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Research Article

Mathematical argumentation ability: error analysis in solving mathematical arguments

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Abstract

Argumentation is a process of constructing an argument consisting of statement and conclusion. When constructing an argument, students frequently mistake proving a valid argument. This study aimed at locating students' mistakes in solving argumentative problems based on Bloom Taxonomy indicators. This study employed the descriptive quantitative design. The subject of this study involved 72 students of the 10th Grade in a public high school in Pasuruan, East Java, Indonesia. Meanwhile, the instrument used to gather the data was a written test whose validity and reliability was tested using the Cronbach Alpha value of 0.874. The result of the test given to the 72 students was analyzed by reducing, displaying the data, and drawing a conclusion. Based on the findings, it could be concluded that most students did not solve the problems thoroughly due to their procedural and conceptual mistakes in proving a valid argument. The majority of students could achieve the cognitive ability of draw conclusion level, shown by the ratio of student with mathematical argumentation ability test to total students by 0.6. It is advised that the achievement of student's cognitive ability be improved through defragmenting of student's thinking structure in solving a valid argumentative problem.

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Introduction

The ability of argumentation is an aspect of thinking skill. It is beneficial for students, especially in mathematics, in which they are required to clearly state a claim in both spoken and written by providing sufficient data and theoretical support of a mathematical problem to provide a correct understanding of mathematical concepts (Indrawatiningsih, 2018; Indrawatiningsih et al. 2019). Each student should be able to engage in argumentation in order to enhance student's self-understanding (Simon, Erduran, & Osborne, 2006; Krummheuer, 2013; Graydon & Holloway, 2017).

The ability of argumentation entails students' ability to construct their own argument, agree or disagree on a given issue, identify a problem, choose a solution plan, and evaluate arguments (Nussbaum & Edwards, 2011; Kuhn, 2010; Indrawatiningsih, 2018, Cottrell, 2017). Students are encouraged to evaluate arguments, ask questions to clarify arguments, and develop arguments that make sense (Krummheuer, 2007; Conner et al. 2014b; Whitenack & Knipping, 2002; Yackel, 2002; Yackel & Cobb, 1996; Yackel, Rasmussen, & King, 2000; Graydon & Holloway, 2017) , formulate questions, describe mechanisms and construct arguments. Furthermore, arguments are statements that are arranged logically and supported by evidence to ensure their validity and reliability.

A key aspect in the learning process include formulating questions, describing mechanisms, and constructing arguments through the argumentation process (Harris, Phillips, & Penuel, 2012). In argumentation, students are able

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to reflect upon, clarify and expand their understanding of mathematical relationships and mathematical arguments (Ontario Ministry of Education, 2005). The ability of argumentation is linked to the ability to construct valid arguments that are supported by sufficient data. An argument is a group of statements and conclusion. Based on the Toulmin's Argumentation Pattern (Toulmin, 2003), an argument involves claims, data, and warrants, while the components of backings, rebuttals, and grounds are the criteria for compiling a complete argument. In addition argument is product or outcome of the argumentation process (Anthony & Walshaw, 2009; Indrawatiningsih, Purwanto, As'ari, Sa'dijah, & Hakim, 2019).

Previous studies suggest when students engage in argumentation, it will be easier for them to learn and reflect upon the learning process (Whitenack & Knipping, 2002; Krummheuer, 2007; Krummheuer, 2013; Conner et al. 2014; Van Ness & Maher, 2018). Argumentation process aims to gather various components in the form of evidence required to prove and construct an argument (Ruggiero, 2012; Simon et al. 2006). It can be a method to rationally answer question or issue, as well as to counter and cope with a problem. An argument consists of a claim (solution) that is supported by a set of principles (warrants), evidence and counter argumentation (Jonassen, 2010). Cross, Hendricks, and Hickey reaffirm that learning to argue can enhance students' conceptual understanding, enable them to acquire new ideas that will expand their knowledge, and eliminate possible misconceptions.

