Ocak 2000, ss.73-75.

A SIMPLE THEOREM ON MINORITY RIGHTS

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***Özet:** Azınlık hakları, değişik koşullar altında, rasyonel ve Pareto-etkin toplumsal tercihlere yol açabilmektedir. Bu makale, sözkonusu koşullardan birini formüle etmektedir.*

I. INTRODUCTION

The twentieth century liberal social theory which so passionately defends and celebrates diversity and pluralism fails to adequately address and resolve the conflicts and dilemmas embodied in that much-valued diversity. This failure has been transparent in the treatment of minority rights, which pose profound difficulties for social choice procedures that respect the principles of economic efficiency and democratic majoritarianism. Considering the fundamental importance contemporary societies attach to rights, efficiency and democracy, the question of how to resolve the potential conflicts between rights and efficiency, or rights and democracy deserves special attention. This paper takes a modest step towards identifying some of the conditions that would resolve one of these conflicts, namely, the conflict between minority rights and Pareto efficiency.

II. THE GENERAL FRAMEWORK

Let S be the set of a finite number of individuals forming a society. Any non-empty subset of S whose elements (i.e. individuals) share some distinguishing common feature(s) and whose cardinality is smaller than (respectively, greater than) half of that of S is called a *minority group* (respectively, a *majority group*) of individuals. Let E be a set of a finite number of such groups in the society, and Z be a set of a finite number of mutually exclusive social alternatives. Assume that the cardinalities of E and Z, denoted by, respectively, $|E|$ and $|Z|$, are such that $|E|>1$, $|Z|>2$. Each group k, $k = 1,...,n$, in the society has a preference relation R^k, which is obtained through a rank-order method of voting as illustrated in Sen (1970b: 39). Each R^k is a binary relation on Z such that $R^k \subseteq \{(x,y): x, y \text{ are in } Z\}$, and $k=1,...,n$. For any x, y in Z, $(x,y) \in R^k$ means the same thing as xR^ky which will be interpreted as "x is preferred to y" by group k. Define strict preference (P^k) and indifference (I^k) relations on $\{x,y\}$ as follows:

$$xP^ky \text{ if and only if } xR^ky \text{ and not } yR^kx,$$

$$xI^ky \text{ if and only if } xR^ky \text{ and } yR^kx.$$

A group preference relation R^k on Z is said to be *complete* if and only if xP^ky or yP^kx or xI^ky for all x, y in Z such that $x \neq y$. R^k on Z is *incomplete* if it is not complete. R^k on Z is *transitive* if and only if for all x, y, z in Z, (xP^ky and yP^kz implies xP^kz), and (xI^ky and yI^kz implies xI^kz). R^k on Z is *intransitive* if it is not transitive. R^k on Z is *acyclical* over an m-set $\{x_1,...,x_m\}$ in Z if and only if the following condition holds: For all $x_1,....,x_m$ in Z, if [$x_1P^kx_2$, and $x_2P^kx_3$, and ... and $x_{m-1}P^kx_m$], then $x_1R^kx_m$. R^k is cyclical over $\{x_1,...,x_m\}$ in Z if $x_1P^kx_2$, and $x_2P^kx_3$, and ... and $x_{m-1}P^kx_m$ and $x_mP^kx_1$. Clearly, if R^k is not cyclical over **any m-set** in Z, none of its subsets is.

$x \in Z$ is said to *dominate* $y \in Z$ with respect to R^k if xP^ky (i.e. if $(x,y) \in R^k$ and $(y,x) \notin R^k$). $x \in Z$ is said to be a *dominated alternative* with respect to R^k if there is a $z \in Z$ such that zP^kx. $x \in Z$ is said to be an *undominated alternative* with respect to R^k if there is no $z \in Z$ such that zP^kx. If R^k over an m-set is not cyclical, there is at least one undominated alternative with respect to R^k in that m-set.

Let h be a collective choice rule, that is, a mapping from the set of group preference relations $R^{group} = \{(R^1,...,R^n)$: R^k is a group preference relation on Z, $k = 1,...,n\}$ into a set of preference relations $R^{soc} = \{R$: R is a social preference relation on $Z\}$ such that for any configuration of group preference relations $R^1,....,R^n$, one and only one social preference relation R is determined, i.e. h: $R^{group} \rightarrow R^{soc}$ such that $R = h(R^1,....,R^n)$. The social preference relation R is a binary relation whose strict preference and indifference parts are P and I.

