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The Development of Teachers' Knowledge of the Nature of Mathematical Modeling Scale

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Abstract: This study addresses a gap in the literature on mathematical modeling education by developing the mathematical modeling knowledge scale (MMKS). The MMKS is a quantitative tool created to assess teachers' knowledge of the nature of mathematical modeling. Quantitative instruments to measure modeling knowledge is scare in the literature partially due to the lack of appropriate instruments developed to assess such knowledge among teachers. The MMKS was developed and validated with a total sample of 364 K–12 teachers from several public-schools using three phases. Phase 1 addresses content validity of the scale using reviews from experts and interviews with knowledgable teachers. Initial psychometric properties and piloting results are presented in phase 2 of the study, and phase 3 reports on the findings during the field test, factor structure, and factor analyses. The results of the factor analyses and other psychometric measures supported a 12-item, one-factor scale for assessing teachers' knowledge of the nature of mathematical modeling. The reliability of the MMKS was moderately high and acceptable ($\alpha = .84$). The findings suggest the MMKS is a reliable, valid, and useful tool to measure teachers' knowledge of the nature of mathematical modeling. Potential uses and applications of the MMKS by researchers and educators are discussed, and implications for further research are provided.

1. INTRODUCTION

For the past 30 years, mathematical modeling or modeling with mathematics education has experienced rapid growth at several educational levels across the world and especially in the USA. With the development and enactment of the Common Core new mathematics standards in the USA (National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010), the assessment guidelines for modeling education report (Consortium for Mathematics and Its Application [COMAP] & Society for Industrial and Applied Mathematics [SIAM], 2016), and modeling standards from other countries across the world including Australia, Germany, Japan, The Netherlands, and Singapore (Ang, 2015; Geiger, 2015; Ikeda, 2015; Kaiser, Blum, Borromeo Ferri, & Stillman, 2011), bring new mathematical practices that accentuate the relevance of mathematical modeling in mathematics education. This new promise of engaging students with mathematical

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modeling fundamentally requires teachers to be effective and well-informed about practices associated with mathematical modeling.

Mathematical modeling enables most of our students to value why we teach and learn mathematics and see the relevance and usefulness of mathematics around us (Asempapa & Foley, 2018; Blum & Borromeo Ferri, 2009). However, sample instruments measuring the knowledge of mathematical modeling among teachers remains scarce, thereby affecting the teaching, learning, and research of mathematical modeling education. The interest in this research study connected to teachers' knowledge of the nature of mathematical modeling stems from the relevance of mathematical modeling to teaching, learning, and doing mathematics not only in the USA, but also elsewhere in the world, where modeling is emphasized heavily in most mathematics curricula. Therefore, creating a tool to examine the know-how of teachers regarding the nature of mathematical modeling remains important considering the growing significance and popularity of mathematical modeling education all over the world.

As already mentioned, evidence of instrument validity and reliability regarding the knowledge of teachers on the nature of mathematical modeling is scant in the literature (Kaiser, Schwarz, & Tiedmann, 2010; Ziebarth, Fonger, & Kratky, 2014). Although a large body of literature exists on mathematical modeling in areas such as (a) the instruction, learning, and studying of modeling (Blum, 2015; Blum & Borromeo Ferri, 2009; Boaler, 2001; Organisation for Economic Co-operation and Development [OECD], 2003; Pollak, 2011); (b) pedagogies of mathematical modeling (Lesh, 2012; Lesh & Doerr, 2003); and (c) assessment of modeling tasks (Asempapa & Foley, 2018; Leong, 2012), the emphasis on theoretical and empirical research about assessment tools on the knowledge of teachers regarding the nature of mathematical modeling practices is limited. Recent emphasis on mathematical modeling has often ignored the important role quantitative measurement instruments play in conducting high quality research.

The need for valid measures and instruments with a clearly defined purpose and supporting validity evidence are fundamental to conducting high quality large-scale quantitative studies (Benjamin et al. 2017). The lack of validated quantitative instruments poses a challenge for most researchers in evaluating if a tool is appropriate for a study and whether it can produce accurate and reliable data (Benjamin et al. 2017; Ziebarth, Fonger, & Krathy, 2014). Thus, the development of the mathematical modeling knowledge scale (MMKS) is necessary and important, and it will provide researchers in the USA and the international community with a validated quantitative tool that is woefully lacking in the mathematics education literature. For these reasons, this current research study was planned to develop the MMKS—a meassuremnt tool—that assesses teachers' knowledge of the nature of mathematical modeling to address a gap in this field. The primary goal in developing the MMKS was to identify questions that would be quicker and more suitable to answer yet would be powerful indicators of teachers' knowledge of the nature of mathematical modeling. Therefore, the purpose of this research was to create, examine the fidelity of, and verify the factor structure related to the development of the MMKS.

