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$\mathcal{I}-$ALMOST LACUNARY VECTOR VALUED SEQUENCE SPACES IN $2-$NORMED SPACES

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Abstract. One of the wide-ranging applications and research areas of Summability theory is the concept of statistical convergence. This concept was studied a related concept of convergence by using lacunary sequence by Fridy and Orhan. At the last quarter of the 20th century, lacunary statistical convergence has been discussed and captured significant aspect of creating the basis of several investigations conducted in many branches of mathematics. On the other hand, in 1961 Krasnoselskii and Rutisky presented the definition of Orlicz function. Also, in 1963 Gähler introduced the notion of 2-normed spaces. The main goal of this article is to introduce $\mathcal{I}-$ almost convergence of lacunary sequences with regard to an Orlicz function in 2-normed spaces and other sequence spaces by considering the concept of ideal that was presented by Kostyrko and others. Additionally, we examine the relationship between these sequence spaces and fundamental inclusion theorems are investigated.

1. Introduction

The concept of $2-$normed spaces was initially introduced by Gähler [3] in the 1960's. Since then, this concept has been studied by many authors (see, for instance ([11], [13], [20], [21]).

Recall in [8] that an Orlicz function $\lambda : [0, \infty) \to [0, \infty)$ is continuous, convex, non-decreasing function such that $\lambda(0) = 0$ and $\lambda(u) > 0$ for $u > 0$, and $\lambda(u) \to \infty$ as $u \to \infty$.

Subsequently the notion of Orlicz function was used by Mursaleen, Khan, Chishti [9], Parashar and B. Choudhary [10], Savaş and Savaş [18], Savaş([16], [17], [19]) and others.

If convexity of Orlicz function λ is replaced by $\lambda(x + y) \le \lambda(x) + \lambda(y)$ then this function is called Modulus function, which was presented and discussed by Ruckle [12] and Maddox [5].

An Orlicz function is said to satisfy Δ_2- condition if there exists a positive constant K such that $\lambda(2u) \le T\lambda(u)$ for all $u \ge 0$.

Note that if λ is an Orlicz function then $\lambda(\psi x) \le \psi\lambda(u)$ for all ψ with $0 < \psi < 1$.

Let E be a real vector space of dimension d, where $2 \le d < \infty$. A $2-$norm on E is a function $\|.,.\| : E \times E \to R$ which satisfies (**i**) $\|u, v\| = 0$ if and only if u and v

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are linearly dependent; **(ii)** $\|u,v\| = \|v,u\|$; **(iii)** $\|\beta u,v\| = |\beta| \|u,v\|$, $\beta \in \mathbb{R}$; **(iv)** $\|u,v+w\| \leq \|u,v\| + \|u,w\|$. The pair $(E, \|.,.\|)$ is then called a 2− normed space [4].

Recall that $(E, \|.,.\|)$ is a 2−Banach space if every Cauchy sequence in E is convergent to some u in E.

The notion of ideal convergence was introduced first by P. Kostyrko et al [6] as a generalization of statistical convergence.

A family $\mathcal{I} \subset 2^F$ of subsets a nonempty set F is said to be an ideal in F if **(i)** $\emptyset \in \mathcal{I}$; **(ii)** $A, B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$; **(iii)** $A \in \mathcal{I}$, $B \subset A$ imply $B \in \mathcal{I}$, while an admissible ideal $\mathcal{I}$ of F further satisfies $\{u\} \in \mathcal{I}$ for each $u \in F$ (see, [6], [7]) .

Given $\mathcal{I} \subset 2^{\mathbb{N}}$ be a nontrivial ideal in $\mathbb{N}$. The sequence $(u_n)_{n\in\mathbb{N}}$ in E is said to be $\mathcal{I}-$convergent to $u \in E$, if for each $\varepsilon > 0$ the set $A(\varepsilon) = \{n \in \mathbb{N} : \|u_n - L\| \geq \varepsilon\}$ belongs to $\mathcal{I}$ ([1, 14, 15]).

