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Research Article

Addendum to "Semi-Hurewicz spaces" [Hacet. J. Math. Stat. 46 (1), 53-66, 2017]

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Abstract

In this addendum we give the correct versions of Theorems 4.5 and 4.6 in the paper "Semi-Hurewicz spaces" [Hacet. J. Math. Stat. 46 (1), 53-66, 2017].

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In [2], the authors introduced $SsH_{\leq n}$ in topological spaces using semi-open sets. We use notation and terminology from [2].

A space X is said to have $SsH_{\leq n}$ if for each sequence $\langle \mathcal{U}_n : n \in \omega \rangle$ of semi-open covers of X there is a sequence $\langle \mathcal{V}_n : n \in \omega \rangle$ such that for each n, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ of cardinality at most n and $\{St(\bigcup \mathcal{V}_n, \mathcal{U}_n) : n \in \omega\}$ is an $s\gamma$-cover of X.

The following two theorems are given in [2]:

Theorem 1 ([2]). *Let X an extremally disconnected space satisfying $SsH_{\leq n}$. Then X satisfies $S_1^\star(s\mathcal{O}, s\mathcal{O}^{gp})$.*

Theorem 2 ([2]). *Let X an extremally disconnected space satisfying the following condition : for each sequence $\langle \mathcal{U}_n : n \in \omega \rangle$ of semi-open covers of X there is a sequence $\langle V_n : n \in \omega \rangle$ of finite subset of X such that for each n, V_n has at most n elements and $\{St(V_n, \mathcal{U}_n) : n \in \omega\}$ is an $s\gamma$-cover of X. Then X satisfies $SS_1^\star(s\mathcal{O}, s\mathcal{O}^{gp})$.*

However, there are gaps in the proofs of these theorems. In order to prove the above theorems we need some additional hypotheses on the space X, that is, we use the principles $CDR^\star_{sub}(\mathcal{A}, \mathcal{B})$ and $CDRF^\star_{sub}(\mathcal{A}, \mathcal{B})$. Our proofs are a slight modifications of the proofs in [2].

Definition 3 ([3]). Let $\mathcal{A}$ and $\mathcal{B}$ be families of subsets of the infinite set S. Then $CDR_{sub}(\mathcal{A}, \mathcal{B})$ denotes the statement that for each sequence $\langle A_n : n \in \omega \rangle$ of elements of $\mathcal{A}$ there is a sequence $\langle B_n : n \in \omega \rangle$ such that for each n, $B_n \subseteq A_n$, for $m \neq n$, $B_m \cap B_n = \emptyset$, and each B_n is a member of $\mathcal{B}$.

Definition 4 ([1]). Let $\mathcal{A}$ and $\mathcal{B}$ be families of subsets of the infinite set S. Then $CDR^\star_{sub}(\mathcal{A}, \mathcal{B})$ denotes the statement that for each sequence $\langle A_n : n \in \omega \rangle$ of elements of $\mathcal{A}$ there is a sequence $\langle B_n : n \in \omega \rangle$ such that for each n, $B_n \subseteq A_n$, for $m \neq n$, $\{St(B, A_m) : B \in B_m\} \cap \{St(B, A_n) : B \in B_n\} = \emptyset$, and each B_n is a member of $\mathcal{B}$.

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Definition 5 ([1]). Let $\mathcal{A}$ and $\mathcal{B}$ be families of subsets of the infinite set S. Then $CDRF^{\star}_{sub}(\mathcal{A},\mathcal{B})$ denotes the statement that for each sequence $\langle A_n : n \in \omega\rangle$ of elements of $\mathcal{A}$ there is a sequence $\langle B_n : n \in \omega\rangle$ such that for each n, $B_n \subseteq A_n$, for $m \neq n$ and for each finite subset F of S, $\{St(x, B_m) : x \in F\} \cap \{St(x, B_n) : x \in F\} = \emptyset$, and each B_n is a member of $\mathcal{B}$.

Theorem 6. *Let X an extremally disconnected space satisfying $SsH_{\leq n}$ and $CDR^{\star}_{sub}(s\mathcal{O}, s\mathcal{O})$. Then X satisfies $S^{\star}_1(s\mathcal{O}, s\mathcal{O}^{gp})$.*

Proof. Let $\langle \mathcal{G}_n : n \in \omega\rangle$ be a sequence of semi-open covers of X. Since X satisfies $CDR^{\star}_{sub}(s\mathcal{O}, s\mathcal{O})$, there is a sequence $\langle \mathcal{U}_n : n \in \omega\rangle$ such that for each n, $\mathcal{U}_n \subseteq \mathcal{G}_n$, for $m \neq n$,

$$\{St(B, \mathcal{G}_m) : B \in \mathcal{U}_m\} \cap \{St(B, \mathcal{G}_n) : B \in \mathcal{U}_n\} = \emptyset,$$

and each $\mathcal{U}_n$ is a semi-open cover of X.

