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Matematik Öğretmenlerinin Radyan Kavramını Anlamlandırma Biçimleri*

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ÖZET

Trigonometri, ilgili temel kavramların öğrenciler tarafından tam olarak anlaşılamamasına bağlı olarak matematikte öğrencilerin anlamakta güçlük çektiği konular arasında ilk sıralarda yer almaktadır. Trigonometriye temel teşkil eden kavramlar arasında yer alan *radyan* kavramı ile ilgili yapılan araştırmalar öğretmenlerin, öğretmen adaylarının ve öğrencilerin bu kavram ile ilgili ortak bir takım kavramsal bilgi eksikliklerine sahip olduğunu ortaya koymaktadır. Dolayısıyla konu ile ilgili olarak yapılacak çalışmalar önem taşımaktadır. Alan yazında radyan kavramı ile ilgili çalışmaların genel olarak öğrenciler ve öğretmen adayları ile yürütüldüğü görülmektedir. Öğretmenlerle yürütülmüş güncel ve ulusal çalışmalara ise rastlanmamaktadır. Bu bağlamda bu araştırmanın amacı matematik öğretmenlerinin radyan kavramını anlamlandırma biçimlerinin incelenmesidir. Çalışma kapsamında öğretmenlerin radyan kavramına yönelik kavramsal bilgi eksikliklerinin altında yatan nedenler araştırılmaya ve bu nedenlerin ortadan kaldırılmasına yardımcı olacak çözüm önerileri sunulmaya çalışılmıştır. Araştırmada durum çalışması yöntemi kullanılmıştır. Çalışmanın örneklem grubunu Ordu ilinde bulunan farklı tür okullarda görev yapmakta olan kırk bir matematik öğretmeni oluşturmaktadır. Çalışmada veri toplama aracı olarak araştırmacılar tarafından geliştirilmiş olan *Genel Bilgi Formu ve Kavram Testi* ile *yapılandırılmamış yüz yüze görüşmeler* kullanılmıştır. Çalışma sonucunda öğretmenlerin çoğunun radyan kavramı ile ilgili olarak derin bir anlayışa sahip olmadıkları, bununla birlikte bir takım kavram yanılgılarına sahip oldukları gözlenmiştir. Çalışmadan elde edilen sonuçlar alan yazınla ilişkili olarak tartışılarak radyan kavramının daha etkili öğretimi adına bir takım önerilerde bulunulmuştur.

Keywords: Radyan kavramı, matematik öğretmenleri, trigonometri, kavramsal anlama

Mathematics Teachers' Understanding of the Concept of Radian

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ABSTRACT

Trigonometry is among the mathematics subjects that students have difficulty in understanding due to the lack of proper understanding of the basic concepts. Research on radian, a concept that is among the fundamental concepts of trigonometry, has revealed that teachers, pre-service teachers and students lack, in some regards, proper conceptual knowledge about the subject term. Accordingly, studies on the issue bear particular importance. In the relevant literature, studies on the concept of radian focus mainly on students and pre-service teachers. However, there are not any national and contemporary studies that are conducted with teachers. In view of these, the current study aims to analyze mathematics teachers' ways of understanding of the concept of radian. In this regard, the study attempts to explore the reasons behind teachers' lack of conceptual knowledge regarding the concept of radian and to propose solutions that would help eliminate these reasons. The study employs case study method. The sample group consists of 41 in-service mathematics teachers who work in different types of schools. *General Information Form* and *Concept Test* developed by the researchers of the study as well as *unstructured face-to-face interviews* were utilized as data collection tools. As a consequence, majority of the participant teachers were found to lack an accurate and complete understanding of the concept of radian and to have some conceptual misunderstandings regarding the subject. In the conclusion section, findings of the study are discussed in relation to the relevant literature and some recommendations are made for teaching the concept of radian more efficiently.

Keywords: The concept of radian, mathematics teachers, trigonometry, conceptual understanding

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1. INTRODUCTION

Mathematics is, by nature, a cumulative and sequential discipline. The words cumulative and sequential here are used to mark the fact that mathematical concepts and systems are built on each other. Accordingly, when studying mathematics, mathematical concepts are used and structured in a way that they are interrelated. As a result, mathematical concepts make sense in relation to other relevant concepts and form a foundation for mathematical structures. In this regard, Baki (2014, p. 259) suggests that mathematical concepts do not have any meaning by themselves; instead, they gain meaning when they are handled in relation to concepts in their relevant groups. Thus, it can be said that understanding mathematical concepts in connection with other relevant concepts forms a basis for conceptual understanding. Conceptual understanding is a process related with mental sense making about mathematical concepts, principles and definitions. Yanık (2016, p. 102) defines this process as a process of transforming new knowledge into a systematic structure with its connections after assessing it in the light of prior knowledge and understanding.

Conceptual understanding has a significant role in mathematics education, hence in the relevant literature, there are many studies (Ay and Başbay, 2017; Çakmak and Durmuş, 2015; Kertil, Erbaş and Çetinkaya, 2017; Siegler and Lortie-Forgues, 2015; Simon, 2017) that explore conceptual understanding. The most important one among the reasons for this is that mathematical concepts are difficult to understand due to their abstract and complex nature (Çiltaş and Işık, 2012). Trigonometry is among the top mathematics subjects that students have difficulty in understanding (Durmuş, 2004; Tatar, Okur and Tuna, 2007) due to the lack of proper understanding of the basic terms (Steckroth, 2007, Thompson, 2008). When investigating the field studies related to trigonometry, it is seen that these studies usually focus on trigonometric functions (Bolte, 1993; Doğan and Şenay, 2000; Even, 1989; Even, 1990; Howald, 1998; İnan, 2013; Kültür, Kaplan and Kaplan, 2008) or use of technology in the teaching of trigonometric functions (Ağaç, 2009; Akkoç, 2007; Blackett and Tall, 1991; Doğan and Abdildaeba, 2013; Emlek, 2007; Lobo da Costa and Magina, 1998; Yılmaz, Ertem and Güven, 2010). Although there are studies in the relevant literature which explore the concept of radian in particular, they are quite limited and are usually conducted either with students or pre-service teachers. Studies concerning mathematics teachers' ways of understanding of the concept of radian, on the other hand, are very few (Akkoç, 2008; Doerr, 1996; Fi, 2003). However, the concept of radian is one of the important concepts that constitute the basis of trigonometry (Erdem and Man, 2018) and the studies on the concept are of great importance.

1.1. Concept of Radian and Related Studies

The use of radian is relatively new, as far as the use of the degree angle unit extends to Babylons (Maor, 1988, as cited in, Akkoç and Gül, 2010). It was Thomas Muir and physicist James Thomson who, in the 1870s, were first to discuss the need for an angular measurement unit which was not based on the division of a full circle into a certain number of equal parts (NCTM, 1971; as cited in Kabaca, 2013, p.180) and the term radian first defined as the ratio of the length of an arc seen by the center angle to the radius of the circle in print in an article authored by Thomson, in 1873. This ratio, which is unchanged in every circle, gives the number of radians of the angle (NCTM, 1971).

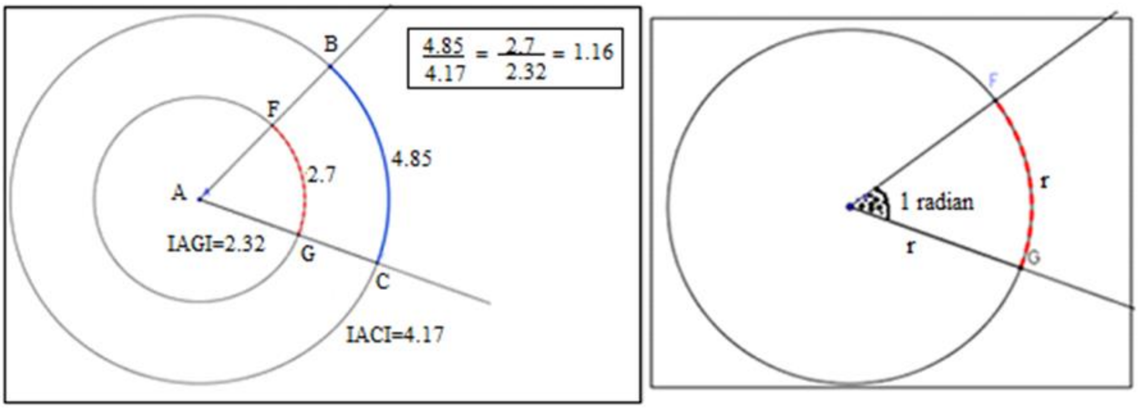


Figure 1. The magnitude of the angle CAB in radians -Figure 2. Representation of 1-radian angle

Thus, the angle CAB given in Figure 1 equals to 1.16 radians. An important point to note here is that radian measure of the arc of BC corresponds to a real number as a mathematical ratio. When the helical function is considered (when we move the real number axis onto the unit circle), it is seen that a radian value is obtained for each real number on the number line. Because the domain and codomain of trigonometric functions are of the radian-type, the concept of radian can be seen as one of the basic concepts underlying trigonometry teaching. However, as a result of teaching trigonometric functions in schools with using the right-angled triangle and degree values of its internal angles, the students do not have a full and relational sense of the subject. Orhun (2004) supports this idea in his work and states that students understand trigonometry as relations between the sides and angles of a right-angled triangle.

