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Hermite-Hadamard Type Inequalities for Multiplicatively h-Convex Functions

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Abstract

In this paper, some Hermite-Hadamard type inequalities for multiplicatively h-convex functions are established. Also, new integral inequalities involving multiplicative integrals are obtained for product and quotient of multiplicatively h-convex and convex positive functions.

Keywords: *Convex functions, multiplicatively h-convex functions, multiplicative calculus, Hermite-Hadamard type inequalities*
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1. Introduction

The concept of multiplicative calculus or non-Newtonian calculus has emerged as a new kind of derivative and integral by changing the roles of addition and subtraction with multiplication and division. One of the initial studies of multiplicative calculus was made by [12] in 1970s. This study modified the classical calculus introduced by Newton and Leibnitz in the 17th century. Since the application area of multiplicative calculus is quite limited, it isn't so popular as the calculus of Newton and Leibnitz. In fact, it only covers positive functions. On the other hands, a number of interesting results has been obtained due to its many applications in various fields. For example, in [5] Bashirov et al. gave a fundamental theorem of multiplicative calculus. In [3], Bashirov and Rıza introduced complex multiplicative calculus. In [8] and [17], some properties of stochastic multiplicative calculus have been studied. For some applications and other aspects of this discipline, see [2, 3, 4, 5, 24, 26] and the references cited therein.
Recall that the concept of multiplicative integral called * integral is denoted by $\int_a^b (f(x))^{dx}$ while the ordinary integral is denoted by $\int_a^b (f(x))\,dx$. This comes from the fact that the sum of the terms of product is used in the definition of a classical Riemann integral of f on $[a,b]$, the product of terms raised to certain powers is used in the definition of multiplicative integral of f on $[a,b]$.
There is the following relation between Riemann integral and * integral [5]:

Proposition 1.1. *If f is Riemann integrable on $[a,b]$, then f is * integrable on $[a,b]$ and*

$$\int_a^b (f(x))^{dx} = e^{\int_a^b \ln(f(x))dx}.$$

In [5], Bashirov et al. show that * integral has the following results and notations:

Proposition 1.2. *If f is positive and Riemann integrable on $[a,b]$, then f is * integrable on $[a,b]$ and*

1. $\int_a^b ((f(x))^p)^{dx} = \int_a^b \left((f(x))^{dx}\right)^p$,
2. $\int_a^b (f(x)g(x))^{dx} = \int_a^b (f(x))^{dx} \cdot \int_a^b (g(x))^{dx}$,
3. $\int_a^b \left(\frac{f(x)}{g(x)}\right)^{dx} = \frac{\int_a^b (f(x))^{dx}}{\int_a^b (g(x))^{dx}}$,
4. $\int_a^b (f(x))^{dx} = \int_a^\mu (f(x))^{dx} \cdot \int_\mu^b (f(x))^{dx}$, $a \le \mu \le b$,
5. $\int_a^a (f(x))^{dx} = 1$ and $\int_a^b (f(x))^{dx} = \left(\int_b^a (f(x))^{dx}\right)^{-1}$.

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2. Preliminaries

The function $f:[a,b]\subset\mathbb{R}\to\mathbb{R}$ is said to be convex in the classical sense if the following inequality holds:

$$f(\lambda x+(1-\lambda)y)\leq\lambda f(x)+(1-\lambda)f(y)$$

for all $x,y\in[a,b]$ and $\lambda\in[0,1]$. The function f is said to be concave if $-f$ is convex.
One of the most famous inequalities related to the integral mean of a convex function is the Hermite-Hadamard inequality. This double inequality is stated as follows (see, [11, 13, 25])
Let $f:I=[a,b]\subset\mathbb{R}\to\mathbb{R}$ is an integrable convex function. Then

$$f\left(\frac{a+b}{2}\right)\leq\frac{1}{b-a}\int_a^b f(x)\,dx\leq\frac{f(a)+f(b)}{2}. \tag{2.1}$$

Both inequalities hold in the reversed direction if f is concave.
Hermite-Hadamard inequality can be considered as a refinement of the concept of convexity. This inequality has been studied extensively by a number of authors, since it is discovered by Hermite (1883) and Hadamard (1896),independently. Especially, over the last twenty years, numerous researchers have focused on to obtain new boundaries for left and right hand sides of the Hermite-Hadamard inequality. For some results which generalize, improve and extend the inequalities (2.1) please refer the monographs [1, 7, 9, 10, 11, 14, 15, 16, 18, 20, 21, 22, 23, 27, 29].
Now it is time to give some basic definitions and results which are used throughout the paper:
In what follows I and $\Im$ be intervals.

