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A Note on the (θ,φ)-Statistical Convergence of the Product Time Scale

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Abstract

In this paper, we introduce the concepts (θ,φ)-density of a subset of the product time scale $\mathbb{T}^2$ and (θ,φ)-statistical convergence of Δ-measurable function f defined on the product time scale $\mathbb{T}^2$ with the help of lacunary sequences. Later, we have discussed the connection between classical convergence and (θ,φ)-statistical convergence. In addition, we have seen that f is strongly (θ,φ)-Cesàro summable on $\mathbb{T}^2$ then f is (θ,φ)-statistical convergent.

Keywords: *Δ-convergence, statistical convergence, density, product time scale, lacunary sequences, p-Cesàro summable.*
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1. Introduction

The concept of statistical convergence which is a generalization of classical convergence was first given by Zygmund [27] and later were introduced independently by Steinhaus [21] and Fast [11]. This concept is discussed under different names in different spaces.

The time scale calculus was first introduced by Hilger in his Ph.D. thesis in 1988 (see[16],[17],[4]). In later years, the integral theory on time scales was given by Guseinov [15], and further studies were developed by Cabada-Vivero [7] and Rzezuchowski [19]. Recently, Seyyidoğlu and Tan [20] defined the density of the subset of the time scale. By using this definition, they gave Δ-convergence and Δ-Cauchy concepts for a real valued function defined on time scale. Turan and Duman [23] introduced the notion of lacunary statistically convergence of a Δ-measurable function by using the lacunary sequence defined by Freedman et.al.(see[12]).
In this paper, our aim is to define (θ,φ)-statistical convergence of Δ- measurable functions defined on the product time scale $\mathbb{T}^2$ by using lacunary sequences $\theta=(k_r)$ and $\varphi=(l_t)$ in light of works of Çınar et al. [10], Seyyidoğlu and Tan [20] and others [3],[9],[14],[15],[2],[22],[25],[26].

2. Prelimineries

The statistical convergence concept is based on the asymptotic (natural) density of a subset B in $\mathbb{N}$ (the set of positive integers) which is defined as

$$\delta(B)=\lim_{n\to\infty}\frac{|\{k\le n: k\in B\}|}{n},$$

where $|B|$ denotes the number of elements in B (see [11],[13]). A measurable (Lebesque) function $f:\mathbb{R}\to\mathbb{R}$ is said to be statistically convergent to a number L if, for every $\varepsilon>0$

$$\delta(\{t\in\mathbb{R}: |f(t)-L|\ge\varepsilon\})=0.$$

In this case, we write $st\text{-}\lim_{t\to\infty} f(t)=L$.

In this study, we shall give the notion of (θ,φ)-statistical convergence on any product time scale $\mathbb{T}^2$ and its some properties. Throughout this paper, we consider the time scales $\mathbb{T}_1$ and $\mathbb{T}_2$ which are unbounded from above and have minimum points.
Lets remember some concepts related to time scale $\mathbb{T}$. A non-empty closed subset of $\mathbb{R}$ is called a time scale and is denoted by $\mathbb{T}$. We suppose that a time scale has the topology inherited from $\mathbb{R}$ with the standart topology. For $t\in\mathbb{T}$, we consider the forward jump operator

