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Application of the Advanced Exp $(-\varphi(\xi))$-Expansion Method to the Nonlinear Conformable Time-Fractional Partial Differential Equations

Md. Habibul Bashar*, Tasnim Tahseen, Nur Hasan Mahmud Shahen

Department of Mathematics, European University of Bangladesh, Dhaka-1216, Dhaka, Bangladesh.

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Abstract. With the assistance of representative calculation programming, the present paper examines the careful voyaging wave arrangements from the nonlinear time fractional modified Kawahara equation by utilizing the advanced $\exp(-\varphi(\xi))$-expansion strategy in-terms of hyperbolic, trigonometric and rational function with some appreciated parameters. The dynamics nonlinear wave solution is examined and demonstrated by maple18 in 3-D, 2-d plots and contour plot with specific values of the intricate parameters are plotted. The advanced $\exp(-\varphi(\xi))$-expansion method is reliable treatment for searching essential nonlinear waves that enrich a variety of dynamic models that arises in engineering fields.

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Keywords: Conformable fractional derivative, advanced $\exp(-\varphi(\xi))$-expansion method, exact solution, time fractional modified Kawahara equation.

1. Introduction

As of late, fractional partial differential equations (FPDEs) is comprehensively used to delineate various huge marvels and dynamic methodology in various fields of science and designing, particularly in liquid mechanics, hydrodynamics, numerical science, dissemination process, strong state material science, plasma material science, neural material science, substance energy and geo-optical filaments. [25, 28, 31]. Numerous researchers arranged through nonlinear evolution equations (NEEs) to build voyaging wave arrangement by executing a few techniques. The methods that are entrenched in ongoing writing, for example, the sub-equation method [26, 33], the improved sub-equation meth-od [18, 38], the modified simple equation method [3],the tanh-coth method [29], sine-cosine method [30],the first integral method [11], the $(G'/G, 1/G)$ −expansion method [23], the exponential rational function method [2], the extended Kudryashov method [39], the modified simple equation method [7], the new extended (G'/G) expansion method [12, 34], the darboux transformation [36], the trial solution method [10], the exp-function Method [19], the multiple simplest equation method [42], $\exp(-\varphi(\xi))$-expansion method [4, 6], Pseudo parabolic model [35, 37], the sine-Gordon expansion method [16] ,the complex solitons in the conformable (2+1)-dimensional Ablowitz-Kaup-Newell-Segur equation [15], the modified auxiliary expansion method [13],the method of line [14],the bernoulli sub-equation function method [8, 9], The modified exponential function method [20], the improved Bernoulli sub equation

**Corresponding Author*
Email addresses: habibul.bashar@eub.edu.bd (M.H. Bashar), tahseentasnim@gmail.com (T. Tahseen), nhmshahenmath@eub.edu.bd (N.H.M. Shahen)

function method [40],the finite difference method [41] and so on. The purpose of this paper is to examine the approximated solution of the nonlinear time fractional modified Kawahara equation in the form

$$D_t^{\delta}u + u^2u_x + \alpha u_{xx} + \beta u_{xxx} = 0, t > 0, \ x \in \mathbb{R} \tag{1.1}$$

where δ is a parameter describing the order of the fractional time derivative and $0 < \delta \leq 1$.The nonlinear time fractional modified Ka-wahara equation was studied by different researchers for in-stance, Atangana et al. [5] studied the numerical solutions of time fractional modified nonlinear Kawahara equation using the homotopy decomposition and the Sumudu transform methods. Kumar and rayand solved the time fractional modified nonlinear Kawahara equation using extended $\exp(-\varphi(\xi))$-expansion method [17].Guner and Hasan [27] solved the time fractional modified nonlinear Kawahara equation using another analytical method namely fractional exp-function method and secured some exact soliton solutions.