Suppose that society has a constitution that defines, among other things, rights to be granted to groups such as minorities. A group k is said to have a constitutional right to an alternative x_i as opposed to (against) x_j, if the constitution stipulates that x_iPx_j whenever $x_iP^kx_j$. Alternatively, we say that group k has a constitutional right over the pair $\{x_i,x_j\}$ in Z.

The rationale for granting minority rights lies in the pluralistic diversity that is so fundamental to the liberal social order. A liberal society committed to the principles of diversity and pluralism may need to protect minorities against the unconstrained power of a majority, which could arbitrarily impose its will on minorities in all social choice contexts. Establishing constitutionally backed minority rights is an effective way of constraining majority rule in such contexts.

III. MAXIMALLY INTENSE MINORITY RIGHTS

The following definitions present the central concepts underlying the result in this paper.

Definition (Pareto efficiency): Let $\{x,y\}$ be any pair contained in Z. If for every k in E xP^ky, then xPy.

Definition (maximally intense minority rights): A minority group k's right over $\{x,y\}$ is said to be *maximally intense* if x is an undominated alternative with respect to R^k. Speaking (informally) in cardinal utility terms, k's right over $\{x,y\}$ is maximally intense if it attributes the highest utility to x, i.e. it 'feels most intensely' for x in Z.

Definition (conflict between minority rights and Pareto efficiency): For a given configuration of group preferences, a *conflict* is said to exist between minority rights and Pareto efficiency with respect to an *m-set*, $m>2$, in Z if 'such rights' and 'Pareto efficiency' jointly (but not individually) produce a social preference relation R that is cyclical over that *m-set*.

The question of a conflict between rights and Pareto efficiency was originally posed by Sen [1,2]. The definition above reformulates and formalizes the conflict in question in the context of minority rights and Pareto efficiency. The following example illustrates this rights-efficiency conflict by demonstrating how minority rights and Pareto efficiency could jointly lead to a cyclical social preference.

Example: Let minority groups 1 and 2 in E have the following preferences over $\{x,y,z,w\}$:

$$R^1 = \{xP^1y,\ yP^1z,\ xP^1z,\ xP^1w,\ yP^1w,\ zP^1w\}$$

$$R^2 = \{wP^2x,\ zP^2x,\ wP^2z,\ zP^2y,\ wP^2y,\ xP^2y\}$$

and suppose that other groups in E have the same preferences as group 2. Let group 1 have rights over $\{y,z\}$ and $\{z,w\}$, and group 2 over $\{w,x\}$. Then, by right assignment,

yP^1z implies yPz,

zP^1w implies zPw,

wP^2x *implies* wPx.

Since, over $\{x,y\}$, all groups strictly prefer x to y, by Pareto efficiency, xPy. Thus, the social preference relation R, induced by minority rights and Pareto efficiency, over $\{x,y,z,w\}$ is:

$$R = \{xPy,\ yPz,\ zPw,\ wPx\}$$

which is cyclical over $\{x,y,z,w\}$. Thus, for this configuration of group preferences, a conflict exists between minority rights and Pareto efficiency.

The conflict exemplified here would, however, cease to exist if minority rights were maximally intense. The formal statement and proof of this result is as follows:

Theorem: There is no conflict between Pareto efficiency and maximally intense minority rights with respect to any *m-set* in Z.

Proof: Let R^p and R^m denote, respectively, the sets of ordered pairs over which social preference relation is determined by Pareto efficiency and maximally intense minority rights. We will prove that, under the condition stated in the theorem, no non-empty subset of $R^p \cup R^m$ is cyclical over any m-set in Z, implying that there is no conflict between maximally intense minority rights and Pareto efficiency with respect to any *m-set* in Z. We will decompose the non-empty subsets of $R^p \cup R^m$ (including itself) into three groups, and prove that no subset of $R^p \cup R^m$ in any of these groups is cyclical over any *m-set* in Z. (The term, "subset", as it is used below, refers to non-empty subsets.)