2. THEORETICAL FRAMEWORK and RELATED LITERATURE

2.1. The Nauture of Mathematical Modeling and Its Process

Mathematical modeling usually means the ability to move back and forth between the real world and the mathematical world (Blum, 2015; Crouch & Haines, 2004; Pollak, 2011). Although mathematical modeling is highlighted and emphasized in most standards and curricula worldwide, missing in the literature is a single agreed-upon approach or definition; rather there are various approaches presented by authors of shared understandings (Lesh & Doerr, 2003; Kaiser & Sriraman, 2006). The various approaches are based on different theoretical frameworks, and there is no consensus on approaches to mathematical modeling in the literature (Kaiser & Sriraman, 2006). For instance, in the GAIMME report modeling is defined as “a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena” (COMMAP & SIAM, 2016, p. 8). According to Borromeo Ferri (2018), mathematical modeling is a process that involves transitioning back and forth between reality and mathematics and using mathematics to understand and solve a specified real-world problem.

Alternatively, the process of mathematical modeling can be described as using several learning situations; from deductively arranged authentic problem modeling activities (English & Sriraman, 2010) to inductively organized inquiry-based problem-solving activities leading the learner to formulate general patterns (Sokolowski & Rackly, 2011). Moreover, Blum and Berromeo Ferri (2009) described mathematical modeling as the “process of translating between the real world and mathematics in both directions (p. 45). Despite the luck of a direct and single agreed approach or definition for mathematical modeling, the convergent view of mathematical modeling can be described as a process that includes the following: (a) identify a problem in real life, (b) make choices and assumptions concerning the problem, (c) utilize a mathematical model, and (d) translate the results into the context of the original problem. A typical mathematical modeling process or procedure adapted for this study is shown in Figure 1.

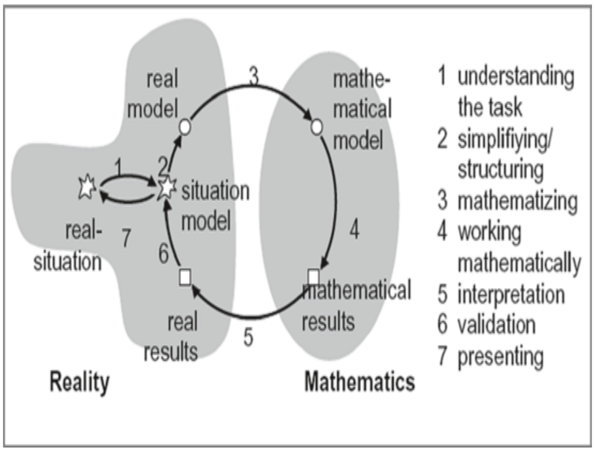

Figure 1. *A typical mathematical modeling process (adapted from Blum & Leiss, 2007, p. 225).*

Most mathematics educators have attempted to teach or communicate the concept of mathematical modeling through the mathematical modeling process. However, Perrenet and Zwaneveld (2012) argued that this is a challenge for instruction on mathematical modeling because of the lack of agreement about the mathematical modeling process regarding its essence, vision, and inherent complexity. For the purpose of this research study, the researcher’s conceptualization of mathematical modeling is based on the definition provided by Blum and Berromeo Ferri (2009). Despite the lack of unanimity on the approaches and definition of mathematical modeling in the literature, the mathematical modeling process demonstrates that individuals must solve a real-life problem utilizing their mathematical knowledge. A possible strategy for testing the efficacy of teaching and learning with mathematical modeling is through the creation of a scale that constitute the knowledge of teachers pertaining to the nature of mathematical modeling practices. In developing the scale, a series of phases were undertaken based on different samples. The phases contributed to construction of items that adequately reflected the domain of interest, relatively free of social desirability bias, and sufficiently represented the underlying construct. Therefore, these phases helped in the initial development and validation of the MMKS using a construct validity approach to scale development (DeVillis, 2017; Messick, 1995, 1998).

2.2. Teachers' Knowledge of the Nature of Mathematical Modeling

According to Ma (1999) "the quality of teachers subject matter knowledge directly affects student learning" (p. 144). Ponte and Chapman (2008) explained that a robust knowledge is insufficient for being an important or valuable teacher, however instructors or teachers with mediocre know-how makes teaching uneasy on students. This implies that it is essentail for us to develop and improve the pedagogies of teaching mathematical modeling. Although there has been several research studies on the content knowledge of teachers in mathematics, the area of mathematical modeling is still scarce. Moreover, research indicates the knowledge of teachers regarding mathematical modeling is deficient, but appropriate and well-timed given the elevated attention on modeling practices in most mathematics standards and reports (COMAP & SIAM, 2016; NGA Center & CCSSO, 2010).

Philosophical and exprerimental knowledge into the pedagogy, instruction, and learning (Ma, 1999; Shulman, 1986, 1987) have highlighted the significance of the understanding of the content in teaching. Additionally, several documents have shown the variation in knowledge of teachers regarding the teaching of mathematics (Ball, 1990; Ma, 1999). The work of Hill, Schilling and Ball (2004), supports this argument, and this warrants a shift and modification in addressing teachers' knowledge and willingnes on mathematical modeling. Because teachers' experiences contribute an important part in instruction and teaching (Lortie, 2002), their actions, dispositions, and attitudes toward mathematics and its relevance in the community, which involves mathematical modeling practices is important. Therefore, it is essentail we design and develp research studies centered on teachers that focus on the content knowledge pertaining to the nature of mathematical modeling.