Definition 2.1. *[28] Let $h:I\to\mathbb{R}$ be a non-negative function. A non-negative function $f:\Im\to(0,\infty)$ is said to be h-convex, or $f\in SX(h,\Im)$, if*

$$f((1-\lambda)x+\lambda y)\leq h(1-\lambda)f(x)+h(\lambda)f(y)$$

holds for all $x,y\in\Im$ and $\lambda\in(0,1)$.

Note that, the concept of h-convex functions is a generalization of several other classes of convex functions. For example, if $h(\lambda)=\lambda$, $h(\lambda)=\lambda^s$ and $h(\lambda)=1$, then Definition 2.1 reduces to definitions of convex functions in [19], s-convex functions in [6], and P-convex functions in [9], respectively.

Definition 2.2. *[25] A function $f:\Im\to(0,\infty)$ is said to be log or multiplicatively convex, if*

$$f((1-\lambda)x+\lambda y)\leq[f(x)]^{1-\lambda}[f(y)]^{\lambda}$$

holds for all $x,y\in\Im$ and $\lambda\in[0,1]$.

Definition 2.3. *[20] Let $h:I\to\mathbb{R}$ be a non-negative function. A function $f:\Im\to(0,\infty)$ is said to be multiplicatively h-convex, if*

$$f((1-\lambda)x+\lambda y)\leq[f(x)]^{h(1-\lambda)}[f(y)]^{h(\lambda)}$$

holds for all $x,y\in\Im$ and $\lambda\in[0,1]$.

Remark 2.4. *If we put $h(\lambda)=\lambda$, then Definition 2.3 reduces to Definition 2.2.*

Definition 2.5. *[30] A function $f:\Im\to(0,\infty)$ is said to be multiplicatively s-convex for $s\in(0,1)$ if*

$$f((1-\lambda)x+\lambda y)\leq[f(x)]^{(1-\lambda)^s}[f(y)]^{\lambda^s}$$

holds for all $x,y\in\Im$ and $\lambda\in[0,1]$.

Remark 2.6. *If we put $h(\lambda)=\lambda^s$, then Definition 2.3 reduces to Definition 2.5.*

Definition 2.7. *[15] A function $f:\Im\to(0,\infty)$ is said to be multiplicatively P-convex, if*

$$f((1-\lambda)x+\lambda y)\leq f(x)f(y)$$

holds for all $x,y\in\Im$ and $\lambda\in[0,1]$.

Remark 2.8. *If we put $h(\lambda)=1$, then Definition 2.3 reduces to Definition 2.7.*

Definition 2.9. *[10] A function $f:\Im\to(0,\infty)$ is said to be quasi convex, if*

$$f((1-\lambda)x+\lambda y)\leq\max\{f(x),f(y)\}$$

holds for all $x,y\in\Im$ and $\lambda\in[0,1]$.

From the above definitions we have the following relations:

$$\begin{aligned}f((1-\lambda)x+\lambda y)&\leq[f(x)]^{1-\lambda}[f(y)]^{\lambda}\\&\leq f(x)+\lambda[f(y)-f(x)]\\&\leq\max\{f(x),f(y)\}.\end{aligned}$$

3. Main results

In this section we obtain some Hermite-Hadamard type integral inequalities in the setting of multiplicative calculus for multiplicatively h-convex and convex positive functions.

Theorem 3.1. *Let f be a multiplicatively h-convex function on $[v_1, v_2]$ such that $h\left(\frac{1}{2}\right) \neq 0$. Then the following inequalities hold:*

$$f\left(\frac{v_1+v_2}{2}\right)^{\frac{1}{2h\left(\frac{1}{2}\right)}} \leq \left(\int_{v_1}^{v_2} (f(x))^{dx}\right)^{\frac{1}{v_2-v_1}} \leq [f(v_1).f(v_2)]^{\int_0^1 h(\lambda)d\lambda}. \tag{3.1}$$

(3.1) is called Hermite-Hadamard integral inequalities for multiplicatively h-convex functions.

