

The Relationship between Stock Returns, Bitcoin Returns, and Risk Aversion: Evidence from a Multivariate GARCH Model

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Hisse Senedi Getirileri, Bitcoin Getirileri ve Riskten Kaçınma Arasındaki İlişki: Çok Değişkenli Bir GARCH Modelinden Kanıtlar

Abstract

This study explores the relationship between the U.S. stock returns, Bitcoin returns and their uncertainties by using a multivariate GARCH model. Specifically, the study compares the reactions of Bitcoin and stock market returns in the presence of global uncertainties and changes in risk appetites. The results show that even though reactions of Bitcoin and stock returns are similar for some highly volatile or risk averse periods, the association between the two returns is not sustainable. Moreover, the U.S. stock market investors are found to be risk averse throughout the entire sample period while Bitcoin investors are not.

Keywords : Multivariate GARCH-M, U.S. Stock Price Return, Bitcoin Price Return, Uncertainty, Volatility, World Risk Aversion Index, World Macroeconomic Uncertainty Index.

JEL Classification Codes : C3, C5, E1.

Öz

Bu çalışma, çok değişkenli bir GARCH modeli kullanarak ABD Dow Jones Borsasında işlem gören hisse senedi getirileri, Bitcoin getirileri ve bunların belirsizlikleri arasındaki ilişkileri araştırmaktadır. Özellikle, yüksek ve düşük olmak üzere farklı risk iştahının ve getirilerde belirsizliğin yüksek olduğu dönemlerde Bitcoin ve ABD hisse senedi getirilerinin verdiği tepkileri karşılaştırmaktadır. Sonuçlar, Bitcoin getirisinin riskten kaçınan veya yüksek belirsizliğin olduğu dönemlerde hisse senedi gibi tepki verdiğini, ancak iki getiri arasındaki ilişkinin sürdürülebilir olmadığını göstermektedir. Öte yandan, ABD borsa yatırımcıları tüm örneklem dönemi boyunca riskten kaçınma davranışını gösterirken, Bitcoin yatırımcıları aynı davranışı göstermemektedir.

Anahtar Sözcükler : Çok Değişkenli GARCH-M, Dow Jones Hisse Senedi Getirisi, Bitcoin Getirisi, Belirsizlik, Volatilité, Dünya Riskten Kaçınma Endeksi, Dünya Makroekonomik Belirsizlik Endeksi.

1. Introduction

In the heat of the global financial crisis, a Japanese computer programmer, Satoshi Nakamoto, introduced Bitcoin in November 2008. In an environment with a lack of confidence in the markets and the financial system, the interest in Bitcoin has increased substantially (ING International Survey, 2019). Bitcoin has been welcomed as an alternative currency and asset by many investors (Bouri et al., 2017b)¹. However, unlike conventional currencies, Bitcoin is not controlled by a central authority. The lack of control in the cryptocurrency markets brings concerns about its use in illegal activities. Moreover, taking into account the volatility of Bitcoin and the bubbles and crashes in the Bitcoin market, Cheah and Fry (2015) argue that Bitcoin is not a store of value and unit of account. Hence, it only provides the medium of exchange function out of three functions of money. If Bitcoin is not commonly considered as money, we need to evaluate it as an alternative investment instrument. This study contributes to this evaluation by comparing the behavior of Bitcoin and the stock market under global uncertainty and changes in risk appetites.