In recent years, the knowledge of teachers regarding mathematical modeling practices has received much discussion in the literature (Borromeo Ferri, 2018; Kaiser, Schwarz, & Tiedmann, 2010; Paolucci & Wessels, 2017). However, within mathematics education, defining the knowledge of mathematical modeling could seem as a complex construct because of the discrepancy in the components associated with the mathematical modeling process usually used as a criterion in teaching mathematical modeling. In conjunction with the above information, it seems important to identify and explain the phrase "knowledge of the nature of mathematical modeling." Teachers knowledge of the nature of mathematical modeling was conceptualized as thier understanding, interpretations, familiarizations, and minimal competencies assocaited with the Common Core standard of mathematical practice—model with mathematics—and teaching and learning of mathematical modeling (Borromeo Ferri, 2018; Blum, 2015; Lesh, & Doerr, 2003; NGA Center & CCSSO, 2010). Based on recent research and literature, the domain of the construct—knowledge of the nature of mathematical modeling—involved the mathematical modeling process, real-world connections, and mathematical modeling tasks (Blum & Leiss, 2007). Because establishing a questionnaire about mathematical modeling knowledge would be too broad and difficult to achieve with a simple scale, the manner in which teachers' comprehend or understand mathematical modeling was conceptualized as the familiarity with mathematical modeling applications, practices, and procedures. Therefore, Blum's and Leiss's (2007) modeling procedure or method was used as a contextual framewrok and domain for the development of the MMKS, which provides educators and researchers a heuristic guideline for exploring mathematical modeling.

3. PHASE 1: GENERATION and DEVELOPMENT OF ITEMS

3.1. Item Generation and Format

Phase 1 addressed issues regarding the evidence on face and content validity for the scale items that has the potential to assess the understanding of teachers about practices that engage students in mathematical modeling. In doing so, the researcher employed DeVellis's (2017) recommendations in scale development. These recommendations include (a) measured construct; (b) generated items; (c) measurement scale format; (d) reviews by experts; and (e) incorporating valid items. Upon examination of relevant literature and standards (Ball, Thames, & Phelps, 2008; Blum & Borromeo Ferri, 2009; English, Fox, & Watters, 2005; Gould, 2013; Lesh, & Doerr, 2003; NGA Center & CCSSO, 2010; Pollak, 2011; Sriraman & English, 2010; Wolfe, 2013), an initial 22 items were generated to constitute the knowledge of teachers regarding practices abouth modeling with mathematics. The intention of this approach to selecting and generating these items was to promote an all-inclusive content-valid construct (Messick, 1995) as a strong content and applicable of the proposed knowledge of the nature of mathematical modeling. Sample scale items are provided in Appendix A.

To identify appropriate questions that fit the identified domain, experts and teachers from the Midwest in the USA were consulted at the inceptive creation of the scale. During the pilot phase, the researcher used 21 items, and the final design of the MMKS was reduced to 12 bunary option (true or false questions), with an open-ended item, and other demographic items. The researcher used the true or false item type because this is the first attempt to develop an instrument of this kind to measure a complex construct—nature of mathematical modeling—which has the potential to generate quick but useful information from participants. Because the focus of this article was on scale development and evaluation of the items, no discussion on the open-ended question was presented. The 12 true or false items were graded with possible scores of 0–12.

3.2. Inclusion of Items and Content Validity

A further important aspect of the scale's development and validation was that the items were reviewed by experts. DeVellis (2017) explained that, the initiative to evaluate things for a newly constructed instrument should be extended to 6–10 experts. The experts evaluated each item's importance and suitability for the domain and offered suggestions and opinions on their view of the products and the MMKS. Ten experts from renowned midwestern universities reviewed the MMKS before the field test phase. These experts comprised three doctoral professors with modeling experience, three professors with analysis, assessment, and measurement skills, and four professors with diverse research interests in mathematical modeling at a reputable research-based university.

In order to assist in the iterative process of qualitative content analysis during the creation of the measure, comprehensive input was received from numerous experts regarding participant directions, scope of item sampling and item quality, and construction of the rating scale. All the experts offered suggestions for the revision of the items. Most of the experts and researcher came together to debate on the inclusion of items based on criteria and theoretical significance. After three iterations, we reached agreement on the final set of items. Before the initial version of the MMKS was submitted to a structured pilot study, a somewhat more detailed evaluation was conducted, using interviews with knowledgable teachers (usually known as cognitive interview). (Fowler, 2014; Tourangeau, Rips, & Rasinski, 2000). During the cognitive interview, four teachers including primary, middle, and high school teachers were used to provide face/content validation for the items. Final design of the MMKS used for the field test demonstrated that the items were logically arranged, reasonable, comprehensible, and truly representative of the construct —knowledge pertaining to the nature of mathematical modeling.