Proof. Let f be a multiplicatively h-convex function. Note that

$$\begin{aligned}
\ln f\left(\frac{v_1+v_2}{2}\right) &= \ln\left(f\left(\frac{(1-\lambda)v_1+\lambda v_2+\lambda v_1+(1-\lambda)v_2}{2}\right)\right) \\
&= \ln\left(f\left(\frac{(1-\lambda)v_1+\lambda v_2}{2}+\frac{\lambda v_1+(1-\lambda)v_2}{2}\right)\right) \\
&\leq \ln\left((f((1-\lambda)v_1+\lambda v_2))^{h\left(\frac{1}{2}\right)}.(f(\lambda v_1+(1-\lambda)v_2))^{h\left(\frac{1}{2}\right)}\right) \\
&= h\left(\frac{1}{2}\right)\ln(f((1-\lambda)v_1+\lambda v_2))+h\left(\frac{1}{2}\right)\ln(f(\lambda v_1+(1-\lambda)v_2)).
\end{aligned}$$

Integrating the above inequality with respect to λ on $[0,1]$, we have

$$\begin{aligned}
\ln f\left(\frac{v_1+v_2}{2}\right) &\leq h\left(\frac{1}{2}\right)\int_0^1 \ln(f((1-\lambda)v_1+\lambda v_2))d\lambda+h\left(\frac{1}{2}\right)\int_0^1 \ln(f(\lambda v_1+(1-\lambda)v_2))d\lambda \\
&= h\left(\frac{1}{2}\right)\left[\frac{1}{v_2-v_1}\int_{v_1}^{v_2}\ln(f(x))dx+\frac{1}{v_1-v_2}\int_{v_2}^{v_1}\ln(f(x))dx\right] \\
&= h\left(\frac{1}{2}\right)\left[\frac{1}{v_2-v_1}\int_{v_1}^{v_2}\ln(f(x))dx+\frac{1}{v_2-v_1}\int_{v_1}^{v_2}\ln(f(x))dx\right] \\
&= 2h\left(\frac{1}{2}\right)\frac{1}{v_2-v_1}\int_{v_1}^{v_2}\ln(f(x))dx,
\end{aligned}$$

which implies that

$$\frac{1}{2h\left(\frac{1}{2}\right)}\ln f\left(\frac{v_1+v_2}{2}\right) \leq \frac{1}{v_2-v_1}\int_{v_1}^{v_2}\ln(f(x))dx.$$

Thus,

$$\begin{aligned}
f\left(\frac{v_1+v_2}{2}\right)^{\frac{1}{2h\left(\frac{1}{2}\right)}} &\leq e^{\left(\frac{1}{v_2-v_1}\int_{v_1}^{v_2}\ln(f(x))dx\right)} \\
&= \left(\int_{v_1}^{v_2}(f(x))^{dx}\right)^{\frac{1}{v_2-v_1}}.
\end{aligned}$$

Hence, we have

$$f\left(\frac{v_1+v_2}{2}\right)^{\frac{1}{2h\left(\frac{1}{2}\right)}} \leq \left(\int_{v_1}^{v_2}(f(x))^{dx}\right)^{\frac{1}{v_2-v_1}}, \tag{3.2}$$

which completes the proof of the first inequality in (3.1).

Now consider the second inequality in (3.1)

$$\begin{aligned}
\left(\int_{v_1}^{v_2}(f(x))^{dx}\right)^{\frac{1}{v_2-v_1}} &= \left(e^{\left(\int_{v_1}^{v_2}\ln(f(x))dx\right)}\right)^{\frac{1}{v_2-v_1}} \\
&= e^{\frac{1}{v_2-v_1}\left(\int_{v_1}^{v_2}\ln(f(x))dx\right)} \\
&= e^{\left(\int_0^1 \ln(f(v_1+\lambda(v_2-v_1)))d\lambda\right)} \\
&\leq e^{\left(\int_0^1 \ln\left((f(v_1))^{h(1-\lambda)}(f(v_2))^{h(\lambda)}\right)d\lambda\right)} \\
&= e^{\left(\int_0^1 (h(1-\lambda)\ln f(v_1)+h(\lambda)\ln f(v_2))d\lambda\right)} \\
&= e^{\left(\ln(f(v_1).f(v_2))^{\int_0^1 h(\lambda)d\lambda}\right)} \\
&= [f(v_1).f(v_2)]^{\int_0^1 h(\lambda)d\lambda}.
\end{aligned}$$

