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On Almost Generalized Weakly Symmetric α-Cosymplectic Manifolds

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Abstract

In the present paper, we study the notions of an almost generalized weakly symmetric α-cosymplectic manifolds and an almost generalized weakly Ricci-symmetrik α-cosymplectic manifolds.

1. Introduction

In 1989, L. Tamassy and T. Q. Binh intruduced the notions of weakly symmetric Riemannian manifold [10]. In the view of [5], a non flat $(2n+1)$-dimensional differantiable manifold, $n>1$, is called almost weakly pseudo symmetric manifold, if there exist A_1,B_1,C_1,D_1, (are non-zero) 1-forms on M such that

$$\begin{aligned}(\nabla_W R)(X_1,X_2,X_3,X_4) &= [A_1(W)+B_1(W)]R(X_1,X_2,X_3,X_4)+C_1(X_1)R(W,X_2,X_3,X_4)\\ &+C_1(X_2)R(X_1,W,X_3,X_4)+D_1(X_3)R(X_1,X_2,W,X_4)+D_1(X_4)R(X_1,X_2,X_3,W),\end{aligned}$$

where $R(X_1,X_2,X_3,X_4)=g(R(X_1,X_2)X_3,X_4)$, R is curvature tensor of type $(1,3)$, A_1,B_1,C_1,D_1 are non-zero 1-forms defined by $A_1(W)=g(W,\sigma_1)$, $B_1(W)=g(W,\rho_1)$, $C_1(W)=g(W,\pi_1)$, $D_1(W)=g(W,\partial_1)$ and σ_1, ρ_1, π_1, ∂_1 are vector fields metrically equivalent to the 1-forms, for all W. Also ∇ denotes Levi-Civita connection with respect to metric tensor g. A $(2n+1)$-dimensional Riemannian manifold of this kind is denoted by $(WS)_{2n+1}$-manifold.
Dubey [8] presented generalized recurrent space. In keeping with this work, we shall call a $(2n+1)$-dimensional α-cosymplectic manifold almost generalized weakly symmetric (briefly $(GWS)_{2n+1}$-manifold) if admits the equation

$$\begin{aligned}(\nabla_W R)(X_1,X_2,X_3,X_4) &= [A_1(W)+B_1(W)]R(X_1,X_2,X_3,X_4)+C_1(X_1)R(W,X_2,X_3,X_4)\\ &+C_1(X_2)R(X_1,W,X_3,X_4)+D_1(X_3)R(X_1,X_2,W,X_4)\\ &+D_1(X_4)R(X_1,X_2,X_3,W)+[A_2(W)+B_2(W)]G(X_1,X_2,X_3,X_4)\\ &+C_2(X_1)G(W,X_2,X_3,X_4)+C_2(X_2)G(X_1,W,X_3,X_4)\\ &+D_2(X_3)G(X_1,X_2,W,X_4)+D_2(X_4)G(X_1,X_2,X_3,W)\end{aligned} \tag{1.1}$$

where

$$G(X_1,X_2,X_3,X_4)=[g(X_2,X_3)g(X_1,X_4)-g(X_1,X_3)g(X_2,X_4)] \tag{1.2}$$

and A_i, B_i, C_i, D_i, $(i=1,2)$, are non-zero 1-forms defined by $A_i(W)=g(W,\sigma_i)$, $B_i(W)=g(W,\rho_i)$, $C_i(W)=g(W,\pi_i)$ and $D_i(W)=g(W,\partial_i)$. There are interesting results of such $(GWS)_{2n+1}$-manifold is that it has kind of

