2 \end{cases} \text{ and } e_G = \begin{cases} \phi & \text{if } e = e_1 \\ \{x_1\} & \text{if } e = e_2 \end{cases},$$
then there is no soft open set (F_i, A), $i \in \{1, 2, 3\}$ in (X, τ, A) such that $e_F \tilde{\in} (F_i, A)$ and $e_G \tilde{\notin} (F_i, A)$. Thus (X, τ, A) is not a soft T_1 space. Also there is no soft open set (F_i, A), $i \in \{1, 2, 3\}$ in (X, τ, A) such that $e_F \tilde{\notin} (F_i, A)$ and $e_G \tilde{\in} (F_i, A)$. Hence (X, τ, A) is not a soft T_0 space too.

Correct example for soft T_1 spae which is not a soft T_2 space is given below.

Example 2.9. *Consider a soft topological space (X, τ, E) discussed in Example: 2.6. It is a soft T_1 space.*

Let x_{e_i} and y_{e_j} be two distinct soft points. Then either $x \neq y$ or $e_i \neq e_j$. Assume that there exists two soft open sets (F, E) and (G, E) such that $x_{e_i} \tilde{\in} (F, E)$ and $y_{e_j} \tilde{\in} (G, E)$. Since (F, E) and (G, E) are soft open sets, $\{e_j : F^c(e_j) \neq \phi\}$ and $\{e_j : G^c(e_j) \neq \phi\}$ are finite sets. Now $E - \{e_j : (F(e_j) \cap G(e_j))^c \neq \phi\} \neq \phi$. For any $e_k \in E - \{e_j : (F(e_j) \cap G(e_j))^c \neq \phi\}$, $F^c(e_k) = \phi$ and $G^c(e_k) = \phi$. That is $F(e_k) \cap G(e_k) = X$ and hence $(F, E) \tilde{\cap} (G, E) \neq \phi$. This proves that (X, τ, E) is not a soft T_2 space.

Theorem 2.9. *Every soft T_2 space is a soft T_1 space*

Proof. Proof is straight forward. □

Theorem 2.10. *Soft subspace of soft T_2-space is a soft T_2-space.*

Proof. Let (X, τ, E) be a soft T_2-space and (Y, τ_Y, E) be a soft subspace. Let $x_{e_i},\ y_{e_j}$ be two soft points in (Y, τ, E). Then $x_{e_i},\ y_{e_j} \tilde{\in} SS(X,\ E)$. Since (X, τ, E) is a soft T_2 space, there exist two soft open sets $(F,\ E)$ and $(G,\ E)$ in (X, τ, E) such that $x_{e_i} \tilde{\in} (F,\ E)$, $y_{e_j} \tilde{\in}\ (G,\ E)$ and $(F,\ E) \tilde{\cap} (G,\ E) = \tilde{\phi}$. Now $(F,\ E) \tilde{\cap} E_Y$ and $(G,\ E) \tilde{\cap} E_Y$ are soft open sets in (Y, τ_Y, E) such that $x_{e_i} \tilde{\in} (F,\ E) \tilde{\cap} E_Y$, $y_{e_j} \tilde{\in} (G,\ E) \tilde{\cap} E_Y$ and $((F,\ E) \tilde{\cap} E_Y) \tilde{\cap} ((G,\ E) \tilde{\cap} E_Y) \tilde{\subseteq}\ (F,\ E) \tilde{\cap} (G,\ E) = \tilde{\phi}$. Thus (Y, τ_Y, E) is a soft T_2 space. □

Lemma 2.11. *Let $(X,\ \tau,\ E)$ be a finite soft T_2 space. Then $(X,\ \tau,\ E)$ is a soft discrete space.*

Proof. Proof follows from theorem:2.9 and theorem:2.5. □

Lemma 2.12. *If $(X,\ \tau,\ E)$ is a countable soft T_2 space and if every soft G_δ set is soft open in $(X,\ \tau,\ E)$, then $(X,\ \tau,\ E)$ is a soft discrete space.*

Proof. Proof follows from theorem:2.9 and theorem:2.6. □

Theorem 2.13. *Let $(X,\ \tau,\ E)$ be a soft topological space. Then $(X,\ \tau,\ E)$ is a soft T_2 space if and only if for any soft points x_{e_i} and y_{e_j}, there exist two soft closed neighbourhoods $(H,\ E)$ and $(K,\ E)$ containing disjoint soft open sets containing x_{e_i} and y_{e_j} respectively such that $(H,\ E)\tilde{\cup}(K,\ E) = \tilde{X}$.*