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$\sigma:\mathbb{T}\to\mathbb{T}$ by $\sigma(t):=\inf\{s\in\mathbb{T}:s>t\}$. In this definition, we take $\inf\emptyset=sup\mathbb{T}$. For $t\in\mathbb{T}$ with $a\leq b$, it is defined the interval $[a,b]$ in $\mathbb{T}$ by $[a,b]=\{t\in\mathbb{T}:a\leq t\leq b\}$. Let $\mathbb{T}$ be a time scale. Denote by $\mathscr{F}$ the family of all left-closed and right-open intervals of $\mathbb{T}$ of the form $[a,b)=\{t\in\mathbb{T}:a\leq t<b\}$ with $a,b\in\mathbb{T}$ and $a\leq b$. It is clear that the interval $[a,a)$ is an empty set, $\mathscr{F}$ is semiring of subsets of $\mathbb{T}$. Let m:$\mathscr{F}\to[0,\infty)$ be the set function on $\mathscr{F}$ that assings to each interval $[a,b)$ its lenght $b-a; m([a,b))=b-a$. Then m is a countably additive measure on $\mathscr{F}$. The Caratheodory extension of the set function m associated with family $\mathscr{F}$ (for the Caratheodory extension, see [20]) is denoted by μ_Δ, the Lebesgue Δ-measure on $\mathbb{T}$, and that is a countably additive measure . In this case, it is known that if $a\in\mathbb{T}-\{max\mathbb{T}\}$, then the single point set $\{a\}$ is Δ-measurable and $\mu_\Delta(a)=\sigma(a)-a$. If $a,b\in\mathbb{T}$ and $a\leq b$ then $\mu_\Delta(a,b)_\mathbb{T}=b-\sigma(a)$. If $a,b\in\mathbb{T}-\{max\mathbb{T}\}$ and $a\leq b$ then $\mu_\Delta(a,b]_\mathbb{T}=\sigma(b)-\sigma(a)$ and $\mu_\Delta[a,b]_\mathbb{T}=\sigma(b)-a$. It can be easily seen that the measure of a subset of $\mathbb{N}$ is equal to its cardinality (see [20],[23],[24]).
Assume that $\theta=\{k_r\}_{r=0}^{\infty}$ is an increasing sequence of non-negative integers with $k_0=0$ and $\sigma(k_r)-\sigma(k_{r-1})\to\infty$ as $r\to\infty$, where $\sigma:\mathbb{R}\to\mathbb{T}$ is the forward jump operator given by $\sigma(x)=inf\{y\in\mathbb{T}:y>x\}$. In this case, we say that θ is a lacunary sequence with respect to $\mathbb{T}$. Throughout this study, Θ is supposed as the set of all such lacunary sequences and by $[a,b]_\mathbb{T}$, we denote the interval in $\mathbb{T}$, i.e., $[a,b]\cap\mathbb{T}$, where $[a,b]$ is the usual real interval and $\mathbb{T}$ is a time scale. Turan and Duman [23] introduced the notion of lacunary statistical convergence (or θ-statistically convergence) on time scale $\mathbb{T}$. A Δ-measurable function $f:\mathbb{T}\to\mathbb{R}$ is said to be lacunary statistically convergent to a number L if, for every $\varepsilon>0$

$$\lim_{r\to\infty}\frac{\mu_\Delta(\{s\in(k_{r-1},k_r]_\mathbb{T}:\ |f(s)-L|\geq\varepsilon\})}{\mu_\Delta((k_{r-1},k_r]_\mathbb{T})}=0 \tag{2.1}$$

where $(k_{r-1},k_r]_\mathbb{T}=(k_{r-1},k_r]\cap\mathbb{T}$. In this case, $st_\mathbb{T}$-$lim_{t\to\infty}f(t)=L$.

In this paper, using $\theta,\varphi\in\Theta$ and taking on the product time scale $\mathbb{T}^2$ in place of the time scale $\mathbb{T}$, we introduce the notion of lacunary statistically convergence on product time scale $\mathbb{T}^2$.

3. Main Results

Let $\theta=(k_r)$, $\varphi=(l_t)\in\Theta,\mathbb{T}_1$ and $\mathbb{T}_2$ time scales such that $t_0=\min\mathbb{T}_1$, $r_0=\min\mathbb{T}_2$ and $\mathbb{T}^2=\mathbb{T}_1\times\mathbb{T}_2$, the set of cartasian product of $\mathbb{T}_1$ and $\mathbb{T}_2$ time scales. Throughout the paper, we denote $A=\{(k_{r-1},k_r]_{\mathbb{T}_1}\times(l_{t-1},l_t]_{\mathbb{T}_2}\}$, $B=\{[t_0,t]_{\mathbb{T}_1}\times[r_0,r]_{\mathbb{T}_2}\}$, where $(k_{r-1},k_r]_{\mathbb{T}_1}=(k_{r-1},k_r]\cap\mathbb{T}_1$, $(l_{t-1},l_t]_{\mathbb{T}_2}=(l_{t-1},l_t]\cap\mathbb{T}_2$. It is easy to see that $\mu_\Delta(A)=\mu_\Delta((k_{r-1},k_r]_{\mathbb{T}_1}).\mu_\Delta((l_{t-1},l_t]_{\mathbb{T}_2})$ and $\mu_\Delta(B)=\mu_\Delta([t_0,t]_{\mathbb{T}_1}).\mu_\Delta([r_0,r]_{\mathbb{T}_2})$ ([5],[6]).
The convergence method in (2.1) can also be defined with respect to the density on product time scales as in the following way.