Group 1. Subsets of $R^p \cup R^m$ that are subsets of R^p. $R^p \subset (\cap^n{}_{k=1} R^k) \subset R^t$, where $t \in E$. Since rank-order method as illustrated by Sen (1970: 39) generates transitive group preferences, R^t is transitive on Z, thus it is not cyclical over any *m-set* in Z, which implies that R^p, which is a subset of R^t, is not cyclical over any *m-set* in Z either. This in turn implies that no subset of R^p is cyclical over any *m-set* in Z.

Group 2. Subsets of $R^p \cup R^m$ that are subsets of R^m. Depending on the way in which minority rights are assigned, R^m could be cyclical or acyclical over an *m-set* in Z. In cases where rights are assigned so as to induce a cyclical R^m, minority rights, by themselves, generate a cyclical social preference, which, by definition, does not constitute a conflict between minority rights and Pareto efficiency. Thus, we need to consider only the cases where minority rights are assigned so that R^m is not cyclical over any *m-set* in Z. Where R^m is not cyclical over any *m-set* in

Z, neither is any of its subsets.

Group 3. Subsets of $R^p \cup R^m$ that are the unions of a subset of R^p and a subset of R^m. Let minority group t have a maximally intense right over a pair $\{x_i, x_j\}$ such that $(x_i, x_j) \in R^m$. By definition of maximal intensity, x_i is an undominated alternative with respect to R^t. Since $R^p \subset (\cap^n_{k=1} R^k) \subset R^t$, x_i, which is an undominated alternative with respect to R^t, is also undominated with respect to R^p (and thus with respect to each of its subsets). Hence, every right-chosen alternative in Z for which there is a minority right is an undominated alternative with respect to the subsets of R^p.

Now, let us examine an arbitrary subset of $R^p \cup R^m$ that is the union of a subset of R^p and a subset of R^m. Let A be a subset of R^p and B be a subset of R^m. In cases we need to consider, no subset of R^m is cyclical over any *m-set* in Z (see the explanation above). Thus, there is at least one undominated alternative with respect to each subset of R^m, and hence with respect to B. Let $x_m \in Z$ be an undominated alternative with respect to B, which is, by the argument above, also an undominated alternative with respect to any subset of R^p, and hence with respect to A. x_m, which is an undominated alternative with respect to A and B, is bound to be an undominated alternative with respect to $A \cup B$. Since we have chosen an arbitrary subset of R^m and formed its union with an arbitrary subset of R^p, the result holds for every subset that is a union of a subset of R^m and a subset of R^p. Thus, there is at least one undominated alternative in Z with respect to each subset of $R^p \cup R^m$ that is a union of a subset of R^p and a subset of R^m, i.e. with respect to each subset in Group 3.

Since no subset in Group 1 and Group 2 above is cyclical over any *m-set* in Z and hence with respect to each of which there is at least one undominated alternative in Z as well, there is at least one undominated alternative in Z with respect to each subset in all three groups, i.e. with respect to every non-empty subset of $R^p \cup R^m$, which implies that neither $R^p \cup R^m$ nor any of its subsets is cyclical over any *m-set* in Z.

QED.

IV. CONCLUDING REMARKS: IMPLICATIONS

The result contained in the theorem is not meant to present a complete list of criteria with respect to which all minority rights should be constitutionally established. Normatively speaking, the result is intended to point out only one possible criterion with respect to which some minority rights could be granted. A liberal society may reasonably want to grant rights to minorities in contexts where maximal intensity criterion does not hold. In such contexts some other conditions may need to be invoked to avoid the conflicts in question.

The result is significant for the rationality, efficiency and stability of social outcomes in liberal societies for the following reason. Social tensions are likely to peak in social contexts where minorities that 'feel most intensely' over certain issues confront the possible tyranny of the majority. As implied by the theorem, however, it is in such contexts that constitutionally established minority rights, which would rule out certain dictates of a majority, would not be in a social irrationality-generating conflict with Pareto efficiency. Hence, establishing minority rights would, in such contexts, constitute a socially rational and Pareto-efficient resolution of potentially costly conflicts that could undermine the stable coexistence of majority and minorities, which is one of the key features of the pluralistic diversity celebrated in liberal social theory.

REFERENCES

[1] Sen, A., "The Impossibility of a Paretian Liberal", **Journal of Political Economy**, 78,1970, s.152-157.

[2] Sen, A., **Collective Choice and Social Welfare**, San Francisco: Holden-Day, 1970.