Hence, we get the inequality

$$\left(\int_{v_1}^{v_2}(f(x))^{dx}\right)^{\frac{1}{v_2-v_1}} \leq [f(v_1).f(v_2)]^{\int_0^1 h(\lambda)d\lambda}. \tag{3.3}$$

Combining the inequalities (3.2) and (3.3), we have

$$f\left(\frac{v_1+v_2}{2}\right)^{\frac{1}{2h\left(\frac{1}{2}\right)}} \leq \left(\int_{v_1}^{v_2}(f(x))^{dx}\right)^{\frac{1}{v_2-v_1}} \leq [f(v_1).f(v_2)]^{\int_0^1 h(\lambda)d\lambda}.$$

So, the proof is completed. □

Corollary 3.2. *Let f and g be multiplicatively h-convex functions on $[v_1,v_2]$ such that $h\left(\frac{1}{2}\right)\neq 0$. Then the following inequalities hold:*

$$\left[f\left(\frac{v_1+v_2}{2}\right)g\left(\frac{v_1+v_2}{2}\right)\right]^{\frac{1}{2h\left(\frac{1}{2}\right)}} \leq \left(\int_{v_1}^{v_2}(f(x))^{dx}\int_{v_1}^{v_2}(g(x))^{dx}\right)^{\frac{1}{v_2-v_1}} \leq [(f(v_1)f(v_2))(g(v_1)g(v_2))]^{\int_0^1 h(\lambda)d\lambda}$$

Proof. Since f and g are multiplicatively h-convex functions, then fg is a multiplicatively h-convex function. Thus if we apply Theorem 3.1 to the function fg, then we obtain the desired result. □

Corollary 3.3. *Let f and g be multiplicatively h-convex functions on $[v_1,v_2]$ such that $h\left(\frac{1}{2}\right)\neq 0$. Then the following inequalities hold:*

$$\left[\frac{f\left(\frac{v_1+v_2}{2}\right)}{g\left(\frac{v_1+v_2}{2}\right)}\right]^{\frac{1}{2h\left(\frac{1}{2}\right)}} \leq \left(\frac{\int_{v_1}^{v_2}(f(x))^{dx}}{\int_{v_1}^{v_2}(g(x))^{dx}}\right)^{\frac{1}{v_2-v_1}} \leq \left[\frac{f(v_1).f(v_2)}{g(v_1).g(v_2)}\right]^{\int_0^1 h(\lambda)d\lambda}.$$

Proof. Since f and g are multiplicatively h-convex functions, then $\frac{f}{g}$ is a multiplicatively h-convex function. Thus, if we apply Theorem 3.1 to the function $\frac{f}{g}$, then we obtain the required result. □

Theorem 3.4. *Let f be a convex positive function and g be a multiplicatively h-convex function such that $h\left(\frac{1}{2}\right)\neq 0$. Then, we have*

$$\left(\frac{\int_{v_1}^{v_2}(f(x))^{dx}}{\int_{v_1}^{v_2}(g(x))^{dx}}\right)^{\frac{1}{v_2-v_1}} \leq \frac{\left(\frac{(f(v_2))^{f(v_2)}}{(f(v_1))^{f(v_1)}}\right)^{\frac{1}{f(v_2)-f(v_1)}}}{(g(v_1).g(v_2))^{\int_0^1 h(\lambda)d\lambda}.e}.$$