By a lacunary sequence $\theta = (l_s)$; $s = 0, 1, 2, ...$ where $l_0 = 0$, we shall mean an increasing sequence of non-negative integers with $l_s - l_{s-1} \to \infty$ as $s \to \infty$. The intervals determined by θ will be denoted by $I_s = (l_{s-1}, l_s]$ and $\mu_s = l_s - l_{s-1}$ ([2]).

2. Main Results

Let $\mathcal{I}$ be an admissible ideal, λ be an Orlicz function, $(E, \|.,.\|)$ be a 2−normed space and $r = (r_i)$ be a sequence of positive real numbers. By $S(2-E)$ we denote the space of all sequences defined over $(E, \|.,.\|)$. Now we define the following sequence spaces:

$$\hat{w}^{\mathcal{I}}(\lambda, r, \|.,.\|)_\theta = \left\{ \begin{array}{c} x \in S(2-X) : \left\{ s \in \mathbb{N} : \frac{1}{\mu_s^\gamma} \sum\limits_{i\in I_s} \left[\lambda \left(\left\| \frac{u_{i+j}-L}{\rho}, z \right\| \right) \right]^{r_i} \geq \varepsilon \right\} \in \mathcal{I} \\ \text{for some } \rho > 0, L > 0 \text{ and each } z \in E, \text{ uniformly in } j \end{array} \right\},$$

$$\hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_\theta = \left\{ \begin{array}{c} x \in S(2-E) : \left\{ s \in \mathbb{N} : \frac{1}{\mu_s^\gamma} \sum\limits_{i\in I_s} \left[\lambda \left\| \frac{u_{i+j}}{\rho}, z \right\| \right]^{r_i} \geq \varepsilon \right\} \in \mathcal{I} \\ \text{for some } \rho > 0 \text{ and each } z \in E, \text{ uniformly in } j \end{array} \right\},$$

$$\hat{w}_\infty(\lambda, r, \|.,.\|)_\theta = \left\{ \begin{array}{c} x \in S(2-E) : \exists K > 0 \text{ s.t. } \sup\limits_{s\in\mathbb{N}} : \dfrac{1}{\mu_s^\gamma} \sum\limits_{i\in I_s} \left[\lambda \left(\left\| \dfrac{u_{i+j}}{\rho}, z \right\| \right) \right]^{r_i} \leq K \\ \text{for some } \rho > 0, \quad \text{and each } z \in E \end{array} \right\},$$

$$\hat{w}_\infty^{\mathcal{I}}(\lambda, r, \|.,.\|)_\theta = \left\{ \begin{array}{c} x \in S(2-E) : \exists K > 0 \ni \left\{ s \in \mathbb{N} : \frac{1}{\mu_s^\gamma} \sum\limits_{i\in I_s} \left[\lambda \left\| \frac{u_{i+j}}{\rho}, z \right\| \right]^{r_i} \geq K \right\} \in \mathcal{I} \\ \text{for some } \rho > 0 \text{ and each } z \in E, \text{ uniformly in } j \end{array} \right\}.$$

The following inequality will be used in the study which is well known.

$$0 \leq r_i \leq \sup r_i = H, \ D = \max\left(1, 2^{H-1}\right)$$

then

$$|u_i + v_i|^{r_i} \leq D\left\{|u_i|^{r_i} + |v_i|^{r_i}\right\}$$

for all i and $u_i, v_i \in C$

Theorem 2.1. $\hat{w}^{\mathcal{I}}(\lambda, r, \|.,.\|)_\theta$, $\hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_\theta$, $\hat{w}_\infty^{\mathcal{I}}(\lambda, r, \|.,.\|)_\theta$ *are linear spaces.*