The target of this article is to apply the advanced $\exp(-\varphi(\xi))$-expansion strategy [32] to build the precise voyaging wave answers for nonlinear advancement conditions in scientific material science by means of the time fractional nonlinear modified Kawahara equations. . The article is set up as pursues: In section 2, the description of the conformable fractional derivative and method are discussed, In section 3, the advanced $\exp(-\varphi(\xi))$-expansion scheme has been talked about. In segment 4, we apply this plan to then nonlinear modified Kawahara equations. In section 5, represents Results and Discussion, In section 6, ends are given.

2. Preliminaries and Methods

2.1. **Definition and Some Features of Conformable Fractional Derivative.** The conformable derivative with a limit operator which was initially introduced by Khalil et al. [24].

Definition 2.1. $f : (0, \infty) \to \mathbb{R}$, then, the conformable derivative of f of order δ is defined as

$$D_t^{\delta} f(t) = \lim_{\varepsilon \to 0} \left(\frac{f\left(t + \varepsilon t^{1-\delta}\right) - f(t)}{\varepsilon} \right) for\ all\ t > 0, \ 0 < \delta \leq 1.$$

Later, Abdeljawad [1] has also offered chain rule, exponential functions, Gronwalls inequality, integration by parts, Taylor power series expansions and Laplace transform for conformable derivative in fractional versions. The definition of conformable derivative can easily overcome the difficulties of exiting modified Riemann-Liouville derivative definition [21].

Theorem 2.2. *Let $\delta \in (0, 1]$ and $f = f(t), g = g(t)$ be δ conformable differentiable at a point $t > 0$ then:*

(1) $D_t^{\delta}(cf + dg) = cD_t^{\delta} f + dD_t^{\delta} g \ \forall\ c, d \in \mathbb{R}$.
(2) $D_t^{\delta}(t\gamma) = \gamma t^{\gamma-\delta} \forall\ \gamma \in \mathbb{R}$.
(3) $D_t^{\delta}(fg) = gD_t^{\delta}(f) + fD_t^{\delta}(g)$.
(4) $D_t^{\delta}(f/g) = \frac{gD_t^{\delta}(f) - fD_t^{\delta}(g)}{g^2}$.

Furthermore, if f is differentiable, then $D_t^{\delta}(f(t)) = t^{1-\delta}\frac{df}{dt}$.

Theorem 2.3. *Let $f : (0, \propto) \to R$ be a function such that f is differentiable and alpha-conformable differentiable. Also, let g be a differentiable function defined in the range of f. Then*

$$D_t^{\delta}(fog)(t) = t^{1-\delta} g(t)^{\delta-1} g'(t),$$
$$D_t^{\delta}(f(t))_t = g(t).$$

3. The Advanced $\exp(-\varphi(\xi))$ -Expansion Method

In this section, we will precis $\exp(-\varphi(\xi))$-expansion method step by step. Consider a nonlinear partial differential equation in the following form,

$$P(U, U_{xx}, U_{xz}, U_{xx}, U_{xy}, U_{xtt}, \ldots \ \ldots) = 0 \tag{3.1}$$

where $U = U(x, y, z, t)$ is an unknown function, R is a polynomial of U, its different type partial derivatives, in which the nonlinear terms and the highest order derivatives are involved.
Step 1: Now we consider a transformation variable to convert all independent variable into one variable, such as,

$$U(x,t) = u(\xi), \xi = kx + ly + mz \pm Vt. \tag{3.2}$$

By implementing this variable Eq. (3.2) permits us reducing Eq.(3.1) in an ODE for $u(x,t) = u(\xi)$

$$P(u,\ u',\ u'', \cdots\ \ \cdots\ \cdots) = 0. \tag{3.3}$$

Step 2: Suppose that the solution of ODE Eq. (3.3) can be expressed by a polynomial in $\exp(-\varphi(\xi))$ as follows

$$u = \sum_{i=0}^{m} a_i \exp(-\phi(\xi))^i,\ a_m \neq 0 \tag{3.4}$$

where the positive integer m can be determined by balancing the highest order derivatives to the highest order nonlinear terms appear in Eq. (3.3). And the derivative of $\phi(\xi)$ satisfies the ODE in the following form