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i) (for $A_i = B_i = C_i = D_i = 0$), locally symmetric space in the sense of Cartan
ii) (for $A_1 \neq 0, B_i = C_i = D_i = 0$), recurrent space by Walker [13],
iii) (for $A_i \neq 0$, $B_i = C_i = D_i = 0$), generalized reccurent space by Dubey [8],
iv) (for $A_1 = B_1 = C_1 = D_1 \neq 0$ and $A_2 = B_2 = C_2 = D_2 = 0$), pseudo symmetric space by Chaki [6],
v) (for $A_1 = -B_1$, $C_1 = D_1$ and $A_2 = B_2 = C_2 = D_2 = 0$), semi-pseudo symmetric space in the sense of Tarafdar et al. [11],
vi) (for $A_1 = -B_1$, $C_1 = D_1$ and $A_2 = -B_2, C_2 = D_2 = 0$), generalized semi-pseudo symmetric space in the sense of Baishya [3],
vii) (for $A_i = B_i = C_i = D_i \neq 0$), generalized pseudo symmetric space, by Baishya [3]
viii) (for $B_1 \neq 0, A_1 = C_1 = D_1 \neq 0$ and $A_2 = B_2 = C_2 = D_2 = 0$), almost pseudo symmetric space in the sprite of Chaki et al [5],
ix) (for $B_i \neq 0, A_i = C_i = D_i \neq 0$), almost generalized pseudo symmetric space in the sense of Baishya,
x) (for $A_2 = B_2 = C_2 = D_2 = 0$), weakly symmetric space by Tamassy and Binh [10].

Recently, α-cosymplectic manifolds and almost α-cosymplectic manifolds have been studied by many different researchers ([1], [2] [4], [9]). Motivated by the above studies, we consider an almost generalized weakly symmetric α-cosymplectic manifolds and an almost generalized weakly Ricci-symmetric α-cosymplectic manifold also obtain some interesting results.

2. Preliminaries

Let M^{2n+1} be a connected almost contact metric manifold with an almost contact metric structure (φ, ξ, η, g), that is, φ is a tensor field, ξ is a vector field, η is a 1-form and g is a compatible Riemannian metric such that

$$\varphi\xi = 0, \quad \eta(\varphi W) = 0, \quad \eta(\xi) = 1, \tag{2.1}$$

$$\varphi^2 W = -W + \eta(W)\xi, \quad g(W,\xi) = \eta(W), \tag{2.2}$$
$$g(\varphi W, \varphi X_1) = g(W, X_1) - \eta(W)\eta(X_1),$$

for any vector fields W and X_1 on M^{2n+1} [7].
If moreover

$$\nabla_W \xi = -\alpha\varphi^2 W, \tag{2.3}$$
$$(\nabla_W \eta)X_1 = \alpha[g(W,X_1) - \eta(W)\eta(X_1)],$$

where ∇ denotes the Riemannian connection of hold and α is a real number, then $(M^{2n+1}, \varphi, \xi, \eta, g)$ is called an α-cosymplectic manifold [12]. In this case, it is well know that [9]

$$R(W,X_1)\xi = \alpha^2[\eta(W)X_1 - \eta(X_1)W], \tag{2.4}$$

$$S(W,\xi) = -2n\alpha^2\eta(W), \tag{2.5}$$

$$S(\xi,\xi) = -2n\alpha^2, \tag{2.6}$$

where S denotes the Ricci tensor. From (2.4), it easily follows that

$$R(W,\xi)X_1 = \alpha^2[g(W,X_1)\xi - \eta(X_1)W] \tag{2.7}$$
$$R(W,\xi)\xi = \alpha^2[\eta(W)\xi - W],$$

for any vector fields W, X_1, Z where R is the Riemannian curvature tensor of the manifold. An α-cosymplectic manifold is said to be an η-Einstein manifold if Ricci tensor S satisfies condition

$$S(W,X_1) = \lambda_1 g(W,X_1) + \lambda_2\eta(W)\eta(X_1),$$

where λ_1, λ_2 are certain scalars.

