Research Article

Cognitive adaptation of pre-service mathematics teachers in construction and reconstruction of quadrilateral relations

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Article Info	Abstract
	This is qualitative research aimed at exploring cognitive adaptation in the construction and reconstruction of quadrilateral relations. The participants were chosen using the Quadrilateral Relations Construction Test given to 20 pre-service mathematics teachers. Based on the test results, 4 types of quadrilateral relations are obtained, namely type 1, type 2, type 3, and type 4. Next, 1 pre-service mathematics teachers with type 1 (P1) and 1 pre-service mathematics teachers with type 2 (P2) are chosen as participants. Next, participants were given a Quadrilateral Relations Reconstruction Test then an in-depth interview was conducted. The data in this study were analyzed through data condensation, data display, and conclusion drawing. This result illustrated that having a scheme is not enough to successfully solve problems. One of the important factors that determine the success of problem solving is the success of cognitive adaptation process. Cognitive adaptation in the process of construction and reconstruction quadrilateral relations takes place through assimilation and accommodation. When the information obtained is in accordance with the scheme that is owned, the cognitive adaptation that occurs is called assimilation. Meanwhile, when the information obtained is not in accordance with the scheme owned, the adaptation that occurs is called accommodation. Thus, pre-service mathematics teachers must be trained to make cognitive adaptations in problem solving process.

Introduction

Knowledge in the form of concepts is not obtained as a result of transfers from the environment of learning resources but is constructed by individuals. Constructivism views knowledge as the result of mental formation (Olusegun, 2015; Pritchard & Woollard, 2010) that is carried out actively by individuals when learning (Bächtold, 2013; Mattar, 2018) by connecting the knowledge they already have (Badie, 2016) with the experience they get. Someone who learns must be proactive in receiving information (Aljohani, 2017) modifying existing knowledge or experience, then representing it in the form of mental networks (schemes) according to their understanding. In learning, interactions occur between internal factors, namely knowledge already possessed and external factors, namely the reality faced in the environment (Dyankova, 2018), for example in the learning process or problem-solving. These interactions occur on an ongoing basis and are an important aspect of the process of cognitive development and concept formation (Singh & Yaduvanshi, 2015).

The concepts that have been formed can be used to understand things that exist in the environment (Norton, 2018), even in solving problems (Singer & Voica, 2013; Trigueros, 2019), and will continue to develop with new

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experiences (Čadež & Kolar, 2015). However, the concept does not always correspond to what is faced. We know that mathematics is a hierarchical science, meaning that concepts in mathematics are presented from the simplest to the most complex (Rohaeti, Bernard & Primandhika, 2019). The scheme that we have about a simple concept is not enough for us to use to understand more complex concepts, so the concepts we already have must be reconstructed to accept and be able to understand new concepts.

Every person has prior knowledge in the form of cognitive structures (schemes) which guide us in doing something (Čadež & Kolar, 2015), understand things that are obtained (Weber, Walkington & Mcgalliard, 2015) and solve problems faced (Moll, Trigueros, Badillo, & Rubio, 2015). All living, thinking beings have a set of rules which are variously known as "scripts", "schemes" and, as we will be using, "schemas" that are used to interpret their everyday surroundings (Pritchard & Woollard, 2010). Schemes owned by someone will continue to develop along with the increasing experience gained (An, 2013). New experiences gained from the process of interaction with the environment will cause the scheme to develop. The development of cognitive structures (schemes) occurs when the information obtained can be integrated directly or indirectly into the existing scheme, so there is a need for cognitive adaptation.

In response to changes in the environment, children adapt to adjust their behavior, cognition, or scheme. The process is carried out through assimilation and accommodation (modification of the scheme to include new information) (Bjorklund, 2015). The development of cognitive structures will only occur if a person can adjust his cognitive structure to the conditions at hand. This shows that the development of one's knowledge is caused by the process of adaptation. Cognitive adaptation results in changes in cognitive structure, which in turn leads to cognitive development that is characterized by an increase in qualitative aspects of thinking. Adaptation is the process by which a person develops his scheme when facing new experiences. The process takes place through assimilation and accommodation.

Problem of Research

In learning someone constructs his own (or with the help of the teacher) the concepts learned then store in the cognitive structure they have in the form of a scheme. These schemes are interrelated to form knowledge of the concepts being studied. When knowledge is needed again at a different time or situation, a person must be able to reconstruct the knowledge he has. This condition requires cognitive adaptation. In construction and reconstruction of quadrilateral relations, a cognitive adaptation process occurs to connect the quadrilateral concept. So that this research will reveal the occurrence of these cognitive adaptations.