4. PHASE 2: PILOT STUDY and PRELIMINARY PSYCHOMETRICS

4.1. Testing Items with a Development Sample

Trying out items is the exclusive approach of ensuring that the written survey items connect to the participants as expected (DeVellis, 2017). The goals of pre-testing guarantee that single items follow all the fundamental principles for quality questionnaire design. These goals include the holistic testing of the questionnaire, ensuring smooth cohesion of procedures, maintaining appropriate survey routines, and developing excellent questionnaire codes (DeVellis, 2017). As a result, a try out for the MMKS was conducted via a pilot study with teachers from a big public-school in the midwestern part of the United States. After determining which relevant items to be used, the scale was then tried out or tested on a sample similar to the target population. The target population for this current study was K–12 teachers of mathematics, which included elementary (primary) middle and high school teachers. This population was suitable and appropriate for the current study because mathematical modeling is a standard of mathematical practice for these group of teachers. Table 1 demonstrates the MMKS design stages from the initial phase to the field-test stage.

Table 1. *MMKS from the Initial Phase to the Field-Test Phase*

	Development Stages		
Domain(s)	MMKS–Initial Version	MMKS–Pilot Study	MMKS–Field-Test
No. of items During (After)	22 (22)	21(13)	13
Demographic Items During (After)	18	19 (14)	14
Total items During (After)	40	40 (27)	27
Authenticity and quality	Items reviewed and conducting interviews.	Items revised and psychometric analyses.	Further psychometric analyses.

As per DeVellis (2017), the sample composition should be broad enough to remove the heterogeneity of the sample and aid with the appropriateness of the items. Experts have suggested several sample sizes for scale model pilot studies. Sample size from 25 to 75 was proposed by Converse and Presser (1986); Fowler (2014) suggested a size between 15–35; and when asking for a single point calculation, Johanson and Brooks (2010) suggested a size of 30 for the sample. While there are some risks involved with small sample size, pre-testing is better than not. Therefore, a size of the sample between 15 to 75 was considered appropriate during this phase.

Phase 1 findings resulted in the creation of a proposed collection of 21 items to evaluate the knowledge of teachers on the nature of mathematical modeling. These 21 items were produced by interviewing scholars knowledgable and with theoretical and experimental experiences in survey production and mathematical modeling. Consequently, the next step was to investigate some of the psychometric measures of these 21 questions or items. Phase 2 therefore investigated whether these 21 items could reliably capture or operationalize the factor—knowledge of modeling—as suggested and conceptualized by the researcher. Phase 2 of this analysis was motivated by the following research questions.

> **Research Question 1 (RQ1)**: Depending on the eligible questions or items produced, which ones created maximum level of understanding on teachers' knowledge of the nature of mathematical modeling, and should be part of the scale?
>
> **Research Question 2 (RQ2)**: Could the current 21 questions or items established via RQ1 and content validity processes reliably and validly operationalize the nature of mathematical modeling knowledge as suggested and conceptualized by the researcher?

4.2. Methods

4.2.1. *Site and Participants*

Participants enlisted for this investigation were mathematics teachers from a large government-funded school site in the U.S. Midwest. Maximum responses checked were 102, but 71 completed all survey items on the MMKS once data has been filtered and formatted. The response rate in the school district was about 19.6 percent compared to the number of mathematics teachers (n = 520). According to Converse and Presser (1986) having a size for the sample between 25 to 75 is adequate for trying out items, and Johanson and Brooks (2010) suggested a size of 30 for a sample, so the 71 respondents in this phase was considered adequate at this phase of the study. The majority of the 71 completed surveys were K–5 elementary teachers (n = 36, 50.7%) and were master's degree holders (n = 25, 35.2%). The age range of respondents varied, about 77% were 35 years of age and older, and about 60% were Caucasian or White. As far as gender was concerned, 15% were classified as males and 85% as females. Such demographics represent a general trend in the USA of K–12 teachers of mathematics.

4.2.2. *Data Collection and Analysis*

Phase 2 utilized purposeful sampling, a non-probabilistic method of sampling. Data were gathered via a self-administered internet-based questionnaire This started the procedure of recognizing defined items, conceptual framework on modeling, applicable literature, and conceptual modeling information description. Surveys were sent by email to the study respondents and their answers were gathered and downloaded via the Qualtrics program. The researcher utilized both qualitative and quantitative methods such as elimination of redundant elements or items, measures of tendency and varaiblity, reliability, and factor analyses to identify and evaluate the selected questions or items. Respondents responses were coded as incorrect response = 0 and correct response = 1. The total scale score was determined and the reliability of the internal consistency was evaluated by computing item-total-correlations.

4.3. Results

4.3.1. *Item Analysis*

Item review of the formatted data was carried out to determine the quality and authenticity of the items. The analyses involved evaluating the matrix of association or correlation, the overall correlations and the scale accuracy, quality and consistency. Established associations or correlations under .30 were supposed to be excluded (Field, 2009; Osterlind, 2010). Additionally, items which reduced the overall consistency in reliability in general should excluded if conceptual deletion was appropriate. The outcome of the item analyses resulted in the retention of 12 items. All the items retained had theoretical and statistical significance with .30 and higher associations or correlations and, if removed, could not have increased Cronbach's alpha as a whole. Phase 2 was intended to offer proof supporting the establishment of the MMKS. The 71 surveys containing the 12 items therefore produced a .80 Cronbach's coefficient alpha, indicating that the MMKS offered accurate and functional measuring questions or items.