Definition 3.1. *Suppose that Ω be a Δ-measurable subset of $\mathbb{T}^2=\mathbb{T}_1\times\mathbb{T}_2$. Then, one defines the set $\Omega(t,r,\theta,\varphi)$ by $\Omega(t,r,\theta,\varphi)=:\{(s,u)\in A:(s,u)\in\Omega\}$ for $(t,r)\in\mathbb{T}^2$. That is $\Omega(t,r,\theta,\varphi)=\Omega\cap A$. In this case, the (θ,φ)-density of Ω on $\mathbb{T}^2$ is defined as*

$$\delta_{\mathbb{T}^2}^{(\theta,\varphi)}(\Omega)=\lim_{(t,r)\to\infty}\frac{\mu_\Delta(\Omega(t,r,\theta,\varphi))}{\mu_\Delta(A)}$$

provided that the limit exists.
In case of $\mathbb{T}_1$=$\mathbb{T}_2=\mathbb{N}$, this reduces to the classical concept of the product asymptotic density, which in the case of $\mathbb{T}=\mathbb{N}$ was first introduced by Fast. If $\mathbb{T}_1$=$\mathbb{T}_2=[a,\infty), a>0$, this reduces to the product asymptotic density of measurable functions that the asymptotic density of measurable functions was studied by Moricz [18]. Finally, if $\mathbb{T}=q^\mathbb{N}$, $q>1$, then we get the notion of (q,q)-statistical convergence which q-statistical convergence introduced by Aktuğlu and Bekar [1].

Definition 3.2. *Let $f:\mathbb{T}^2\to\mathbb{R}$ be a Δ- measurable function. It is said that f is (θ,φ)-statistically convergent to a real number L on $\mathbb{T}^2$ if*

$$\lim_{(t,r)\to\infty}\frac{\mu_\Delta(\{(s,u)\in A:\ |f(s,u)-L|\geq\varepsilon\})}{\mu_\Delta(A)}=0 \tag{3.1}$$

for every $\varepsilon>0$. In this case, we can write $st_{\mathbb{T}^2}^{(\theta,\varphi)}$-$\lim\limits_{(t,r)\to\infty}f(t,r)=L$. The set of all (θ,φ)- statistically convergent functions on $\mathbb{T}^2$ will be denoted by $S_{\mathbb{T}^2}^{(\theta,\varphi)}$.
If one take $k_0=l_0=0$, $k_{-1}=t_0$ and $l_{-1}=r_0$ in (3.1), we get the Δ- statistically convergent function to a real number L on $\mathbb{T}^2$, for the function f, which is defined as

$$\lim_{(t,r)\to\infty}\frac{\mu_\Delta(\{(s,u)\in B:\ |f(s,u)-L|\geq\varepsilon\})}{\mu_\Delta(B)}=0.$$

In this case, we can writes $st_{\mathbb{T}^2}$-$\lim\limits_{(t,r)\to\infty}f(t,r)=L$. The set of all Δ- statistically convergent functions on $\mathbb{T}^2$ will be denoted by $S_{\mathbb{T}^2}$.
Finally, if $\mathbb{T}^2=q^\mathbb{N}\times q^\mathbb{N}$, $q>1$, then we get the notion of the product $q-$statistical convergence which in the case of $\mathbb{T}=q^\mathbb{N}$ is introduced by Aktuğlu and Bekar [1].

Proposition 3.3. *Let $f,g:\mathbb{T}^2\to\mathbb{R}$ be a Δ- measurable functions such that $st_{\mathbb{T}^2}^{(\theta,\varphi)}$-$\lim\limits_{(t,r)\to\infty}f(t,r)=L_1$ and $st_{\mathbb{T}^2}^{(\theta,\varphi)}$-$\lim\limits_{(t,r)\to\infty}g(t,r)=L_2$. Then the following statements hold:*

i) $st_{\mathbb{T}^2}^{(\theta,\varphi)}$- $\lim\limits_{(t,r)\to\infty}(f(t,r)+g(t,r))=L_1+L_2$,

ii) $st_{\mathbb{T}^2}^{(\theta,\varphi)}$- $\lim\limits_{(t,r)\to\infty}(cf(t,r))=cL_1$.