Proof. We shall prove the assertion for $\hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_\theta$only and the others can be proved similarly. Suppose that $u, v \in \hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_\theta$ and $\alpha, \beta \in \mathbb{R}$. So

$$\left\{ s \in \mathbb{N} : \frac{1}{\mu_s^\gamma} \sum_{i \in I_s} \left[\lambda \left(\left\| \frac{u_{i+j}}{\rho_1}, z \right\| \right) \right]^{r_i} \geq \varepsilon \right\} \in \mathcal{I} \text{ for some } \rho_1 > 0$$

and

$$\left\{ s \in \mathbb{N} : \frac{1}{\mu_s^\gamma} \sum_{i \in I_s} \left[\lambda \left(\left\| \frac{u_{i+j}}{\rho_2}, z \right\| \right) \right]^{r_i} \geq \varepsilon \right\} \in \mathcal{I} \text{ for some } \rho_2 > 0.$$

uniformly in j. Since $\|.,.\|$ is a 2−norm, and λ is an Orlicz function the following inequality holds: for all j

$$\begin{aligned} & \frac{1}{\mu_s^\gamma} \sum_{i \in I_s} \left[\lambda \left(\left\| \frac{(\alpha u_{i+j} + \beta v_{i+j})}{(|\alpha| \rho_1 + |\beta| \rho_2)}, z \right\| \right) \right]^{r_i} \\ \leq \;& D \frac{1}{\mu_s^\gamma} \sum_{i \in I_s} \left[\frac{|\alpha|}{(|\alpha| \rho_1 + |\beta| \rho_2)} \lambda \left(\left\| \frac{u_{i+j}}{\rho_1}, z \right\| \right) \right]^{r_i} \\ + \;& D \frac{1}{\mu_s^\gamma} \sum_{i \in I_s} \left[\frac{|\beta|}{(|\alpha| \rho_1 + |\beta| \rho_2)} \lambda \left(\left\| \frac{v_{i+j}}{\rho_2}, z \right\| \right) \right]^{r_i} \\ \leq \;& DF \frac{1}{\mu_s^\gamma} \sum_{i \in I_s} \left[\lambda \left(\left\| \frac{u_{i+j}}{\rho_1}, z \right\| \right) \right]^{r_i} \\ + \;& DF \frac{1}{\mu_s^\gamma} \sum_{i \in I_s} \left[\lambda \left(\left\| \frac{v_{i+j}}{\rho_2} v, z \right\| \right) \right]^{r_i} \end{aligned}$$

where

$$F = \max \left[1, \left(\frac{|\alpha|}{(|\alpha| \rho_1 + |\beta| \rho_2)} \right)^H, \left(\frac{|\beta|}{(|\alpha| \rho_1 + |\beta| \rho_2)} \right)^H \right].$$

From the above inequality we get

$$\begin{aligned} & \left\{ s \in \mathbb{N} : \frac{1}{\mu_s^\gamma} \sum_{i \in I_s} \left[\lambda \left(\left\| \frac{(\alpha u_{i+j} + \beta v_{i+j})}{(|\alpha| \rho_1 + |\beta| \rho_2)}, z \right\| \right) \right]^{r_i} \geq \varepsilon \right\} \\ \subseteq \;& \left\{ s \in \mathbb{N} : DF \frac{1}{\mu_s^\gamma} \sum_{i \in I_s} \left[\lambda \left(\left\| \frac{u_{i+j}}{\rho_1}, z \right\| \right) \right]^{r_i} \geq \frac{\varepsilon}{2} \right\} \\ & \cup \left\{ s \in \mathbb{N} : DF \frac{1}{\mu_s^\gamma} \sum_{i \in I_s} \left[\lambda \left(\left\| \frac{v_{i+j}}{\rho_2}, z \right\| \right) \right]^{r_i} \geq \frac{\varepsilon}{2} \right\}, \end{aligned}$$

uniformly in j. Two sets on the right hand side belong to $\mathcal{I}$ and this completes the proof. □

It is also easy to verify that the space $\hat{w}_\infty(\lambda, r, \|.,.\|)_\theta$ is also a linear space.