The most comprehensive study in the literature which deals with the concept of radian is Fi's (2003) doctoral thesis. In the thesis, Fi investigated pre-service teachers' subject matter content knowledge of trigonometry as well as their pedagogical content knowledge and focused on their understanding of the concept of radian for this purpose. Fi concluded that pre-service teachers were able to convert radians to degrees or vice versa; however, they were unable to define radian as a ratio of two lengths and lacked a deep understanding as to what radian measure meant. In the subject study, it was also observed that pre-service teachers were more comfortable with degree measurements of angles and considered π as the unit used in radian measurement while also considering 1 radian as equivalent to 1800. Other than this, Topçu, Kertil, Yılmaz and Önder (2006) carried out a study with 37 pre-service teachers and 14 in-service teachers. In their study, they addressed the concept images of pre-service and in-service teachers regarding the concept of radian and explored the origins of these concept images. As a consequence of their study, participants were found to have rather limited concept images regarding the concept of radian and the concept of degree was determined as the origin of these images. They used the formula D/180 = R/π for defining radian and none of the participants defined radian as the ratio of two lengths. It was also found out that participants did not consider radian and π as real numbers. Tuna (2013) conducted a conceptual analysis of 93 pre-service teachers' knowledge on the concepts of degree and radian in his study and had similar results. It was concluded that only 8.6% of pre-service teachers were able to define the concept of radian correctly. Almost 90% of pre-service teachers used phrases such as *"The expression of degree in terms of π"*, "*The unit of length of degree*", "*I just know the formula of D/180 = R/π*", "*I do not know what radian is*" when defining radian. Akkoç (2008) obtained parallel results from her study conducted on 42 pre-service teachers with the aim of revealing pre-service teachers' concept images of radian and the possible origins of these images. In the end, she concluded that pre-service teachers had more concept images of degree than concept images of radian and therefore, they had difficulty in making sense of trigonometric functions of real numbers. In the same study, it was observed that pre-service teachers tended to use variables of trigonometric functions not in real numbers but in degrees and had two distinct concept images of the number π. Of them, the first one involved π as an angle value while the second one included π as a real number. As a direct consequence of this, pre-service teachers were found to show the position of the number π on x axis as the one that corresponds to 180. In a similar study of Kang (2003) which was conducted on 33 pre-service teachers, it's concluded that teachers did not know the domain of trigonometric functions was all real numbers. As the reason for this lack of knowledge, it was suggested that the angle is not clearly indicated to be a real number in the definition of trigonometric functions. Besides, Kang stated that defining 1 radian without first defining radian itself and expressing the relation between radian and degree only through the formula D/180 = R/π inhibited learning radian conceptually and grasping its connection with real numbers.

When the above studies are examined, it is seen that the elements which obstruct to understand the radian concept in general are similar. These elements can be expressed as; the radian cannot be defined as the ratio of two lengths and accordingly cannot be predicted to correspond to a real number, the radian cannot be related to unit circle and trigonometric functions, and π cannot be interpreted related with the concept of radian. It appears that the elements mentioned here are also presented as basic points for the correct understanding of the radian concept in different studies (Akbaş, 2008; Akkoç, 2008; Akkoç ve Gül, 2010; Fi, 2003; Maor, 1998; Moore, 2012; Tuna, 2013; Topçu, Kertil, Yılmaz, Akkoç ve Önder, 2006; Thompson, Carlson & Silverman, 2007). These elements were used for the construction of sub-problems of this study.

Apart from the above-mentioned studies, it is revealed in different studies (Akkoç ve Gül, 2010; Kang, 2003; Orhun, 2004; Steckroth, 2007; Turanlı, Keçeli ve Türker, 2007) that students and prospective teachers have lack of conceptual knowledge about the concept of radian. Therefore, it seems that there are some obstacles in understanding the subject concept. So, there is a need to make current assessment studies about how mathematics teachers understand the concept of radian. With the results of the study, suggestions for the solutions of observed problems can be developed by clear up the nature of today's learning environments. So, the aim of this study is to investigate how mathematics teachers make sense of the radiance concept. The sub problems of the study are as follows.

- How do mathematics teachers define the concept of radian?
- How do the mathematics teachers connect the radian concept with the unit circle?
- How do the mathematics teachers connect the radian concept with the trigonometric functions?
- How do mathematics teachers make sense of π?

2. METHODOLOGY

In the present study, case study method was preferred since the aim is to conduct an in-depth analysis of how mathematics teachers make sense of a special mathematics concept and how they connect it with other concepts. Case study emerges as a research method that allows facts and phenomena to be investigated deeply from a holistic perspective and within their natural settings (Yin, 2003; Zainal, 2007). The aim in case studies is not to generalize to the target population, but to generate theoretical propositions by deriving analytical generalizations about a case (Yıldırım and Şimşek, 2013). Considering the study sample, the study will attempt to identify factors that pose an obstacle in the process of making a correct sense of the concept of radian and propose recommendations for the elimination of these factors.

In the study, *holistic multiple-case study design* was employed. In this design, a series of phenomena, each of which lends itself to holistic interpretation, are addressed and these phenomena are compared to each other (Yıldırım and Şimşek, 2013). Since the study was conducted with in-service mathematics teachers from different schools using the same data collection tools and the resultant data were compared to each other, the subject design was considered appropriate for this study.

2.1. Participants

The sample group consisted of 41 in-service mathematics teachers from different types of schools in Ordu province. Maximum diversity sampling from purposive sampling methods was utilized in the selection of the participants. In purposive sampling, the main concern is to select cases that offer abundance of information so that it is possible to carry out a more in-depth research (Patton, 2014; p. 230). As this study was carried out with mathematics teachers in particular, purposive sampling method was used. The idea behind maximum diversity sampling, on the other hand, is not making generalizations. It aims to ensure maximum representation of the characteristics of individuals who the research question seeks to understand (Yıldırım and Şimşek, 2013. In accordance with this aim, 41 in-service voluntary teachers were selected from mathematics teachers working in different types of schools in Ordu province of Turkey. Demographic information about the participants of the study is given in Table 1.

Table 1.
Demographic Information About the Participant Mathematics Teachers

Demographic Information		Frequency
School Type	Social Sciences/Science High School	7
	Anatolian High School	10
	Common High School	13
	Anatolian Religious High School	4
	Anatolian Vocational High School	7
Gender	Female	17
	Male	24
Professional Seniority	5-9	3
	10-14	6
	15-20	22
	20 years or longer	10
Educational Attainment	Bachelor's Degree	31
	Master's Degree	6
	Doctor's Degree	4
Total		41

2.2. Data Collection Tools

Three data collection tools were utilized in the study. One was the *General Information Form* (GIF) which was prepared to determine the demographic characteristics of the participant teachers. Questions in the GIF were set to identify types of schools teachers worked in as well as their professional seniority and educational attainment. Other data collection tools, on the other hand, were intended to define teachers' ways of understanding of the concept of radian. They were *Concept Test* (CT) *and Unstructured Interviews* (UI).

The categories created at the end of the literature review were used in the determination of the questions in CT. In view of these, the concept of radian was handled under four categories within the scope of the study, namely *definition of the concept of radian (DR), connection of the concept of radian to the unit circle (CUC), connection of the concept of radian to trigonometric functions (CTF), the meaning of the number π* (*MNP*). CT was designed by researchers in accordance with the theoretical framework of the study and involved one open-ended question about each category. For setting the questions of the CT, studies by Fi (2003), Topçu, Kertil, Yılmaz and Önder (2006) and by Akkoç (2008) were utilized. CT included a total of four open-ended questions. Two academics (1 Assoc. Prof. and 1 Assist. Prof.) were consulted in determining the validity of the questions in CT and final versions of the questions were established in the light of their feedback. In this process, the opinions of experts were discussed and some of the regulations were made in expressions used in the questions at agreed points by the results of this discussion. As to UI, it was adopted for an in-depth analysis and interpretation of the responses given to CT. Indeed, Chadwick, Bahr and Albrecht (1984) defined UI as a discovery-oriented interview process without any prior expectation about questions and hence responses. In UI, researchers attempt to discover some specific issues with interviewees and if they discover some special areas about their research question, they can also try to investigate these areas in detail by means of more elaborate questions (Brannigan, 1985; Chadwick et al., 1984). Accordingly, UI were considered appropriate for the purposes of the present study.

After necessary permission was taken from Ordu Provincial Directorate of National Education, different types of schools in Ordu province were visited and teachers were informed about the research via face-to-face interviews. Following this process, research plan was designed with teachers who volunteered to take part in the research. In accordance with the plan, research data were collected in two steps. In the first step, with the help of school authorities, times when all voluntary mathematics teachers were available for meeting up were determined for each school. At these times, data collection tools, which were in the form of written documents, were administered to all voluntary teachers. In the process of implementation, teachers were expected to answer the questions in CT individually with no time restriction. Next, each teacher was contacted and the most appropriate UI time for each was determined. Later on, interviews were conducted at the previously organized times. This constituted the second step of the data collection process. Some teachers were available for UI just after CT while others suggested having their UI at later times. With the permission of teachers, UIs were recorded throughout the interview processes. UIs lasted 30 to 60 minutes and all the content was transcribed following the interviews. The objective of UIs undertaken within the scope of this study was to correctly understand the content and meaning of the responses that could not be coded easily and to make these responses viable for data analysis. In this context, UI emerged as an important factor that ensured the validity of the research. Thus, not all questions in the CT were directed to teachers during UIs; instead, more emphasis was laid on some specific questions. Three teachers (T_5, T_7 and T_{28}) did not participate in UIs due to their personal choices.

2.3. Analysis of the Data

Participant teachers' responses to the questions in the CT were analyzed and interpreted within the theoretical framework defined by the researchers. Each response that was relevant to a category in the theoretical framework was expressed under the components *Response (R), Type of Response (TR), Quality of Response (QR), Expression Used (EU), Respondent Teachers (RT) Frequency of Response (f)* and *Total Frequency (tf).* In the interpretation of the findings, frequency (f) values of the relevant components were used. The frequency values are given in parentheses in addition to the teachers' statements in the discussion section.

Of these components, Response (R) indicated the conclusion teachers reached in the calculation processes and hence, named as the solution of the question. Most of the time, it was just a brief statement. Type of Response (TR) was used only for the first question of the CT, because it allowed the responses in the respective category to be interpreted under different themes. In the category of Quality of Response (QR), content analysis was undertaken and teachers' written responses to the CT and the explanations they proposed for these responses in UIs were interpreted together. In this process, codes were used in accordance with the evaluation criteria provided in Table 2, and with some guidance from studies by Peterson and Treagust (1989), Abraham, Williamson and Westbrook (1994), Marek (1986), Ayas and Coştu (2002) and Karataş (2002). During the coding process, transcriptions of the UIs conducted with the participant teachers were read line-by-line in an attempt to prepare a structure and code list that could be used for all the responses given to the questions in the CT. Due to a lack of a conceptual structure that would provide guidance during the analysis of the data, such a structure was designed by the researcher. Therefore, this coding system can be named coding based on the concepts derived from the data (Yıldırım and Şimşek, 2013, p.264).