3. Almost generalized weakly symmetric α-cosymplectic manifold

An α-cosymplectic manifold (M^{2n+1}, g) is said to be an almost generalized weakly symmetric if admits the relation (1.1), $(n \geq 1)$. Now, contracting X_1 over X_4 in both sides of (1.1), we obtain

$$\begin{aligned}(\nabla_W S)(X_2,X_3) = {} & [A_1(W)+B_1(W)]S(X_2,X_3) + C_1(R(W,X_2)X_3) + C_1(X_2)S(W,X_3) + D_1(X_3)S(W,X_2) + D_1(R(W,X_3)X_2) \\ & + 2n[A_2(W)+B_2(W)]g(X_2,X_3) + C_2(G(W,X_2)X_3) + 2nC_2(X_2)g(W,X_3) + 2nD_2(X_3)g(X_2,W) + D_2(G(W,X_3)X_2).\end{aligned} \tag{3.1}$$

Putting $X_3 = \xi$ in (3.1) and using (1.2), (2.4), (2.5), (2.7), we have

$$\begin{aligned}(\nabla_W S)(X_2,\xi) = {} & (-2n\alpha^2)[A_1(W)+B_1(W)]\eta(X_2) + (-2n+1)\alpha^2 C_1(X_2)\eta(W) \\ & - \alpha^2 C_1(W)\eta(X_2) + D_1(\xi)S(X_2,W) + \alpha^2 g(W,X_2)D_1(\xi) - \alpha^2 D_1(W)\eta(X_2) \\ & + 2n[A_2(W)+B_2(W)]\eta(X_2) + C_2(W)\eta(X_2) - C_2(X_2)\eta(W) \\ & + 2nC_2(X_2)\eta(W) + 2nD_2(\xi)g(W,X_2) + D_2(W)\eta(X_2) - D_2(\xi)g(W,X_2).\end{aligned} \tag{3.2}$$

Taking $X_3 = \xi$ in the below identity

$$(\nabla_W S)(X_2,X_3) = \nabla_W S(X_2,X_3) - S(\nabla_W X_2,X_3) - S(X_2,\nabla_W X_3)$$

and then using (2.2), (2.3), (2.5), we obtain

$$(\nabla_W S)(X_2,\xi) = 2n\alpha^2 g(X_2,W) - \alpha^2 S(X_2,W). \tag{3.3}$$

Now, using (3.3) in (3.2), we have

$$\begin{aligned} 2n\alpha^2 g(X_2,W) - \alpha^2 S(X_2,W) = {} & -2n\alpha^2[A_1(W)+B_1(W)]\eta(X_2) + (-2n+1)\alpha^2 C_1(X_2)\eta(W) \\ & - \alpha^2 C_1(W)\eta(X_2) + D_1(\xi)S(X_2,W) + \alpha^2 g(W,X_2)D_1(\xi) \\ & - \alpha^2 D_1(W)\eta(X_2) + 2n[A_2(W)+B_2(W)]\eta(X_2) + C_2(W)\eta(X_2) \\ & - C_2(X_2)\eta(W) + 2nC_2(X_2)\eta(W) + 2nD_2(\xi)g(W,X_2) \\ & + D_2(W)\eta(X_2) - D_2(\xi)g(W,X_2). \end{aligned} \tag{3.4}$$

Then replacing W and X_2 by ξ in (3.4) and (2.1), (2.6), we get

$$\alpha^2[A_1(\xi)+B_1(\xi)+C_1(\xi)+D_1(\xi)] = A_2(\xi)+B_2(\xi)+C_2(\xi)+D_2(\xi). \tag{3.5}$$

In particular, if $A_2(\xi) = B_2(\xi) = C_2(\xi) = D_2(\xi) = 0$, formula (3.5) turns into

$$\alpha^2[A_1(\xi)+B_1(\xi)+C_1(\xi)+D_1(\xi)] = 0.$$

Theorem 3.1. *In an almost generalized weakly symmetric α-cosymplectic manifold (M^{2n+1},g), $n \geq 1$, the relation (3.5) hold good.*