Since X is extremally disconnected, for each n, define $\langle \mathcal{V}_n : n \in \omega\rangle$ a semi-open cover of X by putting

$$\mathcal{V}_n = \bigwedge\{\mathcal{U}_i : (n-1)n/2 < i \leq n(n+1)/2\}.$$

By applying $SsH_{\leq n}$ property to the sequence $\langle \mathcal{V}_n : n \in \omega\rangle$, we get a sequence $\langle \mathcal{W}_n : n \in \omega\rangle$ such that for each n, $|\mathcal{W}_n| \leq n$, $\mathcal{W}_n \subseteq \mathcal{V}_n$ and $\{St(\bigcup \mathcal{W}_n, \mathcal{V}_n) : n \in \omega\}$ is an $s\gamma$-cover of X.

Now write

$$\mathcal{W}_n = \{W_i : (n-1)n/2 < i \leq n(n+1)/2\}.$$

For each W_i take also the set $U_i \in \mathcal{U}_i$ which is a term in the representation of W_i given above. Now to prove that the set $\{St(U_n, \mathcal{G}_n) : n \in \omega\}$ is a semi-open groupable cover of X. Define a sequence $n_1 < n_2 < ... < n_k < ...$ of natural numbers by $n_k = k(k-1)/2$. Then, for each $x \in X$,

$$\{k \in \omega : x \notin \textstyle\bigcup_{n_k < i \leq n_{k+1}} St(W_i, \mathcal{G}_i)\} \subseteq \{k \in \omega : x \notin St(\bigcup \mathcal{W}_k, \mathcal{V}_k)\},$$

that is, $x \in St(\bigcup \mathcal{W}_k, \mathcal{V}_k)$ for all but finitely many k and hence $x \in \bigcup_{n_k < i \leq n_{k+1}} St(W_i, \mathcal{G}_i)$ for all but finitely many k. □

Theorem 7. *Let X an extremally disconnected space satisfying $CDRF^{\star}_{sub}(s\mathcal{O}, s\mathcal{O})$ and the following condition : for each sequence $\langle \mathcal{U}_n : n \in \omega\rangle$ of semi-open covers of X there is a sequence $\langle V_n : n \in \omega\rangle$ of finite subset of X such that for each n, V_n has at most n elements and $\{St(V_n, \mathcal{U}_n) : n \in \omega\}$ is an $s\gamma$-cover of X. Then X satisfies $SS^{\star}_1(s\mathcal{O}, s\mathcal{O}^{gp})$.*

Proof. Let $\langle \mathcal{W}_n : n \in \omega\rangle$ be a sequence of semi-open covers of X. Since X satisfies $CDRF^{\star}_{sub}(\mathcal{O}, \mathcal{O})$, there is a sequence $\langle \mathcal{U}_n : n \in \omega\rangle$ such that for each n, $\mathcal{U}_n \subseteq \mathcal{W}_n$, for $m \neq n$ and for each finite subset F of X,

$$\{St(x, \mathcal{W}_m) : x \in F\} \cap \{St(x, \mathcal{W}_n) : x \in F\} = \emptyset,$$

and each $\mathcal{W}_n$ is a semi-open cover of X.

Since X is extremally disconnected, for each n, define $\langle \mathcal{V}_n : n \in \omega\rangle$ a semi-open cover of X by putting

$$\mathcal{V}_n = \textstyle\bigwedge_{(n-1)n/2 < i \leq n(n+1)/2} \mathcal{U}_i.$$

By applying hypothesis of the theorem to the sequence $\langle \mathcal{V}_n : n \in \omega\rangle$, we get a sequence $\langle A_n : n \in \omega\rangle$ of subsets of X such that for each n, $|A_n| \leq n$ and $\{St(A_n, \mathcal{V}_n) : n \in \omega\}$ is an $s\gamma$-cover of X.

Now write

$$A_n = \{x_i : (n-1)n/2 < i \leq n(n+1)/2\}.$$

Now to prove that the set $\{St(x_n, \mathcal{W}_n) : n \in \omega\}$ is a semi-open groupable cover of X. Define a sequence $n_1 < n_2 < ... < n_k < ...$ of natural numbers by $n_k = k(k-1)/2$. Then, for each $x \in X$,

$$\{k \in \omega : x \notin \bigcup_{n_k < i \leq n_{k+1}} St(x_i, \mathcal{W}_i)\} \subseteq \{k \in \omega : x \notin St(A_k, \mathcal{V}_k)\},$$

that is, $x \in St(A_k, \mathcal{V}_k)$ for all but finitely many k and hence $x \in \bigcup_{n_k < i \leq n_{k+1}} St(x_i, \mathcal{W}_i)$ for all but finitely many k. □

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