Table 2.
Evaluation Criteria Used in the Analysis of Teachers' Responses

Quality of Response	Code of Response	Explanation
Correct Response	CR	• Includes all parts of the valid response/explanation
Partially Correct Response	PCR	• Includes a part of the valid response/explanation, but not all parts • Includes some parts of the valid response/explanation, but also some misunderstandings
Incorrect Response	IR	• Responses with illogical or incorrect content
No Response	NR	• Left blank • Responses such as I do not know or I cannot make any comments

For the aforementioned coding, two academics who were experts in the area were consulted and the analyses were verified. In the Expression Used (EU) section, main expressions that teachers wanted to explain and underlined in their responses during the UIs were detected. In the Respondent Teachers (RT) section, numerical codes were used for referring to the participant teachers and the symbol T_n was used to indicate the n^{th} *teacher in the study* and *R* for the *researcher*. While Frequency of Response (f) denoted the number of teachers who gave a given response, Total Frequency *(tf)* indicated the total number of teachers who gave the responses in a given category.

Presentation of the results was based on the theoretical framework defined by the researcher. Considering the categories that constituted the basis for making sense of the concept of radian in connection with other concepts, a comparison was made between the frequency and percentage values of the different types of responses in these categories. After the reasons and results of the findings of the study were discussed, findings of the present study were compared and contrasted with those of past studies.

3. FINDINGS

3.1. Findings and Comments Regarding the DR Category

As a result of the study, findings given in Table 3 were obtained when teachers' responses to the first question read *"Please define the term radian, an angular measurement unit"* was examined.

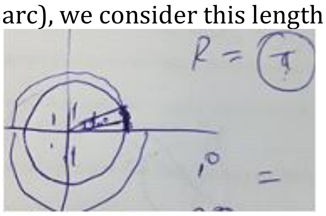
Table 3.
Responses to the First Question in the Concept Test

TR	QR	EU	RT	f
Responses involving the definition of 1 radian	PCR	The measure of a central angle subtended by an arc that is equal in length to the radius of the circle is called 1 radian.	T_1, T_2, T_{11}, T_{12}, T_{13}, T_{14}, T_{15}, T_{17}, T_{18}, T_{19}, T_{20}, T_{27}, T_{36}, T_{39}, T_{40}, T_{41}	16
	IR	The length of an arc that subtends a one-degree angle is equal to 1 radian.	T_3, T_7, T_9	5
		The measure of an arc, length of which is equal to the length of the radius is called 1 radian.	T_{23}	
		We define the area which subtends a one-unit arc and radius of which is equal to one unit as 1 radian.	T_{37}	
Responses involving the definition of radian	PCR	It is an angular measurement unit that divides a 360-degree circular angle into 2π equal parts.	T_4	4
		It is the length of the arc that is formed by an angle.	T_{30}, T_{32}, T_{35}	
	IR	It is a measurement unit and is known as π.	T_5, T_{24}, T_{25}, T_{28}, T_{38}	7
		It equals to the ratio of circumference to diameter. It is indicated by π.	T_{10}, T_{22}	
No Response	NR	-	T_6, T_8, T_{16}, T_{21}, T_{26}, T_{29}, T_{31}, T_{33}, T_{34}	9
Total				41

According to data presented in Table 3, none of the teachers provided a correct response to the first question of the concept test. If the other responses are examined, it is seen that a total of 20 teachers provided partially correct responses while 12 teachers gave incorrect responses and 9 of the teachers did not provide any response. PCR category responses about the definition of 1 radian were used for teachers who defined 1 radian instead of the term radian itself. These codes were also verified via UIs. UIs with those teachers whose responses about the definition of radian were in PCR category revealed that teachers were not able to fully define radian and that they usually regarded it as the length of an arc. Although the concept of radian is related to arc length, teachers who were able to respond correctly to the question could not actually explain this relation. Excerpts from the interviews conducted with the teachers who provided partially correct responses are presented below.

<u>*PCR-Sample Case 1:*</u> An excerpt from the interview conducted with the participant teacher who was assigned the code T_{30} is provided below.

T_{30}: Radian is the arc length of the circumscribed circle in a unit circle. What we call radian, we defined it as π. We defined the arc length in a unit circle as π. Therefore, expressing an arc length in π was called radian.
R: Could you please show me a one-radian angle?
T_{30}: Instead of a one-radian angle, you know we defined radian as length… it is not an angle… so your question should be like… a one-radian length…
R: Isn't radian an angular measurement unit?
T_{30}: It is an angular measurement unit, but as a length defined by that degree…
R: O.K. Then let's show it…
T_{30}: Look, assume that this equals to 20 degrees (points to the central angle that is subtended by the shaded arc), we consider this length that subtends this 20-degree angle as radian (points to the shaded arc).



R: If I asked you what is 20-degrees in radians, would you be able to calculate it?
T_{30}: Yes, of course. It has a standard formula D/180=R/ π. We get the equation R= π/9 from the previous one. Here, π =180 degrees, however, it scans an area and it's the length π/9. So if you come towards this point it equals to π (draws a semi-circle) and if you come towards this point, it equals to 2π.
R: What exactly equals to π/9?
T30: It is π/9 in radians.

R: Is it possible to that we use π as 3.14?
T_{30}: Yes, it is. That is, π/9=3.14/9=0.some value… So we've found the unit.
R: O.K. In a circle with a radius of 5 units, what is the measure of a central angle which is subtended by a 10-unit arc in radians? How would you calculate this?
T_{30}: We can do it like this. In the formula we use for arc length l= (2πrα) / 360, we put l =10 and r= 5 units and we calculate α in degrees. Then we can convert degrees to radians.

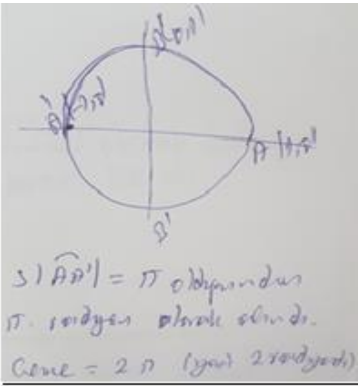
In the interview presented above, it is clear that T_{30} connected the concept of radian only with the unit circle and was not able to define radian as the ratio of two lengths. This teacher perceived radian as the length of an arc that subtends a central angle in a unit circle. Although this knowledge is accurate, it is seen that the same teacher had difficulty in using radian as an angular measurement unit and could not understand and make sense of a one-radian angle.

PCR-Sample Case 2: An excerpt from the interview conducted with the participant teacher who was assigned the code T_{35} is provided below.

T_{35}: To be honest, I don't know exactly, but I think it is the length of an arc that is scanned by an angle.
R: In a unit circle or in all circles?
T_{35}: I don't know.

In the interview with T_{35}, the teacher defined radian as arc length; however, this teacher was not able to provide any further explanations. In a similar way, T_{32} who provided a partially correct response to the first question of the CT was also not quite sure about their response. Excerpts from the interviews with 12 teachers whose responses to the first question of CT were incorrect are provided below.

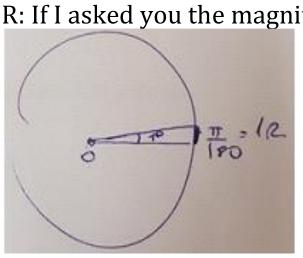
IR-Sample Case 1: An excerpt from the interview conducted with the participant teacher who was assigned the code T_{22} is provided below. It is obvious from the interview with T_{22} that this teacher defined radian as π and believed that the number π equals to 1 radian.



T_{22}: Radian can well be a fixed real number, and 1 radian equals to 180 degrees, another angular measurement unit. π =22/7 and π=180^0. If put as a definition, π= Circumference/ Diameter € R.
R: What is the definition of radian not π?
T_{22}: But π is radian.
R: Would you please draw a circle and show me a one-radian angle?
T_{22}: 1 radian equals to 180 degrees. And so, the magnitude of the arc AA' is 1 radian. That is, there are two radians in a circle.

IR-Sample Case 2: An excerpt from the interview conducted with the participant teacher who was assigned the code T_9 is provided below.

T_9: A central angle that is subtended by an arc with the same length as the radius is equal to 1 radian.
R: Could you please provide a general definition for radian?
T_9: This is the definition of radian; this is what we teach our students when defining radian. In a circle, a central angle that is subtended by an arc with the same length as the radius is equal to 1 radian.
R: Radian or 1 radian?
T_9: This is the way radian can be defined. It doesn't have another definition.
R: If I asked you the magnitude of the arc in the image (points to the scanned arc), what would be your answer?



T9: We can identify it using this angle. Each arc that subtends a one-degree angle equals to 1 radian. Magnitude of a given angle is equal to the same amount of radian That is, if this is 60 degrees, it is also 60 here… (corrects their response) π/3 radian As the equivalent of degree.
R: Would you please draw 1 radian for me?
T_9: This is 1 radian (points to the arc that subtends a one-degree angle). If my degree is 1 unit, I would use D=1 in the formula D/180=R/π and get R= π/180. And so, this equal to 1 degree. Anyway, π is equal to 180 degrees.

As a result of the interview with T_9, it was revealed that this teacher defined radian correctly at the beginning of the interview; however, since this teacher perceived π/180 radians as 1 radian, their response was marked as incorrect. Teacher T_9 tried to define the concept of radian within the context of a unit circle, but failed to provide correct responses as they could not see the connection between radian and degree.

3.2. Findings and Comments Regarding the CUC Category

Analysis of the teachers' responses to the second question of the CT namely "*How many radians are there in a circle?*" yielded the findings presented in Table 4.

