

SITEM for the Conformable Space-Time Fractional (2+1)-Dimensional Breaking Soliton, Third-Order KdV and Burger's Equations

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Abstract

In the present paper, new analytical solutions for the conformable space-time fractional (2+1)-dimensional breaking soliton, third-order KdV and Burger's equations are obtained by using the simplified $\tan(\frac{\phi(\xi)}{2})$ -expansion method (SITEM). Here, fractional derivatives are described in conformable sense. The obtained traveling wave solutions are expressed by the trigonometric, hyperbolic, exponential and rational functions. Simulation of the obtained solutions are given at the end of the paper.

Keywords: Space-time fractional (2+1)-dimensional breaking soliton equations; Space-time fractional third-order KdV equation; Space-time fractional Burger's equation; Simplified $\tan(\frac{\phi(\xi)}{2})$ -expansion method (SITEM).

AMS Subject Classification (2020): Primary: 35C07, 35C08, 35R11.

1. Introduction

Nonlinear fractional partial differential equations have significant applications in various fields of science and engineering such as fluid mechanics, mechanics of materials, biology, plasma physics, finance, chemistry, image processing (see, for example, [1–5]). Traveling wave solutions to nonlinear fractional partial differential equations play an important role in the study of nonlinear physical phenomena. The traveling wave solutions of the nonlinear partial differential equations have been investigated by using various method such as exponential rational function method, (G'/G) -expansion method, Exp-function method, extended sinh-Gordon equation expansion method, modified exponential rational function method, Jacobi elliptic equation method (see, for example, [6–10]).

(2+1)-dimensional breaking soliton equations describe the $(2 + 1)$ -dimensional interaction of a Riemann wave propagating along the y-axis with a long wave along the x-axis. (G'/G) -expansion method, extended tanh-function method, improved Riccati equations method, sine-cosine method, improved extended Fan sub-equation method, generalized (G'/G) -expansion method and extended three wave method have been applied to the (2+1)-dimensional breaking soliton equations [11–18]. The space-time fractional $(2 + 1)$ -dimensional breaking soliton equations with modified Riemann-Liouville derivative have been solved by using new fractional Jacobi elliptic equation method,

new fractional sub-equation method, modified simple equation method, improved fractional sub-equation method, exponential rational function method, new fractional Jacobi elliptic equation method [19–23]. (G'/G) -expansion method has been studied for the space fractional $(2 + 1)$ -dimensional breaking soliton equations with modified Riemann-Liouville derivative [24].

The general projective Riccati equation method, Exp-function method, extended hyperbolic function method and collocation method with the modified exponential cubic B-spline have been applied to the third-order KdV equation [25–28]. Time-fractional generalized third-order KdV equation with modified Riemann-Liouville derivative has been solved by using generalized Kudryashov method [29].

Burger's equation plays a major role in the study of nonlinear waves since it is used as a mathematical model in turbulence problems, in the theory of shock waves, and in continuous stochastic processes [30]. Hopf-Cole transformation and a reproducing kernel function method, a semi-analytical iterative method, $(G'/G, 1/G)$ -expansion method have been applied to the Burger's equation [31–33].

In this paper, the conformable space-time fractional $(2+1)$ -dimensional breaking soliton, third-order KdV and Burger's equations have been solved by using the simplified $\tan(\frac{\phi(\xi)}{2})$ -expansion method (SITEM). SITEM has been applied to the Kundu-Eckhaus equation only for the parameter $p = 0$ in [34]. In our work, SITEM for the nonzero parameter p has been applied to the space-time fractional some evolution equations with conformable fractional derivative. New analytic solutions for these equations have been reported. Note that space-time fractional $(2+1)$ -dimensional breaking soliton, third-order KdV and Burger's equations including conformable derivatives have not yet been solved.

2. Description of the conformable fractional derivative and its properties

For a function $f : (0, \infty) \rightarrow R$, the conformable fractional derivative of f of order $0 < \alpha < 1$ is defined as (see, for example, [35])

$$T_t^\alpha f(t) = \lim_{\varepsilon \rightarrow 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}. \quad (2.1)$$

Some important properties of the the conformable fractional derivative are as follows:

$$\begin{aligned} T_t^\alpha (af + bg)(t) &= aT_t^\alpha f(t) + bT_t^\alpha g(t), \quad \forall a, b \in R, \\ T_t^\alpha (t^\mu) &= \mu t^{\mu-\alpha}, \\ T_t^\alpha (f(g(t))) &= t^{1-\alpha} g'(t) f'(g(t)). \end{aligned} \quad (2.2)$$

3. Analytic solutions to the conformable space-time fractional $(2+1)$ -dimensional breaking soliton equations

The breaking soliton equations can be used to describe the $(2 + 1)$ -dimensional interaction of a Riemann wave propagating along the y -axis with a long wave propagating along the x -axis. The $u(x, y, t)$ and $v(x, y, t)$ represent the physical field and some potential, respectively. This equation was studied by Bogoyavenskii [36].

Conformable space-time fractional $(2+1)$ -dimensional breaking soliton equations are given in the following form[23]

$$T_t^\alpha u + T_x^\beta T_x^\beta T_y^\theta u + 4uT_x^\beta v + 4vT_x^\beta u = 0, \quad (3.1)$$

$$T_y^\theta u = T_x^\beta v, \quad 0 < \alpha \leq 1, \quad 0 < \beta \leq 1, \quad 0 < \theta \leq 1. \quad (3.2)$$

Let us consider the following transformation

$$u(x, y, t) = U(\xi), \quad v(x, y, t) = V(\xi), \quad \xi = k \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta} + n \frac{y^\theta}{\theta}, \quad (3.3)$$

where k, m and n are constants. Using the third property in Eq.(2.2), we can compute the following derivatives

$$\begin{aligned} T_t^\alpha u(x, y, t) &= T_t^\alpha U(\xi) = t^{1-\alpha} \frac{d\xi}{dt} \frac{dU(\xi)}{d\xi} = kU'(\xi), \\ T_x^\beta u(x, y, t) &= T_x^\beta U(\xi) = x^{1-\beta} \frac{d\xi}{dx} \frac{dU(\xi)}{d\xi} = mU'(\xi), \\ T_y^\theta u(x, y, t) &= T_y^\theta U(\xi) = y^{1-\theta} \frac{d\xi}{dy} \frac{dU(\xi)}{d\xi} = nU'(\xi), \\ T_x^\beta T_x^\beta T_y^\theta u(x, y, t) &= nm^2 kU'''(\xi), \\ T_x^\beta v(x, y, t) &= T_x^\beta V(\xi) = x^{1-\beta} \frac{d\xi}{dx} \frac{dV(\xi)}{d\xi} = mV'(\xi). \end{aligned} \quad (3.4)$$