The jumps and high volatility in Bitcoin prices have drawn the attention of media, government, and investors and become the subject for many academic researches. The volatility (Dwyer, 2014; Katsiampa, 2017), informational efficiency (Urquhart, 2016; Nadarajah & Chu, 2017; Tiwari et al., 2018), price discovery (Ciaian et al., 2016), price clustering (Urquhart, 2017), the existence of bubbles in the market (Cheung et al., 2015; Cheah & Fry, 2015) and hedging ability against global uncertainty (Bouri et al., 2017b) are the features of Bitcoin that have mostly been investigated. Recently the literature pays more attention to Bitcoin as an investment asset. For instance, Brière et al. (2015) show the significant diversification benefits of Bitcoin, Baeck and Elbeck (2015) compare Bitcoin with the S&P 500 Index and report that the Bitcoin market is 26 times more volatile than the stock market. They also argue that Bitcoin returns are not influenced by fundamental economic factors, concluding that Bitcoin is a speculative commodity. Dyhrberg (2016a) argues that the hedging ability of Bitcoin is between gold and the U.S. dollar. Dyhrberg (2016b) indicates that Bitcoin is a hedge against the U.K. equities and the U.S. dollar. Additionally, Bouri et al. (2017a) show that Bitcoin is an inadequate hedge but an effective diversifier. Fry and Cheah (2016) document that Bitcoin has a speculative component. All these researches indicate that the role of Bitcoin as an alternative asset is still incomplete, and there is a lack of information about its market behavior.

This study contributes to this debate by comparing the behavior of Bitcoin and stock markets under global uncertainty and changes in risk appetites. First, we examine the link between the stock and Bitcoin returns and their uncertainties using a bivariate GARCH model and the Granger Causality Tests. Then, we investigate whether the results related to the co-movement of the Bitcoin and stock returns depend on the global financial investors' risk perceptions. To this end, we utilize the dataset developed in Bekaert et al. (2017). The

¹ On the other hand, many other investors kept their distance from this new technology. For example, JP Morgan Chase's CEO Jamie Dimon initially called Bitcoin a fraud, later stated that he regretted his earlier stance (WSJ, 9/1/2018). CEO of Berkshire Hathaway Warren Buffett called Bitcoin a "rat poison squared" (WSJ, 14/5/2018).

results suggest that Bitcoin returns have their own volatility and react like stock returns only at certain times. Additionally, this study reveals that Bitcoin investors do not follow the risk perceptions of stock investors.

The remainder of the study is organized as follows. Section 2 presents data and the methodology; Section 3 reports the results, and Section 4 presents concluding remarks.

2. Data and Methodology

This paper utilizes the daily Bitcoin and the U.S. stock market prices. The Bitcoin price (BP) data is extracted from Coindesk and spans from 19 July 2010 to 16 February 2018. Dow Jones Industrial Average Index (DJIA) is used as a proxy for the U.S. stock market prices (SP)².

Let S_t and B_t denote the U.S. Stock Return and Bitcoin Return, respectively. Stock returns are computed as $S_t = \log\left(\frac{SP_t}{SP_{t-1}}\right)$ and Bitcoin returns are computed as $B_t = \log\left(\frac{BP_t}{BP_{t-1}}\right)$.

A VAR model for U.S. Stock Return and Bitcoin Return can be written as in Equation 1.

$$x_t = \phi_j + \sum_{i=1}^{p-1} \varphi_{j,i} x_{t-i} + \varepsilon_t \quad (1)$$

where x_t is a (2×1) column vector given by $x_t = (S_t, B_t)'$, $\phi_j, j = 1, 2$ are (2×1) vector of constants, $\varphi_{j,i}, j = 1, 2, i = 1, \dots, p$ are $(2 \times 2p)$ matrix of parameters, and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})$ is a (2×1) vector of residuals.

We assume that the vector of residuals ε_t is conditionally normal with mean vector 0 and covariance matrix H_t where Ω_{t-1} is the information set available at time $t-1$, $(\varepsilon_t | \Omega_{t-1}) \sim N(0, H_t)$. The conditional covariance matrix H_t has the GARCH(1,1) structure as proposed in Bollerslev (1990)³. In particular, we assume that

$$\begin{aligned} h_{s,t} &= \alpha_s + \beta_s h_{s,t-1} + \gamma_s \varepsilon_{s,t-1}^2 \\ h_{b,t} &= \alpha_b + \beta_b h_{b,t-1} + \gamma_b \varepsilon_{b,t-1}^2 \\ h_{sb,t} &= \rho_{s,b} \sqrt{h_{s,t} h_{b,t}} \text{ Constant Correlation} \\ h_{sb,t} &= \alpha_{s,b} + \beta_{s,b} h_{s,t-1} h_{b,t-1} + \gamma_{s,b} \varepsilon_{s,t-1}^2 \varepsilon_{b,t-1}^2 \text{ BEKK GARCH}(1,1) \end{aligned} \quad (2)$$

² We also utilize Standard & Poors 500 (S&P 500) index. We reach the similar results.