4.3.2. *Exploratory Factor Analysis*

Authenticity of the construct was achieved by examining homogeneity of the item via item-total correlation and factorial validity (DeVellis, 2017; Meyers, Gamst, & Guarino, 2013). Despite the relative small sample size of 71, the ratio was nearly 1:6 (Kline, 2000; Meyers, Gamst, & Guarino, 2013); consequently, during the pilot study, analysis of exploratory factor (EFA) was used to affirm the validity of the 12 MMKS items. The measure of accuracy for the sample (KMO = .81) and Bartlett's test of sphericity ($p < .01$) demonstrated the applicability of exploratory factor analysis (Meyers, Gamst, & Guarino, 2013; Warner, 2013). The factorial validity used principal axis factoring (PAF) with a rotation by varimax appraoch. PAF examines the interrelationship between objects, offers a basis for eliminating items, helps to classify structures and associated domians. (DeVellis, 2017; Meyers, Gamst, & Guarino, 2013).

Analysis of exploratory factor (EFA) was used to determine structures of one and two factors. However, after analyzing the items described in the factor loadings and variances of the component, the one-factor structure produced the best simple fit. Due to the theoretical significance, total variance accounted, the criteria of eigenvalue suggested by Kaiser (> 1.00) and the plot of the eigenvalues of factors "leveling off" of its own values, the one-factor approach was favored. Together the one–factor structures explained about 29.0% of the variance and was labeled *knowledge of modeling*. Using parallel analysis (O'Connor, 2000) as a standard methodology to evaluate the threshold for derived factors provided, a one-factor solution was also achieved explaining approximately 28.5 percent of the total variability. For every question or item from the MMKS, the factor loadings for the one-factor model was moderate to relatively high from .29 to .81.

5. PHASE 3: FIELD-TEST and FURTHER PSYCHOMETRICS

The pilot study and initial findings outlined in Phase 2 resulted in a reasonable collection of items to evaluate the knowledge of the nature of mathematical modeling among teachers. These items were generated by consensus between leading experts with expertise in mathematical modeling methods, modeling pedagogy, and measurement assessment. In this research effort, the next extra logical step was examining the psychometric measures of the 12 questions or items. Consequently, Phase 3 investigated whether these 12 items could effectively and validly operationalize the information collected on the MMKS. The research question in this study's Phase 3 included:

> **Research Question 3 (RQ3)**: Could the current 12 items established via RQ2 and construct validity procedures reliably and validly operationalize knowldge on the nature of mathematical modeling as proposed and conceptualized by the researcher?

5.2. Methods

5.2.1. *Site and Participants*

The field test setting comprised of teachers in midwestern U.S. public school districts. Teachers teaching mathematics from Kindergarten to high school in the U.S. were the target group in this phase of the study. The field test consisted of nine districts that were among the largest in the USA of public schools and the study respondents teach mathematics to students. Additionally, the respondents lived within the identified school districts classified as rural, small-town, suburban, and urban.

A purposeful sampling technique was used during this phase to identify the sample frame and fit the geographic strata. Fourhendred seventy three teacher responses were obtained by the Qualtrics system, but after data cleaning and coding, 364 completed data points were utilized in analyzing the data. This sample size classified 21% as males and 79% as females. The mean age for the respondents was about 40.42 years (SD = 10.84). The oldest respondent was aged 67, and the youngest was aged 22. Roughly 66.5% (n = 242) of respondents were elementary teachers, 17.3% (n = 63) were teachers from middle grades, and the remaining 16.2% (n = 59) were teachers from the high school. The data was split into dual data points for both EFA and confirmatory factor analysis (CFA) because the completed data was large enough, which is a standard procedure for developing scales (Brown, 2015; Costello & Osborne, 2005). The EFA was alloted randomly to one hundred and eighty-two data set, and the remaining data (n = 182) was used for the CFA.

5.2.2. *Data Collection and Analysis*

As defined by Fowler (2014), the field test used a cross-sectional survey design. Data were obtained through a self-managed web-based survey that did not require respondents to exchange responses with an interviewer. This approach is likely to validate the compilation of confidential data (Fowler, 2014). The MMKS used 12 binary (true or false) items, one short answer question, and some demographic information to collect survey data (see Appendix A). The researcher gathered data through Qualtrics system and analyzed it using the statistical packages SPSS and SAS, widely utilized in social science research. The data analysis focused on the evaluation of the MMKS' structure (key factors) and psychometric measures (acuracy, reliability, authenticity, and validity) issues. The analyzes carried out included descriptive analysis, measures of normality, reliability anlysis, item-total-correlation, EFA, and CFA.

5.3. Results

5.3.1. *Item Analysis*

Although the distribution of scores from the respondents was somehow skewed, it was assumed that there would not be much ceiling effect because of the large sample size. Overall, the average score of the respondents was (M = 9.17, SD = 2.81) and the mean female teacher score (M = 9.31) was substantially higher than the mean male teacher score (M = 8.06). An item discrimination index was not performed; however, the observation of the distributions of data between groups on the construct indicated the items correctly differentiated between the respondents. To evaluate the reliability of the questions or items, an item analysis was conducted. Correlations or associations between items estimated and below .30 were supposed to be excluded (Field, 2009; Osterlind, 2010). Additionally, items that usually reduced Cronbach's alpha should be excluded if conceptual deletion was acceptable.