Substituting Eqs.(3.4) into Eqs.(3.1)-(3.2), we obtain the following differential equations

$$kU' + m^2 nU''' + 4mUV' + 4mVV' = 0, \quad (3.5)$$

$$nU' = mV'. \quad (3.6)$$

Integrating of Eqs.(3.5)-(3.6) with zero constant of integration and eliminating V , we have

$$kU + m^2 nU'' + 4nU^2 = 0. \quad (3.7)$$

Let us suppose that the solution of Eq.(3.7) can be expressed in the following form

$$U(\xi) = \sum_{k=0}^N A_k \left[p + \tan\left(\frac{\phi(\xi)}{2}\right) \right]^k + \sum_{k=1}^N B_k \left[p + \tan\left(\frac{\phi(\xi)}{2}\right) \right]^{-k}. \quad (3.8)$$

Here, $\phi(\xi)$ satisfies the following ordinary differential equation

$$\phi'(\xi) = a \sin(\phi(\xi)) + b \cos(\phi(\xi)) + c, \quad (3.9)$$

and $a, b, c, A_k (0 \leq k \leq N)$ and $B_k (1 \leq k \leq N)$ are constants to be determined. The solution of Eq. (3.9) is given as follows:

For $b = c, a = 0$,

$$\tan\left(\frac{\phi}{2}\right) = b\xi + c_1 - p.$$

For $b = c, a \neq 0$,

$$\tan\left(\frac{\phi}{2}\right) = c_1 \exp(a\xi) - \frac{b}{a}.$$

For $b \neq c, \Delta = a^2 + b^2 - c^2 > 0$,

$$\tan\left(\frac{\phi}{2}\right) = \frac{2}{b-c} \frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} - p.$$

For $b \neq c, \Delta = a^2 + b^2 - c^2 = 0$,

$$\tan\left(\frac{\phi}{2}\right) = \frac{a}{b-c} + \frac{2}{b-c} \frac{c_2}{c_1 + c_2 \xi}.$$

For $b \neq c, \Delta = a^2 + b^2 - c^2 < 0$,

$$\tan\left(\frac{\phi}{2}\right) = \frac{a}{b-c} + \frac{\sqrt{-\Delta}}{b-c} \frac{-c_1 \sin\left(\frac{\sqrt{-\Delta}}{2}\xi\right) + c_2 \cos\left(\frac{\sqrt{-\Delta}}{2}\xi\right)}{c_1 \cos\left(\frac{\sqrt{-\Delta}}{2}\xi\right) + c_2 \sin\left(\frac{\sqrt{-\Delta}}{2}\xi\right)},$$

where c_1 and c_2 are arbitrary constants, $r_1 = (a + p(b-c) + \sqrt{\Delta})/2$ and $r_2 = (a + p(b-c) - \sqrt{\Delta})/2$.

Substituting Eq.(3.8) into Eq.(3.7) and then by balancing the highest order derivative term and nonlinear term in result equation, the value of N can be determined as 2. Therefore, Eq.(3.8) reduces to

$$U(\xi) = A_0 + A_1 \left[p + \tan \left(\frac{\phi(\xi)}{2} \right) \right] + A_2 \left[p + \tan \left(\frac{\phi(\xi)}{2} \right) \right]^2 + B_1 \left[p + \tan \left(\frac{\phi(\xi)}{2} \right) \right]^{-1} + B_2 \left[p + \tan \left(\frac{\phi(\xi)}{2} \right) \right]^{-2}. \quad (3.10)$$

Substituting Eq.(3.10) into Eq.(3.7), collecting all the terms with the same power of $\tan(\frac{\phi}{2})$, we can obtain a set of algebraic equations for the unknowns $A_0, A_1, A_2, B_1, B_2, k, m, n$:

$$8nA_2^2 + 3nA_2b^2m^2 - 6nA_2bcm^2 + 3nA_2c^2m^2 = 0, \\ 64npA_2^2 + \dots$$

Solving the algebraic equations in the Mathematica, we obtain the following set of solutions:

Case 1: $A_0 = -\frac{3}{8}(b-c)m^2(-b-c+2ap+bp^2-cp^2)$, $A_1 = 0$, $A_2 = 0$, $B_1 = \frac{3}{4}m^2(-ab-ac+2a^2p-b^2p+c^2p+3abp^2-3acp^2+b^2p^3-2bcp^3+c^2p^3)$, $B_2 = -\frac{3}{8}m^2(-b-c+2ap+bp^2-cp^2)^2$, $k = -\Delta m^2 n$:

For $b = c$ and $a = 0$,

$$U_1(\xi) = -\frac{3}{2}m^2b^2 \left[b\xi + c_1 \right]^{-2}. \quad (3.11)$$

For $b = c$ and $a \neq 0$,

$$U_2(\xi) = \frac{3}{4}m^2(-2ab+2a^2p) \left[p + c_1 \exp(a\xi) - \frac{b}{a} \right]^{-1} - \frac{3}{8}m^2(-2b+2ap)^2 \cdot \left[p + c_1 \exp(a\xi) - \frac{b}{a} \right]^{-2}. \quad (3.12)$$

For $\Delta > 0$ and $b \neq c$,

$$U_3(\xi) = -\frac{3}{8}(b-c)m^2(-b-c+2ap+bp^2-cp^2) + \frac{3}{4}m^2(-ab-ac+2a^2p-b^2p+c^2p+3abp^2-3acp^2+b^2p^3-2bcp^3+c^2p^3) \cdot \left[\frac{2}{b-c} \frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right]^{-1} - \frac{3}{8}m^2(-b-c+2ap+bp^2-cp^2)^2 \left[\frac{2}{b-c} \frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right]^{-2}. \quad (3.13)$$

For $\Delta < 0$ and $b \neq c$,

$$U_4(\xi) = -\frac{3}{8}(b-c)m^2(-b-c+2ap+bp^2-cp^2) + \frac{3}{4}m^2(-ab-ac+2a^2p-b^2p+c^2p+3abp^2-3acp^2+b^2p^3-2bcp^3+c^2p^3) \cdot \left[p + \frac{a}{b-c} + \frac{\sqrt{-\Delta}}{b-c} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right]^{-1} - \frac{3}{8}m^2(-b-c+2ap+bp^2-cp^2)^2 \left[p + \frac{a}{b-c} - \frac{\sqrt{-\Delta}}{b-c} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right]^{-2}. \quad (3.14)$$

Here $\xi = -\Delta m^2 n \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta} + n \frac{x^\theta}{\theta}$.