Again from (3.1), putting $X_2 = \xi$, we have

$$\begin{aligned} -2n\alpha^3 g(X_3,W) - \alpha S(X_3,W) = {} & [A_1(W)+B_1(W)]S(\xi,X_3) + C_1(R(W,\xi)X_3) + C_1(\xi)S(W,X_3) \\ & + D_1(X_3)S(W,\xi) + D_1(R(W,X_3)\xi) + 2n[A_2(W)+B_2(W)]g(\xi,X_3) \\ & + C_2(W)g(\xi,X_3) - C_2(\xi)g(W,X_3) + 2nC_2(\xi)g(W,X_3) \\ & + 2nD_2(X_3)g(\xi,W) + D_2(W)g(\xi,X_3) - D_2(X_3)g(\xi,W). \end{aligned} \tag{3.6}$$

Using (2.4), (2.5), (2.7) in (3.6), we obtain

$$\begin{aligned} -2n\alpha^3 g(X_3,W) - \alpha S(X_3,W) = {} & -2n\alpha^2[A_1(W)+B_1(W)]\eta(X_3) + \alpha^2 C_1(\xi)g(W,X_3) - \alpha^2\eta(X_3)C_1(W) \\ & + C_1(\xi)S(W,X_3) - 2n\alpha^2\eta(W)D_1(X_3) + \alpha^2\eta(W)D_1(X_3) \\ & - \alpha^2\eta(X_3)D_1(W) + 2n[A_2(W)+B_2(W)]\eta(X_3) + C_2(W)\eta(X_3) \\ & + 2nD_2(X_3)\eta(W) + D_2(W)\eta(X_3) - D_2(X_3)\eta(W). \end{aligned} \tag{3.7}$$

Putting $X_3 = \xi$ in (3.7), we get

$$\begin{aligned} \alpha^2[2n(A_1(W)+B_1(W))+C_1(W)+D_1(W)] + (2n-1)\alpha^2[C_1(\xi)+D_1(\xi)]\eta(W) = {} & 2n[A_2(W)+B_2(W)]+C_2(W)+D_2(W) \\ & + (2n-1)[C_2(\xi)+D_2(\xi)]\eta(W). \end{aligned} \tag{3.8}$$

Using $W = \xi$ in (3.7), we obtain

$$\begin{aligned} & 2n\alpha^2[A_1(\xi)+B_1\xi+C_1(\xi)]\eta(X_3) + \alpha^2 D_1(\xi)\eta(X_3) + (2n-1)\alpha^2 D_1(X_3) \\ & = 2n[A_2(\xi)+B_2(\xi)+C_2(\xi)]\eta(X_3) + D_2(\xi)\eta(X_3) + (2n-1)D_2(X_3). \end{aligned}$$

Replacing X_3 by W in the above equation and using (3.5), we have

$$\alpha^2 D_1(\xi)\eta(W) - \alpha^2 D_1(W) = D_2(\xi)\eta(W) - D_2(W). \tag{3.9}$$

Again, putting $W = \xi$ in (3.4), we get

$$\begin{aligned} & 2n\alpha^2[A_1(\xi)+B_1(\xi)+D_1(\xi)]\eta(X_2) + \alpha^2 C_1(\xi)\eta(X_2) + (2n-1)\alpha^2 C_1(X_2) \\ & = 2n[A_2(\xi)+B_2(\xi)+D_2(\xi)]\eta(X_2) + C_2(\xi)\eta(X_2) + (2n-1)C_2(X_2). \end{aligned} \tag{3.10}$$

Replacing X_2 by W in (3.10) and using (3.5), we obtain

$$\alpha^2 C_1(\xi)\eta(W) - \alpha^2 C_1(W) = C_2(\xi)\eta(W) - C_2(W). \tag{3.11}$$

Subtracting (3.9), (3.11) from (3.8)

$$\alpha^2[A_1(W)+B_1(W)+C_1(W)+D_1(W)] = [A_2(W)+B_2(W)+C_2(W)+D_2(W)]. \tag{3.12}$$