The deletion benchmark for items was a correlation value below .30 (Osterlind, 2010), beginning with least correlationsor associations. The correlation values analyzed indicated item Q3 had relatively low values in comparison to other items (see Tables 2 and Table 3). Upon eliminating item Q3, however, the alpha value of Cronbach would only have improved by a value of .001. All 12 questions or items on the scale had item-to-total correlation values that exceeded .30 (r = .30). Therefore, because of their theoretical significance, all items were kept, with item-correlations higher than .30. The 364 surveys comprising the 12 items culminated in a Cronbach's alpa of .84, indicating that the MMKS produced accurate and functional measuring items. Table 2 offers information on the MMKS items regarding Cronbach's alpha and item–total–correlations.

Table 2. *Descriptive statistics on the MMKS scores—Field-Test*

Item	*M*	*SD*	*SE*	ITC	α if Item is Deleted
Q1	.78	0.41	0.02	.51	.83
Q2	.87	0.34	0.02	.62	.82
Q3	.72	0.45	0.03	.39	.84
Q4	.72	0.45	0.02	.45	.83
Q5	.73	0.45	0.02	.41	.83
Q6	.82	0.39	0.02	.50	.83
Q7	.91	0.29	0.01	.77	.81
Q8	.78	0.41	0.02	.48	.83
Q9	.87	0.33	0.01	.67	.82
Q10	.75	0.44	0.02	.46	.83
Q11	.76	0.43	0.02	.47	.83
Q12	.84	0.37	0.02	.46	.83

Note: n = 364; ITC = item–total correlation

5.3.2. *Exploratory Factor Analysis*

An EFA was carried out to ascertain the number of common factors that are acceptable and acceptable MMKS indicators by the amount and scope of the factor loadings (Brown, 2015). The EFA used principal axis factoring (PAF) with a rotation by varimax appraoch. The KMO = .92 tested showed that the sample was appropriate for EFA (Field, 2009). A KMO near 1 with small partial correlation values demonstrate a common factor for the variables. The sphericity test by Bartlett was statistically significant ($p< .001$), which showed that the items were appropriate and suitable for performing EFA using a PAF approach.

An assessment of the extracted factor based on the Kaiser eigenvalue criteria (> 1.00) and the scree plot analysis showed no significant difference in the number of factors. Consequently, for further validity proof, a parallel analysis (O'Connor, 2000) was performed. Parallel analysis is a statistical method for facilitating the choice of factors in the EFA. This is achieved by comparing parallel randomly generated data points representing the number of original data items and factors. Afterwards, one derives eigenvalues from the generated random data points and contrasts it with the original. O'Connor (2000) explained that components or factors are kept provided the original *i*th eigenvalue is higher than the random data. The performed parallel analysis provided a one-factor solution accounting for 47.3% of the explined total variance. Examination of the factors revealed that all item factor loadings surpassed .30. Therefore, the one-factor solution with all 12 items were kept on the scale.

5.3.3. *Factor Structure*

Following Preacher's and MacCallum's (2003) recommendations, several measures were utilized in deciding on the factors to keep. The researcher employed three strategies: scree plot, Kaiser's eigenvalue test (> 1.00), and parallel analysis tests. (Horn, 1965). Visual examination of the factor item content was used for all evaluated solutions to verify that the extracted factor was relevant. The EFA scree plot of the 12 items showed a sharp decline until after the first factor. It supports the parallel analysis for the one-factor solution discussed in the previous paragraph. The factor extracted from the EFA had items with factor loadings exceeding .30 (Tabachnick & Fidell, 2007).

5.3.4. Confirmatory *Factor Analysis*

The factor structure was evaluated using the SAS PROC CALIS analytical technique for CFA. This was done to determine whether the measurement hypothesis was compatible with actual data during the field test using the MMKS scores. The data set had an item-to-respondent ratio of 1:15, ideal for CFA. CFA was performed on the data because CFA could determine the underlying factor structure of the scale and test the validity of the MMKS. According to Brown (2015), CFA's hypothesis-driven existence is a fundamental feature. By previous empirical analysis utlizing EFA during the try out phase, and based on theoretical grounds, a one-factor solution and underlying structure of the MMKS was tentatively defined. All expectations and assumptions for performing a CFA on the MMKS data was met. The assumptions included, adequate sample size, the right definition of a priori model, multivariate normality, multicollinearity, and the items-to-factor ratio.

Because the MMKS was one-dimensional, a CFA was performed for the entire scale of the overall measurement model. Due to the huge lack of agreement in the literature on preferred fit indices, the model fit was evaluated using these goodness-of-fit indices. (Hu & Bentler, 1999; Kline, 2000). The fit indicators also included the chi-square, Tucker-Lewis index (TLI), goodness of fit index (GFI), the root mean square error of approximation (RMSEA), the normed fit index (NFI), the comparative fit index (CFI), and the standardized root mean square residual (SRMR). A one-factor model was established on the basis of previous evidence and theory as well as the results of the EFA. The one-factor CFA model was subsequently carried out on the

12 items during the field test, with 182 valid results. The one-factor model fit measurement produced the following results: chi-square χ^2 (53) = 91.99, $p < .001$; *TLI* = .96; *GFI* = .95; *RMSEA* = .05 and 90% *CI* = [.03, .06]; *NFI* = .92; *CFI* = .97; and *SRMR* = .04.