Case 2: $A_0 = -\frac{1}{8}m^2(2a^2-b^2+c^2+6abp-6acp+3b^2p^2-6bcp^2+3c^2p^2)$, $A_1 = 0$, $A_2 = 0$, $B_1 = \frac{3}{4}m^2(-ab-ac+2a^2p-b^2p+c^2p+3abp^2-3acp^2+b^2p^3-2bcp^3+c^2p^3)$, $B_2 = -\frac{3}{8}m^2(-b-c+2ap+bp^2-cp^2)^2$, $k = \Delta m^2 n$:

For $b = c$ and $a = 0$,

$$U_5(\xi) = -\frac{3}{2}m^2b^2[b\xi + c_1]^{-2}. \quad (3.15)$$

For $b = c$ and $a \neq 0$,

$$U_6(\xi) = -\frac{1}{4}m^2a^2 + \frac{3}{2}m^2(-ab + a^2p)\left[p + c_1 \exp(a\xi) - \frac{b}{a}\right]^{-1} \\ + -\frac{3}{2}m^2(-b + ap)^2\left[p + c_1 \exp(a\xi) - \frac{b}{a}\right]^{-2}. \quad (3.16)$$

For $\Delta > 0$ and $b \neq c$,

$$U_7(\xi) = -\frac{1}{8}m^2(2a^2 - b^2 + c^2 + 6abp - 6acp + 3b^2p^2 - 6bcp^2 + 3c^2p^2) \\ + \frac{3}{4}m^2(-ab - ac + 2a^2p - b^2p + c^2p + 3abp^2 - 3acp^2 + b^2p^3 - 2bcp^3 + c^2p^3) \\ \cdot \left[\frac{2}{b-c} \frac{c_1r_1 \exp(r_1\xi) + c_2r_2 \exp(r_2\xi)}{c_1 \exp(r_1\xi) + c_2 \exp(r_2\xi)}\right]^{-1} \\ - \frac{3}{8}m^2(-b - c + 2ap + bp^2 - cp^2)^2 \left[\frac{2}{b-c} \frac{c_1r_1 \exp(r_1\xi) + c_2r_2 \exp(r_2\xi)}{c_1 \exp(r_1\xi) + c_2 \exp(r_2\xi)}\right]^{-2}. \quad (3.17)$$

For $\Delta < 0$ and $b \neq c$,

$$U_8(\xi) = -\frac{1}{8}m^2(2a^2 - b^2 + c^2 + 6abp - 6acp + 3b^2p^2 - 6bcp^2 + 3c^2p^2) \\ + \frac{3}{4}m^2(-ab - ac + 2a^2p - b^2p + c^2p + 3abp^2 - 3acp^2 + b^2p^3 - 2bcp^3 + c^2p^3) \\ \cdot \left[p + \frac{a}{b-c} + \frac{\sqrt{-\Delta}}{b-c} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)}\right]^{-1} \\ - \frac{3}{8}m^2(-b - c + 2ap + bp^2 - cp^2)^2 \left[p + \frac{a}{b-c} + \frac{\sqrt{-\Delta}}{b-c} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)}\right]^{-2}. \quad (3.18)$$

Here $\xi = \Delta m^2 n \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta} + n \frac{x^\theta}{\theta}$.

Case 3: $A_0 = -\frac{3}{8}(b-c)m^2(-b-c+2ap+bp^2-cp^2)$, $A_1 = \frac{3}{4}(b-c)m^2(a+bp-cp)$, $A_2 = -\frac{3}{8}(b-c)^2m^2$, $B_1 = 0$, $B_2 = 0$, $k = -\Delta m^2 n$:

For $\Delta > 0$ and $b \neq c$,

$$U_9(\xi) = -\frac{3}{8}(b-c)m^2(-b-c+2ap+bp^2-cp^2) \\ + \frac{3}{2}m^2(a+bp-cp) \left[\frac{c_1r_1 \exp(r_1\xi) + c_2r_2 \exp(r_2\xi)}{c_1 \exp(r_1\xi) + c_2 \exp(r_2\xi)}\right] \\ - \frac{3}{2}m^2 \left[\frac{c_1r_1 \exp(r_1\xi) + c_2r_2 \exp(r_2\xi)}{c_1 \exp(r_1\xi) + c_2 \exp(r_2\xi)}\right]^2. \quad (3.19)$$

For $\Delta < 0$ and $b \neq c$,

$$U_{10}(\xi) = -\frac{3}{8}(b-c)m^2(-b-c+2ap+bp^2-cp^2) \\ + \frac{3}{4}m^2(a+bp-cp) \left[p(b-c) + a + \sqrt{-\Delta} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)}\right] \\ - \frac{3}{8}m^2 \left[p(b-c) + a + \sqrt{-\Delta} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)}\right]^2. \quad (3.20)$$

Here $\xi = -\Delta m^2 n \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta} + n \frac{y^\theta}{\theta}$.

Case 4: $A_0 = -\frac{1}{8}m^2(2a^2 - b^2 + c^2 + 6abp - 6acp + 3b^2p^2 - 6bcp^2 + 3c^2p^2)$, $A_1 = \frac{3}{4}(b-c)m^2(a + bp - cp)$, $A_2 = -\frac{3}{8}(b-c)^2m^2$, $B_1 = 0$, $B_2 = 0$, $k = \Delta m^2 n$:

For $b = c$ and $a \neq 0$,

$$U_{11}(\xi) = -\frac{1}{4}m^2a^2. \quad (3.21)$$

For $\Delta > 0$ and $b \neq c$,

$$\begin{aligned} U_{12}(\xi) &= -\frac{1}{8}m^2(2a^2 - b^2 + c^2 + 6abp - 6acp + 3b^2p^2 - 6bcp^2 + 3c^2p^2) \\ &+ \frac{3}{2}m^2(a + bp - cp) \left[\frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right] \\ &- \frac{3}{2}m^2 \left[\frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right]^2. \end{aligned} \quad (3.22)$$

For $\Delta < 0$ and $b \neq c$,

$$\begin{aligned} U_{13}(\xi) &= -\frac{1}{8}m^2(2a^2 - b^2 + c^2 + 6abp - 6acp + 3b^2p^2 - 6bcp^2 + 3c^2p^2) \\ &+ \frac{3}{4}m^2(a + bp - cp) \left[p(b-c) + a + \sqrt{-\Delta} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right] \\ &- \frac{3}{8}m^2 \left[p(b-c) + a + \sqrt{-\Delta} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right]^2. \end{aligned} \quad (3.23)$$

Here $\xi = \Delta m^2 n \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta} + n \frac{y^\theta}{\theta}$. Using formula $V(\xi) = \frac{n}{m}U(\xi)$ the unknown function $V(\xi)$ can be computed.

