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Determination of Multiplication Algorithm with Basis on Pascal Triangle

Oğuzhan Dervişağaoğlu

Nuclear Sciences Institute, Ege University, Turkey
e-mail: promedyum@hotmail.com

Abstract: Multiplication algorithms have a crucial role with occuring of approximately 10% in operating systems such as Windows, Linux, Mac OS, İOS, DOS and etc. Furthermore, these types of algorithms which are related with multiplication have to make an efficient impact on speed of CPU. Nevertheless, a lot of investigators have been making research on both improvement and optimising of methods to make a huge stride on increasing efficiency of processor's speed. In this article, a new multiplication algorithm going to be introduced with basis on the cooperation of pascal triangle and also given specific examples about the other basic multiplication algorithms which are, Karatsuba and Nikhilam. However, results of pascal multiplication algorithm with respect to a factor of complexity analysis have been evaluated and compared to complexity of Karatsuba and classical multiplication.

Keywords: Pascal triangle, multiplication algorithm, karatsuba.

1. Introduction

History of n digit numbers with comlexity of $O(n^2)$ have extended over ancient civilizations such as Egyptians and Babylonians in B.C 3000's. Moreover, first subquadratic approach for multiplication algorithm has been devised by Karatsuba with the complexity of $O(n^{\log 3/\log 2})$ [1]. Karatsuba Algorithm can be useful for larger digit integers by including multiplication, addition and substraction operations [2]. Nikhilam Multiplication algorithm is utterly responsible to divide both integers into minor parts by addition and shifting operations to reach the solution. $O(n^2)$ function complexity contains algorithms such as nested loops algorithms (selection sort algorithms). O(n logn) is linear function complexity can be defined for problems which are divided into minor problems and then minor results can be combined to reach main solution which corresponds divide and conquer logic which refers to merge

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sort algorithms such as karatsuba algorithm. O(logn) complexity contains both multiplication and divide operations.

A few studies on pascal triangle were carried out for evaluate of both fibonacci numbers and triangle numbers. However it is very useful for determination of finding combinations [3]. Vertically and cross-wise methods in vedic mathematics which are also explained in scientific studies are quite useful of solving multiplication operations. Urdhva refers to vertically and tiryak stands for cross-wise. Pascal triangle numbers are used as coefficient related with vedic mathematics [4]. On the other hand, logic of pascal triangle was suitable for determination of new geometric properties of atom models with respect to de broglie waves and heisenberg uncertainty principle that helps to investigate atom models as regular hexagons form by using ordinary pascal triangle form [5].

A new algorithm with the robust proof will be declared as Pascal triangle approach is likely tend to stand for the complexity of $O(n^2)$. Therefore, some action should have been taken to elucidate this new multiplication algorithm complexity as a function and subsequently will be evaluated its own situation by comparison with the other multiplication algorithms. Firstly, karatsuba and nikhilam(vedic mathematics) have been explained briefly. After that, new one is going to be a main character to finalize this paper.

2. Demonstration of Karatsuba Algorithm

X and Y refer to any positive numbers and when we multiply them, both numbers divided into minor ones with the base named as B and m is going to be exponential of base B. Finally all results are combined due to basis on divided and conquer logic [6].

$$X = X_1 * B^m + X_0$$

$$Y = Y_1 * B^m + Y_0$$

$$X * Y = (X_1 * B^m + X_0) * (Y_1 * B^m + Y_0)$$

$$X * Y = (X_1Y_1 * B^{2m}) + B^m * (X_1Y_0 + X_0Y_1) + X_0Y_0$$

$$Z_2B^{2m} + Z_1B^m + Z_0$$

$$Z_2 = X_1Y_1$$

$$Z_1 = X_1Y_0 + X_0Y_1$$

$$Z_0 = X_0Y_0$$

$$Z_1 = (X_1Y_1 + X_1Y_0 + X_0Y_1 + X_0Y_0) - (X_1Y_1) - (X_0Y_0) = X_1Y_0 + X_0Y_1$$

$$Z_1 = (X_1 + X_0)(Y_1 + Y_0) - Z_2 - Z_0$$

We can demonstrate this algorithm with giving some spesific example by choosing both 91 and 99.

Example1.

91*99 = ?

9*9 = 81

1*9 = 9

(9+9) *(9+1) = 180

180-81-9 = 90

(10*81+90)*(10+9) = 9009

3. Demonstration of Nikhilam Algorithm

Nikhilam is one of the 16 sutras in vedic mathematics. This algorithm is accomplished on converting from large digit multiplication to small digits multiplication. We can select both 91 and 94 numbers for understanding of its own logic. We need to determine nearest base for both numbers and proceed on the other steps. The examples about nikhilam have been explained which seems to be efficient for the numbers that are close each other [7].