In mathematics, the ability of argumentation is linked to mathematical reasoning. This capability includes: (a) drawing logical conclusions, (b) providing explanation using models, facts, properties, relationships, or patterns, (c) estimating answers and solution process, (d) using relationship patterns to analyze situations, making analogies, generalizations, and constructing conjectures, (e) formulating counter claims, (f) following the rules of inference, checking the validity of arguments, proving, and constructing valid arguments, and (g) constructing direct and indirect evidence proof by mathematical induction (NCTM, 2000).

Argumentation is the process of arranging an argument comprising of statements and conclusions. In the process of preparing arguments, students are expected to be able to explain and prove that the arguments they make are valid so they can be trusted (Deane, 2014; Ayalon & Hershkowitz, 2018). An argument is a series of statements that lead to inferences. An argument consists of two groups, namely a group of preceding statements called the premises, and a group of last statements or conclusions. An argument is valid if the form is valid. The form of an argument is said to be valid if at the time each premise is substituted with any particular statement, the results of all the premises are true, thus the conclusion is also true (Purwanto, 2012). To reaffirm the validity of an argument, the first step is to write the argument in the form of symbols.

In fact, the main source of difficulty in solving mathematical problems is in transforming the written words into mathematical symbols (Salma & Sherwin, 2012). Such difficulty may lead to errors in proving valid arguments. The types of error can be in writing symbols, drawing conclusions, processing and having misconception (Legutko, 2008). Likewise, Foster (2007) emphasizes the process of learning abstracts without capturing the meaning will be useless for students. Consequently, they will not understand the concept and eventually, will face difficulties in solving mathematical problems. In addition, the mistakes in solving the problems will lead to misconception symptoms. The cause of this misconception is mostly the systematic errors related to students' procedural understanding and conceptual understanding, or the relationship between these two (Li, 2006).

Essentially, identification of errors is crucial to find out the causes of students' learning disabilities and low learning outcomes (Salvia & Ysseldyke, 2004; Lai, 2012). By pointing out specific errors, the teacher can provide direction or guidance targeted to students' special needs. In general, students who encounter difficulties in learning mathematics frequently have lack of conceptual knowledge for several reasons, including the inability to process information at instructional pace, lack of ability to respond, lack of specific feedback from teachers about misconception, anxiety about mathematics, and difficulty in visual and/or auditory processing (Lai, 2012). In this study, the error in question is the fault of students in solving mathematical argument problems. it is very important to know because mathematical arguments are related to logical reasoning in compiling and proving valid arguments. The preparation of valid arguments aims to strengthen understanding concepts, minimizing student misconceptions, and getting new ideas (Wood, 2013; Yee, Boyle, Ko, & Bleiler-Baxter, 2018).

Problem of Study

Based on the explanation above, the present study aims to investigate errors made by students in solving mathematical arguments. It is expected that the findings will be a useful input for teachers to design appropriate learning instruction and enhance students' mathematical argumentation skills. Three research questions (RQs) are formulated to meet the research objectives, namely:

- Research question 1 (RQ1): How many students "pass" and "fail/not pass) the mathematical argumentation ability test based on the criteria of Bloom's Taxonomy?
- Research question 2 (RQ2): What are the most common errors made by students in solving mathematical arguments?
- Research question 3 (RQ3): Based on the Bloom's Taxonomy, at which level is the students' ability in solving mathematical arguments?

Method

Research Design

The method used in this research is descriptive research with a quantitative approach that aims to raise facts, circumstances, variables, and phenomena that occur now (Creswell, 2012). This description method can be used to examine the status, objects, specific conditions, systems of thought, or future events. This research to analyze errors made by students in solving mathematical argument.

Participants

This study was carried out in a state senior secondary school in Pasuruan, East Java, Indonesia. Specifically, this study involved 72 students of natural science program of academic year 2019/2020. The sampling technique used simple random sampling, which is sampling from members of the population that is carried out randomly without regard to strata in the population (Cohen, Manion, & Morrison, 2007). So, the sample in this study were all 10th grade students students taking natural science programs.