$$\varphi'(\xi) + \lambda \exp(-\varphi(\xi)) + \mu \exp(\varphi(\xi)) = 0 \tag{3.5}$$

then the solutions of ODE Eq. (3.5) are
Case I: Hyperbolic function solution (when $\lambda\mu < 0$):

$$\varphi(\xi) = \ln\left(\sqrt{\frac{\lambda}{-\mu}} \tanh(\sqrt{-\lambda\mu}(\xi + C))\right)$$
and
$$\varphi(\xi) = \ln\left(\sqrt{\frac{\lambda}{-\mu}} \coth(\sqrt{-\lambda\mu}(\xi + C))\right).$$

Case II: Trigonometric function solution (when $\lambda\mu > 0$):

$$\varphi(\xi) = \ln\left(\sqrt{\frac{\lambda}{\mu}} \tan(\sqrt{\lambda\mu}(\xi + C))\right)$$
and
$$\varphi(\xi) = \ln\left(-\sqrt{\frac{\lambda}{\mu}} \cot(\sqrt{\lambda\mu}(\xi + C))\right).$$

Case III: When $\mu > 0$ and $\lambda = 0$

$$\varphi(\xi) = \ln\left(\frac{1}{-\mu(\xi + C)}\right).$$

Case IV: When $\mu = 0$ and $\lambda \in \Re$

$$\varphi(\xi) = \ln(\lambda(\xi + C)).$$

Where C is integrating constants and $\lambda\mu < 0$ or $\lambda\mu > 0$ depends on sign of μ.
Step 3: By substituting Eq. (3.4) into Eq. (3.3) and using the ODE (3.5), collecting all same order of $\exp(-m\varphi(\xi)), m = 0, 1, 2, 3...$ together, then we execute an polynomial form of $\exp(-m\varphi(\xi))$. Equating each coefficients of this polynomial to zero, yields a set of algebraic system.
Step 4: Assume the estimation of the constants can be gotten by fathoming the mathematical conditions got in step 4. Substituting the estimations of the constants together with the arrangements of Eq. (3.5), we will acquire new and far reaching precise traveling wave arrangements of the nonlinear development Eq. (3.1).

4. Application of the Method

In this subsection we implement advanced $\exp(-\phi(\xi))$ expansion method in Eq. (1.1) And now using the following transformation:
$u(x,t) = u(\xi), \xi = kx - \frac{\omega t^{\delta}}{\delta}$. where k and ω are nonzero constants. We get ODE from Eq. (1.1)

$$-\omega u' + ku^2u' + \alpha k^2 u'' + \beta k^3 u''' = 0. \tag{4.1}$$

Now we integrating the Eq. (4.1) with respect to ξ and we get

$$-\omega u + \frac{k}{3}u^3 + \alpha k^2 u' + \beta k^3 u'' = 0. \tag{4.2}$$

where symbolize prime represent the derivative with respect to ξ. Now we compute the balance number of Eq. (4.2) between the linear term u'' and the nonlinear term u^3 is $m = 1$so the solution Eq. (4.2) takes the form

$$u(\xi) = A_0 + A_1 \exp(-\varphi(\xi)). \tag{4.3}$$

Differential Eq. (4.3) with respect to ξ and substituting the value of u, u', u'' into the Eq. (4.2) and equating the coefficients of $e^{i\varphi(\xi)}$ is equal to zero where $i = 0, \pm1, \pm2...$ Solving those the system of equations, we attain the only two set solutions.