The solutions $u_2(x, y, t)$, $u_3(x, y, t)$ and $u_4(x, y, t)$ of the Eqs.(3.1)-(3.2) are simulated as traveling wave solutions for various values of the physical parameters in Fig.1-Fig.6. Figs.1, 2 show kink waves solutions, Figs.3 and 4 show solitary waves solutions, Figs.5, 6 show periodic waves solutions of Eqs.(3.1)-(3.2). Figs.1 and 2 are 3D and 2D plots of the traveling wave solution $u_2(x, 1, t)$ and $u_2(x, 1, 1)$ in Eq.(3.12) for parameters $\alpha = 0.75$, $\beta = 1$, $\theta = 0.5$, $m = -0.05$, $n = 0.5$, $a = 1$, $b = 5$, $c = 5$, $c_1 = 1$, $c_2 = 2$ and $p = 0.1$. Figs.3 and 4 are 3D and 2D plots of the traveling wave solution $u_3(x, 1, t)$ and $u_3(x, 1, 1)$ in Eq.(3.13) for $\alpha = 0.75$, $\beta = 1$, $\theta = 0.5$, $m = 0.5$, $n = 0.2$, $a = 0.1$, $b = 0.5$, $c = 0.02$, $c_1 = 1$, $c_2 = 1$ and $p = 2$. Figs.5 and 6 are 3D and 2D plots of the traveling wave solution $u_4(x, 1, t)$ and $u_4(x, 1, 1)$ in Eq.(3.14) for $\alpha = 0.5$, $\beta = 1$, $\theta = 0.5$, $m = 0.5$, $n = 0.2$, $a = 0.05$, $b = 0.2$, $c = 0.6$, $c_1 = 1$, $c_2 = 1$ and $p = 1$. Note that the 3D graphs describe the behavior of u in space x and time t at fixed $y = 1$, which represents the change of amplitude and shape for each obtained traveling wave solutions. 2D graphs describe the behavior of u in space x at fixed time $t = 1$ and fixed $y = 1$. All graphics in figures are drawn by the aid of Mathematica 10.

4. Analytic solutions to the conformable space-time fractional Korteweg-de Vries (KdV) equation

Conformable space-time fractional KdV equation is given in the following form[25]

$$T_t^\alpha u + T_x^\beta T_x^\beta T_x^\beta u + 6uT_x^\beta u = 0, 0 < \alpha \leq 1, 0 < \beta \leq 1. \quad (4.1)$$

Let us consider the following transformation

$$u(x, t) = U(\xi), \quad \xi = k \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta}, \quad (4.2)$$

where k, m are constants. Substituting (4.2) into Eq.(4.1) we obtain the following differential equations

$$kU' + m^3U''' + 6mUU' = 0. \quad (4.3)$$

Integrating of Eq.(4.3)with zero constant of integration, we have

$$kU + m^3U'' + 3mU^2 = 0. \quad (4.4)$$

Let us suppose that the solution of Eq.(4.4) can be expressed in the form Eq.(3.8). Substituting Eq.(3.8) into Eq.(4.4) and then by balancing the highest order derivative term and nonlinear term in result equation, the value of N can be determined as 2. Therefore, Eq.(3.8) reduces to Eq.(3.10). Substituting Eq.(3.10) into Eq.(4.4), collecting all the terms with the same power of $\tan(\frac{\phi}{2})$, we can obtain a set of algebraic equations for the unknowns $A_0, A_1, A_2, B_1, B_2, k, m$:

$$\begin{aligned} 6A_2^2m + 3A_2b^2m^3 - 6A_2bcm^3 + 3A_2c^2m^3 &= 0, \\ 48pA_2^2m + \dots \end{aligned}$$

Solving the algebraic equations in the Mathematica, we obtain the following set of solutions:

Case 1: $A_0 = -\frac{1}{2}(b-c)m^2(-b-c+2ap+bp^2-cp^2)$, $A_1 = 0$, $A_2 = 0$, $B_1 = m^2(-ab-ac+2a^2p-b^2p+c^2p+3abp^2-3acp^2+b^2p^3-2bcp^3+c^2p^3)$, $B_2 = -\frac{1}{2}m^2(-b-c+2ap+bp^2-cp^2)^2$, $k = -\Delta m^3$:
For $b = c$ and $a = 0$,

$$U_1(\xi) = -2m^2b^2 \left[b\xi + c_1 \right]^{-2}. \quad (4.5)$$

For $b = c$ and $a \neq 0$,

$$U_2(\xi) = 2m^2(a^2p - ab) \left[p + c_1 \exp(a\xi) - \frac{b}{a} \right]^{-1} - 2m^2(ap - b)^2 \left[p + c_1 \exp(a\xi) - \frac{b}{a} \right]^{-2}. \quad (4.6)$$

For $\Delta > 0$ and $b \neq c$,

$$\begin{aligned} U_3(\xi) &= -\frac{1}{2}(b-c)m^2(-b-c+2ap+bp^2-cp^2) \\ &+ m^2(-ab-ac+2a^2p-b^2p+c^2p+3abp^2-3acp^2+b^2p^3-2bcp^3+c^2p^3) \\ &\cdot \left[\frac{2}{b-c} \frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right]^{-1} \\ &- \frac{1}{2}m^2(-b-c+2ap+bp^2-cp^2)^2 \left[\frac{2}{b-c} \frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right]^{-2}. \end{aligned} \quad (4.7)$$

For $\Delta < 0$ and $b \neq c$,

$$\begin{aligned} U_4(\xi) &= -\frac{1}{2}(b-c)m^2(-b-c+2ap+bp^2-cp^2) \\ &+ m^2(-ab-ac+2a^2p-b^2p+c^2p+3abp^2-3acp^2+b^2p^3-2bcp^3+c^2p^3) \\ &\cdot \left[p + \frac{a}{b-c} + \frac{\sqrt{-\Delta}}{b-c} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right]^{-1} \\ &- \frac{1}{2}m^2(-b-c+2ap+bp^2-cp^2)^2 \left[p + \frac{a}{b-c} + \frac{\sqrt{-\Delta}}{b-c} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right]^{-2}. \end{aligned} \quad (4.8)$$

Here $\xi = -\Delta m^3 \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta}$.

Case 2: $A_0 = -\frac{1}{6}m^2(2a^2 - b^2 + c^2 + 6abp - 6acp + 3b^2p^2 - 6bcp^2 + 3c^2p^2)$, $A_1 = 0$, $A_2 = 0$, $B_1 = m^2(-ab-ac+2a^2p-b^2p+c^2p+3abp^2-3acp^2+b^2p^3-2bcp^3+c^2p^3)$, $B_2 = -\frac{1}{2}m^2(-b-c+2ap+bp^2-cp^2)^2$, $k = \Delta m^3$:
For $b = c$ and $a = 0$,

$$U_5(\xi) = -2m^2b^2 \left[b\xi + c_1 \right]^{-2}. \quad (4.9)$$

For $b = c$ and $a \neq 0$,

$$\begin{aligned} U_6(\xi) &= -\frac{1}{3}m^2a^2 + 2m^2(a^2p - ab) \left[p + c_1 \exp(a\xi) - \frac{b}{a} \right]^{-1} \\ &- 2m^2(ap - b)^2 \left[p + c_1 \exp(a\xi) - \frac{b}{a} \right]^{-2}. \end{aligned} \quad (4.10)$$