Proof. It is easy to prove and we omit it. □

Theorem 3.4. $S_{\mathbb{T}^2}\subseteq S_{\mathbb{T}^2}^{(\theta,\varphi)}$ *if and only if*

$$\liminf_{(t,r)\to\infty}\frac{\mu_\Delta(A)}{\mu_\Delta(B)}>0.$$

Proof. For given $\varepsilon>0$, we have

$$\mu_\Delta(\{(s,u)\in B:\ |f(s,u)-L|\geq\varepsilon\})\supset\mu_\Delta(\{(s,u)\in A:\ |f(s,u)-L|\geq\varepsilon\})).$$

Then

$$\frac{\mu_\Delta(\{(s,u)\in B:\ |f(s,u)-L|\geq\varepsilon\})}{\mu_\Delta(B)}\geq\frac{\mu_\Delta(\{(s,u)\in A:\ |f(s,u)-L|\geq\varepsilon\})}{\mu_\Delta(B)}$$

$$=\frac{\mu_\Delta(A)}{\mu_\Delta(B)}\frac{1}{\mu_\Delta(A)}\ .\ \mu_\Delta(\{(s,u)\in A:\ |f(s,u)-L|\geq\varepsilon\})\tag{3.2}$$

Hence by using (3.2) and taking the limit as $(t,r)\to\infty$, we get $st_{\mathbb{T}^2}$- $\lim\limits_{(t,r)\to\infty}f(t,r)\to$L implies $st_{\mathbb{T}^2}^{(\theta,\varphi)}$- $\lim\limits_{(t,r)\to\infty}f(t,r)$=L . □

The definition of strongly *p-Cesàro* summability on time scale $\mathbb{T}$ was given by Turan and Duman [23],[24]. Using it, we get the following.

Definition 3.5. *[24] Let $f:\mathbb{T}^2\to\mathbb{R}$ be a Δ-measurable function and $0<p<\infty$. Then, f is strongly p-Cesàro summable function on $\mathbb{T}^2$ if there exists some $L\in\mathbb{R}$ such that*

$$\lim_{(t,r)\to\infty}\frac{1}{\mu_\Delta(B)}\iint\limits_B|f(s,u)-L|^p\ \Delta s\ \Delta u=0.$$

The set of all strongly *p-Cesàro* summable functions on $\mathbb{T}^2$ is denoted by $[W_p]_{\mathbb{T}^2}$.
We need to emphasize that measure theory on time scale was first constructed by Guseinov [15] and *Lebesque* Δ-*integral* on time scales introduced by Cabada and Vivero [7],[8]. Using this we have following definition.

Definition 3.6. *Let $f:\mathbb{T}^2\to\mathbb{R}$ be a Δ- measurable function, $\theta,\varphi\in\Theta$ and $0<p<\infty$. We say that f is strongly $(\theta,\varphi)_p$-Cesàro summable function on $\mathbb{T}^2$ if there exists some $L\in\mathbb{R}$ such that*

$$\lim_{(t,r)\to\infty}\frac{1}{\mu_\Delta(A)}\iint\limits_A|f(s,u)-L|^p\ \Delta s\ \Delta u=0.$$

In this case we write $(W,(\theta,\varphi)_p)_{\mathbb{T}^2}$- $\lim\limits_{(t,r)\to\infty}f(t,r)=L$. The set of all strongly $(\theta,\varphi)_p$-*Cesàro* summable functions on $\mathbb{T}^2$ will be denoted by $[W,(\theta,\varphi)_p)]_{\mathbb{T}^2}$.

Lemma 3.7. *Let $f:\mathbb{T}^2\to\mathbb{R}$ be a Δ- measurable function and $\Omega(t,r,\theta,\varphi)=\{\ (s,u)\in A:|f(s,u)-L|\geq\varepsilon\ \}$ for $\varepsilon>0$. In this case, we have*

$$\mu_\Delta(\Omega(t,r,\theta,\varphi))\leq\frac{1}{\varepsilon}\iint\limits_{\Omega(t,r,\theta,\varphi)}|f(s,u)-L|^p\ \Delta s\ \Delta u\leq\frac{1}{\varepsilon}\iint\limits_A|f(s,u)-L|^p\ \ \Delta s\ \Delta u$$