Set 1:

$$k = \pm\frac{\sqrt{-\frac{1}{36\lambda\mu}}\alpha}{\beta}, \omega = \mp\frac{2}{9}\frac{\sqrt{-\frac{1}{36\lambda\mu}}\alpha^3}{\beta^2},\ A_0 = \alpha\sqrt{-\frac{1}{6\beta}},\quad A_1 = \pm\frac{1}{6}\frac{\alpha\sqrt{-\frac{1}{6\beta}}}{\sqrt{-\frac{1}{36\lambda\mu}}\mu}.$$

Set 2:

$$k = \pm\frac{\sqrt{-\frac{1}{36\lambda\mu}}\alpha}{\beta}, \omega = \mp\frac{2}{9}\frac{\sqrt{-\frac{1}{36\lambda\mu}}\alpha^3}{\beta^2},\quad A_0 = -\alpha\sqrt{-\frac{1}{6\beta}}, A_1 = \mp\frac{1}{6}\frac{\alpha\sqrt{-\frac{1}{6\beta}}}{\sqrt{-\frac{1}{36\lambda\mu}}\mu}.$$

Case I: When $\lambda\mu < 0$ we get following hyperbolic solution

Family-1

$$u_{1,2}(x,t) = \frac{\alpha}{6}\sqrt{-\frac{6}{\beta}} \pm \frac{1}{6}\frac{\alpha\sqrt{-\frac{6}{\beta}}}{\sqrt{-\frac{1}{\lambda\mu}}\mu\sqrt{\frac{-\lambda}{\mu}}\tanh\left(\sqrt{-\lambda\mu}\,(\xi + C)\right)},$$

$$u_{3,4}(x,t) = \frac{\alpha}{6}\sqrt{-\frac{6}{\beta}} \pm \frac{1}{6}\frac{\alpha\sqrt{-\frac{6}{\beta}}}{\sqrt{-\frac{1}{\lambda\mu}}\mu\sqrt{\frac{-\lambda}{\mu}}\coth\left(\sqrt{-\lambda\mu}\,(\xi + C)\right)},$$

Family-2

$$u_{5,6}(x,t) = -\frac{\alpha}{6}\sqrt{-\frac{6}{\beta}} \mp \frac{1}{6}\frac{\alpha\sqrt{-\frac{6}{\beta}}}{\sqrt{-\frac{1}{\lambda\mu}}\mu\sqrt{\frac{-\lambda}{\mu}}\tanh\left(\sqrt{-\lambda\mu}\,(\xi + C)\right)},$$

$$u_{7,8}(x,t) = -\frac{\alpha}{6}\sqrt{-\frac{6}{\beta}} \mp \frac{1}{6}\frac{\alpha\sqrt{-\frac{6}{\beta}}}{\sqrt{-\frac{1}{\lambda\mu}}\mu\sqrt{\frac{-\lambda}{\mu}}\coth\left(\sqrt{-\lambda\mu}\,(\xi + C)\right)},$$

where $\xi = \pm\frac{\sqrt{-\frac{1}{36\lambda\mu}}\alpha}{\beta}\ x\ \pm \frac{\frac{2}{9}\frac{\sqrt{-\frac{1}{36\lambda\mu}}\alpha^3}{\beta^2}t^{\delta}}{\delta}$.

Case II: When $\lambda\mu > 0$ we get following trigonometric solution

Family-3

$$u_{9,10}(x,t) = \frac{\alpha}{6}\sqrt{-\frac{6}{\beta}} \pm \frac{1}{6}\frac{\alpha\sqrt{-\frac{6}{\beta}}}{\sqrt{-\frac{1}{\lambda\mu}}\mu\sqrt{\frac{\lambda}{\mu}}\tan\left(\sqrt{\lambda\mu}\,(\xi + C)\right)},$$

$$u_{11,12}(x,t) = \frac{\alpha}{6}\sqrt{-\frac{6}{\beta}} \mp \frac{1}{6}\frac{\alpha\sqrt{-\frac{6}{\beta}}}{\sqrt{-\frac{1}{\lambda\mu}}\mu\sqrt{\frac{\lambda}{\mu}}\cot\left(\sqrt{\lambda\mu}\,(\xi + C)\right)}$$