➢ How cognitive adaptation of preservice mathematic teachers in the process of construction and reconstruction of quadrilateral relations?

Method

Research Design

This study was designed to uncover cognitive adaptations of pre-service mathematics teachers in constructing and reconstructing quadrilateral relations, so the research design used was qualitative research. This research was conducted naturally without any settings in the process of taking data to maintain the originality of the data obtained.

Participants

This research involved 20 pre-service mathematics teachers who had studied geometry in the Department of Mathematics Education at a university in East Java, Indonesia. The selection of participants was carried out by giving the Quadrilateral Relations Construction Test (Figure 1) to 20 pre-service mathematics teachers. The Test given to find out the schemes of pre-service mathematics teachers about the concept of quadrilateral relations.

Data Collection Tools

The data collection tools used in this study were the quadrilateral relations construction test (QRCT) and the quadrilateral relations reconstruction test (QRRT). The data collection tools were developed by researchers and tailored to the objectives to be achieved in this study. QRCT and QRRT were validated by two experts consisting of a professor of mathematics education and a doctor of mathematics. After the validator declared QRCT and QRRT valid, then QRCT and QRRT were used in this study.
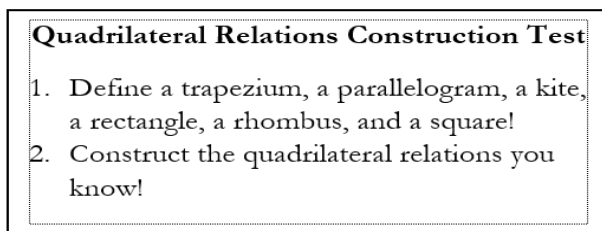
> **Quadrilateral Relations Construction Test**
>
> 1. Define a trapezium, a parallelogram, a kite, a rectangle, a rhombus, and a square!
> 2. Construct the quadrilateral relations you know!

Figure 1.
Quadrilateral Relations Construction Test

The results are that there are four types of quadrilateral relations are obtained, namely type 1 consisting of 7 pre-service mathematics teachers, type 2 consisting of 10 pre-service mathematics teachers, type 3 consisting of 1 pre-service mathematics teachers and type 4 consists of 2 pre-service mathematics teachers. The types of quadrilateral relations and the number of pre-service mathematics teachers in each type are presented in Table 1. From Table 1, it appears that the quadrilateral relations type 1 and type 2 are more dominant and complex than type 3 and type 4. In addition to information from Table 1, communication factors are also a consideration of researchers in determining participants because credible data are obtained through in-depth interviews. Thus, the researcher chose 1 pre-service mathematics teachers with 1 type as participants 1 (P1) and 1 pre-service mathematics teachers with type 2 as participants 2 (P2).

Table 1.
Types of Quadrilateral Relations and the Number of Pre-Service Mathematics Teachers in Each Type

Types	Quadrilateral Relations	Pre-service Mathematics Teachers
1		7 (35%)
2		10 (50%)
3		1 (5%)

Types	Quadrilateral Relations	Pre-service Mathematics Teachers
4		2 (10%)

Next, the chosen participants, P1 and P2, were given the Quadrilateral Relations Reconstruction Test (Figure 2) that contained a different definition of a trapezium with the previous test (Quadrilateral Relations Construction Test). This test aims to reveal the process of cognitive adaptation that occurs.
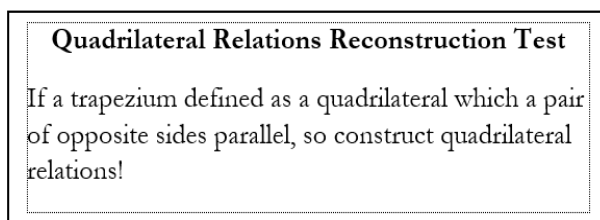

Figure 2.
Quadrilateral Relations Reconstruction Test

Data collection using tests methods and in-depth interviews. The test in this study consisted of a quadrilateral relations construction test and a quadrilateral relations reconstruction test. Data collection begins with the provision of a quadrilateral relations construction test to the participants to find out the schemes owned by the participants regarding quadrilateral relations. Then the participants was given a quadrilateral relations reconstruction test and continued with in-depth interviews to find out the cognitive adaptations carried out in the process of reconstructing quadrilateral relations. To ensure the credibility of the data collected, checking the credibility of the data using member check techniques, peer checks, and referential adequacy is carried out.