Theorem 2.2. *If λ is an Orlicz function and (r_i) is bounded sequence of strictly positive real numbers then $\hat{w}_\infty(\lambda, r, \|.,.\|)_\theta$ is a paranormed space with respect to paranorm g defined by*

$$g(x) = \sum_{i \in I_s} \|u_{i+j}, z\| + \inf \left\{ \rho^{\frac{r_t}{H}} : \sup_k \left[\lambda \left(\left\| \frac{u_{i+j}}{\rho}, z \right\| \right) \right]^{r_i} \leq 1, \ \rho > 0, \ t = 1, 2, ... \right\}, \ \textit{each } z \in E$$

Corollary 1. If one considers the sequence space $\hat{w}_{\infty}^{\mathcal{I}}(\lambda, r, \|.,.\|)_{\theta}$ which is larger than the space $\hat{w}_{\infty}(\lambda, r, \|.,.\|)_{\theta}$ the construction of the paranorm is not clear and we leave it as an open problem.

Theorem 2.3. *Let λ, λ_1 and λ_2 be Orlicz functions. Then we have $\hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_{\theta} \subseteq \hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_{\theta}$ provided (r_i) is such that $H_0 = \inf r_i > 0$.*

Proof. (i) For given $\varepsilon > 0$, first choose $\varepsilon_0 > 0$ such that $\max\{\varepsilon_0^H, \varepsilon_0^{H_0}\} < \varepsilon$. Now using the continuity of λ choose $0 < \delta < 1$ such that $0 < t < \delta \Rightarrow \lambda(t) < \varepsilon_0$. Let $(u_i) \in \hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_{\theta}$. Now from the definition,

$$A(\delta) = \left\{ s \in \mathbb{N} \ : \frac{1}{\mu_s^{\gamma}} \sum_{i \in I_s} \left[\lambda_1 \left(\left\| \frac{u_{i+j}}{\rho}, z \right\| \right) \right]^{p_i} \geq \delta^H \right\} \in \mathcal{I},$$

uniformly in j. Thus if $s \notin A(\delta)$ then

$$\frac{1}{\mu_s^{\gamma}} \sum_{i \in I_s} \left[\lambda_1 \left(\left\| \frac{u_{i+j}}{\rho}, z \right\| \right) \right]^{r_i} < \delta^H$$

$$\text{i.e.} \sum_{i \in I_s} \left[\lambda_1 \left(\left\| \frac{u_{i+j}}{\rho}, z \right\| \right) \right]^{r_i} < \mu_s^{\gamma} \delta^H$$

$$\text{i.e.} \left[\lambda_1 \left(\left\| \frac{u_{i+j}}{\rho}, z \right\| \right) \right]^{r_i} < \delta^H \text{ for all } i \in I_s$$

$$\text{i.e.} \lambda_1 \left(\left\| \frac{u_{i+j}}{\rho}, z \right\| \right) < \delta \text{ for all } i \in I_s.$$

Hence from above using the continuity of λ we must have

$$\lambda \left(\lambda_1 \left(\left\| \frac{u_i}{\rho}, z \right\| \right) \right) < \varepsilon_0 \text{ for all } i \in I_s$$

which consequently implies that

$$\sum_{i \in I_s} \left[\lambda \left(\lambda_1 \left(\left\| \frac{u_{i+j}}{\rho}, z \right\| \right) \right) \right]^{p_k} < \mu_s^{\gamma} \max\{\varepsilon_0^H, \varepsilon_0^{H_0}\} < \mu_s^{\gamma} \varepsilon,$$

$$\text{i.e.} \frac{1}{\mu_s^{\gamma}} \sum_{i \in I_s} \left[\lambda \left(\lambda_1 \left(\left\| \frac{u_{i+j}}{\rho}, z \right\| \right) \right) \right]^{r_i} < \varepsilon, \text{ uniformly in } j.$$