Table 1.
Structures of Participants

		f	%
Gender			
	Male	19	26
	Female	53	74
Grade			
	10th Grade	72	100

Data Collection Tools

Mathematical Argumentation Ability Test (MAAT)

The *MAAT* as referred to the cognitive domain of Bloom's Taxonomy (C1 s/d C6). The indicators used to evaluate the ability are: 1) identifying argument/not argument; 2) explaining argument/not argument; 3) drawing conclusion after the premises; 4) adding/reducing premises to support conclusion; 5) proving valid/invalid arguments; and 6) constructing valid argument. The type of the test was an essay test with duration of 90 minutes. The instrument used in this study was in the form of a 5-item essay test. The test aims to measure students' mathematical argumentation abilities by using modified Bloom's Taxonomy indicators (Anderson & Krathwohl, 2010).

The questions had been validated by experts prior to the implementation. Two experts were involved, namely a mathematician and a mathematics education expert. The materials included absolute value linear equations and inequalities of one variable and solved using a mathematical argument model with connectives such as "if", "then", "so", and "in fact". The same method had been exemplified by Krummheuer (2007). Testing the reliability of the test instrument using the Cronbach alpha test. said to be reliable if the reliability coefficient of Alfa Cronbach is more than 0.70 (Cohen et al. 2007).

Based on the Cronbach alpha test it was found that the instruments used in this study were declared reliable. The following Table 1 presents the results of the Cronbach alpha test.

Table 1.
Instrument Test Results with the Cronbach Alpha Test

	N	%	Cronbach's Alpha	N of Items
Cases Valid	72	100.0	.874	5
Excleded[a]	0	0		
Total	7	100.0		

a Listwise deletion based on all variables in the procedure

Table 1 above provides information about the number of samples (N) analyzed using a computer program, which is 72 N students. because there is no data blank, then the amount is 100% valid. from the output table above shows that N is known of items (number of items or item test items). there are 5 test questions with a Cronbach alpha value of 0.874. because the value of Cronbach alpha is 0.874> 0.7, the reliability test of the essay question instrument can be concluded to be reliable.

Data Analysis

The procedure used in analyzing data in this study there are three stages, namely reduction, presenting of descriptive form, and concludes conceptual understanding of mathematical arguments (Creswell, 2012).

Student test score are analyzed based on graduation criteria at school and explained based on predetermined indicators and assessment rubrics (Anderson & Krathwohl, 2010). Scores obtained by students are also used to find out the mistakes made by students in completing mathematical arguments.

Results

Based on the results of Mathematical Argumentation Ability Test *(MAAT)*, the average score of 72 participants is 56.25. This score is classified "fail" since the passing score is 70. To be specific, 69 percent of students failed in the test, indicating the low ability of the participants. The reason for this low average score relates to the students' lack ability in understanding the concept (conceptual understanding). Table 2 presents the result of Mathematical Argumentation Ability Test *(MAAT)*.

Table 2.
Result of Students' Mathematical Argumentation Ability Test (MAAT)

Cognitive Domain in Bloom's Taxonomy	Indicator	Ratio of student with *KAM* to total students	Percentage	
			Pass	Not Pass
Identify argument (C1)	Identifying argument/not argument	1	31%	69%
Explain argument (C2)	Explaining argument	0.9		
Draw conclusion (C3)	Drawing conclusion after the premises	0.6		
Select (C4)	Reducing/adding premises to support conclusion	0.4		
Prove (C5)	Prove valid/invalid argument	0.2		
Construct (C6)	Constructing valid argument	0.08		

Note: School criteria: Not Pass (< 70)
MAAT: Mathematical Argumentation Ability Test

Table 2 shows the ratio of students with mathematical argumentation ability *(MAAT)* to total students on the indicator of identification (C1) is 1, all participants are able to identify which one is an argument and not. Specifically, 72 students pass the C1 indicator. On the indicator of explanation (C2), the ratio of students with mathematical argumentation ability (*MAAT*) to total students is 0.9, in which 88% or 63 students are able to explain argument and not argument. In the indicator of draw conclusions (C3), the ratio of students with mathematical argumentation ability (*MAAT*) to total students is 0.6, 61% or 44 students are able to draw conclusions after the premises. In the indicator of selection (C4), the ratio of students with mathematical argumentation ability (*MAAT*) to total students is 0.4, 42% or 30 students are able to reduce/add premises to support the conclusion. In the indicator of prove (C5), the ratio of students with mathematical argumentation ability (*MAAT*) to total students is 0.2, 19% or 14 students are able to prove valid/invalid arguments. In the indicator of construct valid argument (C6), the ratio of students with mathematical argumentation ability (*MAAT*) to total students is 0.08, 8% or six students are able to construct valid arguments.