Table 4.
Responses to the Second Question of Concept Test

R	QR	EU	RT	*f*	*tf*
There are 2π radians.	CR	2πr/r=2π.	T_{11}, T_{14}, T_{20}, T_{39}, T_{40}, T_{41}	6	10
		A circle is made up of 2π arcs with each with the same length as r.	T_{17}, T_{18}, T_{27}, T_{36}	4	
	PCR	There isn't any reason, I know it by heart.	T_4, T_6, T_9,T_{16}, T_{21}, T_{29}, T_{32}, T_{34}, T_{35}	9	17
		A circle equals to 360^0. In the formula D/180=R/π, it is calculated by using D=360.	T_2, T_{31}	2	
		Since π=180^0, there are 2π radians.	T_5, T_{33}	2	
		Circumference=2π.1=2π radians.	T_{26}	1	
		Because the number of one-unit arcs is 2π.	T_{37}	1	
		1 degree is equal to 0.017 radians; hence if we calculate 360/0.017, it makes 6.28.	T_8	1	
		If we assume that 1 radian is approximately 56^0, then we get 6.28 radians.	T_{19}	1	
There are 6.28 radians.	CR	Because circumference=2πr=2. 3.14.r =6.28. r	T_{12}, T_{15}	2	2
There are π radians.	IR	In a unit circle, the number of radians equals to π=Circumference/2.	T_{22}, T_{23}	2	3
		Because π equals to radian.	T_{25}	1	
There are 2r radians.	IR	Since the circumference of a circle equals to 2πr, we can write π.2r. In brief, there are 2r radians.	T_{10}, T_{30}	2	2
There are 2 radians.	IR	Two semi-circles put together make 2π radians. In brief, the number of radians is 2.	T_1	1	1
There is 1 radian.	IR	It is the ratio of circumference to diameter. That is, it equals to π.	T_{38}	1	1
There are 360 radians.	IR	The length of an arc that subtends a one-degree angle is 1 radian.	T_3	1	1
-	NR	-	T_7, T_{13}, T_{24}, T_{28}	4	4
Total					41

If Table 4 is examined, it is clear that 10 teachers in the sample gave correct responses to the second question of the CT. Responses of 25 teachers; on the other hand, were either incorrect or partially correct. Finally, 4 teachers failed to respond to the question.

As a result of the interviews with those whose responses to the 2nd question were correct, it was revealed that T_2 and T_{31} stated that there were 2π radians in a circle, however, they could calculate this only through the formula D/180=R/π and could not offer any further comments. Therefore, their responses were coded as ICR. As to the teachers who were assigned the codes T_8 and T_{19}, they could respond correctly only after making some calculations although there was no need for calculations. These two teachers' calculations are presented below.

Teacher T_{19}'s Response:

T_{19}: Considering the formula 2πr.α/360=r, we get α=360/2. 3.14. That is, α=360/6.28. And it can be accepted approximately as 56 degrees. And considering this, since an arc of circumference equals to 360 degrees, there are 350/56=6. … radians.

Teacher T_8's Response:

T_8: If the length of a circle is calculated using 2πr, we find 1^0=2π/360= π/180=0.017 radians. There are 360^0/0.017=6.282 radians.

16 of the 17 teachers (except for T_5) whose responses to the 2^{nd} question of the concept test were partially correct said that *there are 2π radians in a circle*; however, this was only memorized knowledge and they could not explain the reason for this response. Relevant interview excerpts are presented below.

PCR-Sample Case 1:

T_{26}: As r=1 in a unit circle, a full circular angle is called 2π radians. Therefore, its half equals to π radian.
R: Why 2π radians?
T_{26}: 1 π corresponds to 180 degrees, and so 2π = 360 degrees. But, I do not remember why it was named π or why π was defined as 180.

If the response of the teacher T_{26} is examined, it is seen that this response was correct, yet this teacher was not able to explain the reasons for their answer. The fact that the teacher calculated the circumference of a unit circle could be taken to imply that this teacher perceived radian as the length of an arc; however, it was revealed in the interview that this teacher had actually memorized that a full circular angle equals to 2π.

PCR-Sample Case 2:

$T_{34:}$ The unit circle is 360 degrees and equals to 2π radians. And 1 radian equals to 180 degrees.
R: Then is it possible to say that there are 2 radians in a circle?
$T_{34:}$ There are two radians in a unit circle. Because handling it in relation to a unit circle is different from handling it from a trigonometric perspective.
R: So you mean radian is a concept that is relevant only to a unit circle, right?
$T_{34:}$ Yes, this is my opinion.

Although teacher T_{34} responded correctly to the question, this was only memorized knowledge; because, the teacher could not justify their response and gave some wrong explanations about the concept through the end of the interview.

In the concept test, 8 teachers' responses to the 2^{nd} question was incorrect, interviews with these 8 teachers revealed that they actually had different misconceptions about the concept of radian. Teachers T_{22}, T_{23}, T_{24}, T_{25}, T_{10}, T_1 and T_{38} were seen to perceive radian as π. Therefore, it is possible to say that teachers whose responses were incorrect had quite similar misconceptions about the concept of radian. Some relevant interview excerpts are presented below.

IR-Sample Case 1: Relevant excerpt from the interview with teacher T_{30} is given below. Interview with the teacher assigned the code T_{30} indicates that this teacher perceived radian as π.

$T_{30:}$ A full circle is 2π radians.
R: How is it calculated?
$T_{30:}$ If we accept a full angle as 360 degrees, 360 degrees equal to 2π in a unit circle, because circumference equals to 2πr=2π.1=2π in a unit circle. Therefore, we get $2\pi=360^0$.
R: So there are 2π radians, right?
$T_{30:}$ There are 2π radians in a unit circle. But, if you handle it as a length, you can put 3.14 in the formula.
R: I couldn't understand what you've said. If I put the question differently, how many radians, do you think, are there in a circle?
$T_{30:}$ In a circle or unit circle?
R: That doesn't make any difference.
$T_{30:}$ It does make a difference, because we handle it as a length and as the radius changes, so does the length.
R: O.K. Then we take a general circle.
$T_{30:}$ In any given circle, it will be equal to 2πr. And radian will be equal to π times this measure. I mean. We can say that 2πr= π. 2r and hence, the result will be 2r. In this case, I change my answer. If we handle radian in a unit circle as π, there are 2 radians. But, only in a unit circle. In other circles, it varies depending on the radius.

IR-Sample Case 2: Relevant excerpt from the interview with teacher T_3 is given below. Interview conducted with the teacher who was assigned the code T_3 demonstrated that this teacher was able define radian only in relation to a unit circle and defined it as the length of an arc that subtended a one-degree angle.

$T_{3:}$ A one-radian angle in a unit circle equals to the length of an arc that subtends a one-degree angle. Therefore, there are 360 radians in a circle.
R: Is it true for all circles or only for a unit circle?
$T_{3:}$ Only in relation to a unit circle.
R: Then, if I asked you how many radians are there in a circle other than a unit circle, what would be your answer?
$T_{3:}$ I don't know. We teach the concept of radian as part of the trigonometry curriculum and thus, this is the only way I can explain it.

3.3. Findings and Comments Regarding the CTF Category

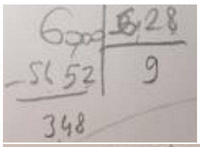
As a result of the study, analysis of the teachers' responses to the third question of the CT namely *"$f: R \rightarrow R$; f(x)=x.cosx function is what you have. Please calculate f(60) (60 is given as a pure number and not as an amount with units of degrees)"* yielded the findings presented in Table 5.

Table 5.
Responses to the Third Question in the Concept Test

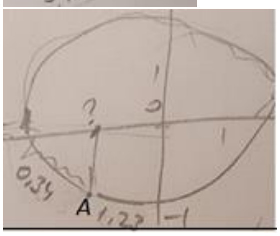
R	QR	EU	RT	tf
60.cos (11π/10)	CR	60≅19.10.π≅ 18π+11π/10	T_{12}, T_{15}	6
The interval is (-60,-30.√3).		60 radians equal to 3436⁰. Its principal value is 196⁰. Cos 196⁰ ϵ (-1, -√3/2)	T_{22}, T_{23}	
-57.25		60 radians equal to 197.4⁰. Cos (197.4⁰)=-0.9542, 60.(-0.9542)=-57.25	T_{27}	
Cos 60≅ -0.…		If we assume 60 is in radians, the value is around -0.…	T_{41}	
60	PCR	We have 60 ≅ 60/3 =20π ≅ 0⁰. cos 60 ≅ cos 0⁰ =1. f(60)=60.	T_1, T_3	3
		Cos 60≅cos3600⁰ ≅ 1, f(60)=60	T_2	
30	IR	Cos60= 1/2, f(60)=60.1/2 =30	T_4, T_8, T_9, T_{10}, T_{11}, T_{16}, T_{19}, T_{26}, T_{31}, T_{33}, T_{34}, T_{36}, T_{38}	13
-	NR	60 should be used in degrees for the calculation to be done.	T_6, T_{20}	19
		There is a real number in a trigonometric expression, this is wrong by concept.	T_{30}	
		As 60 is not given in units of degrees (as radian or gradian units are not mentioned at all), it is undefined.	T_{37}	
		-	T_5, T_7, T_{13}, T_{14}, T_{17}, T_{18}, T_{21}, T_{24}, T_{25}, T_{28}, T_{29}, T_{32}, T_{35}, T_{39}, T_{40}	
Total				41

If Table 5 is examined, it is clear that 6 teachers' responses to the 3rd question of the CT were correct. However, 5 of those teachers converted radian to degrees so as to be able to calculate the results. Only one teacher (T_{41}) was able to answer the question in radians.

<u>*CR-Sample Case:*</u> Relevant excerpt from the interview with teacher T_{41} is given below.



T_{41}: We can calculate it by converting 60 into degrees.
R: Isn't it possible without conversion? I mean, is it possible we handle it in radians?
T_{41}: Yes, it is possible via calculations like proportionality. Is the number 60 a real one here?
R: Yes, it is a real number.



T_{41}: Then, is it radian?... Yes… If we handle it in relation to a unit circle, what do 60 radians equal to (the teacher performs mathematical operations given on the left). For an arc of 3.48 units… If we subtract it from 3.14… The result is 3.48-3.14=0.34.
T_{41}: We take 9 rotations… It corresponds to this part (points to the point A in the drawing on the left). It is possible to use proportions to calculate the cosine value of the point A.
If the cosine value of an arc of 1.57 units is 1, an arc of 1.23 units has a cosine value of… The result is 1.23/1.57= 0 dot something. This is Cos 60.

Teacher T_{41} indicated that they could solve the problem by converting radian into degree. However, with the guidance of the interviewing researcher, this teacher was able to find the correct solution of the problem without conversion. Teacher T_{41} was also able to solve the problem by means of the alternative solution they proposed, yet it was not considered necessary to present it here.