Proof. Since $(X,\ \tau,\ E)$ is a soft T_2 space, for any two distinct soft points x_{e_i} and y_{e_j}, there exist two soft open sets $(F,\ E)$ and $(G,\ E)$ such that $x_{e_i}\tilde{\in}(F,\ E)$ and $y_{e_j}\tilde{\in}(G,\ E)$ such that $(F,\ E)\tilde{\cap}(G,\ E) = \tilde{\phi}$. Now $x_{e_i}\tilde{\in}(G^c,\ E)$, $y_{e_j}\tilde{\in}(F^c,\ E)$ and $(F^c,\ E)\tilde{\cup}(G^c,\ E) = \tilde{X}$. Note that $(F,\ E)\tilde{\subseteq}(G^c,\ E)$ and $(G,\ E)\tilde{\subseteq}(F^c,\ E)$. Let $(F^c,\ E) = (K,\ E)$ and $(G^c,\ E) = (H,\ E)$. Then we have two soft closed neighbourhoods $(H,\ E)$ and $(K,\ E)$ containing disjoint soft open sets (F,E) and (G,E) respectively, such that $x_{e_i}\tilde{\in}(F,E)$, $y_{e_j}\tilde{\in}(G,E)$ $(H,\ E)\tilde{\cup}(K,\ E) = \tilde{X}$.

Conversely let x_{e_i} and y_{e_j} be two distinct soft points. Then there exist two soft closed neighbourhoods $(H,\ E)$ and $(K,\ E)$ and two soft open sets (L,E) containing x_{e_i} and (M,E) containing y_{e_j} such that $(L,E)\tilde{\subseteq}(H,E)$, $(M,E)\tilde{\subseteq}(K,E)$, $(L,E)\tilde{\cap}(M,E) = \tilde{\phi}$ and $(H,\ E)\tilde{\cup}(K,\ E) = \tilde{X}$. This proves that (X,τ,E) is a soft T_2 space □

Theorem 2.14. *Product of soft T_2-spaces is a soft T_2-space*

Proof. Let $\{(X_i,\ \tau_i,\ E_i) : i \in I\}$ be the collection of soft topoloogical spaces and $(\prod X_i,\ \prod \tau_i,\ \prod E_i)$ be their product soft toplogical space. Suppose $\mathbf{x_e}$ and $\mathbf{y_f}$ be two distinct soft points, where $\mathbf{x} =< x_i >_{i\in I}$, $\mathbf{y} =< y_i >_{i\in I}$ $x_i, y_i \in X_i$ and $\mathbf{e} =< e_i >_{i\in I}$, $\mathbf{f} =< f_i >_{i\in I}, e_i, f_i \in E_i$. Then there exists atleast one $\beta \in I$ such that $x_\beta \neq y_\beta$ or there exist $e_{i_k},\ e_{i_m} \in E_i$ such that $e_{i_k} \neq e_{i_m}$.

Case: 1

If $x_\beta \neq y_\beta$, $(p_q)_\beta$ $(\mathbf{x_e}) = (p_{\beta_{q_\beta}})$ $(\mathbf{x_e}) = p_\beta(\mathbf{x})_{\mathbf{q}_\beta(\mathbf{e})} = x_{\beta_{e_\beta}}$ and $(p_q)_\beta$ $(\mathbf{y_f}) = (p_{\beta_{q_\beta}})$ $(\mathbf{y_f}) = p_\beta(\mathbf{y})_{\mathbf{q}_\beta(\mathbf{f})} = y_{\beta_{f_\beta}}$. Since X_β is a soft T_2 space, there are disjoint soft open sets (F_β, E_β) and $(G_\beta,\ E_\beta)$ such that $x_{\beta_{e_\beta}}$ $\tilde{\in}$ (F_β, E_β) and $y_{\beta_{f_\beta}}$ $\tilde{\in}(G_\beta,\ E_\beta)$. Then the subbasic members $(p_q)_\beta^{-1}(F_\beta,\ E_\beta)$ and $(p_q)_\beta^{-1}$ $(G_\beta,\ E_\beta)$ are the soft open sets such that $\mathbf{x_e}$ $\tilde{\in}$ $(p_q)_\beta^{-1}$ (F_β, E_β) and $\mathbf{y_f}$ $\tilde{\in}$ $(p_q)_\beta^{-1}$ $(G_\beta,\ E_\beta)$. Let $\mathbf{z_g}$ $\tilde{\in}$ $(p_q)_\beta^{-1}$ $(F_\beta,$ $E_\beta)\tilde{\cap}$ $(p_q)_\beta^{-1}$ $(G_\beta,\ E_\beta)$, where $\mathbf{z} =< z_i >_{i\in I}, z_i \in X_i$ and $\mathbf{g} =< g_i >_{i\in I}, g_i \in E_i$. Then $\mathbf{z_g}\tilde{\in}$ $(p_q)_\beta^{-1}(F_\beta,\ E_\beta)$ and $\mathbf{z_g}\tilde{\in}$ $(p_q)_\beta^{-1}$ (G_β, E_β). $(p_q)_\beta$ $(\mathbf{z_g})$ $\tilde{\in}$ $(p_q)_\beta((p_q)_\beta^{-1}$ $(F_\beta, E_\beta))$ and $(p_q)_\beta$ $(\mathbf{z_g})$ $\tilde{\in}$ $(p_q)_\beta((p_q)_\beta^{-1}$ $(G_\beta, E_\beta))$. That is $z_{\beta_{g_\beta}}$ $\tilde{\in}$ (F_β, E_β) and $z_{\beta_{g_\beta}}$ $\tilde{\in}$ (G_β, E_β) which is a contradiction to our assumption of soft T_2 space.