Family-4

$$u_{13,14}(x,t) = -\frac{\alpha}{6}\sqrt{-\frac{6}{\beta}} \mp \frac{1}{6}\frac{\alpha\sqrt{-\frac{6}{\beta}}}{\sqrt{-\frac{1}{\lambda\mu}}\mu\sqrt{\frac{\lambda}{\mu}}\tan\left(\sqrt{\lambda\mu}\,(\xi + C)\right)},$$

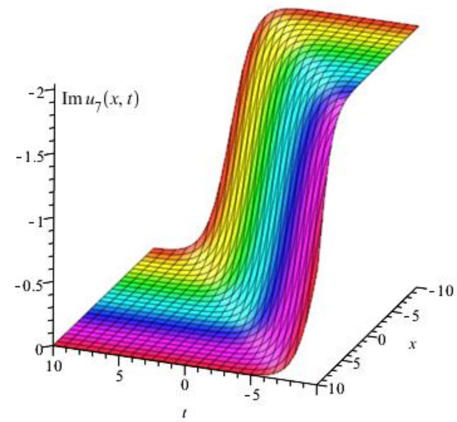
$$u_{15,16}(x,t) = -\frac{\alpha}{6}\sqrt{-\frac{6}{\beta}} \pm \frac{1}{6}\frac{\alpha\sqrt{-\frac{6}{\beta}}}{\sqrt{-\frac{1}{\lambda\mu}}\mu\sqrt{\frac{\lambda}{\mu}}\cot\left(\sqrt{-\lambda\mu}\,(\xi + C)\right)}$$

where $\xi = \pm\frac{\sqrt{-\frac{1}{36\lambda\mu}}\alpha}{\beta}\,x \pm \frac{\frac{2}{9}\,\frac{\sqrt{-\frac{1}{36\lambda\mu}}\alpha^3}{\beta^2}t^\delta}{\delta}$.

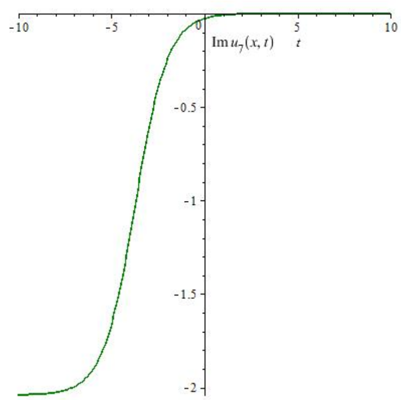
Case III and Case IV: When $\lambda = 0$ the executing value of A_1 undefined. So the solution cannot be obtained. For this purpose this case is rejected. Similarly when $\mu = 0$ the executing value of A_1 undefined. So the solution cannot be obtained. So this case is also rejected.

5. Results and Discussions

5.1. **Graphical Explanation:** This sub-section represents the graphical representation of the time fractional modified nonlinear Kawahara equations. By using mathematical software Maple 18, Contour, 3D and 2D plots of some achieved solutions have been shown in Fig. 1 - Fig. 6 to envisage the essential instrument of the original equations.

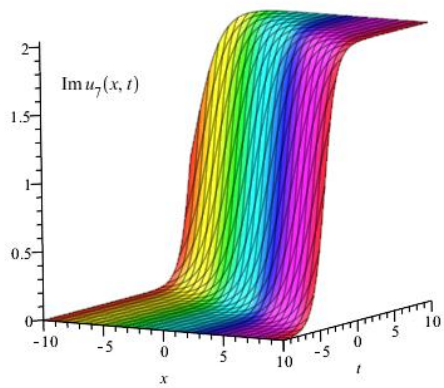


(A) 3D plot

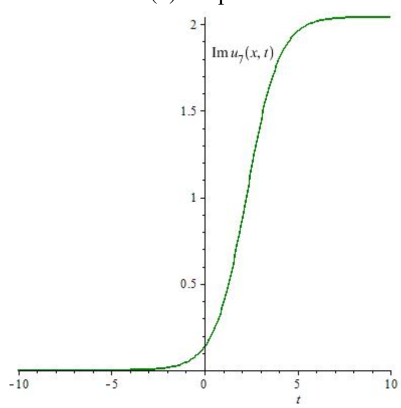


(B) 2D plot

Figure 1. Anti bright kink shape solution of u_7 for $\alpha = 2.5, \beta = 1, \delta = 1, C = 1, \lambda = 3, \mu = -1$ within $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$.