³ In addition to diagonal CCC GARCH (1,1) model of Bollerslev (1990), we estimated other types of multivariate GARCH models and found similar results. The AIC criteria suggests that the suitable model is CCC GARCH (1,1). Estimation results with other specifications are available upon request.

where $h_{s,t}$ and $h_{b,t}$ are the conditional variances of stock and Bitcoin returns, respectively. $h_{sb,t}$ is the conditional covariance between stock price return residuals $\varepsilon_{s,t}$ and Bitcoin return residuals $\varepsilon_{b,t}$. We use the estimated variance $h_{s,t}$ and $h_{b,t}$ as proxies for stock and Bitcoin returns uncertainties, respectively. It is assumed that $\gamma_i > 0$, $\alpha_i \geq 0$ for $i = s, b$ and $-1 \leq \rho \leq 1$.

3. Results

The econometric methodology assumes that both the U.S. stock return (S_t) and Bitcoin price return (B_t) rates are $I(0)$ processes. The results of Augmented Dickey-Fuller Tests (ADF), Phillips-Perron Tests (PP), and Kwiatkowski-Phillips-Schmidt-Shin Tests (KPSS)⁴ indicate that both variables are stationary in level. As a first step in the specification procedure, we estimate a linear VAR model for the U.S. stock return (S_t) and Bitcoin return (B_t). Therefore, we continue with the lag and model selection stages of the multivariate GARCH model.

Table: 1
Selection of the GARCH Model

	AIC	SC
GARCH (1,1)	23809	24312
Constant Correlation	23793	24252
Full VECH parameterization	23905	24670
BEKK	23833	24401
Dynamic Conditional Constant Correlation	23811	24382

Table 1 provides the Akaike Information Criteria (AIC) and Schwarz Criteria (SC) values of several different models. Based on AIC and SC values, the most appropriate model seems to be the constant conditional correlation model (CCC). However, the CCC model assumes that the covariance between the two variables is constant. Considering the dynamic correlations of the Bitcoin and the U.S. stock returns, we further estimate the second-best alternative, GARCH (1,1), as well. Table 2 reports the bivariate-GARCH model estimates.

⁴ Results are available upon request.

Table: 2
Bivariate-GARCH -Model

	S_t	B_t
<i>Mean Eq.</i>		
<i>Intercept</i>	0,782*** (3,857)	3,336*** (2,747)
S_{t-1}	-0,053** (-2,297)	0,133 (1,176)
S_{t-2}	-0,008 (-0,320)	0,012 (0,110)
S_{t-3}	-0,027 (-1,080)	0,097 (0,822)
B_{t-1}	0,007** (2,570)	0,037* (1,807)
B_{t-2}	-0,002 (-0,795)	-0,006 (-0,230)
B_{t-3}	0,003 (0,999)	0,032 (1,158)
<i>Variance equation</i>		
α	6,000 (6,150)	78,067 (4,469)
β	0,177 (8,148)	0,126 (7,377)
γ	0,769 (30,443)	0,873 (58,479)
<i>Covariance equation</i>		
ρ		-0,001 (-0,071)
<i>Log Lik.</i>		-16040,171

Note: *, **, *** denote significance at 1%, 5%, and 10% significance levels, respectively. The lag selection is obtained by SBC. The figures in the parentheses are t-statistics of the tests.