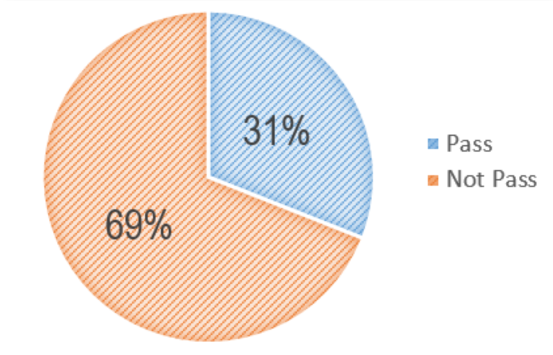


Figure 1.
Percentage of Students' Argumentation Ability Level

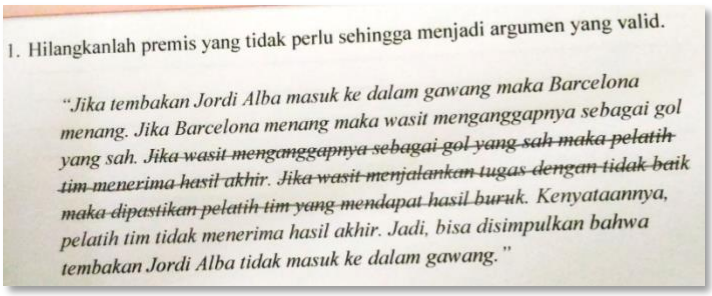
Figure 1 shows the percentage of students who complete all the indicators of mathematical argumentation ability is 31% with the score of ≥ 70. Meanwhile, 69% of students fail in completing the indicators with the score less than 70. Based on the data, the majority of students didn't pass the indicators due to errors in solving problems related to mathematical arguments. The analysis of errors made by students in solving problems related to mathematical arguments is presented in Table 3. The errors are started from the ability in explaining arguments and not arguments (C2). In the ability to identify argument (C2), all students meet the indicators.

Table 3.
Analysis of Students' Errors in Solving Mathematical Argumentation Problems

Indicator of Argumentation Ability based on Bloom's Taxonomy	Analysis of Error
Explain (C2)	Student makes errors in explaining argument/not argument due to the lack of conceptual understanding.
Draw conclusion (C3)	Student makes error in determining argument from mathematical model devised by the student; thus the conclusion is invalid.
Select (C4)	Student fails in reducing or adding premises to support the conclusion.
Prove (C5)	Student is not able to model mathematical model, create mathematical model and prove valid/invalid argument based on truth tables.
Construct (C6)	Student makes error in constructing valid/invalid arguments.

Students are not able not determine the correct conclusion (C3) after the premises that have been predetermined. It occurs because students have not been able to model mathematical argument. In addition, students are not able to convert mathematical sentences into mathematical models. Furthermore, students are not able to explain (C2) which one is argument/not argument due to their lack of conceptual understanding related to mathematical arguments. The subsequent indicators of students' cognitive abilities (selecting (C4), proving (C5) and constructing (C6)) are based on preceding abilities (C1 and C2). If the preceding indicators are not fulfilled, it is most likely the students fail in the subsequent indicators, confirming the hierarchical cognitive abilities of Bloom's Taxonomy with C1 as the lowest and C6 as the highest.

It is shown by the data that in solving mathematical argumentation problems, students frequently make errors in the indicators of selecting (C4), concluding (C5), and constructing (C6). As a consequence, they fail to pass the test with score less than 70. As presented in Figure 2, a student fails in the indicator of C4, which is the ability to select in the cognitive domain of Bloom's Taxonomy.