For $\Delta > 0$ and $b \neq c$,

$$\begin{aligned}
 U_7(\xi) &= -\frac{1}{6}m^2(2a^2 - b^2 + c^2 + 6abp - 6acp + 3b^2p^2 - 6bcp^2 + 3c^2p^2) \\
 &+ m^2(-ab - ac + 2a^2p - b^2p + c^2p + 3abp^2 - 3acp^2 + b^2p^3 - 2bcp^3 + c^2p^3) \\
 &\cdot \left[\frac{2}{b-c} \frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right]^{-1} \\
 &- \frac{1}{2}m^2(-b - c + 2ap + bp^2 - cp^2)^2 \left[\frac{2}{b-c} \frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right]^{-2}.
 \end{aligned} \tag{4.11}$$

For $\Delta < 0$ and $b \neq c$,

$$\begin{aligned}
 U_8(\xi) &= -\frac{1}{6}m^2(2a^2 - b^2 + c^2 + 6abp - 6acp + 3b^2p^2 - 6bcp^2 + 3c^2p^2) \\
 &+ m^2(-ab - ac + 2a^2p - b^2p + c^2p + 3abp^2 - 3acp^2 + b^2p^3 - 2bcp^3 + c^2p^3) \\
 &\cdot \left[p + \frac{a}{b-c} + \frac{\sqrt{-\Delta} - c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right]^{-1} \\
 &- \frac{1}{2}m^2(-b - c + 2ap + bp^2 - cp^2)^2 \left[p + \frac{a}{b-c} \right. \\
 &\left. + \frac{\sqrt{-\Delta} - c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right]^{-2}.
 \end{aligned} \tag{4.12}$$

Here $\xi = \Delta m^3 \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta}$.

Case 3: $A_0 = -\frac{1}{2}(b-c)m^2(-b-c+2ap+bp^2-cp^2)$, $A_1 = (b-c)m^2(a+bp-cp)$, $A_2 = -\frac{1}{2}(b-c)^2m^2$, $B_1 = 0$, $B_2 = 0$, $k = -\Delta m^3$:

For $\Delta > 0$ and $b \neq c$,

$$\begin{aligned}
 U_9(\xi) &= -\frac{1}{2}(b-c)m^2(-b-c+2ap+bp^2-cp^2) \\
 &+ 2m^2(a+bp-cp) \left[\frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right] \\
 &- 2m^2 \left[\frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right]^2
 \end{aligned} \tag{4.13}$$

For $\Delta < 0$ and $b \neq c$,

$$\begin{aligned}
 U_{10}(\xi) &= -\frac{1}{2}(b-c)m^2(-b-c+2ap+bp^2-cp^2) \\
 &+ (b-c)m^2(a+bp-cp) \left[p + \frac{a}{b-c} + \frac{\sqrt{-\Delta} - c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right] \\
 &+ -\frac{1}{2}(b-c)^2m^2 \left[p + \frac{a}{b-c} + \frac{\sqrt{-\Delta} - c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right]^2
 \end{aligned} \tag{4.14}$$

Here $\xi = -\Delta m^3 \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta}$.

Case 4: $A_0 = -\frac{1}{6}m^2(2a^2 - b^2 + c^2 + 6abp - 6acp + 3b^2p^2 - 6bcp^2 + 3c^2p^2)$, $A_1 = (b-c)m^2(a+bp-cp)$, $A_2 = -\frac{1}{2}(b-c)^2m^2$, $B_1 = 0$, $B_2 = 0$, $k = \Delta m^3$:

For $b = c$ and $a \neq 0$,

$$U_{11}(\xi) = -\frac{1}{3}m^2a^2. \tag{4.15}$$

For $\Delta > 0$ and $b \neq c$,

$$\begin{aligned} U_{12}(\xi) &= -\frac{1}{6}m^2(2a^2 - b^2 + c^2 + 6abp - 6acp + 3b^2p^2 - 6bcp^2 + 3c^2p^2) \\ &+ 2m^2(a + bp - cp) \left[\frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right] \\ &- 2m^2 \left[\frac{c_1 r_1 \exp(r_1 \xi) + c_2 r_2 \exp(r_2 \xi)}{c_1 \exp(r_1 \xi) + c_2 \exp(r_2 \xi)} \right]^2. \end{aligned} \quad (4.16)$$

For $\Delta < 0$ and $b \neq c$,

$$\begin{aligned} U_{13}(\xi) &= -\frac{1}{6}m^2(2a^2 - b^2 + c^2 + 6abp - 6acp + 3b^2p^2 - 6bcp^2 + 3c^2p^2) \\ &+ m^2(a + bp - cp) \left[p(b - c) + a + \sqrt{-\Delta} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right] \\ &- \frac{1}{2}m^2 \left[p(b - c) + a + \sqrt{-\Delta} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right]^2. \end{aligned} \quad (4.17)$$

Here $\xi = \Delta m^3 \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta}$.

The solution $u_{13}(x, t)$ of the Eq.(4.1) is simulated in Fig.7-Fig.8 as periodic waves solutions. 3D plot of the obtained solution $u_{13}(x, t)$ is given for parameters $\alpha = 0.5$, $\beta = 1$, $m = 0.5$, $a = 0.5$, $b = 0.25$, $c = 1$, $c_1 = 1$, $c_2 = 3$ and $p = 1$ in Fig.7. Fig.8 demonstrate the same solution with 2D plot for $-40 \leq x \leq 40$ at $t = 1$.

5. Analytic solutions to the conformable space-time fractional Burger's equation

Conformable space-time fractional Burger's equation is given in the following form[31]

$$T_t^\alpha u + u T_x^\beta u - T_x^\beta T_x^\beta u = 0, 0 < \alpha \leq 1, 0 < \beta \leq 1. \quad (5.1)$$

Let us consider the following transformation

$$u(x, t) = U(\xi), \quad \xi = k \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta}, \quad (5.2)$$

where k, m are constants. Substituting Eq.(5.2) into Eq.(5.1) we obtain the following differential equations

$$kU' + mUU' - m^2U'' = 0. \quad (5.3)$$

Integrating of Eq.(5.3)with zero constant of integration, we have

$$kU + \frac{m}{2}U^2 - m^2U' = 0. \quad (5.4)$$

Let us suppose that the solution of Eq.(5.4) can be expressed in the form Eq.(3.8). Substituting Eq.(3.8) into Eq.(5.4) and then by balancing the highest order derivative term and nonlinear term in result equation, the value of N can be determined as 1. Therefore, Eq.(3.8) reduces to

$$U(\xi) = A_0 + A_1 \left[p + \tan \left(\frac{\phi(\xi)}{2} \right) \right] + B_1 \left[p + \tan \left(\frac{\phi(\xi)}{2} \right) \right]^{-1}. \quad (5.5)$$