As reported in Table 2, the Bitcoin returns affect the U.S. stock returns positively but not vice versa. Besides, the covariance between stock returns and the Bitcoin returns is insignificant, which indicates that the two assets are not connected. Since we take the U.S. stock return as a benchmark asset for understanding the reaction of Bitcoin, we can conclude that there is no co-movement of the two assets. This finding suggests that Bitcoin does not behave like a generally accepted asset.

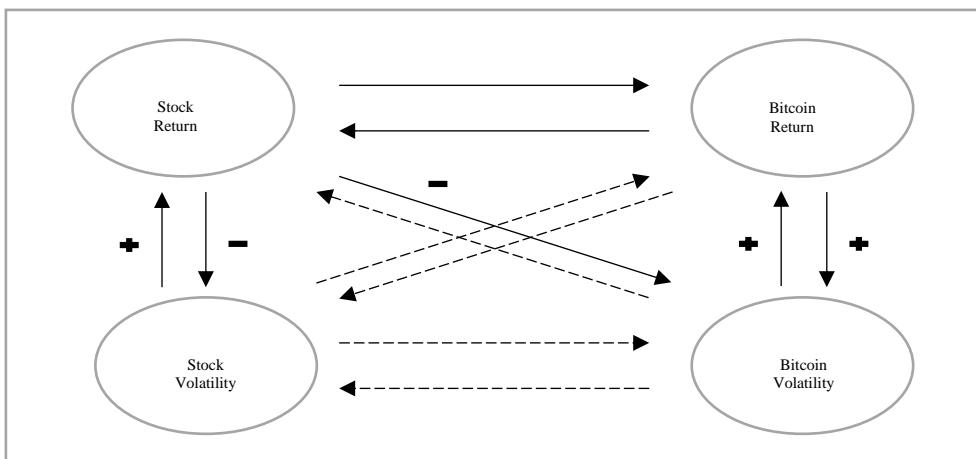
To gain more insight into the relationship between these two markets and understand if their volatilities affect each other, we employ Granger causality tests. The results of the Granger causality analysis are tabulated in Table 3 and pictured in Figure 1. They show that stock volatility significantly increases stock returns but has an insignificant effect on Bitcoin returns. Moreover, Bitcoin volatility significantly increases Bitcoin returns. Bitcoin returns have a significant positive effect on Bitcoin volatility but an insignificant effect on stock volatility. Stock return, however, significantly decreases both Bitcoin and stock market volatility. Bitcoin volatility has an insignificant effect on stock return volatility. In sum, Granger causality analysis reveals that the co-movement of the two assets can only be observed from stock returns to Bitcoin volatility.

Table: 3
Granger Causality Analysis

Hypothesis	Sign	
	(-)	(+)
$h_{st} \rightarrow S_t$ Stock volatility to stock return		5.203 ** (0.022)
$h_{Bt} \rightarrow S_t$ Bitcoin volatility to stock return	0.024 (0.876)	
$h_{st} \rightarrow B_t$ Stock volatility to Bitcoin return	2.044 (0.153)	
$h_{Bt} \rightarrow B_t$ Bitcoin volatility to Bitcoin return		6.588** (0.010)
$B_t \rightarrow h_{Bt}$ Bitcoin return to Bitcoin volatility		3.839*** (0.050)
$B_t \rightarrow h_{st}$ Bitcoin return to stock volatility	2.696 (0.101)	
$S_t \rightarrow h_{Bt}$ Stock return to Bitcoin volatility	3.717** (0.054)	
$S_t \rightarrow h_{st}$ Stock return to stock volatility	99.780 * (0.000)	
$h_{Bt} \rightarrow h_{st}$ Bitcoin volatility to stock volatility		2.178 (0.140)
$h_{st} \rightarrow h_{Bt}$ Stock volatility to Bitcoin volatility	0.013 (0.907)	

Note: The figures in the parentheses are t-statistics of the tests.