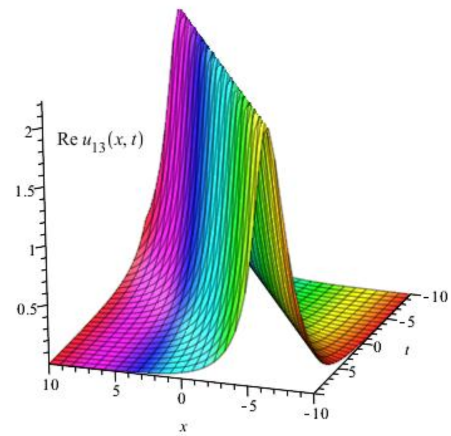


(A) 3D plot

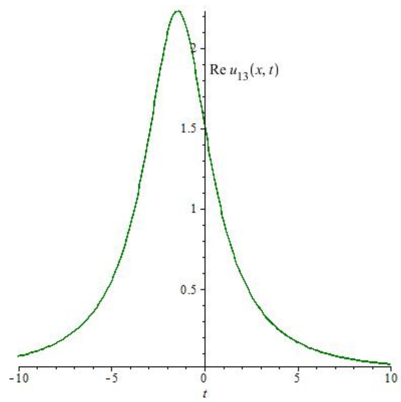


(B) 2D plot

FIGURE 2. Anti bright kink shape solution of u_7 for $\alpha = -2.5, \beta = 1, \delta = 1, C = 1, \lambda = 3, \mu = -1$ within $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$.

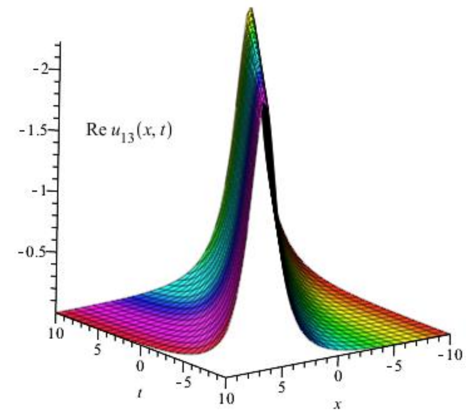


(A) 3D plot

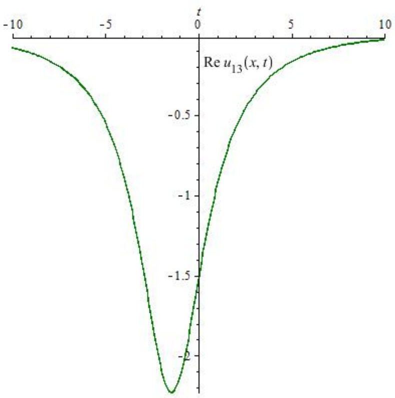


(B) 2D plot

FIGURE 3. Bright bell shape solution of u_{13} for $\alpha = 2.5, \beta = 2, \delta = 1, C = 1, \lambda = 4, \mu = 2$ within $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$.