This shows that

$$\left\{ s \in \mathbb{N} : \frac{1}{\mu_s^{\gamma}} \sum_{i \in I_s} \left[\lambda \left(\lambda_1 \left(\left\| \frac{u_{i+j}}{\rho}, z \right\| \right) \right) \right]^{p_k} \geq \varepsilon \right\} \subset A(\delta), \text{ uniformly in } j$$

and so belongs to $\mathcal{I}$. This proves the result. □

Theorem 2.4. *Let the sequence (r_i) be bounded, then $\hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_{\theta} \subseteq \hat{w}^{\mathcal{I}}(\lambda, r, \|.,.\|)_{\theta} \subseteq \hat{w}_{\infty}^{\mathcal{I}}(\lambda, r, \|.,.\|)_{\theta}$.*

Proof. Let $u = (u_i) \in \hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_{\theta}$. Then given $\varepsilon > 0$ we have

$$\left\{ s \in \mathbb{N} \ : \frac{1}{\mu_s^{\gamma}} \sum_{i \in I_s} \left[\lambda \left(\left\| \frac{u_{i+j}}{\rho}, z \right\| \right) \right]^{r_i} \geq \varepsilon \right\} \in \mathcal{I} \text{ for some } \rho > 0, \text{ uniformly in } j.$$

Since λ is non-decreasing and convex it follows that, for all j,

$$\frac{1}{\mu_s^\gamma}\sum_{i\in I_s}\left[\lambda\left(\left\|\frac{u_i}{2\rho},z\right\|\right)\right]^{r_i} \leq \frac{D}{\mu_s^\gamma}\sum_{i\in I_s}\frac{1}{2^{p_i}}\left[\lambda\left(\left\|\frac{u_{i+j}-u_0}{\rho},z\right\|\right)\right]^{r_i}+\frac{D}{\mu_s^\gamma}\sum_{i\in I_s}\frac{1}{2^{r_i}}\left[\lambda\left(\left\|\frac{u_0}{\rho},z\right\|\right)\right]^{r_i}$$

$$\leq \frac{D}{\mu_s^\gamma}\sum_{i\in I_s}\left[\lambda\left(\left\|\frac{u_{i+j}-u_0}{\rho},z\right\|\right)\right]^{r_i}+Dmax\left\{1, sup\left[\lambda\left(\left\|\frac{u_0}{\rho},z\right\|\right)\right]^{r_i}\right\}.$$

Hence we have

$$\begin{aligned}&\left\{s\in\mathbb{N}\ :\frac{1}{\mu_s^\gamma}\sum_{i\in I_s}\left[\lambda\left(\left\|\frac{u_i}{2\rho},z\right\|\right)\right]^{r_i}\geq\varepsilon\right\}\\ \subseteq\quad&\left\{s\in\mathbb{N}\ :\frac{D}{\mu_s^\gamma}\sum_{i\in I_s}\left[\lambda\left(\left\|\frac{u_{i+j}}{\rho},z\right\|\right)\right]^{r_i}\geq\frac{\varepsilon}{2}\right\}\\ \cup\quad&\left\{s\in\mathbb{N}\ :max\left\{1,sup\left[\lambda\left(\left\|\frac{u_0}{\rho},z\right\|\right)\right]^{r_i}\right\}\geq\frac{\varepsilon}{2}\right\},\end{aligned}$$

uniformly in j. Since the set on the right hand side belongs to $\mathcal{I}$ so does the left hand side. The inclusion $\hat{w}^{\mathcal{I}}(\lambda,r,\|.,.\|)_\theta\subseteq\hat{w}_\infty^{I}(\lambda,r,\|.,.\|)_\theta$ is obvious. □

Theorem 2.5. (1) *Let* $0<\inf r_i\leq r_i<1$. *Then*

$$\hat{w}^{\mathcal{I}}(\lambda,r,\|.,.\|)_\theta\subseteq\hat{w}^{\mathcal{I}}(\lambda,r,\|.,.\|)_\theta\,.$$

(2) *Let* $1\leq r_i\leq\sup r_i<\infty$. *Then*

$$\hat{w}^{\mathcal{I}}(\lambda,r,\|.,.\|)_\theta\subseteq\hat{w}^{\mathcal{I}}(\lambda,r,\|.,.\|)_\theta\,.$$