Example1.	**Example 2.**
91*94 =?	113*125=?
X= 100 – 91 = 9	200 – 113 = 87
Y= 100 – 94 = 6	200 – 125 = 75
Z= 9*6 = 54	87*75 = 6525
91 – 6 = 94 – 9 = 85	113 – 75 = 125 – 87 = 38
100*85 + 54 = 8554	200*38 +6525 = 14125

Karatsuba and nikhilam algorithms are more efficient than the classical multiplication method. In following section, new multiplication algorithm by guiding of Pascal triangle is introduced and furthermore, it derived a new formula seems like a logic of gaussian's sum for numbers. Nikhilam algorithm have an affirmative advantage when both multiplicand and multiplier are near each other and are close to same base [8]. New (pascal) algorithm is precisely independent on the subject of base which is being independent by contrast with the other multiplication methods. One of those sutras of vedic mathematics is related with Urdhva- tiryak method on multiplication which has a good impact on developing of memory power for human beings [9]. Most of schools have an ambition on helping pupils to learn some multiplication methods which are useful to calculate multiplication in a short time [10].

4. New Multiplication Algorithm with Pascal Approach

1

0 1 1 0 --- **components of 1 are (0, 1)**

1 2 1 **--- components of 2 are (1, 3)**

1 3 3 1 **--- components of 3 are (3, 6)**

1 4 6 4 1 **--- components of 4 are (6, 10)**

1 5 10 10 5 1 **--- components of 5 are (10, 15)**

1 6 15 20 15 6 1 **--- components of 6 are (15, 21)**

21 7 1 **--- components of 7 are (21, 28)**

28 8 1 **---components of 8 are (28, 36)**

36 9 1 **--- 9 (36, 45)**

45 10 1 **--- 10 (45, 55)**

These components of main numbers in pascal triangle represent triangle numbers. After addition operation of both consecutive numbers that allows us to see square of main numbers. When we look at the scheme, it is quite simple to drift the components;

Main number 1 ; $0 + 1 = 1 = 1^2$

Main number 2 ; $1+3 = 4 = 2^2$

Main number 3 ; $3+6 = 9 = 3^2$

Main number 4 ; $6+10 = 16 = 4^2$

Main number 5 ; $10+15 = 25 = 5^2$

Main number 6; $15+21 = 36 = 6^2$

Main number 7; $21+28 = 49 = 7^2$

Main number 8; $28+36 = 64 = 8^2$

Main number 9; $36+45 = 81 = 9^2$

Main number 10; $45 + 55 = 100 = 10^2$

Example 1.

3*5=?

This simple multiplication will be resulted by a new method.

3=(3,6)=3*3;   5=(10,15)= 5*5

3, 6   **10, 15**

Inner numbers are canceled and we have **3** and **15,** which correspond to **Ax_1, By_2** ; 3+15=**18** , main numbers put into substraction operation which is 5- 3 = 2 , Number 2 (difference number is C=B-A) which has components are (1,**3**) , finally second component is selected which is **3** (3 is refers to Cy which is called as difference number's second component), 18-3 =15 is going to be main result of 3*5 operation.

$$A * B = Ax + By - Cy = 3 + 15 - 3 = 15$$

Example 2.

Dealing with large digit numbers on multiplication operations, determination of components of main number may be tough when we use Pascal triangle. So we need to find a way which is efficient to get components of main number that we choose in pascal triangle.

113 *125=?

When bigger number is first main one and is written first, algorithm logic will be a bit changed. In the next section this situation has been interpreted.

A= 113 and B = 125, B-A= 12 is difference number and we need to take second component of 12. First component of A and second component of B are selected to finalize the operation.

Ax, Ay are first and second components of A =113.

Bx, By are first and second components of B=125.

$$Ax + Ay = A^2 \text{ and } Ay - Ax = A \quad (1)$$

$$Bx + By = B^2 \text{ and } By - Bx = B \quad (2)$$

These equations will be shown as below,

$$Ax = (A * (A + 1))/2 - A \quad (3)$$

$$Ay = (A * (A + 1))/2 \quad (4)$$

$$Bx = (B * (B + 1))/2 - B \quad (5)$$

$$By = (B * (B + 1))/2 \quad (6)$$

C= B - A and components of C are,

$$Cx = (C * (C + 1))/2 - C \quad (7)$$

$$Cy = (C * (C + 1))/2 \quad (8)$$

and basic formula is

$$A * B = Ax + By - Cy \quad (9)$$

When all components of A, B and B-A(C) are combined, finally we get that formula which is resulted as following mathematical approach;

$$\boldsymbol{A} * \boldsymbol{B} = (\frac{A*(A+1)}{2} - A) + (\frac{B*(B+1)}{2}) - (C * \frac{(C+1)}{2})$$

Ax_1 By_2 Cy_2 (10)

This formula has given an oppurtunity to require the components of main numbers (multiplicant, multiplier and difference number components). (3), (6), (8) formulas are combined and result is shown as below;

$$113 * 125 = \left(\frac{113*114}{2}\right) - 113) + (125 * 126)/2) - (12 * 13)/2 = 6441 - 113) + 7875 - 78) = 14125$$