Figure: 1
Granger Causality Analysis



Note: Solid arrows indicate a significant relationship, while the dashed arrows show an insignificant relationship. The arrows' direction and the sign show the effect of that variable to directed variable.

Next, we focus on whether the results related to the co-movement of the Bitcoin and stock returns depend on the global financial investors' risk perceptions. To answer this, we use risk aversion and macroeconomic uncertainty indicators obtained from Bekaert et al. (2017). We aim to see the reaction of stock return and Bitcoin return volatilities to the time

variation in risk aversions (the price of risk) and the time variation in economic uncertainties (the amount of risk). In contrast to competent measures of risk appetite levels and economic uncertainty, these newly developed measures allow us to incorporate daily *good or bad* volatility into calculations. Daily data has the advantage of capturing some significant variations that are not available with monthly data. We regress risk aversion and macroeconomic uncertainty indices on both variance and covariance of the multivariate GARCH model. However, risk aversion and macroeconomic uncertainty data end on 30 December 2016. To balance the data, we re-estimate the model by using the sample until the end of 2016. The results are very similar to the results portrayed in Table 1, 2, and 3.

Table: 4
Dependent Variables Risk Aversion and the Macroeconomic Uncertainty

Independent	Risk aversion	Macroeconomic uncertainty
h_{St}	0.010 (27.404)	332.440 (19.687)
R^2	0.318	0.194
SSR	0.082	0.097
h_{Bt}	0.442 (13.657)	5101.205 (3.789)
R^2	0.101	0.008
SSR	557.686	616.798
Covariance	-0.0006 (-5.151)	-15.277 (-3.115)
R^2	0.016	0.005
SSR	0.008	0.0081
Correlation	0.089 (21.933)	1798.065 (10.077)
R^2	0.230	0.059
SSR	8.868	10.835

Note: The figures in the parentheses are t-statistics of the tests. SSR stands for the sum of square residuals. The goodness of the fit is indicated by SSR and R^2 .

Table 4 shows that stock return volatility has a more significant and robust relationship with the world risk aversion index (R^2 and Sum of Square Residuals-SSR 0.318 and 0.082, respectively) than the Bitcoin return volatility (R^2 and SSR are 0.101 and 557.686, respectively). This result is not surprising since the stock price volatility is affected by the investors' overall risk perception. Therefore, when the risk perception index increases, the volatility of the stock returns are also expected to increase. On the other hand, Bitcoin return volatility exhibits similar behavior, but the positive association is very low compared to stock return volatility. These results support the finding of the multivariate GARCH analysis, which is that the covariance between these two assets is found to be insignificant. Besides, Granger causality analysis shows that the volatilities of these two assets are not affecting each other. Based on that conclusion, we use the GARCH (1,1) model to obtain the dynamic correlations of the stock return and the Bitcoin return⁵. When we run regression between this correlation and the world risk aversion index, we see that both assets can be seen as alternative investment instruments to be included in a portfolio. Similar results are

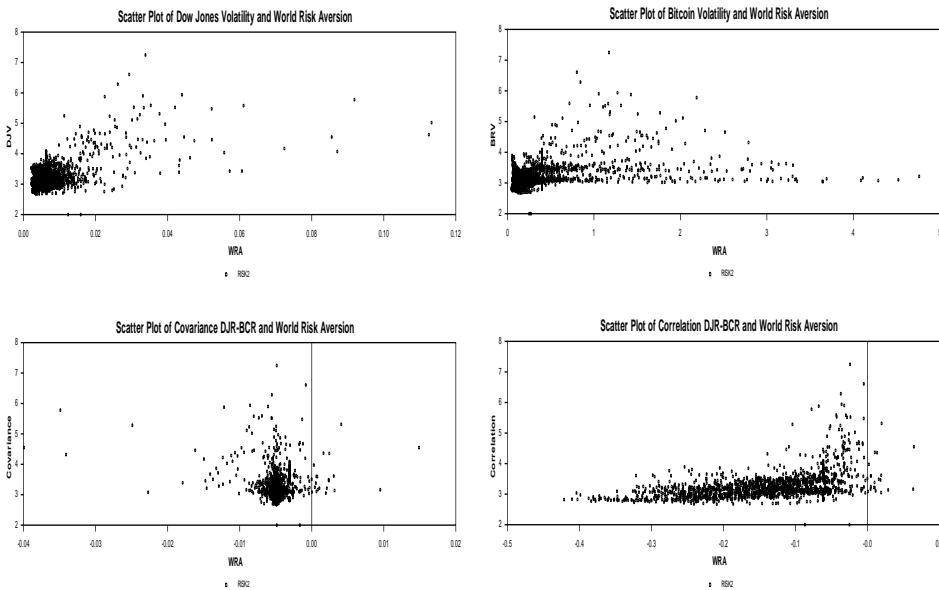
⁵ The estimation results of the multivariate GARCH(1,1) model is available upon request.