3 of the participant teachers provided partially correct responses to the 3rd question of the CT. Those teachers tried to convert radian to degree, yet they failed to do so due to calculation errors. The fact that participant teachers utilized approximate values for the solution of the problem led to considerable changes in the results. Nevertheless, teachers did not notice this change and could not answer the question correctly. Interview processes are not presented in this section due to the fact that most of the expressions that those teachers in question used in their solutions are demonstrated in Table 5.

13 teachers' responses to the subject question were incorrect. Although those teachers indicated that in the question, 60 was not given in the form of degrees, they used this value as if it was given in the form of degrees. Despite the fact that they were reminded of this fact, they replied *"I cannot think of another solution"* (T_{10}, T_{31}), *"I don't how I can do it in another way"* (T_4, T_9,

T_{26}), *“No comments”* (T_{11}), *“I cannot calculate it unless the angle unit is provided”* (T_{19})“, *“f is a function from R to R, therefore it is true that cos 60=1/2.”* (T_8, T_{36}), *“It cannot be solved using the other method*.” (T_{33}).

IR-Sample Case: Relevant excerpt from the interview with teacher T_{38} is given below.

R: It is underlined in the question that 60 is not in degrees.
T_{38}: If a number is provided in a question, we automatically handle it as if it was given in degrees. If it was π, we would assume it was 180 degrees and solve the problem that way. This is why I assumed it was an angle.

Teacher with the code T_{38} whose response to the 3rd question of the CT was incorrect stated that they perceived any trigonometric function variable that had a numerical value other than π as degree and that they handled the calculation that way. 19 of the participant teachers preferred not to respond and justified this act of theirs with different explanations. Relevant excerpts from the interviews with teachers who did not give any response are presented below so that their explanations can be studied.

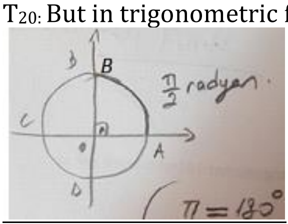
NR-Sample Case 1: Relevant excerpt from the interview with teacher T_{37} is given below.

T_{37}: As 60 is not specified as an amount with units of degrees, this expression is undefined. Cosine function can be defined only via the concept of angle.
R: So you think 60 does not denote an angle here, right?
T_{37}: Yes, exactly.
R: O.K. We assume cos π= -1, do you think this value denotes an angle?
T_{37}: Yes, it does… (?) I don’t think it has another equivalent… Π is an angle here, because cosine is a ratio, it is a special ratio in triangles.
R: What do you mean by ratio?
T_{37}: Cosine is a special ratio. It is a special ratio in right triangles. Π is not given in degrees, but… it is undefined anyway… (cannot respond)… I don’t know.

According to their statements, teacher T_{37} had the misperception that trigonometric functions could be defined only when variables were specified in degrees and T_{37} had hesitation in accepting that π denoted an angle. In conclusion, T_{37} believed that angles are denoted only in degrees and could not make sense of alternative cases.

NR-Sample Case 2: Relevant excerpt from the interview with teacher T_{20} is given below.

T_{20}: What does 60 correspond to in terms of its unit?
R: It is a real number and does not have any unit.
T_{20}: But in trigonometric functions this expression should be provided in the form of angles.



R: Do you mean degrees?
T_{20}: Yes.
R: In cos π, what does π represent?
T_{20}: Π is a radian. We can show its measure in degrees on the unit circle. The point B in the drawing denotes π/2 and the point C denotes π.
R: But why does B denote π/2 and why does C denote π?
T_{20}: I don’t know.

Teacher with the code T_{20} had a misperception like T_{37}. More clearly, T_{20} was of the opinion that trigonometric functions could be defined only when variables were provided in degrees.

3.4. Findings and Comments Regarding the MNP Category

As a result of the study, findings given in Table 6 were obtained when teachers’ responses to the fourth question which read *“What is the meaning of the number π that we use in trigonometry? Please explain its connection with the real number π=3.14.”* was examined.

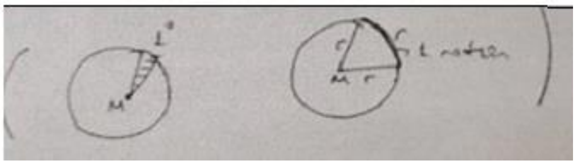
Table 6.
Responses to the Fourth Question in the Concept Test

R	QN	EU	RT	tf
It is the same π.	CR	It is the same π.	T_{27}, T_{41}	2
	PCR	Both have the same meaning. If it is about angles, it is used as 180^0, while it is used as 3.14 in calculations of area, volume etc.	T_9, T_{30}	3
		It is the same π. Measurement unit of radian is π.	T_{13}	
	IR	Π means radian in trigonometry. And the equation is π≅ 3.14.	T_{25}	1
π radian= 180^0.	PCR	It is a constant that equals to the ratio of the circumference of a circle to its perimeter. It equals to 180 degrees in a unit circle.	T_1, T_2, T_3, T_8, T_{10}, T_{14}, T_{15}, T_{17}, T_{19}, T_{33}, T_{36}	11

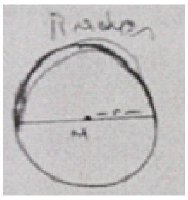
They are different terms.	IR	They are not the same. There are two different π numbers. One is for angles and the other is for length.	T_{20}, T_{22}, T_{23}, T_{26}, T_{38}, T_{39}, T_{40}	11
		Although their symbols are the same, the one that is used in trigonometry is $\pi=180^0$.	T_{31}, T_{37}, T_{34}	
		Π is a value expressed in radians. As to π=3.14, it is a different approximate value that is used in the area.	T_6	
-	NR	-	T_4, T_5, T_7, T_{12}, T_{16}, T_{24}, T_{28}, T_{29}, T_{32}	13
		No comments.	T_{11}, T_{18}, T_{21}, T_{35}	
Total				41

If the data in Table 6 are analyzed, it is seen that 2 teachers' responses to the fourth question were correct while 14 teachers' responses were partially correct. 12 of the teachers gave incorrect responses and 13 teachers did not responded to the question. Teachers whose responses were partially correct generally were of the opinion that a π-radian angle equaled to 180^0. Although this explanation is true, it was seen in the interviews that teachers failed to justify their responses.

PCR-Sample Case: Relevant excerpt from the interview with teacher T_{14} is given below.



T_{14}: A central angle subtended by 1/360 of the circumference of a circle equals to 1^0 in magnitude. A central angle that is subtended by an arc with the same length as the radius of a circle equals to 1 radian. The formula for the circumference of a circle is C=2.π.r. And thus, half of the circumference of a circle is equal to π radians. That is, π radians equal to 180^0.



In this case, π=180 is wrong. It should be Π=3.14.
R: We say that cos π= -1, is the number π here equals to 3.14?
T_{14}: No. It is $\pi=180^0$ in that example. Alternatively, we can say that it is π radians.
R: How do we determine that?
T_{14}: In a high school setting, we always use it as degrees.
R: Do you think the expression cos 3.14= -1 right?
T_{14}: I don't know the reason actually. I need to check the course book, but in course books, it is always handled as degrees. Radian is not used. I don't know why it is so.
R: Do we always use π in expressions with radians?
T_{14}: Yes. For example, we say 2π radians.

In the interview excerpt shown above, teacher with the code T_{14} responded to the fourth question of the CT correctly by stating that there are not two different π numbers. Yet, the same teacher found it difficult to accept radian as a real number and believed that an angle measured in radians should always involve some π expressions.

If teachers' incorrect responses to the fourth question of the CT are analyzed, it is clear that they all had different misconceptions.

IR-Sample Case: Relevant excerpt from the interview with teacher T_6 is given below.

T_6: π is a value in radians. For example, π radians mean 180 degrees. 2π radians mean a full circle i.e. 360^0.
R: Do you think it has a connection with the real number π=3.14?
T_6: I don't think so. I guess there isn't any connection, because we always use radian as degrees. Actually, this is the way this topic is explained in course books. For example, in Sin π/3, we assume $\pi=180^0$ and say that this angle which is a π/3-radian angle equals to 60^0. This is what I think.
R: O.K. Why is not radian represented by an alternative symbol but by the symbol π?
T_6: I have never thought its reason before…. The number π is the number that equals to the ratio of the circumference of a circle to its perimeter. Its connection with this… I have no idea. This is how we teach it in our classes.

In the interview process explained above, teacher T_6 had incorrect knowledge on the issue. More clearly, this teacher believed that the number π in trigonometry was a special number which equals to 180^0. Besides, the same teacher also thought that the number π did not have any connection with the real number 3.14 which is the number that expresses the ratio of the circumference of a circle to its perimeter.

13 teachers who did not respond to the fourth question of the CT were unable to provide any opinion regarding the question.

NR-Sample Case: Relevant excerpt from the interview with teacher T_{18} is given below.

T_{18}: The number π equals to the ratio of the circumference of a circle to its perimeter.
R: Why do we assume that a number which equals to 3.14 is equal to 180^0? Or do they represent two different numbers?
T_{18}: Circumference of a circle is equal to 2.π.r. Thus, π.r equals to half the circumference. And so, π radians equal to 180^0.
R: Which π do you mean when you say π radians? Is it the real number that equals to 3.14 or is it 180^0?
T_{18}: I don't know.

4. RESULTS, DISCUSSION AND RECOMMENDATIONS

As a result of the present study which aimed to discover mathematics teachers' ways of understanding of the concept of radian, teachers of mathematics were found to be unable to make a complete and correct sense of radian. Teachers were also unable to connect radian to the unit circle and trigonometric functions except for some specific cases. Finally, it was revealed that they failed to make a connection between radian and the real number form of the number π. In general, teachers utilized the formula D/180 = R/π for converting the measure of an angle given in degrees into one that is expressed in radians. As a consequence of this; however, they held that radian is a concept that always used in relation to π. When teachers were asked to explain the meaning of π, they usually responded 180^0 and pointed to the point (-1,0) on the unit circle. Teachers, in general, knew that π radians equal to 180^0, yet they could not explain the reason for this. Therefore, it is possible to say that teachers had some memorized knowledge about the concept of radian; however, they did not have a deep conceptual understanding of it. Below, results obtained from each category used for the preparation of the CT will be discussed one-by-one.