Proof. Let $u\in\hat{w}^{\mathcal{I}}(\lambda,r,\|.,.\|)_\theta$, since $0<\inf r_i\leq 1$, we obtain the following:

$$\left\{s\in\mathbb{N}\ :\frac{1}{\mu_s^\gamma}\sum_{i\in I_s}\left[\lambda\left(\left\|\frac{u_{i+j}-L}{\rho},z\right\|\right)\right]\geq\varepsilon\right\}\subseteq\left\{s\in\mathbb{N}\ :\frac{1}{\mu_s^\gamma}\sum_{i\in I_s}\left[\lambda\left(\left\|\frac{u_{i+j}-L}{\rho},z\right\|\right)\right]^{r_i}\geq\varepsilon\right\}\in\mathcal{I},$$

uniformly in j. Thus $u\in\hat{w}^{I}(\lambda,\|.,.\|)_\theta$. Let us establish part (2). Let $r_i>1$ for each i, and $\sup r_i<\infty$. Let $x\in\hat{w}^{\mathcal{I}}(\lambda,\|.,.\|)_\theta$. Then for each $0<\epsilon<1$ there exists a positive integer N such that

$$\mu_s^\gamma\sum_{i\in I_s}\left[\lambda\left(\left\|\frac{u_i-L}{\rho},z\right\|\right)\right]\leq\epsilon<1,$$

uniformly in j, for all $s\geq N$. This implies that

$$\left\{s\in\mathbb{N}\ :\frac{1}{\mu_s^\gamma}\sum_{i\in I_s}\left[\lambda\left(\left\|\frac{u_{i+j}-L}{\rho},z\right\|\right)\right]^{p_{k,l}}\geq\varepsilon\right\}\subseteq\left\{s\in\mathbb{N}\ :\frac{1}{\mu_s^\gamma}\sum_{i\in I_s}\left[\lambda\left(\left\|\frac{u_{i+j}-L}{\rho},z\right\|\right)\right]\geq\varepsilon\right\}\in\mathcal{I},$$

uniformly in j. Therefore $u\in\hat{w}^{\mathcal{I}}(\lambda,r,\|.,.\|)_\theta$. This completes the proof. □

Definition 2.1. *Let E be a sequence space. Then E is called solid if $(\alpha_iu_i)\in E$ whenever $(u_i)\in E$ for all sequences (α_i) of scalars with $|\alpha_i|\leq 1$ for all $i\in N$.*

We now have

Theorem 2.6. *The sequence spaces $\hat{w}_0^{\mathcal{I}}(\lambda,r,\|.,.\|)_\theta$, $\hat{w}_\infty^{\mathcal{I}}(\lambda,r,\|.,.\|)_\theta$ are solid.*

Proof. We give the proof for $\hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_\theta$. Let $(u_i) \in \hat{w}_0^{I}(\lambda, r, \|.,.\|)_\theta$ and (α_i) be sequences of scalars such that $|\alpha_i| \leq 1$ for all $i \in N$. Then we have,

$$\left\{r \in \mathbb{N} \ : \frac{1}{\mu_s^{\gamma}} \sum_{i \in I_s} \left[\left(\lambda \left\| \frac{(\alpha_i u_{i+j})}{\rho}, z \right\|\right)\right]^{r_i} \geq \varepsilon\right\} \subseteq \left\{r \in \mathbb{N} \ : \frac{1}{\mu_s^{\gamma}} \sum_{i \in I_s} \left[\left(\lambda \left\| \frac{u_{i+j})}{\rho}, z \right\|\right)\right]^{r_i} \geq \varepsilon\right\} \in \mathcal{I},$$

uniformly in j. Hence $(\alpha_i u_i) \in \hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_\theta$ for all sequences of scalars (α_i) with $|\alpha_i| \leq 1$ for all $i \in N$ whenever $(u_i) \in \hat{w}_0^{\mathcal{I}}(\lambda, r, \|.,.\|)_\theta$. □

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