obtained when the macroeconomic uncertainty index is used, indicating a positive impact of macroeconomic uncertainty on both the stock return and the Bitcoin return volatility.

The estimation of the initial GARCH model indicates that stocks and Bitcoin are not strongly related assets. However, the Granger causality analysis and the regression results of the model including the world risk aversion index have shed more light on the relationship. The results reveal that even though Bitcoin returns behave like stock returns in some periods, the association between these two assets are weak.

To investigate the matter more closely, we illustrate the volatility and the risk aversion data. Figure 2 shows the scatter plots of stock returns, Bitcoin returns, and the risk aversion indices. As seen in the figure, the association between the stock return volatility and the Bitcoin return volatility is rarely positive.

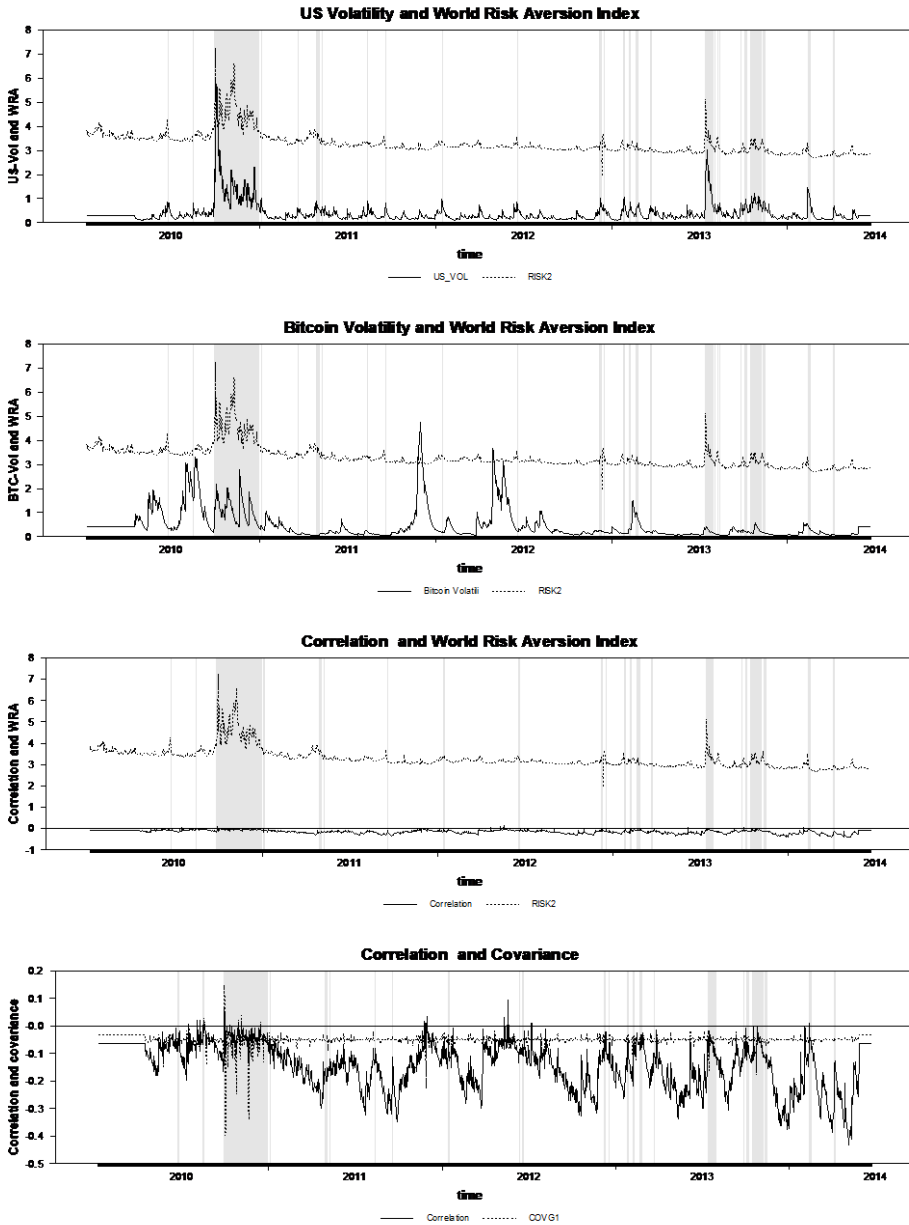
Figure: 2
Scatter Plots of Stock Return, Bitcoin Return and Risk Aversion Index