In relation to the question in the category DR, it was seen that none of the participant teachers were able to provide a response that involved all the aspects of the valid response. The other correct responses with low frequency were questions in the categories MNP, CTF and CUC, respectively. In the category DR, majority of the teachers defined 1 radian, but none of them were able to define radian as a ratio of two lengths. Although this also replicates the results of various past studies (Fi, 2003; Topçu, Kertil, Yılmaz & Önder, 2006), it demonstrates the origins of the different types of misconceptions. In that, teachers who did not know the subject meaning of radian connected it with arc length and the number π. Besides, they had difficulty in considering radian as a real number. In the category DR, some incorrect responses with the highest frequency levels were "*It is a measurement unit and is known as π.*" (5), "*The length of an arc that subtends a one-degree angle is equal to 1 radian.*" (3) and "*It is the ratio of circumference to diameter. It is expressed as π.*" (2). In view of these responses, it is seen that 17.07% of the participant teachers defined radian as π. In the interviews, it was discovered that those teachers perceived π as a unit of radian due to the term π-radian. This misunderstanding of teachers was also confirmed by the findings of various past studies (Fi, 2003; Tuna, 2013). Responses in the category DR, 10 teachers were found to define radian as arc length. In the subsequent interviews, it was seen that teachers made explanations that contradicted their responses and were not consistent. In general, teachers expressed that this was how the concept of radian remained in their minds and they were not able to properly connect radian to arc length. This might imply that teachers actually had learnt the concept of radian accurately, but they probably forgot this knowledge through time.

An analysis of the responses in the category CUC reveals that responses with the highest frequency were partially correct ones. As a result of the interviews concerning this section, 9 teachers replied the question about the reason for which there are 2πs in a circle as *"There is no specific reason, I know it by heart."* As to the others, they knew that a π-radian angle equals to the half of a unit circle; however, they could not justify this fact by explanations. Keçeli & Turanlı (2013) obtained similar findings in their study. Responses with the second highest frequency were the correct ones. If correct responses are analyzed, it is seen that the majority of the teachers used the formula 2πr/r=2π. Teachers explained its reason as *"If an angle that is subtended by an arc of r length equals to 1 radian, then an angle that is subtended by an arc of 2πr, which is equal to the circumference of a circle, is equal to 2π radians."* As this explanation is right, this response was assigned the code CR, but what did the participant teachers mean by the expression *"2π"*? Interviews with the 3 teachers who responded this way demonstrated that they used the expression 2π to mean approximately the real number 6.28 and that they perceived the concept of radian as a real number. Other teachers whose responses were categorized as CR, on the other hand, had difficulty in perceiving π as a real number. Similarly, in a study by Topçu, Kertil, Yılmaz and Önder (2006), participants were found to perceive π as a real number confirming also the findings of the present study. Incorrect responses in the category CUC included a variety of misunderstandings. Nevertheless, 6 teachers who responded as "there are π radians in a circle", "there are 2r radians in a circle" or "there is 1 radian in a circle" were found to perceive radian as π, as it was also the case in the DR category. In a similar way, a study by Fi (2003) also yielded similar conclusions.

In the category CTF, responses with the highest frequency were those categorized as no response. 15 teachers could not express any opinion regarding the question and left the response section blank. Those with the second highest frequency, on the other hand, were incorrect ones. Interviews with teachers to whom those responses belonged demonstrated that 8 teachers handled the number 60 in the form of degrees because, in general, they were not able to find an alternative solution. Teachers whose responses were partially correct considered the number 60 as radians and attempted to express it in the form of π; however, they failed to find the correct answer. In the interviews with those teachers, it was discovered that they believed in order for an expression specified in radians to be used in trigonometric functions, it should be expressed in the form of π. In the category CTF, it is seen that in the majority of the correct responses, the value that was originally in radians was converted either into degrees or into π. As a result of the interviews with teachers to whom these responses belonged, it was seen that except for one teacher, the others could not provide an alternative solution to the question. This is an indicator of the fact that teachers had difficulty in considering radian as a real number and connecting it with the unit circle and trigonometric functions. In a study by Akkoç (2008), pre-service teachers were found to have difficulty in considering radian as a real number. In view of these, findings of that study and the present one can be considered to be similar.

In the category MNP, responses with the highest frequency were those categorized as no response, partially correct responses, incorrect responses and correct responses, respectively. The number of teachers who responded correctly to this question was 2. If partially correct and incorrect responses are analyzed, 10 teachers stated that "*π radian units equal to 180 degrees in a unit circle*", although they could not fully justify their explanations, 7 teachers, on the other hand, believed that there were two types of π. According to the participant teachers, of these two types, one was equal to the real number 3.14 while the other equaled to 180 degrees. In a study by Akkoç (2008), pre-service teachers also suggested that there were two types of π. In this regard, findings of the afore-mentioned study confirm the findings of the present study.

If all the results that have been presented up to this point are analyzed, the following general remarks can be made about the results.

- Teachers know the definition of the concept of radian; however, they are not able to describe it as the ratio of two lengths. This is one of the major factors that lie behind their incomplete understanding of the concept of radian.
- In general, teachers know that there are 2π radians in a circle, yet they cannot interpret it properly since they are unable to use the number π as a real number.
- Some teachers perceive radian as π due to the use of the common expression "π radians." In such cases, the number π is usually perceived as the measurement unit of radian.
- In general, teachers are incapable of recognizing the concept of radian as a real number. Due to this fact, they have difficulty in understanding the connection expressions which are put in radians without using the number π has with trigonometric functions or the unit circle.

These results have been replicated by some other studies conducted with students and pre-service teachers. Therefore, the fact that teachers' misunderstandings are similar to those of students and pre-service teachers is an indicator of their incomplete understanding about the concept of radian. Limited use of the concept in learning environments might be a reason for this. In our current education system, students follow a question-oriented study method due to their concerns about the university entrance examination they are expected to take. From past to present, curricula followed in our education system have always addressed angles in relation to trigonometry in the form of π radians. Therefore, students have never felt the need to use the concept of radian in different settings. As to teachers, it can be said that they had not been able to develop a sound conceptual understanding of radian since they teach to meet their students' needs in accordance with the curriculum. Some teachers emphasized this fact during the UIs as well. Those teachers indicated that they did not use the type of questions directed during the interviews since they did not need them for their classes and questions of subject type were not included in university entrance exams. Apart from these, some teachers stated that course books did not cover information and questions of that type. If a high school course book for 11th grade mathematics course (MoNE, 2016) is examined, it is seen that the only definition about radian is "a central angle that is subtended by an arc with the same length as the radius of the circle is equal to 1 radian" and it is stated in the course book that this definition means that a full circular arc equals to 2π radians. In the subject course book, it is also explained that 1 radian equals to 360/2π degrees that is to 57.30. This, in a way, explains teachers' inability to provide a general definition for radian; however, cannot justify their failure to depict radian as a real number. A brief examination of the exercises in the course book reveals that students are usually asked to convert degrees to π radians and exercises focus mainly on the formula D/180 =R/π. This can constitute one of the reasons for the inability to connect radian as a real number with trigonometric functions. Similarly, Kang (2003) argued that the way trigonometric functions and particularly the concept of radian were handled in curricula and course books made it harder for students to fully understand the subject. Briefly explained, some findings of the study might be explained by the way the concept of radian is addressed in course books. Nevertheless, in relation to the negative results, participant teachers are expected to recognize their potential in relation to their subject matter knowledge and to develop themselves in areas where there is need for improvement. Then, Orhun (2004) states that learning difficulties that students experience when studying the concept of radian stem not only from curricula but also from teacher-centered and rote learning-based instruction methods.

Although, initially teachers from Science/Social Sciences High schools and Anatolian High Schools were expected to be more competent in terms of their conceptual knowledge as per radian; the study has not detected a relationship between school type and conceptual knowledge. In a similar fashion, senior teachers were expected to have a richer repertoire of conceptual knowledge, yet no such finding was obtained. Some teachers with bachelor's degree were more proficient than their peers with master's degree. Yet, the participant teacher whose responses to the research questions were mostly correct was a PhD student. According to these findings, it is clear that variables that were observed so as to ensure maximum diversity did not exert the expected influence on research results.

Depending on the discussion of this study so far, two important points can be underlined in the meaningful learning of the concept of radian. Firstly, it should be conveyed that the real number which is not specified by a unit type and denotes the ratio of the length of an arc that subtends a central angle to the radius of a given circle indicates the magnitude of the subject angle in radians. This piece of knowledge plays a particular role in teaching that radian is a real number as well as in explaining the reason for which it does not have a unit. Considering this piece of knowledge, an individual will be able to grasp that the magnitude of a central angle in radians equals to the length of the arc it is subtended by. In this way, it will be possible to calculate the images of variables which are provided in real numbers in trigonometric functions. The second point to be underlined entails the fact that there are $2\pi \cong 6.28\ldots$ radians in a circle. If possible, this fact should be reinforced via visualization. It can benefit from different teaching methods, computer aided environments and different software. Steckroth (2007) mentions the positive effects of lessons on meaningful learning by visualizing the radian concept in computer-assisted environments. It is believed that these strategies would promote a complete understanding of the concept while also eliminating various misunderstandings to a certain extent. Considering that the participant teachers used course books for reference, it would be recommendable to cover important aspects about the concept of radian in course books and curricula. Besides, inclusion of concept-based open-ended questions, which resemble to those in the present study, in course books or teachers' books would enable teachers to recognize their own competence in relation to the subject concept and to offer their students concept-based learning processes.

Another step might be to organize concept-based seminars for teachers. These seminars can be designed as projects by various academics and can be held in the regions of service of those academics. This, as a result, would help teachers recognize their repertoires of conceptual knowledge and enhance teachers' competence in terms of subject matter knowledge. Findings of the present study correspond to the results of other studies in the relevant literature. Accordingly, there is actually a clear picture of the aspects in which conceptual knowledge problems are/would be experienced in relation to the concept of radian. In the light of the findings of this study and other studies in the literature, quantitative studies might also be conducted concerning the connection of the concept of radian with other trigonometric concepts.

5. REFERENCES

Abraham, M. R., Williamson, V. M., & Westbrook, S. L. (1994). A cross-age study of the understanding of five chemistry concepts. *Journal of Research in Science Teaching*, *31*(2), 147-165.