Proof. Note that

$$\begin{aligned}
\left(\frac{\int_{v_1}^{v_2}(f(x))^{dx}}{\int_{v_1}^{v_2}(g(x))^{dx}}\right)^{\frac{1}{v_2-v_1}} &= \left(\frac{e^{\int_{v_1}^{v_2}\ln(f(x))dx}}{e^{\int_{v_1}^{v_2}\ln(g(x))dx}}\right)^{\frac{1}{v_2-v_1}}\\
&= \left(e^{\int_{v_1}^{v_2}\ln(f(x))dx-\int_{v_1}^{v_2}\ln(g(x))dx}\right)^{\frac{1}{v_2-v_1}}\\
&= e^{\int_0^1\ln(f(v_1+\lambda(v_2-v_1)))d\lambda-\int_0^1\ln(g(v_1+\lambda(v_2-v_1)))d\lambda}\\
&\leq e^{\int_0^1\ln(f(v_1)+\lambda(f(v_2)-f(v_1)))d\lambda-\int_0^1\ln\left((g(v_1))^{h(1-\lambda)}(g(v_2))^{h(\lambda)}\right)d\lambda}\\
&= e^{\ln\left(\left(\frac{(f(v_2))^{f(v_2)}}{(f(v_1))^{f(v_1)}}\right)^{\frac{1}{f(v_2)-f(v_1)}}\right)-1-\ln(g(v_1).g(v_2))^{\int_0^1 h(\lambda)d\lambda}}\\
&= \frac{\left(\frac{(f(v_2))^{f(v_2)}}{(f(v_1))^{f(v_1)}}\right)^{\frac{1}{f(v_2)-f(v_1)}}}{(g(v_1).g(v_2))^{\int_0^1 h(\lambda)d\lambda}.e}.
\end{aligned}$$

Thus, we have

$$\left(\frac{\int_{v_1}^{v_2}(f(x))^{dx}}{\int_{v_1}^{v_2}(g(x))^{dx}}\right)^{\frac{1}{v_2-v_1}} \leq \frac{\left(\frac{(f(v_2))^{f(v_2)}}{(f(v_1))^{f(v_1)}}\right)^{\frac{1}{f(v_2)-f(v_1)}}}{(g(v_1).g(v_2))^{\int_0^1 h(\lambda)d\lambda}.e},$$

which completes the proof. □

Theorem 3.5. *Let f be a multiplicatively h-convex function such that $h\left(\frac{1}{2}\right)\neq 0$ and g be a convex positive function. Then, we have*

$$\left(\frac{\int_{v_1}^{v_2}(f(x))^{dx}}{\int_{v_1}^{v_2}(g(x))^{dx}}\right)^{\frac{1}{v_2-v_1}} \leq \frac{(f(v_1).f(v_2))^{\int_0^1 h(\lambda)d\lambda}.e}{\left(\frac{(g(v_2))^{g(v_2)}}{(g(v_1))^{g(v_1)}}\right)^{\frac{1}{g(v_2)-g(v_1)}}}.$$

Proof. Note that

$$\begin{aligned}
\left(\frac{\int_{v_1}^{v_2}(f(x))^{dx}}{\int_{v_1}^{v_2}(g(x))^{dx}}\right)^{\frac{1}{v_2-v_1}} &= \left(\frac{e^{\int_{v_1}^{v_2}\ln(f(x))dx}}{e^{\int_{v_1}^{v_2}\ln(g(x))dx}}\right)^{\frac{1}{v_2-v_1}}\\
&= \left(e^{\int_{v_1}^{v_2}\ln(f(x))dx-\int_{v_1}^{v_2}\ln(g(x))dx}\right)^{\frac{1}{v_2-v_1}}\\
&= e^{\int_0^1\ln(f(v_1+\lambda(v_2-v_1)))d\lambda-\int_0^1\ln(g(v_1+\lambda(v_2-v_1)))d\lambda}\\
&\leq e^{\int_0^1\ln\left((f(v_1))^{h(1-\lambda)}(f(v_2))^{h(\lambda)}\right)d\lambda-\int_0^1\ln(g(v_1)+\lambda(g(v_2)-g(v_1)))d\lambda}\\
&= e^{\ln(f(v_1).f(v_2))^{\int_0^1 h(\lambda)d\lambda}-\ln\left(\left(\frac{(g(v_2))^{g(v_2)}}{(g(v_1))^{g(v_1)}}\right)^{\frac{1}{g(v_2)-g(v_1)}}\right)+1}\\
&= \frac{(f(v_1).f(v_2))^{\int_0^1 h(\lambda)d\lambda}.e}{\left(\frac{(g(v_2))^{g(v_2)}}{(g(v_1))^{g(v_1)}}\right)^{\frac{1}{g(v_2)-g(v_1)}}}.
\end{aligned}$$