Next, in view of $A_2 = B_2 = C_2 = D_2 = 0$, the relation (3.12) yields

$$\alpha^2[A_1(W)+B_1(W)+C_1(W)+D_1(W)] = 0.$$

This motivates us to state the followings

Theorem 3.2. *In an almost generalized weakly symmetric α-cosymplectic manifold (M^{2n+1},g) $(n \geq 1)$, the sum of the associated 1-forms is given by (3.12).*

Theorem 3.3. *There does not exist an α-cosymplectic manifold which is*
(i) recurrent,
(ii) generalized recurrent provided the 1-forms are collinear,
(iii) pseudo symmetric,
(iv) generalized semi-pseudo symmetric provided the 1-forms are collinear,
(v) generalized almost-pseudo symmetric provided the 1-forms are collinear.

4. Almost generalized weakly Ricci-symmetric α-cosymplectic manifold

An α-cosymplectic manifold (M^{2n+1},g) $(n\geq 1)$, is said to be almost generalized weakly Ricci-symmetric if there exist 1-forms, $\tilde{A}_i$, $\tilde{B}_i$, $\tilde{C}_i$ and $\tilde{D}_i$ which satisfy the condition

$$(\nabla_W S)(X_2,X_3)=[\tilde{A}_1(W)+\tilde{B}_1(W)]S(X_2,X_3)+\tilde{C}_1(X_2)S(W,X_3)+\tilde{D}_1(X_3)S(X_2,W)+[\tilde{A}_2(W)+\tilde{B}_2(W)]g(X_2,X_3)+\tilde{C}_2(X_2)g(W,X_3)+\tilde{D}_2(X_3)g(X_2,W). \tag{4.1}$$

Putting $X_3=\xi$ in (4.1), and using (2.1), (2.5), we get

$$(\nabla_W S)(X_2,\xi)=-2n\alpha^2[\tilde{A}_1(W)+\tilde{B}_1(W)]\eta(X_2)-2n\alpha^2\tilde{C}_1(X_2)\eta(W)+\tilde{D}_1(\xi)S(X_2,W)+[\tilde{A}_2(W)+\tilde{B}_2(W)]\eta(X_2)+\tilde{C}_2(X_2)\eta(W)+\tilde{D}_2(\xi)g(X_2,W). \tag{4.2}$$

Using equation (3.3) in (4.2) we get,

$$-2n\alpha^3 g(X_2,W)-\alpha S(X_2,W)=-2n\alpha^2[\tilde{A}_1(W)+\tilde{B}_1(W)]\eta(X_2)-2n\alpha^2\tilde{C}_1(X_2)\eta(W)+\tilde{D}_1(\xi)S(X_2,W)+[\tilde{A}_2(W)+\tilde{B}_2(W)]\eta(X_2)+\tilde{C}_2(X_2)\eta(W)+\tilde{D}_2(\xi)g(X_2,W). \tag{4.3}$$

Putting $W=X_2=\xi$ in (4.3), we have

$$2n\alpha^2[\tilde{A}_1(\xi)+\tilde{B}_1(\xi)+\tilde{C}_1(\xi)+\tilde{D}_1(\xi)]=\tilde{A}_2(\xi)+\tilde{B}_2(\xi)+\tilde{C}_2(\xi)+\tilde{D}_2(\xi). \tag{4.4}$$

Then, taking $W=\xi$ in (4.3), we obtain

$$2n\alpha^2[\tilde{A}_1(\xi)+\tilde{B}_1(\xi)+\tilde{D}_1(\xi)]\eta(X_2)+2n\alpha^2\tilde{C}_1(X_2)=[\tilde{A}_2(\xi)+\tilde{B}_2(\xi)+\tilde{D}_2(\xi)]\eta(X_2)+\tilde{C}_2(X_2). \tag{4.5}$$