Substituting Eq.(5.5) into Eq.(5.4), collecting all the terms with the same power of $\tan(\frac{\phi}{2})$, we can obtain a set of algebraic equations for the unknowns A_0, A_1, B_1, k, m :

$$\begin{aligned} &A_1^2 m + A_1 b m^2 - A_1 c m^2 = 0, \\ &2A_1 k + 2A_0 A_1 m - 2a A_1 m^2 + 4A_1^2 m p + 2A_1 b m^2 p - 2A_1 c m^2 p = 0, \\ &2A_0 k + A_0^2 m + 6A_1^2 m p^2 + 2A_1 B_1 m + 6A_1 k p - A_1 b m^2 - A_1 c m^2 - b B_1 m^2 \\ &+ B_1 c m^2 - 4a A_1 m^2 p + A_1 b m^2 p^2 - A_1 c m^2 p^2 + 6A_0 A_1 m p = 0, \\ &2B_1 k + 4A_1^2 m p^3 + 2A_0 B_1 m + 4A_0 k p + 2a B_1 m^2 + 6A_1 k p^2 + 2A_0^2 m p \\ &+ 6A_0 A_1 m p^2 - 2A_1 b m^2 p - 2A_1 c m^2 p - 2a A_1 m^2 p^2 + 4A_1 B_1 m p = 0, \\ &B_1^2 m + A_0^2 m p^2 + A_1^2 m p^4 + 2B_1 k p + b B_1 m^2 + B_1 c m^2 + 2A_0 k p^2 + 2A_1 k p^3 \\ &+ 2A_0 A_1 m p^3 + 2A_1 B_1 m p^2 - A_1 b m^2 p^2 - A_1 c m^2 p^2 + 2A_0 B_1 m p = 0. \end{aligned}$$

Solving the algebraic equations in the Mathematica, we obtain the following set of solutions:

Case 1: $A_0 = -am \pm \sqrt{m^2\Delta} - mp(b-c)$, $A_1 = 0$, $B_1 = m(-b-c+2ap+bp^2-cp^2)$, $k = \mp m\sqrt{m^2\Delta}$:

For $b = c$ and $a = 0$,

$$U_1(\xi) = -2bm [b\xi + c_1]^{-1}. \quad (5.6)$$

For $b = c$ and $a \neq 0$,

$$U_{2,3}(\xi) = -am \pm \sqrt{m^2a^2} + 2m(ap-b) \left[p + c_1 \exp(a\xi) - \frac{b}{a} \right]^{-1}. \quad (5.7)$$

For $\Delta > 0$ and $b \neq c$,

$$U_{4,5}(\xi) = -am \pm \sqrt{m^2\Delta} - mp(b-c) + m(-b-c+2ap+bp^2-cp^2) \left[\frac{2}{b-c} \frac{c_1 r_1 \exp(r_1\xi) + c_2 r_2 \exp(r_2\xi)}{c_1 \exp(r_1\xi) + c_2 \exp(r_2\xi)} \right]^{-1}. \quad (5.8)$$

For $\Delta < 0$ and $b \neq c$,

$$U_{6,7}(\xi) = -am \pm \sqrt{m^2\Delta} - mp(b-c) + m(-b-c+2ap+bp^2-cp^2) \left[p + \frac{a}{b-c} + \frac{\sqrt{-\Delta}}{b-c} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right]^{-1}. \quad (5.9)$$

Here $\xi = \mp m\sqrt{m^2\Delta} \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta}$.

Case 2: $A_0 = am \pm \sqrt{m^2\Delta} + mp(b-c)$, $A_1 = -m(b-c)$, $B_1 = 0$, $k = \mp m\sqrt{m^2\Delta}$:

For $\Delta > 0$ and $b \neq c$,

$$U_{8,9}(\xi) = am \pm \sqrt{m^2\Delta} + mp(b-c) - 2m \left[\frac{c_1 r_1 \exp(r_1\xi) + c_2 r_2 \exp(r_2\xi)}{c_1 \exp(r_1\xi) + c_2 \exp(r_2\xi)} \right]. \quad (5.10)$$

For $\Delta < 0$ and $b \neq c$,

$$U_{10,11}(\xi) = am \pm \sqrt{m^2\Delta} + mp(b-c) - m \left[p(b-c) + a + \sqrt{-\Delta} \frac{-c_1 \sin(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \cos(\frac{\sqrt{-\Delta}}{2}\xi)}{c_1 \cos(\frac{\sqrt{-\Delta}}{2}\xi) + c_2 \sin(\frac{\sqrt{-\Delta}}{2}\xi)} \right]. \quad (5.11)$$

Here $\xi = \mp m\sqrt{m^2\Delta} \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta}$.

The solution $u_5(x, t)$ in Eq.(5.8) is simulated in Fig.9-Fig.10. These figures show kink wave solutions. Figs.9 and 10 are 3D and 2D plots of the traveling wave solution $u_5(x, t)$ and $u_5(x, 1)$ in Eq.(5.8) for $\alpha = 0.75$, $\beta = 1$, $\theta = 0.5$, $m = 0.5$, $a = 2$, $b = 5$, $c = 2$, $c_1 = 1$, $c_2 = 1$ and $p = 0.2$.

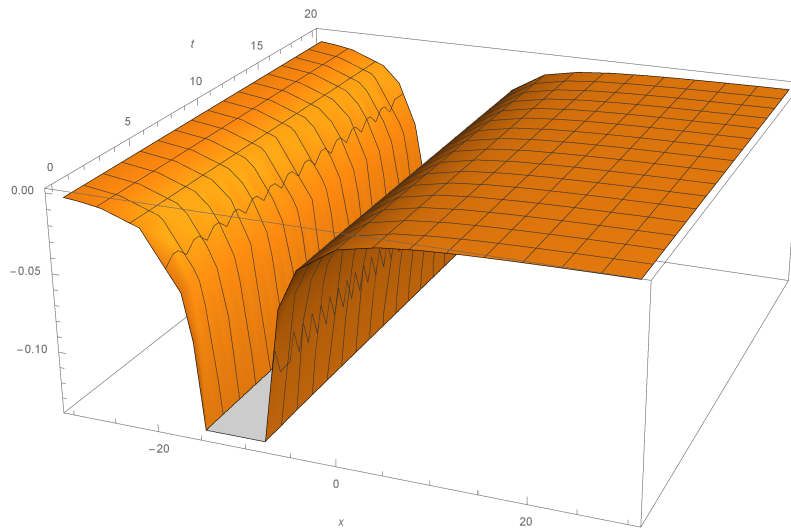


Figure 1. 3D plot of the obtained traveling wave solution $u_2(x, 1, t)$ in Eq.(3.12).