Hence,

$$\left(\frac{\int_{v_1}^{v_2}(f(x))^{dx}}{\int_{v_1}^{v_2}(g(x))^{dx}}\right)^{\frac{1}{v_2-v_1}} \leq \frac{(f(v_1).f(v_2))^{\int_0^1 h(\lambda)d\lambda}.e}{\left(\frac{(g(v_2))^{g(v_2)}}{(g(v_1))^{g(v_1)}}\right)^{\frac{1}{g(v_2)-g(v_1)}}},$$

which is the desired result. □

Theorem 3.6. *Let f be a convex positive function and g be a multiplicatively h-convex function such that $h\left(\frac{1}{2}\right) \neq 0$. Then, we have*

$$\left(\int_{v_1}^{v_2}(f(x))^{dx}\cdot\int_{v_1}^{v_2}(g(x))^{dx}\right)^{\frac{1}{v_2-v_1}} \leq \frac{\left(\frac{(f(v_2))^{f(v_2)}}{(f(v_1))^{f(v_1)}}\right)^{\frac{1}{f(v_2)-f(v_1)}}\cdot(g(v_1)\cdot g(v_2))^{\int_0^1 h(\lambda)d\lambda}}{e}.$$

Proof. Note that

$$\begin{aligned}
\left(\int_{v_1}^{v_2}(f(x))^{dx}\cdot\int_{v_1}^{v_2}(g(x))^{dx}\right)^{\frac{1}{v_2-v_1}} &= \left(e^{\int_{v_1}^{v_2}\ln(f(x))dx+\int_{v_1}^{v_2}\ln(g(x))dx}\right)^{\frac{1}{v_2-v_1}} \\
&= \left(e^{(v_2-v_1)\left(\int_0^1\ln(f(v_1+\lambda(v_2-v_1)))d\lambda+\int_0^1\ln(g(v_1+\lambda(v_2-v_1)))d\lambda\right)}\right)^{\frac{1}{v_2-v_1}} \\
&= e^{\int_0^1\ln(f(v_1+\lambda(v_2-v_1)))d\lambda+\int_0^1\ln(g(v_1+\lambda(v_2-v_1)))d\lambda} \\
&\leq e^{\int_0^1\ln(f(v_1)+\lambda(f(v_2)-f(v_1)))d\lambda-\int_0^1\ln\left((g(v_1))^{h(1-\lambda)}(g(v_2))^{h(\lambda)}\right)dt} \\
&= e^{\ln\left(\left(\frac{(f(v_2))^{f(v_2)}}{(f(v_1))^{f(v_1)}}\right)^{\frac{1}{f(v_2)-f(v_1)}}\right)-1+\ln(g(v_1)\cdot g(v_2))^{\int_0^1 h(\lambda)d\lambda}} \\
&= \frac{\left(\frac{(f(v_2))^{f(v_2)}}{(f(v_1))^{f(v_1)}}\right)^{\frac{1}{f(v_2)-f(v_1)}}\cdot(g(v_1)\cdot g(v_2))^{\int_0^1 h(\lambda)d\lambda}}{e}.
\end{aligned}$$

Hence,

$$\left(\int_{v_1}^{v_2}(f(x))^{dx}\cdot\int_{v_1}^{v_2}(g(x))^{dx}\right)^{\frac{1}{v_2-v_1}} \leq \frac{\left(\frac{(f(v_2))^{f(v_2)}}{(f(v_1))^{f(v_1)}}\right)^{\frac{1}{f(v_2)-f(v_1)}}\cdot(g(v_1)\cdot g(v_2))^{\int_0^1 h(\lambda)d\lambda}}{e}.$$

This completes the proof. □

Remark 3.7. *Now we point out some special cases which are included in our main results.*

1. *If $h(\lambda)=\lambda$, then our results reduce to the results for multiplicatively convexity given in [1].*
2. *If If $h(\lambda)=\lambda^s$, then our results reduce to the results for multiplicatively s-convexity.*
3. *If If $h(\lambda)=1$, then our results reduce to the results for multiplicatively P-convexity.*

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