To better understand the periods of association between stock return volatility and Bitcoin volatility with the world risk aversion index, we obtain the time plots of the risk aversion and the volatility of the stock returns and Bitcoin returns, respectively. The results are exhibited in Figure 3. The shaded areas in Figure 3 show when the stock return volatility is above the 50 percent threshold. The first panel of Figure 3 shows the one-to-one association between stock return volatility and the world risk aversion index. The second panel of the figure provides the Bitcoin return volatility with respect to world risk aversion.

As easily observed from the shaded areas, Bitcoin behaves like stock in mid-2010 for a long period and in some other periods, such as the beginning of 2013, for a short term. In sum, Figure 3 supports the findings of the multivariate GARCH, Granger causality test, and the regression analysis. Bitcoin behaves like stocks period by period, but it has its own volatility that separates its data generating process from other assets on average. Moreover, if the association of the volatility of the stock return and the Bitcoin returns increases, this can be interpreted as the rise in the positive association between these two assets, hence, a decrease in the negative correlation between the stock and Bitcoin returns. The last panel of Figure 3 suggests that the increase in the risk aversion index is associated with negative and low correlations between stock returns and the Bitcoin returns.

Figure: 3
Time Plots of Stock Return, Bitcoin Return and Risk Aversion Index



4. Conclusion

Despite Bitcoin's growing popularity among investors, regulators, and market players, there is still a lack of empirical knowledge on its role as an investment instrument. This study contributes to the literature by evaluating Bitcoin's role as an alternative asset. For that purpose, we compare the response of the Bitcoin to the U.S. stock market returns during the periods when there exist high and low-risk appetites and uncertainties. The results show that in some periods, Bitcoin returns reacts like the stock returns, but the association between the two assets is not sustainable. Bitcoin investors do not follow the risk perceptions of stock investors, and they have different dynamics in making their investment decisions.

A further avenue of research would be to investigate these different dynamics using different asset classes as a comparison. Besides, there is a need for more studies on other cryptocurrencies. Although Bitcoin is the most popular cryptocurrency with the highest market capitalization, there are also some other strong alternative cryptocurrencies such as Ripple, Litecoin, Ethereum. Future studies will be needed to see if results obtained using Bitcoin data can be generalized for other cryptocurrencies.