1. Hilangkanlah premis yang tidak perlu sehingga menjadi argumen yang valid.

"Jika tembakan Jordi Alba masuk ke dalam gawang maka Barcelona menang. Jika Barcelona menang maka wasit menganggapnya sebagai gol yang sah. ~~Jika wasit menganggapnya sebagai gol yang sah maka pelatih tim menerima hasil akhir.~~ ~~Jika wasit menjalankan tugas dengan tidak baik maka dipastikan pelatih tim yang mendapat hasil buruk.~~ Kenyataannya, pelatih tim tidak menerima hasil akhir. Jadi, bisa disimpulkan bahwa tembakan Jordi Alba tidak masuk ke dalam gawang."

Translation

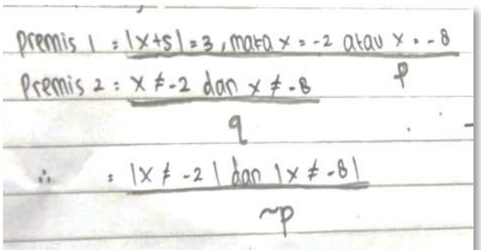
1. Eliminate unnecessary places so that they become valid arguments.
   "If Jordi Alba's shot goes into the goal then Barcelona wins. If Barcelona win the referee considers it a legitimate goal. If the referee considers it a legitimate goal then the team coach accepts the final result. If the referee performs an erroneous task then it is certain that the team coach will get poor results. In fact, the team coach did not receive the final results. So, it can be concluded that Jordi Alba's shot did not enter the goal."

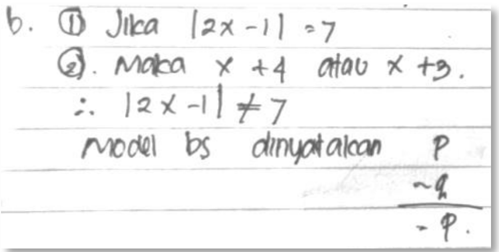
Figure 2.
Student's Worksheet in Selecting the Premises to Generate Valid Argument

From the above worksheet, it can be claimed that the student made error in selecting/omitting premise to arrange a valid argument. The student should merely omit the fourth premise but instead the third and fourth premises were omitted. Hence, it can be concluded that the student does not completely grasp the conceptual understanding of drawing conclusions through modus ponens, modus tollens, and syllogism. Shortly, the indicator C4 of Bloom's Taxonomy, namely the ability to select, is not fulfilled.

In the indicators of the ability to draw conclusion (C5), the students made errors in modeling the problems. The errors are described in Table 4.

Table 4.
Error Analysis for the Indicator of C5

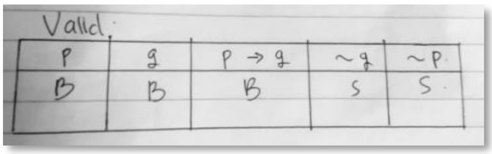
Students' Worksheets	Description
 Premis 1 : |x+5| = 3, maka x = -2 atau x = -8 (p) Premis 2 : x ≠ -2 dan x ≠ -8 (q) ∴ : |x ≠ -2| dan |x ≠ -8| (~p) Translation Premise 1 : $\|x+5\| = 3$ so $x = -2$ or $x = -8$ (p) Premise 2: $x \neq -2$ and $x \neq -8$ (q) ∴ $x \neq -2$ and $x \neq -8$ ($\sim p$)	Mathematical model An error in creating mathematical model. As the first premise, the student expresses it as $\|x+5\| = 3$ so $x = -2$ or $x = -8$ as p. As the second premise, $x \neq -2$ and $x \neq -8$ as q. As the conclusion, $x \neq -2$ and $x \neq -8$ as $\sim p$.


b. ① Jika |2x − 1| = 7
② Maka x + 4 atau x + 9.
∴ |2x − 1| ≠ 7
model bs dinyatakan p
~q
~p.