4.1. Proof

If A and B ε N^+ , B>A and B-A=C is difference number,

$$\boldsymbol{A} * \boldsymbol{B} = (\frac{A * (A + 1)}{2} - A) + (\frac{B * (B + 1)}{2}) - (C * (C + 1))/2)$$

$$\frac{1}{2} * ((A * (A + 1) - 2A + \left(B * (B + 1)\right) - \left(C * (C + 1)\right) = A * B$$

$$A^2 + A - 2A + B^2 + B - C^2 - C = 2(A * B)$$

$$A^2 - A + B^2 + B - C^2 - C = 2(A * B)$$

$$A^2 + B^2 - C^2 - A + B - C = 2(A * B)$$

B-A refers to C,

$$A^2 + B^2 - C^2 + C - C = 2(A * B)$$

$$\frac{\boldsymbol{A^2+B^2-C^2}}{\boldsymbol{2}} = \boldsymbol{A} * \boldsymbol{B}$$

and C is replaced with B-A ,

$$A^2 + B^2 - (B - A)^2 = 2(A * B)$$

$$A^2 + B^2 - \left(B^2 + A^2 - 2(A * B)\right) = 2(A * B)$$

$$A^2 + B^2 - B^2 - A^2 + 2(A * B) = 2(A * B)$$

$$A * B = A * B$$

$$\boldsymbol{A} * \boldsymbol{B} = (\frac{A * (A + 1)}{2} - A) + (\frac{B * (B + 1)}{2}) - (C * (C + 1)/2) = \frac{\boldsymbol{A^2} + \boldsymbol{B^2} - \boldsymbol{C^2}}{\boldsymbol{2}}$$

$$= Ax + By - Cy$$

If we write bigger number firstly that changes array of formula on pascal triangle algorithm.

When **A>B**, we should suppose that A first main (bigger than B) number; the other one corresponds to second main number. Multiplication algorithm logic is going to become as below;

$$\boldsymbol{A} * \boldsymbol{B} = (\frac{A * (A + 1)}{2}) + (\frac{B * (B + 1)}{2} - B) - (C * \frac{(C + 1)}{2})$$

When **A< B** and if smaller number is written first, formula was determined in the previous section as following.

$$\boldsymbol{A} * \boldsymbol{B} = (\frac{A * (A + 1)}{2} - A) + (\frac{B * (B + 1)}{2}) - (C * \frac{(C + 1)}{2})$$

5. Results

1+2+3+4…..+n= n*(n+1)/2 complexity is O(n^2) and gaussian formula is same with combined formula of pascal traingle which is mentioned as new algorithm. Moreover, there are 3 division operations on new multiplication algorithm. If there are divide or multiplication operations, responsible complexity is O(logn) and pascal algorithm produce a formula which supplies the main numbers split into minor parts which are called as components of main numbers and then, all result are combined. So, other responsible complexity function notation is O(nlogn). Furthermore, this algorithm cannot be worse than O(n^2) when we compile these data, new algorithm complexity function is standing among O(n^2) and O(logn) notations. However, we note these knowledgements for this algorithm and compare with classical method of multiplication and karatsuba. This will be just an approximation of Pascal multiplication algorithm. Classical Multiplication method's asymtoticly comlexity is due to a time of O(n^2) with 4 multiplication of n/2 bit integer and 3 addition of integers at most 2n bits and 2 shifts operation . Time(n) = 4. Time (n/2) + cn (c is constant and cn refers to addition and shifts). Karatsuba algorithm asymtoticly time complexity is the Time (n) = 3 Time (n/2)+cn with three multiplications, four additions and two substractions let us to see O($n^{\log 3/\log 2}$). Karatsuba is faster than classic method. Pascal algorithm has three multiplication, three division operations. Addition – substraction operations which are correspond to dn. Pascal method quite likely, can not be worse than classical method O(n^2) and also can not be much more faster than karatsuba O($n^{1.58}$). In the previous studies, pascal triangle and multiplication algorithms such as karatsuba, nikhilam vedic mathematics sutras are evaluated. However, in previous studies about multiplication algorithm, all numbers divided into small ones to reach solution that makes easier to reach main solution. But these old methods applied on bigger numbers that would contain more complexity and need more effort to learn as practice for anyone. In the new method no need to do exercise and just necessary to know gaussian formula. Pascal algorithm has likely possessed less advantage on big digit numbers for computer scientists who are working in programming of multiplication algorithm which is compared to the others in this paper. In short time period that algorithm is aidful and beneficial with standing among both classical method and karatsuba. Nikhilam will be efficient in both numbers multiplication which are close to same base such as 94 and 96.

6. Conclusions

In this study, pascal multiplication algorithm has been evaluated by considering both karatsuba and classical multiplication method and furthermore nikhilam algorithm has been explained with some spesific examples. Pascal algorithm will be providing a new point of view on computer arithmetic's future. Pascal algorithm can be used in the studies of CPU for the last version computers by examining of speed which is definitely crucial issue for technology facilities or industries. Pascal algorithm approach in this manuscript has been devised 17 years ago by author.

Authorship contribution statement

Oğuzhan Dervişağaoğlu: Conceptualization, Methodology, Data curation, Writing, Investigation, Writing, review and editing.

Declaration of Competing Interest

The author declares that there is no competing financial interests or personal relationships that influence the work in this paper.

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