Case: 2

If $e_{i_k} \neq e_{i_m}$, there are disjoint soft open sets (F_{i_k}, E_i) and (F_{i_m}, E_i) such that $x_{e_{i_k}}$ $\tilde{\in}$ (F_{i_k}, E_i) and $x_{e_{i_m}}(= y_{e_{i_m}})$ $\tilde{\in}$ (F_{i_m}, E_i). Then $(p_q)_i^{-1}$ (F_{i_k}, E_i) and $(p_q)_i^{-1}$ $(F_{i_m},$ $E_i)$ are disjoint soft open sets containing $\mathbf{x_e}$ and $\mathbf{y_f}$ respectively. Let $\mathbf{z_g}$ $\tilde{\in}$ $(p_q)_i^{-1}$ $(F_{i_k}, E_i)\tilde{\cap}$ $(p_q)_i^{-1}$ (F_{i_m}, E_i), where $\mathbf{z} =< z_i >_{i\in I}, z_i \in X_i$ and $\mathbf{g} =< g_i >_{i\in I}, g_i \in E_i$. Then $\mathbf{z_g}$ $\tilde{\in}$ $(p_q)_i^{-1}(F_{i_k}, E_i)$ and $\mathbf{z_g}\tilde{\in}$ $(p_q)_i^{-1}$ (F_{i_m}, E_i). $(p_q)_i$ $(\mathbf{z_g})$ $\tilde{\in}$ $(p_q)_i((p_q)_i^{-1}$ $(F_{i_k}, E_i))$ and $(p_q)_i$ $(\mathbf{z_g})$ $\tilde{\in}$ $(p_q)_i((p_q)_i^{-1}$ $(F_{i_m}, E_i))$. That is $z_{i_{g_i}}$ $\tilde{\in}$ (F_{i_k}, E_i) and $z_{i_{g_i}}$ $\tilde{\in}$ (F_{i_m}, E_i) which is a contradiction to our assumption of soft T_2 space. □

Definition 2.7. Let $(X,\ \tau,\ E)$ be a soft topological space. Then $(X,\ \tau,\ E)$ is a soft Urysohn space or soft $T_{2\frac{1}{2}}$ space if for any two soft points x_{e_i} and y_{e_j}, there exist two soft open sets $(F,\ E)$ and $(G,\ E)$ such that $x_{e_i}\tilde{\in}(F,\ E)$, $y_{e_j}\tilde{\in}(G,\ E)$ and $Cl(F,\ E)\tilde{\cap}Cl(G,\ E) = \tilde{\phi}$.

Theorem 2.15. *Every soft $T_{2\frac{1}{2}}$-space is a soft T_2-space.*

Proof. Proof is straight forward ☐

Theorem 2.16. *Soft subspace of soft $T_{2\frac{1}{2}}$-space is a soft $T_{2\frac{1}{2}}$-space.*

Proof. Proof is similar to theorem 2.10 ☐

Theorem 2.17. *Let $(X,\ \tau,\ E)$ be a soft topological space. Then $(X,\ \tau,\ E)$ is a soft $T_{2\frac{1}{2}}$ space if and only if for any two soft points x_{e_i} and y_{e_j}, there exist two soft open sets $(H,\ E)$ and $(K,\ E)$ such that $x_{e_i}\tilde{\in}(H,\ E)$, $y_{e_j}\tilde{\in}(K,\ E)$ and $(H,\ E)$ and $(K,\ E)$ containing the disjoint closed soft neighbourhoods of x_{e_i} and y_{e_j} respectively with $(H,\ E)\tilde{\cup}(K,\ E)=\tilde{X}$.*