Ağaç, G. (2009). *Lise öğrencilerinin trigonometri öğrenme alanında grafik hesap makinesi kullanımının akademik başarıya ve problem çözme becerisine etkisi*. Unpublished PhD thesis, Dokuz Eylül University, İzmir.

Akbaş, N. (2008). *10. sınıf öğrencilerinin radyan kavramına ilişkin sahip olduğu yanılgıların giderilmesine yönelik bir öğretim sürecinin incelenmesi.* Unpublished master's thesis, Marmara University, İstanbul.

Akkoç, H. (2007). Matematik öğretiminde bilgisayar kullanımının sınıf pratiğine entegrasyon süreci: İntegral kavramı. *Yeditepe Üniversitesi Eğitim Fakültesi Dergisi (EDU 7), 2*(2), 1-15.

Akkoç, H. (2008). Pre-service mathematics teachers' concept images of radian. *International Journal of Mathematical Education in Science and Technology*, *39*(7), 857-878.

Akkoç, H., & Gül, N. A. (2010). Analysis of a teaching approach aiming at eliminating student difficulties with radian. *Eğitim Bilimleri Fakültesi Dergisi*, *43*(1), 97.

Ay, Y. ve Başbay, A. (2017). Çokgenlerle ilgili kavram yanılgıları ve olası nedenler. *Ege Eğitim Dergisi*, *18*(1), 83-104.

Ayas, A., & Coştu, B. (2002). Levels of understanding of the evaporation concept at secondary stage. *The First International Education Conference, Changing Times Changing Needs,* Gazimagusa-Northern Cyprus: Eastern Mediterranean University.

Baki, A. (2014). *Kuramdan uygulamaya matematik eğitimi* (5 th ed.). Ankara: Harf Publishing.

Blackett, N., & Tall, D. (1991). Gender and versatile learning of trigonometry using computer software, F. Furinghetti (Ed.), Proceedings: *Fifteenth PME Conference* içinde. Vol 1 (pp. 144-151), Italy.

Brannigan, G. G. (1985). The research interview. A. Tolor, (Ed.), *Effective interviewing* (pp. 196-205). Illinois: Charles C. Thomas Publisher.
Bolte, L. (1993). *Preservice teachers' content knowledge of functions.* Unpublished PhD thesis, University of Missouri, Columbia.

Chadwick, B. A., Bahr, H. M., & Albrecht, S. L. (1984). *Social science research methods.* Englewood Cliffs, NJ: Prentice Hall.

Çakmak, Z. T. ve Durmuş, S. (2015). İlköğretim 6-8. sınıf öğrencilerinin istatistik ve olasılık öğrenme alanında zorlandıkları kavram ve konuların belirlenmesi. *Abant İzzet Baysal Üniversitesi Eğitim Fakültesi Dergisi*, 15 (2), 27-58.

Çiltaş, A. ve Işık, A. (2012). Matematiksel modelleme yönteminin akademik başarıya etkisi. *Çağdaş Eğitim Dergisi Akademik*, 2, 57-67.

Doerr, H. (1996). Integrating the study of trigonometry, vectors, and force through modelling. *School Science and Mathematics, 96* (8), 407–418.

Doğan, A. ve Şenay, H. (2000). Genel liselerde trigonometri öğretimi üzerine matematik öğretmenlerinin görüşleri. *IV. Fen Bilimleri Eğitimi Kongresi'*nde sunulan bildiri. Hacettepe Üniversitesi, Ankara.

Doğan, A. ve Abdildaeva, E. (2013). Trigonometrik denklem sistemlerinin çözümünde görsel ve analitik uygulama üzerine bir çalışma. *MANAS Journal of Engineering (MJEN), 1*(1), 51-58.

Durmuş, S. (2004). Matematikte öğrenme güçlüklerinin saptanması üzerine bir çalışma. *Kastamonu Eğitim Dergisi, 12*(1), 125-128.

Emlek, B. (2007). *Dinamik modelleme ile bilgisayar destekli trigonometri öğretimi.* Unpublished master's thesis, Selçuk University, Konya.

Erdem, E., ve Man, S. (2018). Ortaokul matematik öğretmenlerinin radyan ve özelde π sayısına ilişkin kavramsal bilgileri. *Ege Eğitim Dergisi*, *19*(2), 488-504.

Even, R. (1989). *Prospective secondary mathematics teachers' knowledge and understanding about mathematical functions.* Unpublished PhD thesis, Michigan State University, Michigan.

Even, R. (1990). Subject matter knowledge for teaching and the case of functions. *Educational Studies in Mathematics, 21*(6), 521 - 544.

Fi, C. (2003). Preservice *secondary school mathematics teachers' knowledge of trigonometry: subject matter content knowledge, pedagogical content knowledge and envisioned pedagogy.* Unpublished PhD thesis, University of Iowa, Iowa.

Howald, C. L. (1998). *Secondary teachers' knowledge of functions: Subject matter knowledge, pedagogical content knowledge, and classroom practice.* Unpublished PhD thesis, University of Iowa, Iowa.

Inan, C. (2013). Influence of the constructivist learning approach on students' levels of learning trigonometry and on their attitudes towards mathematics. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 28* (3), 219-234.

Kabaca, T. (2013). Açı ölçüsü birimleri: Derece – radyan – grad. *Tanımları ve tarihsel gelişimleriyle matematiksel kavramlar* içinde (s. 165-183). Ankara: Pegem A.

Kang, O.K. (2003). *A new way to teach trigonometric functions.* [Available online at http://www.icme-organisers.dk/tsg09/OkKiKang.pdf], Retrieved on May 12, 2017.

Karataş, F.Ö. (2002). *Lise 2 kimyasal denge konusunun öğretiminde bilgisayar paket programları ile klasik yöntemlerin etkililiğinin karşılaştırılması*. Unpublished master's thesis, Karadeniz Teknik University, Trabzon.

Keçeli, V. ve Turanlı, N. (2013). Karmaşık sayılar konusundaki kavram yanılgıları ve ortak hatalar. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *28*(1), 223-234.

Kertil, M., Erbaş, A. K. ve Çetinkaya, B. (2017). İlköğretim matematik öğretmen adaylarının değişim oranı ile ilgili düşünme biçimlerinin bir modelleme etkinliği bağlamında incelenmesi. *Türk Bilgisayar ve Matematik Eğitimi Dergisi (TURCOMAT)*, *8*(1), 188-217.

Kültür, M. N., Kaplan, A. ve Kaplan, N. (2008). Ortaöğretim öğrencilerinde trigonometri öğretiminin değerlendirilmesi. *Atatürk Üniversitesi Kazım Karabekir Eğitim Fakültesi Dergisi*, 17, 202-211.

Lobo da Costa, N. L., & Magina, S. (1998). Making sense of sine and cosine functions through alternative approaches: computer and experimental world contexts. Proceedings of the *Conference of the International Group for the Psychology of Mathematics Education* içinde. Vol 2 (s.224). Stellenbosch, South Africa.

Marek, E. A. (1986). They misunderstand, but they'll pass. *The Science Teacher, 53*(9), 32-35.

Moore, T. (2012). *Questioning practices and students' mathematical justifications.* Unpublished PhD thesis, Evergreen State College, Olympia, Washington.

Orhun, N. (2004). Students' mistakes and misconceptions on teaching of trigonometry. *Journal of Curriculum Studies*, *32*(6), 797-820.

MoNE (2016). *Ortaöğretim ileri düzey matematik 11. sınıf ders kitabı*. Ankara: MoNE.

National Council of Teaching of Mathematics [NCTM] (1971). *Historical topics for the mathematics classroom: Thirty first yearbook (*2nd ed.). Reston, VA: National Council of Teachers of Mathematics.

Patton, Q. M. (2014). *Nitel araştırma ve değerlendirme yöntemleri*. (M. Bütün and S. B. Demir, translated from third edition). Ankara: PegemA.

Peterson, R.F., & Treagust, D.F. (1989). Grade-12 students' misconceptions of covalent bonding and structure. *Journal of Chemical Education, 66* (6), 459- 460.

Siegler, R. S., & Lortie-Forgues, H. (2015). Conceptual knowledge of fraction arithmetic. *Journal of Educational Psychology*, *107*(3), 909.

Simon, M. A. (2017). Explicating mathematical concept and mathematical conception as theoretical constructs for mathematics education research. *Educational Studies in Mathematics*, *94*(2), 117-137.

Steckroth, J. J. (2007). *Technology-enhanced mathematics instruction: effects of visualization on student understanding of trigonometry.* Unpublished PhD thesis, University of Virginia, Virginia.

Tatar, E., Okur, M. ve Tuna, A. (2008). Ortaöğretim matematiğinde öğrenme güçlüklerinin saptanmasına yönelik bir çalışma. *Kastamonu Eğitim Dergisi*, *16*(2), 507-516.

Thompson, T. D. (2008). *Growth, precision, and cat: An examination of gain score conditional sem.* New York: National Council on Measurement in Education.

Thompson, P. W., Carlson, M. P., & Silverman, J. (2007). The design of tasks in support of teachers' development of coherent mathematical meanings. *Journal of Mathematics Teacher Education*, *10*(4-6), 415-432.

Topçu, T., Kertil, M., Akkoç, H., Yılmaz, K., & Önder, O. (2006). Pre-service and in-service mathematics teachers' concept images of radian. *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* içinde (Vol: 5, p. 281-288). Prague, Czech Republic: PME.

Tuna, A. (2013). A conceptual analysis of the knowledge of prospective mathematics teachers about degree and radian. *World Journal of Education, 3*(4), 1-9.

Turanlı, N., Keçeli, V., Türker, N.K. (2007). Ortaöğretim ikinci sınıf öğrencilerinin karmaşık sayılara yönelik tutumları ile karmaşık sayılar konusundaki kavram yanılgıları ve ortak hataları. *Balıkesir Üniversitesi FBE Dergisi*, 9(2), 135- 149.

Yanık, H. B. (2016). Kavramsal ve işlemsel anlama. E. Bingölbali, S. Arslan, İ. Ö. Zembat (Ed.). *Matematik eğitiminde teoriler* içinde (s. 102-116). Ankara: Pegem Academy.

Yıldırım, A. ve Şimşek, H. (2013). *Sosyal bilimlerde nitel araştırma yöntemleri* (Ninth Edition). Ankara: Seçkin Publishing.