Using $X_2=\xi$ in (4.3), we get

$$2n\alpha^2[\tilde{A}_1(\xi)+\tilde{B}_1(\xi)+\tilde{D}_1(\xi)]\eta(W)+2n\alpha^2\tilde{C}_1(W)=[\tilde{A}_2(\xi)+\tilde{B}_2(\xi)+\tilde{D}_2(\xi)]\eta(W)+\tilde{C}_2(W). \tag{4.6}$$

Replacing X_2 by W in (4.5) and adding with (4.6), we have

$$2n\alpha^2[\tilde{A}_1(W)+\tilde{B}_1(W)+\tilde{C}_1(W)]-[\tilde{A}_2(W)+\tilde{B}_2(W)+\tilde{C}_2(W)]=-2n\alpha^2[\tilde{A}_1(\xi)+\tilde{B}_1(\xi)+\tilde{C}_1(\xi)+\tilde{D}_1(\xi)]\eta(W)+[\tilde{A}_2(\xi)+\tilde{B}_2(\xi)+\tilde{C}_2(\xi)+\tilde{D}_2(\xi)]\eta(W)-2n\alpha^2\tilde{D}_1(\xi)\eta(W)-\tilde{D}_2(\xi)\eta(W). \tag{4.7}$$

In view of (4.4) the relation (4.7) becomes

$$2n\alpha^2[\tilde{A}_1(W)+\tilde{B}_1(W)\tilde{C}_1(W)]+2n\alpha^2\tilde{D}_1(\xi)\eta(W)=[\tilde{A}_2(W)+\tilde{B}_2(W)+\tilde{C}_2(W)]-\tilde{D}_2(\xi)\eta(W). \tag{4.8}$$

Then, taking $W=X_2=\xi$ in (4.1), we obtain

$$2n\alpha^2[\tilde{A}_1(\xi)+\tilde{B}_1(\xi)+\tilde{C}_1(\xi)]\eta(X_3)+2n\alpha^2\tilde{D}_1(X_3)=[\tilde{A}_2(\xi)+\tilde{B}_2(\xi)+\tilde{C}_2(\xi)]\eta(X_3)+\tilde{D}_2(X_3). \tag{4.9}$$

In view of (4.4), replacing X_3 by W in (4.9) and then adding the resultant with (4.8),

$$2n\alpha^2\{[\tilde{A}_1(W)+\tilde{B}_1(W)+\tilde{C}_1(W)+\tilde{D}_1(W)]+[\tilde{A}_1(\xi)+\tilde{B}_1(\xi)+\tilde{C}_1(\xi)+\tilde{D}_1(\xi)]\eta(W)\}=[\tilde{A}_2(W)+\tilde{B}_2(W)+\tilde{C}_2(W)+\tilde{D}_2(W)]+[\tilde{A}_2(\xi)+\tilde{B}_2(\xi)+\tilde{C}_2(\xi)+\tilde{D}_2(\xi)]\eta(W). \tag{4.10}$$

Next, putting (4.4) in (4.10), we get

$$2n\alpha^2[\tilde{A}_1(W)+\tilde{B}_1(W)+\tilde{C}_1(W)+\tilde{D}_1(W)]=\tilde{A}_2(W)+\tilde{B}_2(W)+\tilde{C}_2(W)+\tilde{D}_2(W). \tag{4.11}$$

Theorem 4.1. *In an almost generalized weakly Ricci-symmetric α-cosymplectic manifold (M^{2n+1},g), $n\geq 1$, the relation (4.11) hold good.*

Theorem 4.2. *There does not exist an almost generalized weakly Ricci symmetric α-cosymplectic manifold which is*

i) recurrent,
ii) generalized recurrent provided the 1-forms are collinear,
iii) pseudo symmetric,
iv) generalized semi-pseudo symmetric provided the 1-forms are collinear,
v) generalized almost-pseudo symmetric provided the 1-forms are collinear.

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