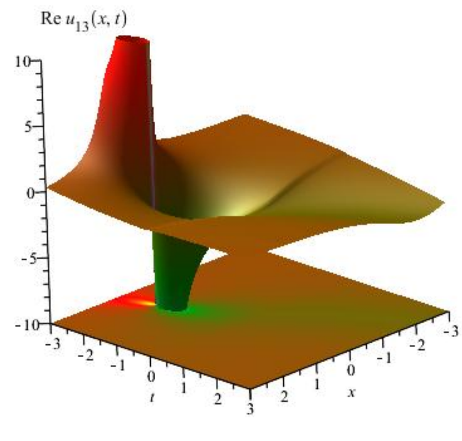


(A) 3D plot

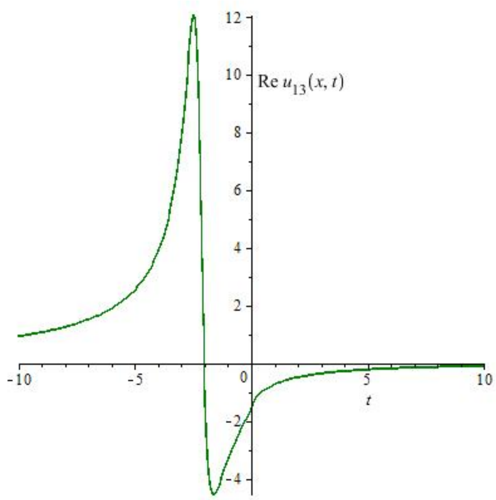


(B) 2D plot

Figure 4. Dark bell shape solution of u_{13} for $\alpha = -2.5, \beta = 2, \delta = 1, C = 1, \lambda = 4, \mu = 2$ within $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$.

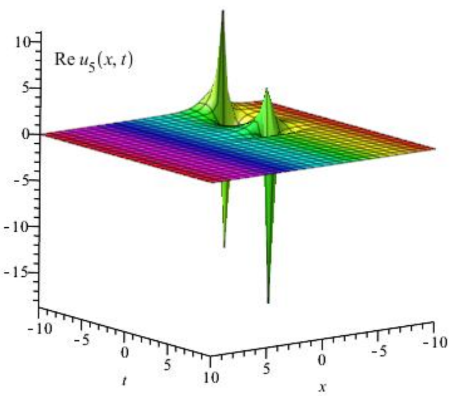


(A) Contour plot

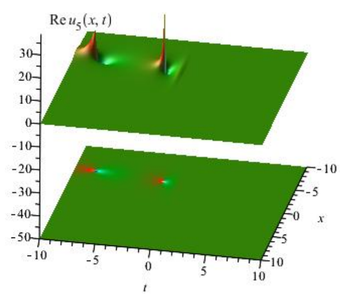


(B) 2D plot

FIGURE 5. Lump shape solution of u_{13} for $\alpha = -2.5, \beta = 2, \delta = .65, C = 1, \lambda = 4, \mu = 2$ within $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$.



(A) 3D plot



(B) Contour plot

FIGURE 6. Lump shape solution of u_5 for $\alpha = 2.9, \beta = .9, \delta = .45, C = 1, \lambda = 3, \mu = -1$ within $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$.

6. CONCLUSIONS

In this segment, we have seen that two kinds of traveling wave arrangements as far as hyperbolic and trigonometric capacities for the time fractional modified nonlinear Kawahara equations is effectively discovered by utilizing the advancedexp $(-\varphi(\xi))$-expansion method under some parametric conditions. From our outcomes got in this paper, we finish up the advanced exp $(-\varphi(\xi))$-expansion method strategy is amazing, powerful and helpful. The exhibition of this technique is dependable, basic and gives numerous new arrangements. This equation could be a model equation of pulse extension in communication systems. We investigated the stability of the solitary wave solutions on the parametric conditions. We also investigated to examine the approximated solution of the nonlinear time fractional modified Kawahara equation. As an outcomes, the progressed exp $(-\varphi(\xi))$- extension technique shows a significant method to discover novel voyaging wave arrangements. We here included six types of solutions. Anti bright kink shape, Bright Bell kink shape, Dark bell shape, lump shape and other Soliton shape solutions are found. The got arrangements in this paper uncover that the technique is a powerful and effectively material of defining more definite voyaging wave arrangements than others strategy for the nonlinear advancement conditions emerging in numerical physical science.

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Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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