Yılmaz, G. K., Ertem, E. ve Güven, B. (2010). Dinamik geometri yazılımı Cabri'nin 11. sınıf öğrencilerinin trigonometri konusundaki öğrenmelerine etkisi. *Türk Bilgisayar ve Matematik Eğitimi Dergisi, 1*(2), 200-216.

Yin, R. K. (2003). *Case study research design and methods* (Third Edition). London: Sage.

Zainal, Z. (2007). Case study as a research method. *Jurnal Kemanusiaan, 9*, 1-6.

6. GENİŞ ÖZET

Kavramsal anlama, matematik eğitiminde oldukça önemli bir yere sahiptir ve alan yazında kavramsal anlama üzerine yapılmış çokça araştırma mevcuttur. Bu durumun nedenlerinden en önemlisi matematiksel kavramların genel olarak soyut ve karmaşık yapıları itibariyle anlaşılmalarının zor oluşudur. Trigonometri ise ilgili temel kavramların öğrenciler tarafından tam olarak anlaşılamamasına bağlı olarak (Steckroth, 2007, Thompson, 2008) öğrencilerin anlamakta güçlük çektiği konular arasında ilk sıralarda yer almaktadır (Tatar, Okur & Tuna, 2008; Durmuş, 2004). Trigonometri ile ilgili olarak alan yazında yer alan araştırmalar incelendiğinde bunların genel olarak trigonometrik fonksiyonlar (İnan, 2013; Kültür, Kaplan ve Kaplan, 2008; Doğan ve Şenay, 2000; Even, 1989; Even, 1990; Bolte, 1993; Howald, 1998) veya trigonometrik fonksiyonların öğretiminde teknolojinin kullanımı (Doğan ve Abdildaeba, 2013; Ağaç, 2009; Akkoç, 2007; Emlek, 2007; Yılmaz, Ertem ve Güven, 2010; Blacket & Tall, 1991; Lobo da Costa & Magina, 1998) ile ilgili oldukları görülmektedir. Özel olarak radyan kavramı ile ilgili olarak alan yazında yer alan araştırmalar olmakla birlikte bunların sınırlı olduğu ve öğrenciler veya öğretmen adaylarından oluşan benzer çalışma grupları ile gerçekleştirildiği görülmektedir. Matematik öğretmenlerinin radyan kavramını anlamlandırma biçimlerine yönelik yapılan çalışmaların ise oldukça sınırlı olduğu görülmektedir. Halbuki radyan kavramı, trigonometriye temel teşkil eden önemli kavramlardan biridir ve bu nedenle radyan kavramı üzerine yapılmış ve yapılacak olan çalışmalar büyük önem taşımaktadır. Bu noktadan hareketle bu çalışmada matematik öğretmenlerinin radyan kavramını anlamlandırma biçimlerinin incelenmesi amaçlanmıştır. Çalışmanın alt problemleri aşağıdaki gibidir.

- Matematik öğretmenleri radyan kavramını nasıl tanımlamaktadır?
- Matematik öğretmenlerinin radyan kavramını birim çemberle ilişkilendirme durumları nasıldır?
- Matematik öğretmenleri radyan kavramını trigonometrik fonksiyonlarla ilişkilendirme durumları nasıldır?
- Matematik öğretmenlerinin π sayısını anlamlandırma durumları nasıldır?

Bu araştırmada özel bir matematik kavramının, matematik öğretmenleri tarafından nasıl anlamlandırıldığı ve ilgili diğer kavramlarla nasıl ilişkilendirildiği analiz edilmiş olduğundan durum çalışması yönteminin kullanımı tercih edilmiştir. Bu araştırmada durum çalışması desenlerinden *bütüncül çoklu durum deseni kullanılmıştır.* Bu desende birden fazla kendi başına bütüncül olarak algılanabilecek durumun ele alınması ve birbirleriyle karşılaştırılması söz konusudur (Yıldırım ve Şimşek, 2013). Bu araştırmada da farklı tür okullarda görev yapmakta olan matematik öğretmenleriyle aynı veri toplama araçları kullanılarak çalışıldığından ve elde edilen veriler birbiriyle karşılaştırılarak sunulduğundan ötürü bu desenin kullanımı tercih edilmiştir. Araştırmanın çalışma grubunu Ordu ilinde bulunan farklı tür okullarda görev yapmakta olan 41 matematik öğretmeni oluşturmaktadır. Katılımcıların seçiminde amaçlı örnekleme yöntemlerinden biri olan maksimum çeşitlilik örneklemesi kullanılmıştır. Araştırmada üç tür veri toplama aracı kullanılmıştır. Bunlardan biri çalışma grubunda yer alan öğretmenlerin demografik bilgilerini belirlemeye yönelik olarak hazırlanan *Genel Bilgi Formu* (GBF)'dur. GBF'de yer alan sorular öğretmenlerin çalıştıkları okul türü, mesleki kıdemleri ve öğrenim durumlarını belirlemeye yönelik olarak hazırlanmıştır. Diğer veri toplama araçları ise, çalışma grubunda yer alan öğretmenlerin radyan kavramını anlamlandırma biçimlerini ortaya çıkarmayı amaçlamaktadır. Bunlardan biri *Kavram Testi* (KT), bir diğeri ise *Yapılandırılmamış Görüşme* (YG) lerdir. Çalışmada yer alan öğretmenlerin KT'nde yer alan sorulara verdikleri yanıtlar, çalışma kapsamında araştırmacılar tarafından oluşturulmuş olan kuramsal çerçeveye bağlı olarak analiz edilmiş ve yorumlanmıştır. Kuramsal çerçevede yer alan her bir kategoriye ilişkin yanıtlar, *Yanıt (Y), Yanıtın Türü (YT), Yanıtın Niteliği (YN), Kullanılan İfade (Kİ), Yanıt Veren Öğretmenler (YVÖ), Yanıtın Frekansı (f) ve Toplam Frekans (tf)* bileşenleri ile ifade edilmiştir. Bulguların yorumlanmasında ise ilgili bileşenlerin frekans (f) değerleri kullanılmıştır.

Matematik öğretmenlerinin radyan kavramını anlamlandırma biçimlerini analiz etmeyi amaçlayan bu araştırma sonucunda, öğretmenlerin genel olarak radyanı tam ve doğru biçimde anlamlandıramadıkları, belirli durumlar dışında birim çemberle ve trigonometrik fonksiyonlarla ilişkilendiremedikleri ve π sayısını radyan kavramı ile ilişkilendirmedikleri görülmüştür. Öğretmenler genel olarak derece türünden verilen bir açıyı radyana çevirmek için D/180 = R/π eşitliğini kullanmakta ve buna bağlı olarak radyanın her zaman π ile birlikte kullanılan bir kavram olduğunu düşünmektedirler. Buradaki π'nin anlamı sorulduğunda 180^0 yanıtını vermekte ve birim çemberde (-1,0) noktasını işaret etmektedirler. Öğretmenler genel olarak π radyanın 180^0'ye karşılık geldiğini bilmekte lakin bunun gerekçesini ifade edememektedirler. Dolayısıyla öğretmenlerin radyan kavramı ile ilgili ezberi bir takım bilgilere sahip oldukları ve derin bir kavramsal anlamaya sahip olmadıkları söylenebilir. Çalışma sonuçları özel olarak aşağıdaki şekilde ifade edilebilir.

- Öğretmenler genel olarak 1 radyanın tanımını bilmekte fakat radyan kavramını iki uzunluğun birbirine oranı biçiminde ifade edememektedirler. Bu durum onların radyan kavramını tam olarak anlamlandıramamalarındaki en büyük nedenlerden birisidir.
- Öğretmenler genel olarak bir çemberde 2π radyan olduğunu bilmekte fakat, burada kullandıkları π sayısını reel sayı olarak kullanamadıklarından ötürü bu ifadeyi tam anlamıyla yorumlayamamaktadırlar.
- Öğretmenlerin bazıları "π radyan" kullanımından ötürü radyanı π olarak algılamaktadırlar. Burada π sayısının radyanın ölçü birimi olduğu düşünülmektedir.
- Öğretmenler genel olarak radyan kavramını bir reel sayı olarak görememekte, bu durumdan ötürü π sayısı kullanılmaksızın radyan türünden verilen ifadeleri, trigonometrik fonksiyonlarla ve birim çemberle ilişkilendirmekte güçlük çekmektedirler.

Araştırmaya katılan öğretmenlerden Fen/ Sosyal Bilimler Liseleri ile Anadolu Liselerinde görev yapanların, radyan kavramı ile ilgili olarak kavramsal bilgilerinin diğer okullarda görev yapan öğretmenlere nazaran daha zengin olması beklenen bir durum iken, okul türü ile kavramsal bilgi arasında bu tür bir ilişki belirlenememiştir. Benzer şekilde mesleki kıdemleri yüksek olan öğretmenlerin diğer öğretmenlere nazaran kavramsal bilgilerinin daha zengin olması beklenirken, böyle bir bulguya da rastlanmamıştır. Lisans mezunu bazı öğretmenlerin kavramsal bilgilerinin yüksek lisans mezunu öğretmenlerden daha zengin olduğu görülmekle birlikte, araştırmada yer alan tüm soruları büyük ölçüde doğru olarak yanıtlayan öğretmenin doktora öğrencisi olduğu görülmüştür. Dolayısıyla bu araştırmada maksimum çeşitliliğin sağlanması için göz önüne alınan değişkenlerin araştırma sonuçları üzerinde beklenen bir etkiye sahip olmadıkları söylenebilir.

Her ne kadar bu araştırma, sınırlı sayıda matematik öğretmeni ile gerçekleştirilse de, ortaya çıkan kavramsal bilgi eksiklikleri ve kavram yanılgıları, söz konusu anlamlandırma biçimlerinin diğer matematik öğretmenlerinde de de var olabileceğine işaret etmektedir. Ayrıca bu araştırmada belirlenen kavramsal bilgi eksiklikleri, araştırmada yer alan öğretmenlerin yetiştirmekte oldukları öğrencilerde de büyük ölçüde görülecek olduğundan, söz konusu olumsuz tablonun giderilmesine yönelik tedbirlerin belirlenerek bir an evvel uygulamaya konulması oldukça önemlidir.