Translation
b. 1. If $|2x - 1| = 7$
2. So $x + 4$ or $x + 9$
$\therefore |2x - 1| \neq 7$
mathematics modelling: p
$\underline{\sim q}$
$\sim p$

The form of mathematical model

The error in forming mathematical model, in the form of argument, namely:
p (first premise)
$\sim p$ (second premise)
$\therefore \sim p$ (conclusion)


Valid.

p	q	p → q	~q	~p
B	B	B	S	S

*B : Right
S : False

Proving valid argument

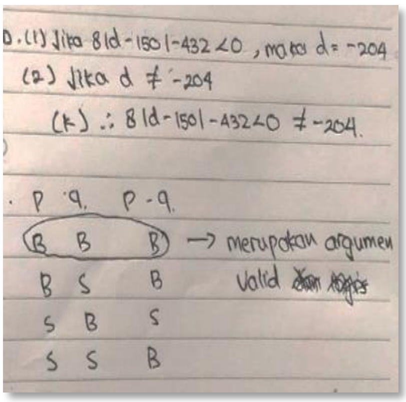
The error in proving valid argument using truth tables.

Table 3 demonstrates three types of error in solving the problems in the indicator of C5. *First*, the student made error in formulating mathematical model (equation). The first premise should consist of two equations, namely $|x + 5| = 3$ as p and $x = -2$ or $x = -8$ as q. The second premise, the equations are $x \neq -2$ and $x \neq -8$ as $\sim q$. Meanwhile, the conclusion is $\sim p$. *Second*, the student made errors in formulating mathematical model in which it should be $((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$. *Third*, the student made error in proving the validity of an argument using truth tables. The following Figure 3 is the correct truth tables that should be devised by the student to prove valid arguments.

p	q	~ p	~ q	p ⇒ q	(p ⇒ q) ∧ ~ q	[(p ⇒ q) ∧ ~q] ⇒ ~ p
B	B	S	S	B	S	B
B	S	S	B	S	S	B
S	B	B	S	B	S	B
S	S	B	B	B	B	B

Figure 3.
Truth Tables of the Equation $((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$

In the indicator of the ability to construct (C6), the students frequently make errors in constructing mathematical model (equation) from the problem. An example of such error is displayed in Figure 4.


D. (1) Jika 8|d − 150| − 432 < 0, maka d = −204
(2) Jika d ≠ −204
(k) ∴ 8|d − 150| − 432 < 0 ≠ −204.

p	q	p → q
B	B	B
B	S	B
S	B	S
S	S	B

→ merupakan argumen valid

Translation
(1) If $8|d - 150| - 432 < 0$, then $d = -204$
(2) If $d \neq -204$
(k) $\therefore 8|d - 150| - 432 < 0 \neq -204$

p	q	$p \rightarrow q$
B	B	B
B	S	B
S	B	S
S	S	B

→ Valid Argument

Figure 4
Student' Worksheet in Constructing Valid Arguments

Figure 4 shows the student does not understand the concept of mathematical argument. As a result, when solving mathematical problems, students are not able to construct valid arguments. Meanwhile, this ability is the highest indicator in the cognitive domain of Bloom's Taxonomy. In Figure 4, the student states the first premise: if $8|d - 150| - 432 < 0$ then $d = -204$, second premise: if $d \neq -204$, it can be concluded: $8|d - 150| - 432 < 0 \neq -204$. From this statement, it is clear that the student does not understand the concept of mathematical argumentation, in which a valid argument requires modus ponens, modus tollens, and syllogism (Purwanto, 2012). In addition, the student has misconception about the variable's absolute value linear equations and inequalities. It is indicated by the determination of d is not in accordance with absolute value linear equation. Essentially, the student has not fulfilled the indicator of C6 in solving mathematical arguments.