References

- Baek, C. & M. Elbeck (2015), "Bitcoins as an investment or speculative vehicle? A firstlook", *Appl. Econom. Lett.*, (22), 30-34.
- Bekaert, G. & E. Engstrom & N.R. Xu (2017), "The time variation in risk appetite and uncertainty", Columbia Business School *Research Paper* No. 17-108, Available at SSRN: <<https://ssrn.com/abstract=3069078>>, 23.08.2019.
- Bollerslev, T. (1990), "Modelling the coherence in short-run nominal exchange rates: A multivariate generalized arch model", *The Rev. of Econom. and Statistics*, 72(3), 498-505.
- Bouri, E. & P. Molnár & G. Azzi & D. Roubaud & L.I. Hagfors (2017a), "On the hedge and safe haven properties of Bitcoin: is it really more than a diversifier?", *Finance Res. Lett.*, (20), 192-198.
- Bouri, E. & R. Gupta & A.K. Tiwari & D. Roubaud (2017b), "Does Bitcoin hedge global uncertainty? Evidence from wavelet-based quantile-in-quantile regressions", *Finance Res. Lett.*, (23), 87-95.
- Brière, M. & K. Oosterlinck & A. Szafarz (2015), "Virtual currency, tangible return: Portfolio diversification with Bitcoin", *J. Asset Manag.*, 16(6), 365-373.
- Cheah, E.-T. & J. Fry (2015), "Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin", *Econom. Lett.*, (130), 32-36.
- Cheung, A. & E. Roca & J.-J. Su (2015), "Crypto-currency bubbles: An application of the Phillips-Shi-Yu (2013) methodology on Mt.Gox Bitcoin prices", *Appl. Econom.*, (47), 2348-2358.
- Ciaian, P. & M. Rajcaniova & D.A. Kancs (2016), "The economics of Bitcoin price formation", *Appl. Econ.*, 48(19), 1799-1815.
- Dwyer, G.P. (2014), "The economics of Bitcoin and similar private digital currencies", *J. Financ. Stab.*, (17), 81-91.

- Dyhrberg, A.H. (2016a), "Bitcoin, gold and the dollar - a GARCH volatility analysis", *Finance Res. Lett.*, (16), 85-92.
- Dyhrberg, A.H. (2016b), "Hedging capabilities of Bitcoin. Is it the virtual gold?", *Finance Res. Lett.*, (16), 139-144.
- Fry, J. & E.-T. Cheah (2016), "Negative bubbles and shocks in cryptocurrency Markets", *Int. Rev. Financ. Anal.*, (47), 343-352.
- ING International Survey (2019),
<https://think.ing.com/uploads/reports/IIS_New_Tech_Cryptocurrencies_report_18092019.pdf>, 23.08.2019.
- Katsiampa, P. (2017), "Volatility estimation for Bitcoin: A comparison of GARCH Models", *Econom. Lett.*, (158), 3-6.
- Nadarajah, S. & J. Chu (2017), "On the inefficiency of Bitcoin", *Econom. Lett.*, (150), 6-9.
- Tiwari, A.K. & R.K. Jana & D. Das & D. Doubaud (2018), "Informational efficiency of Bitcoin", *Econom. Lett.*, (163), 106-109.
- Urquhart, A. (2016), "The inefficiency of Bitcoin", *Econom. Lett.*, (148), 80-82.
- Urquhart, A. (2017), "Price clustering in Bitcoin", *Econom. Lett.*, (159) (Suppl. C), 145-148.
- Wall Street Journal (14/5/2018), *Steve Eisman of 'The Big Short' bashes cryptocurrency: 'I don't see the purpose of it'*, Retrieved from: <<https://www.wsj.com/articles/steve-eisman-of-the-big-short-bashes-cryptocurrency-i-dont-see-the-purpose-of-it-1526292273>>, 23.08.2019.
- Wall Street Journal (9/1/2018), *Jamie Dimon: I 'regret' calling Bitcoin a fraud*, Retrieved from: <<https://www.wsj.com/video/jamie-dimon-i-regret-calling-Bitcoin-a-fraud/A633471B-A251-440E-A959-B72CEFC01E99.html>>, 23.08.2019.