Proof. Since $(X,\ \tau,\ E)$ be a soft $T_{2\frac{1}{2}}$ space, for any soft points x_{e_i} and y_{e_j}, there exist two soft open sets $(F,\ E)$ and $(G,\ E)$ such that $x_{e_i}\tilde{\in}(F,\ E)$ and $y_{e_j}\tilde{\in}(G,\ E)$ such that $Cl(F,\ E)\tilde{\cap}Cl(G,\ E)=\tilde{\phi}$. Now $x_{e_i}\tilde{\in}[Cl(G,\ E)]^c$ and $y_{e_j}\tilde{\in}[Cl(F,\ E)]^c$ and $[Cl(F,\ E)]^c\tilde{\cup}[Cl(G,\ E)]^c=\tilde{X}$. Note that $Cl(F,\ E)\tilde{\subseteq}[Cl(G,\ E)]^c$ and $Cl(G,\ E)\tilde{\subseteq}[Cl(F,\ E)]^c$. Let $[Cl(F,\ E)]^c=(K,\ E)$ and $[Cl(G,\ E)]^c=(H,\ E)$. Then we have two soft open sets $(H,\ E)$ and $(K,\ E)$ containing x_{e_i} and y_{e_j} respectively, such that $x_{e_i}\tilde{\in}(F,\ E)\tilde{\subseteq}Cl(F,\ E)\ \tilde{\subseteq}(H,\ E)$, $y_{e_j}\tilde{\in}(G,\ E)\tilde{\subseteq}\ Cl(G,\ E)\ \tilde{\subseteq}(K,\ E)$ and $(H,\ E)\tilde{\cup}(K,\ E)=\tilde{X}$. Thus $(H,\ E)$ and $(K,\ E)$ are soft open sets containing the disjoint closed neighbourhoods $Cl(F,\ E)$ and $Cl(G,\ E)$, respectively such that $x_{e_i}\tilde{\in}Cl(F,\ E)$, $y_{e_j}\tilde{\in}Cl(G,\ E)$ and $(H,E)\tilde{\cup}(K,E)=\tilde{X}$.

Conversely, let x_{e_i} and y_{e_j} be two distinct soft points. By our assumption, there exist two soft open sets (H,E) and (K,E) containing disjoint closed neighbourhoods (L,E) and (M,E) of x_{e_i} and y_{e_j} respectively such that $(H,E)\tilde{\cup}(K,E)=\tilde{X}$. Note that there are soft open sets (F,E) and (G,E) such that $(F,E)\tilde{\subseteq}Cl(F,E)\tilde{\subseteq}(L,E)\tilde{\subseteq}(H,E)$, $(G,E)\tilde{\subseteq}Cl(G,E)\tilde{\subseteq}(M,E)\ \tilde{\subseteq}(K,E)$ and $(L,E)\tilde{\cap}(M,E)=\tilde{\phi}$. So that $Cl(F,E)\tilde{\cap}Cl(G,E)=\tilde{\phi}$. That is (F,E) and (G,E) are soft open sets containing x_{e_i} and y_{e_j} such that $Cl(F,E)\tilde{\cap}Cl(G,E)=\tilde{\phi}$. Thus (X,τ,E) is a soft $T_{2\frac{1}{2}}$ space. ☐

Soft single point space discussed in [5] is not a soft T_0 or T_1 or T_2 or $T_{2\frac{1}{2}}$ space. Because for the soft points x_{e_i} and x_{e_j}, there is no soft open set containing x_{e_i} not containing x_{e_j}.

Theorem 2.18. *Product of soft $T_{2\frac{1}{2}}$-spaces is a soft $T_{2\frac{1}{2}}$-space*

Proof. Proof is similar to theorem 2.14 ☐

Lemma 2.19. *Let (X,τ,E) be a finite soft $T_{2\frac{1}{2}}$ space. Then (X,τ,E) is a soft discrete space.*

Proof. Proof follows from theorem:2.15, theorem:2.9 and theorem:2.5. ☐

Lemma 2.20. *If (X,τ,E) is a countable soft $T_{2\frac{1}{2}}$ space and if every soft G_δ set is soft open in $(X,\ \tau,\ E)$, then (X,τ,E) is a soft discrete space.*

Proof. Proof follows from theorem:2.15, theorem:2.9 and theorem:2.6. ☐

3. Conclusion

For the soft separation axioms of soft points defined on soft topological space, we discuss the characterizations and properties of soft $T_0,\ T_1,\ T_2$ and soft $T_{2\frac{1}{2}}$ spaces. Also it is verified that the product of soft T_i spaces, $i = 1, 2, 2\frac{1}{2}$ is a soft T_i space. But there is an example given here for the product of soft T_0 spaces need not be a soft T_0 space. Also we provide correct examples for the wrong examples example:1, example:2 and example:3 given in article [8].

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