Kenny (2015) stated that for CFA or structural equation models (SEM), CFI, TLI, RMSEA and SRMR are at the moment the most famous fit of measurements or statistics commonly reported. Additionally, the following are the recommended cut-offs that indicate a good model fit: CFI ≥ .90; TLI ≥ .95; RMSEA < 0.08; and SRMR < 0.08 (Kenny 2015; Kline, 2016). Thus, in comparison with the fit statistics commonly reported and as recommended by Kenny (2015), the construct's one-factor model fits the data from the above CFA results. This provided validity proof for the MMKS and validated the scale. The moderate to relatively high standardized factor loadings in Table 3 provided additional proof of validity for the MMKS items. This yielded extra inherent or intrinsic proof of construct authenticity for the instrument. The 12 items accounted for about 47% of the total MMKS variation, and all factor loadings were > .30.

Table 3. *The standardized factor loading values on the MMKS—Field-Test*

Items	*SE*	FL	*p*
Q1	0.05	.53	.00
Q2	0.04	.67	.01
Q3	0.05	.43	.00
Q4	0.05	.46	.01
Q5	0.04	.47	.00
Q6	0.05	.57	.01
Q7	0.03	.86	.01
Q8	0.05	.54	.00
Q9	0.03	.76	.01
Q10	0.04	.48	.00
Q11	0.05	.51	.01
Q12	0.05	.52	.01

Note: n = 364; FL = factor loadings; each FL value in the table was more than .30

6. DISCUSSION

Mathematical modeling is now a highly crucial component of mathematics education at different levels around the world and especially in the USA. Implementing modeling tasks and lessons during mathematics class have important influence on students doing mathematics. Recent literature indicates that an increasing number of teachers and researchers are involved in using and involving students in classroom mathematical modeling activities (COMAP & SAIM, 2016; Doerr, Ärlebäck, & Costello, 2014). Nonetheless, involving students with classroom activities and events that incorporate mathematical modeling practices is challenging for most teachers of mathematics. In this context, and to help comprehend the understanding teachers have about the nature of mathematical modeling, it became necessary to develop this instrument. Since there are no current instruments assessing the knowledge of teachers on mathematical modeling and in the spirit of creating a useful, reliable and credible scale, Messick's (1995, 1998) unified assessment of the legitimacy of validating a construct was implemented. Proof of validity in the Messick model implies gathering data for accurate analysis of scores or results that are intended for a particular purpose and at a specified time point (Downing, 2003).

The validity model of Messick illustrates construct validity because almost all social science evaluations deal with constructs — "intangible collections of abstract concepts and principles"

(Downing, 2003, p. 831)—such as the knowledge of the nature of mathematical modeling. Establishing the legitimacy of the construct requires a continuous procedure of collecting evidence. This indicates that the scores of the measurement procedure represent the anticipated structure. Cronbach (1998) defined the process as a justification for validation, which provides evidence for score interpretation. In this study, the validity of the construct was demonstrated utlizing content validity, consequential, factor structure, and factor analyses evidence. This was accomplished through the three phases to justify the worthiness and validity of the MMKS for future applications.

Although the development of the MMKS was evidently supported by theoretical significance, reliability, and factorial validity, and all 12 items were well correlated, only item (Q3) did not perform optimally under psychometric measures. The goal of item Q3 was to determine whether teachers could identify the difference between the modeling and problem-solving processes. Teachers' responses to this item was poor and this could have resulted in the weak correlations between item Q3 and the other items. However, the final MMKS's model retained 12 items because of their theoretical relevance. The Cronbach's alpha ($\alpha = .84$) of the MMKS was fairly decent for the unidimensional prototype during the field test. This means that the model determined 84% of the variation in the MMKS scores to reflect the construct being examined and an error rate of approximately 16% in the scores associated or identified with the MMKS. Therefore, based on these values, the proportion of variance on the scores in the MMKS that is due to extraneous or measurement error was relatively small, and it is within acceptable range (Field, 2009; Meyers, Gamst, & Guarino, 2013).

Additionally, this study investigated what the MMKS revealed about how teachers conceptualize the nature of mathematical modeling practices. Based on their MMKS scores, most of the teachers demonstrated reasonable levels of professional knowledge of the nature of mathematical modeling in this data set. In terms of gender, the researcher found female teachers to be relatively more knowledgeable about the nature of mathematical modeling practices than their male colleagues. Overall, the final one-dimensional model results of the MMKS showed a great model that suits the underlying proposed prototype by the one-factor and 12-item structure. The findings obtained from the content and construct validity works showed that the MMKS was reliable and useful. This research is the only first step in developing a quantitative measure to evaluate the knowledge of teachers regarding the nature of mathematical modeling. As far as the psychometric characteristics of MMKS are concerned, the supporting evidence confirms the proposed dimension, quality, and credibility of the construct. Although the study does not provide adequate specifics on convergent and discriminaant validity, the MMKS was initially developed to achieve greater applicability with acceptable sample size.