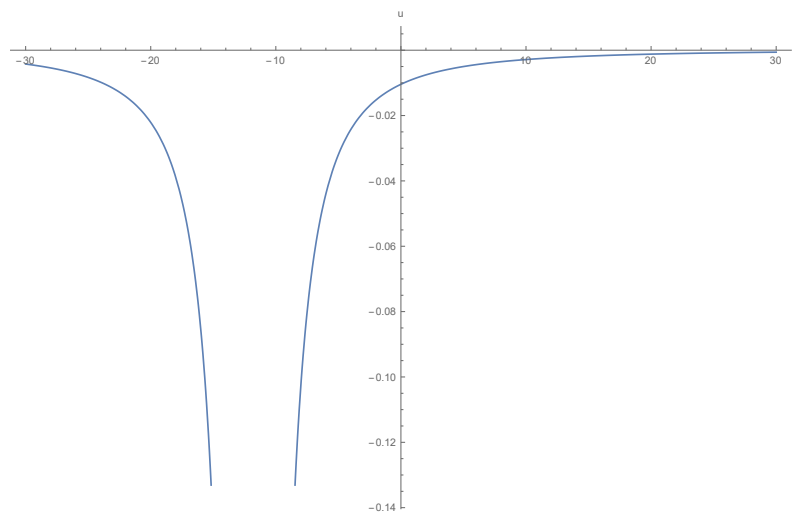


Figure 2. 2D plot of the obtained traveling wave solution $u_2(x, 1, 1)$ in Eq.(3.12).

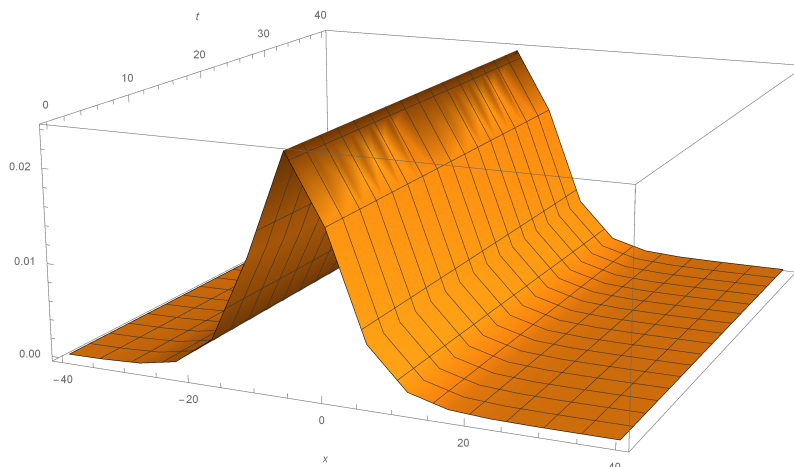


Figure 3. 3D plot of the obtained traveling wave solution $u_3(x, 1, t)$ in Eq.(3.13).

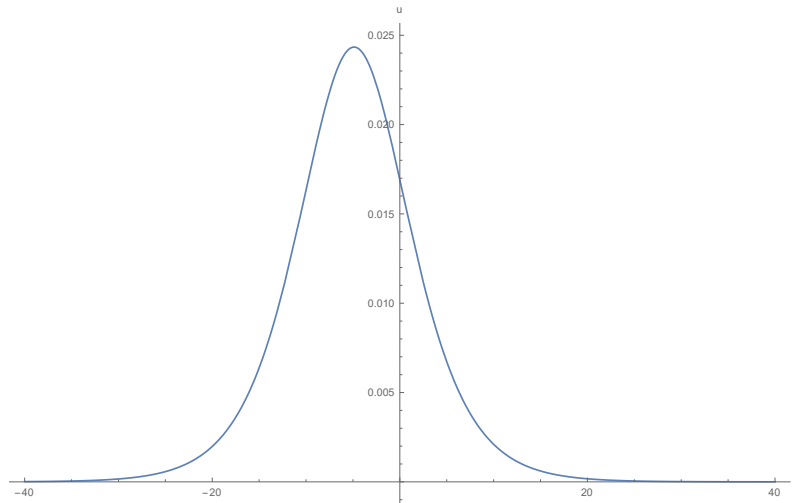


Figure 4. 2D plot of the obtained traveling wave solution $u_3(x, 1, 1)$ in Eq.(3.13).

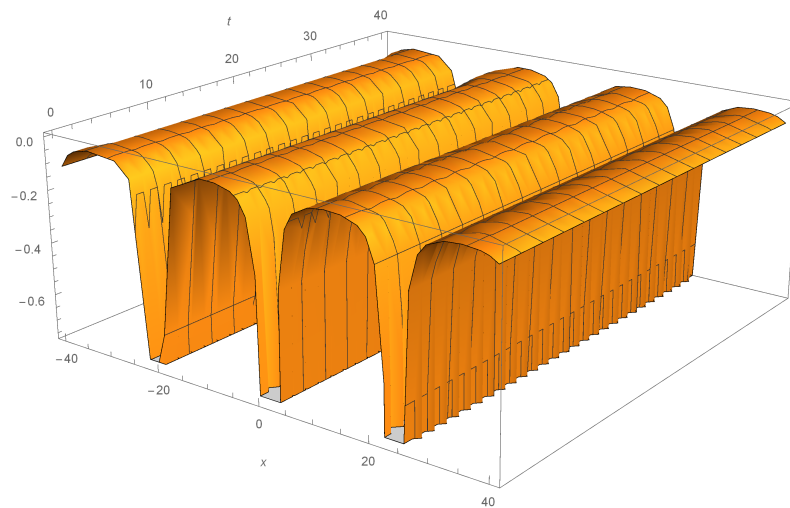


Figure 5. 3D plot of the obtained traveling wave solution $u_4(x, 1, t)$ in Eq.(3.14).

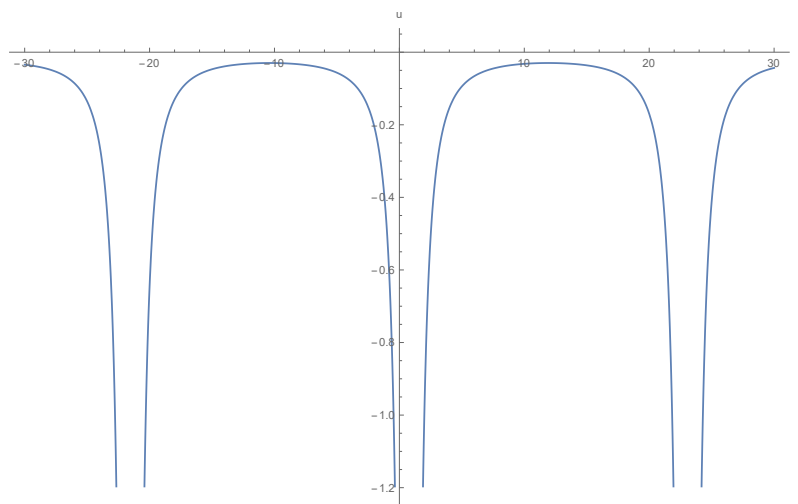


Figure 6. 2D plot of the obtained traveling wave solution $u_4(x, 1, 1)$ in Eq.(3.14).