Proof. It can be proved by using similar method in [24]. □

Theorem 3.8. *Let $f:\mathbb{T}^2\to\mathbb{R}$ be a Δ- measurable function, $\theta,\varphi\in\Theta$, $L\in\mathbb{R}$ and $0<p<\infty$. Then we get:*

(i) If f is strongly $(\theta,\varphi)_p$-*Cesàro* summable to L, then $st_{\mathbb{T}^2}^{(\theta,\varphi)}$- $\lim\limits_{(t,r)\to\infty}f(t,r)=L$

(ii) If $st_{\mathbb{T}^2}^{(\theta,\varphi)}$- $\lim\limits_{(t,r)\to\infty}f(t,r)=L$ and f is a bounded function, then f is strongly $(\theta,\varphi)_p$-*Cesàro* summable to L.

Proof. (i) Let f is strongly $(\theta,\varphi)_p$-*Cesàro* summable to L. For given $\varepsilon>0$, let $\Omega(t,r,\theta,\varphi)=\{\, s\in A: |f(s,u)-L|\geq\varepsilon \,\}$ on time scale $\mathbb{T}^2$. Then, it follows from lemma 3.7

$$\varepsilon\ \mu_\Delta(\Omega(t,r,\theta,\varphi))\leq\iint_A |f(s,u)-L|^p\ \ \Delta s\ \Delta u. \tag{3.3}$$

Dividing both sides of the last equality by $\mu_\Delta(A)$ and taking limit as $(t,r)\to\infty$, we obtain

$$\lim_{(t,r)\to\infty}\frac{\mu_\Delta(\Omega(t,r,\theta,\varphi))}{\mu_\Delta(A)}\leq\frac{1}{\varepsilon}\lim_{(t,r)\to\infty}\frac{1}{\mu_\Delta(A)}\iint_A |f(s,u)-L|^p\ \ \Delta s\Delta u=0$$

which yields that $st_{\mathbb{T}^2}^{(\theta,\varphi)}\text{-}\lim\limits_{(t,r)\to\infty} f(t,r)=L.$

(ii) Let f be bounded and (θ,φ)-statistically convergent to L on $\mathbb{T}^2$. Then, there exists a positive number M such that $|f(s,u)-L|\leq M$ for all $(s,u)\in\mathbb{T}^2$ and also

$$\lim_{(t,r)\to\infty}\frac{\mu_\Delta(\Omega(t,r,\theta,\varphi))}{\mu_\Delta(A)}=0 \tag{3.4}$$

where $\Omega(t,r,\theta,\varphi)=\{\,(s,u)\in A: |f(s,u)-L|^p\geq\varepsilon\,\}$ as stated before. Since

$$\begin{aligned}\int_A |f(s,u)-L|^p\ \ \Delta s\ \Delta u &= \iint_{\Omega(t,r,\theta,\varphi)} |f(s,u)-L|^p\ \ \Delta s\ \Delta u \iint_{A/\Omega(t,r,\theta,\varphi)} |f(s,u)-L|^p\ \ \Delta s\ \Delta u\\ &\leq M^p\iint_{\Omega(t,r,\theta,\varphi)}\Delta s\ \Delta u+\varepsilon\iint_{A/\Omega(t,r,\theta,\varphi)}\Delta s\ \Delta u,\end{aligned}$$

we obtain

$$\lim_{(t,r)\to\infty}\frac{1}{\mu_\Delta(A)}\iint_A |f(s,u)-L|\ \ \Delta s\ \Delta u\leq M^p\lim_{(t,r)\to\infty}\frac{\mu_\Delta(\Omega(t,r,\theta,\varphi))}{\mu_\Delta(A)}+\varepsilon. \tag{3.5}$$

Since $\varepsilon>0$ is arbitrary, the proof follows from (3.4) and (3.5). □

Theorem 3.9. *Let f be a Δ-measurable function. Then, $st_{\mathbb{T}^2}^{(\theta,\varphi)}\text{-}\lim\limits_{(t,r)\to\infty} f(t,r)=L$ if and only if there exists a Δ-measurable set $\Omega\subseteq\mathbb{T}^2$ such that $\delta_{\mathbb{T}^2}^{(\theta,\varphi)}(\Omega)=1$ and $\lim\limits_{(t,r)\to\infty}|f(t,r)-L|=0,\ \ ((t,r)\in\Omega(t,r,\theta,\varphi))$.*

Proof. It can be easily proved by using similar way in Theorem 3.9 of Turan and Duman [24]. □

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