7. CONCLUSION and IMPLICATIONS

The goal of this study was to generate reliable items and evaluate the factor structure of the MMKS in measuring teachers' knowledge of the nature of mathematical modeling. The approaches used in this work could be used in conjunction with other techniques such as dimensionality analysis, convergent and discriminant analyses. This can provide further confirmation evidence to boost awareness and implementation of the findings of this research to educational research. Future work should concentrate on how to build certain subscales that can capture or classify a specific contribution of different factors to explaining the knowledge of teachers in mathematical modeling practices. Additional collection of data must continue, particularly for convergent and discriminant validity. Other and future studies must analyze settings with a larger population of both public and private schools. Such data would help philosophically endorse the theoretical concepts of mathematical modeling and be more inclusive in the variety of measures and respondents.

Although the content, internal structure, and construct validity were determined during this study, establishing and defining certain aspects of the validity evidence for future research (generalizability and external validity) would be helpful and important. Because the MMKS has been developed with binary options, an item response theory (IRT) technique can be a wonderful complement to help establish the validity eveidence of MMKS items in future research. The IRT methodology is based on the use of specific scale items to evaluate the construct being examined. The IRT approach claims that the characteristics of both the respondent and the item affect a person's reaction to an item. (Furr & Bacharach, 2014). Finally, future research can improve the MMKS using a Likert scale with multiple options for enough knowledge retention and interpretation.

Taking into account the information gathered from this research and provided in this article, the MMKS appears to be valuable in addressing interesting research concerns and information creation to expand the reach of mathematical modeling education. It is important that we build teacher's mathematical modeling knowledge to fulfill the school mathematics vision set out by the Common Core, national council of teachers of mathematics (NCTM), COMAP, SIAM, and other international standards. The finalized MMKS presented in this study represents a reliable and adaptable survey with which educators and researchers can monitor and assess both practicing and preservice teachers' development of their knowledge on the nature of mathematical modeling practices. Furthermore, for the successful integration and application of mathematical modeling into teaching school mathematics, the MMKS has the potential to support practicing teachers feel comfortable in their teaching.

This scale will allow researchers and mathematics educators to undertake mathematical modeling research using different methods for teacher programs and preservice courses. Although some work needs to be done with the MMKS in capturing teachers'comprehensive knowledge on mathematical modeling practices, the MMKS in its current form represents a useful and reliable tool for mathematics educators and researchers. The scale provides users with valuable information regarding the pedagogical content knowledge of mathematical modeling and its practices. This article offers a first step in the development of a quantitative tool that evaluates teachers' knowledge of the nature of mathematical modeling. It is a promising tool to guide researchers and educators as well as to inform teachers which areas they need to improve in their mathematical modeling practices. It is hoped that this scale will provide researchers and mathematics educators with the opportunity to accurately assess the knowledge of teachers about the nature of mathematical modeling practices.

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Declaration of Conflicting Interests and Ethics

The authors declare no conflict of interest. This research study complies with research publishing ethics. The scientific and legal responsibility for manuscripts published in IJATE belongs to the author(s).

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APPENDIX A

SECTION 1: This section focuses on asseing teachers' knowledge of the nature of mathematical modeling. Consider how they can be used in the classroom. The items below describe the nature of mathematical modeling. Please respond to these items to the best of your ability.

Q1. The practice of mathematical modeling involves a single-step process.

- ❍ True
- ❍ False

Q2. Mathematical modeling is a process of translation between the real world and mathematics.

- ❍ True
- ❍ False

Q3. The mathematical modeling process is the same as mathematical problem solving

- ❍ True
- ❍ False

Q4. Mathematical modeling discourages students' interest in mathematics

- ❍ True
- ❍ False

Q5. Mathematical modeling involves problem posing before problem solving

- ❍ True
- ❍ False

Q6. Mathematical modeling connects mathematical representations.

- ❍ True
- ❍ False

Q7. Solving mathematical modeling tasks always require the use of technology

- ❍ True
- ❍ False

Q8. Mathematical modeling assists students in their social interactions

- ❍ True
- ❍ False

Q9. Mathematical modeling supports productive struggle in learning mathematics

- ❍ True
- ❍ False

Q10. Mathematical modeling tasks are of low cognitive demand.

- ❍ True
- ❍ False

Q11.Mathematical modeling facilitates meaningful mathematical discourse, which elicits evidence of student thinking.

- ❍ True
- ❍ False

Q12. Mathematical modeling is accomplished by simply covering the content standards in the Common Core State Standards for Mathematics (2010) marked with a ★

- ❍ True
- ❍ False

Q13. Write a brief definition of mathematical modeling.

SECTION 2: Demographic Information and Experience with Mathematical Modeling.

Q14. What is your gender?

- ❍ Male
- ❍ Female
- ❍ Other

Q15. What is your age in years? ___________________

Q16. What is your race or ethnicity? ________________

Q19. In which grade level(s) do you teach? ________________

Q20. What is your highest degree earned? _____________________

Q23. Do you teach mathematical modeling activities? ___________________

Q27. Please comment on your experiences with mathematical modeling.

Thank you for taking time out of your busy schedule to complete this questionnaire!