Discussion and Conclusion

Errors in mathematics are divided into 2 types, namely conceptual errors, procedural errors and technical errors (Veloo, Krishnasamy, Shahida, & Abdullah, 2017; Elbrink, 2007). Error Conceptual is a mistake made by students in interpreting terms, properties, facts, concepts and principle. Procedural errors are errors in preparing symbols, steps hierarchical and systematic rules in answering a problem. Error analysis is an analysis to identify common patterns of errors made by students in solving mathematical problems and to find the cause of the errors (Herholdt & Sapire, 2014). This method has been claimed to be an effective diagnostic tool to bridge the gap between the expected outcome and the performance. This method has been claimed to be an effective diagnostic tool to bridge the gap between the expected outcome and the performance. The students' mistakes in making the argument model included (a) students were wrong in writing an example in a given problem, (b) students were wrong in choosing the form of a mathematical argument model (did not use the rules of modus ponens, tollens mode, and syllogism), (c) students cannot prove an argument with the correct steps in the process, for example students prove a valid argument using only one row of truth tables. Thus, it allows teachers to perform remedial. Errors caused by formulating incorrect mathematical model and later invalid argument as shown in Table 3 are categorized into procedural errors as well as conceptual errors. Procedural error is an error in the preparation of the steps to solve the problem. In accordance with research conducted by Lai (2012) that procedural errors are mistakes because students do not do the process correctly. Riccomini (2010) argues that procedural error is the most common type of error frequently made by students. Meanwhile, procedural understanding has the secondary role and supports the conceptual understanding (Kirshner, 2014).

Besides procedural error, students also make conceptual error. Conceptual errors made by students i.e students are wrong in understanding and applying the concept of the equation of absolute linearity of one variable i.e students cannot find the value of an equation that has been given. This greatly impedes the work of future students because students must understand about the concept of the equation of absolute linearity of one variable. In addition, students do not understand the concept of arguments, especially the rules of drawing conclusions regarding modus ponens, tollens mode, and syllogism. Conceptual errors occur when students have misconception in understanding mathematical argument itself, hence it is complicated for them to model a problem. As presented in Table 3 of the argumentation model. Furthermore, students frequently make errors in proving valid argument. They attempt to prove valid argument by arranging truth tables. This method indicates student's conceptual understanding in proving valid argument. Nevertheless, the students are not able to comprehend the concept of truth tables. In accordance with research conducted by Chamundeswari (2014) that one of the most misconceptions made by students is students difficulty in determining the value of an equation that is known.

It is common for students making logical error in providing valid proof for their arguments (Lee, Kosze & Smith III, 2009). In addition, to enhance students' logical reasoning related to mathematical argument, truth tables can also be useful (Stylianides & Bieda, 2016; Zazkis & Chernoff, 2015). Meanwhile, most students are accustomed to using inductive reasoning instead of valid proof. Furthermore, they tend to ignore the basic principles of mathematical logic in setting theory and consequently fail to prove the theorem correctly (Nasional, 2005). Valid proof is obtained after several stages are carried out, namely: identifying problem, modeling problem, formulating mathematical model, and proving it using truth tables (Cottrell, 2005; Schwarz & Asterhan, 2010). Students can develop their ability of argumentation by proving their arguments (Stylianides, 2007) . While the active role in proving can support students to explore mathematical arguments, thus providing them with a solid basis for improving conceptual understanding (Stylianides, 2007; Ball, Thames, Phelps, & Ball, 2008).

Krathwohl (2002) suggests the Taxonomy Table is useful for educators to classify the instructional and learning activities used to achieve the objectives, as well as the assessments to determine the cognitive skills of the students. One of the cognitive skills of students is preparing valid arguments. In terms of constructing valid arguments, the ability of C6 is needed, that is to compile, or create (Zazkis & Chernoff, 2015). C6 is a cognitive ability at the highest level in taxonomy bloom.

Based on the results of the above research it can be concluded as follows: (1) as many as 31% of students pass the argumentation ability test while 69% didn't pass the test, (2) those who fail in the test mostly make errors related to procedural and conceptual understanding of mathematical argumentation, and (3) based on the cognitive domain of Bloom's Taxonomy, students who pass the test have only reached the C3 level of cognitive domain, namely the ability to draw conclusion.

Recommendations

Such conditions must immediately receive attention from schools and teachers, especially mathematics teachers. Teachers should be able to carry out the learning process that can improve students' mathematical argumentation abilities as stated in the Peraturan Pemerintah RI (2013) that one of the abilities that must be developed in mathematics learning is the ability of argumentation. The teacher needs to implement strategies, methods, and learning models that is intended to enhance students' mathematical argumentation ability. It is suggested for further studies to design an instructional learning model for teacher that is intended to enhance students' mathematical argumentation ability.

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