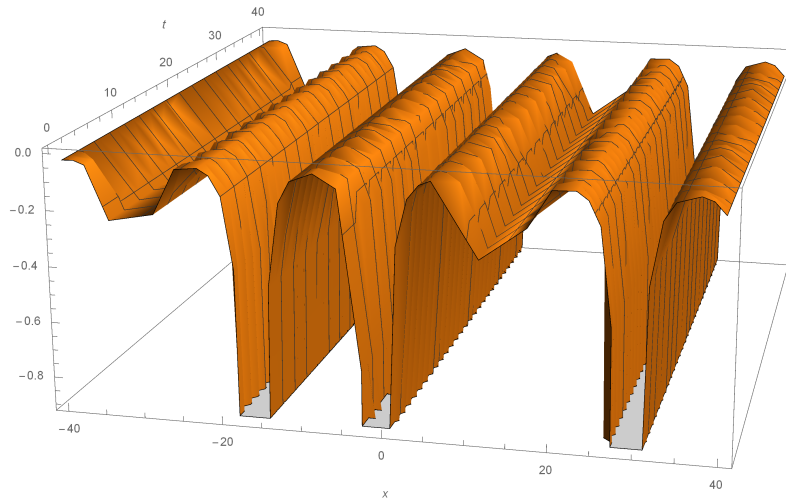


Figure 7. 3D plot of the obtained traveling wave solution $u_{13}(x, t)$ in Eq.(4.17).

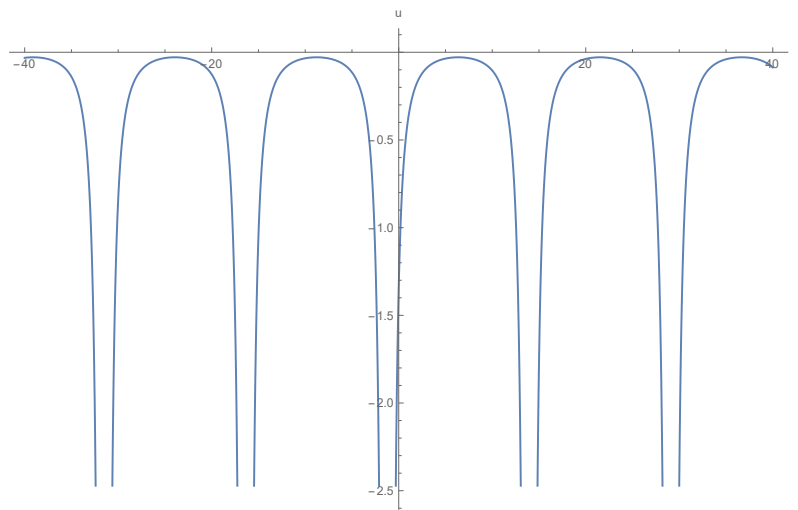


Figure 8. 2D plot of the obtained traveling wave solution $u_{13}(x, 1)$ in Eq.(4.17).

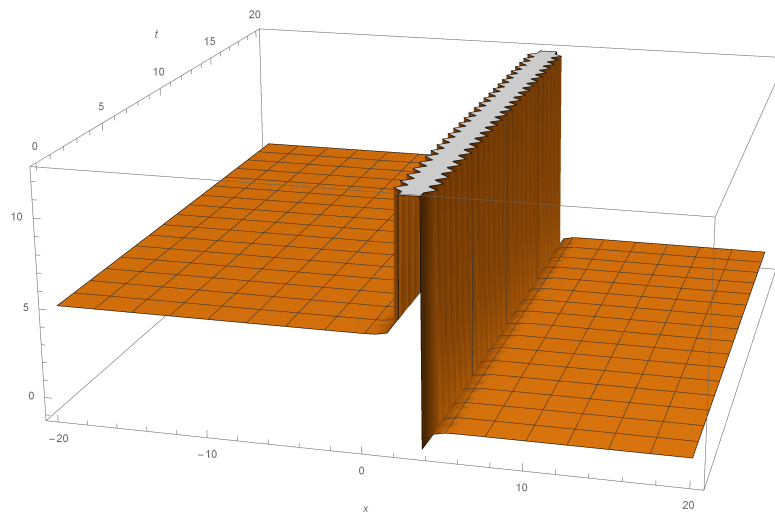


Figure 9. 3D plot of the obtained traveling wave solution $u_5(x, t)$ in Eq.(5.8).

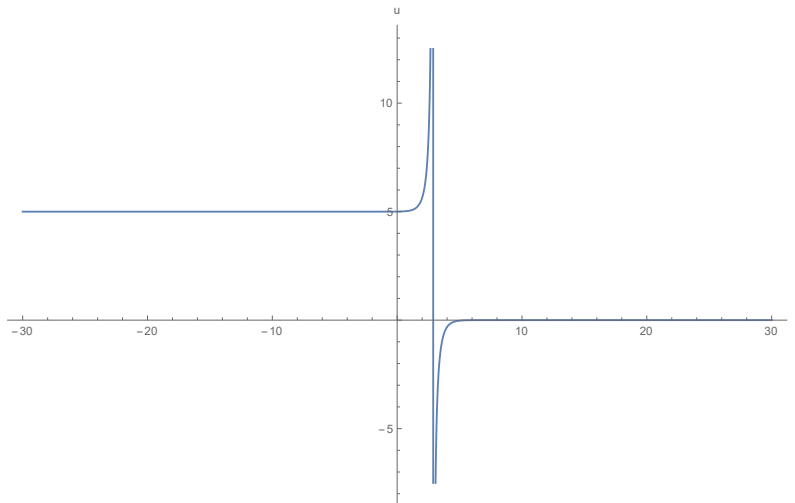


Figure 10. 2D plot of the obtained traveling wave solution $u_5(x, 1)$ in Eq.(5.8).

6. Conclusion

The fundamental goal of the paper has been to construct an approximation to the solution of the conformable space-time fractional (2+1)-dimensional breaking soliton, third-order KdV and Burger's equations by SITEM. The obtained solutions are traveling wave solutions of the conformable space-time fractional (2+1)-dimensional breaking soliton, third-order KdV and Burger's equations. These equations have been converted into its equivalent nonlinear ordinary differential equation by using fractional complex transformation. Solutions of the obtained nonlinear ordinary differential equation have been seek in the form of the summation of the function $p + \tan(\frac{\phi(\xi)}{2})$. Substituting the summation of the function $p + \tan(\frac{\phi(\xi)}{2})$ into the nonlinear ordinary differential equation and equalizing coefficients of the term with the same degree, nonlinear algebraic system is obtained. Solving the nonlinear algebraic system, we have the traveling wave solutions.

There are many types of traveling waves that are of particular interest in solitary wave theory. Three of these types are the solitary waves, the periodic waves and the kink waves. The solitary waves are asymptotically zero at large distances, the periodic waves have periodicity, the kink waves rise or descend from one asymptotic state to another. The 3D and 2D graphics of the obtained solutions have been presented in the paper. Figs.1- 2, Figs.9-10 show kink waves solutions, Figs.5- 6, Figs.7-8 have periodic waves solutions and Figs.3-4 give solitary waves solutions. This method changes the given difficult problems into simple one and solve easily by using MATLAB programming. The obtained solutions are new and have not been reported in former literature. The method can also be applied to other nonlinear fractional partial differential equations.

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Author's contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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