Data Analysis
After collecting data, it is obtained the old scheme data, namely the concept of quadrilateral relations constructed by the participants and new schema data, which are the results of the reconstruction of quadrilateral relations. The old schema data were obtained through the Quadrilateral Relations Construction Test, while the new schema data were obtained through the Quadrilateral Relations Reconstruction Test and in-depth interviews based on answers. In-depth interviews were conducted to uncover the process of cognitive adaptation that occurs when the participants constructs and reconstructs quadrilateral relations. In this study, the data analysis process starts from the time the researcher collects the data until when the research is complete. The data obtained in this study were analyzed using a data analysis model consisting of data condensation, data display, and conclusion drawing (Milles, Huberman & Saldana, 2014).

Result and Discussion
Description Process of Construction and Reconstruction Quadrilateral Relations by Participants 1 (P1)
Quadrilateral relations had constructed by Participants 1 (P1) based on a scheme of definitions of the quadrilateral. A trapezium is a quadrilateral with only a parallel side. Besides that, P1 also had a scheme about the definitions of a parallelogram, kite, rectangle, rhombus, and square. A parallelogram is a quadrilateral with two pairs of parallel sides. A kite is a quadrilateral with two distinct pairs of congruent adjacent sides. A rectangle is a parallelogram that has four right angles. A rhombus is a parallelogram with all sides congruent. A square is a rectangle that has all sides congruent.

The definitions of the quadrilateral mentioned P1 is a type of analytic definition because they contain the immediate family and special distinction (Budiarto, Khabibah, & Setianingsih, 2017). The P1 defines the trapezium by mentioning the immediate family which is a quadrilateral and a special distinction that is only a parallel side. The subject defines

the parallelogram by mentioning the immediate family, namely quadrilateral and special distinction which two pairs parallel sides. The P1 defines the kite by mentioning the immediate family, namely quadrilateral and special distinction which two distinct pairs of congruent adjacent sides. The P1 defines a rectangle by mentioning the immediate family, namely the parallelogram and special distinction that has four right angles. The P1 defines rhombus by mentioning the immediate family, namely the parallelogram and special distinction that has four congruent sides. The P1 defines a square by mentioning the immediate family, namely the rectangle and special distinction that has four congruent sides.

In the process of constructing quadrilateral relations, the P1 conducts assimilation by integrating the information obtained, namely constructing the relationship between quadrilaterals with the scheme that is owned, namely the definition of quadrilaterals. Assimilation is the process of integrating information obtained into an appropriate scheme (Zhiqing, 2015; Di Paolo, Barandiaran, Beaton & Buhrmann, 2014). The integration of the information is carried out in response to the information obtained (Castillo, Heredia, & Gallardo, 2017; Kaasila, Pehkonen, & Hellinen, 2010). The response given is a form of knowledge or experience that is associated with the stimulus received.

Based on the definitions mentioned, trapezium, parallelogram, and kite had directly connected with quadrilateral. Furthermore, the parallelogram had connected to the rectangle because of the parallelogram which given an additional condition of having a right angle transform into a rectangle. The parallelogram also connected with the rhombus because the parallelogram which gave an additional condition that all sides congruent transform to rhombus. A rectangle had given an additional condition that all sides congruent transform into a square, so connected to a square. Besides that, a rhombus had given an additional condition a right angle transform to a square, so connected to the square. The schematic of quadrilateral relations that had been constructed by P1 is presented in Figure 3 below.
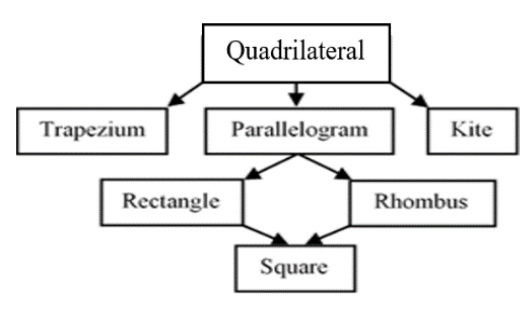

Figure 3.
The Schematic of Quadrilateral Relations Construction Results by P1

When given a different definition of a trapezium with the definition he knows, then the concept of quadrilateral relations owned by P1 becomes "disturbed". So, P1 must reconstruct the concepts they have. The process of reconstructing the concept of quadrilateral relations by P1 was revealed in the interview between Researcher (R) and Participants 1 (P1) below.

> *R : If the definition of trapezium is changed to "a trapezium is a quadrilateral with a pair of opposite sides is parallel", is the quadrilateral relations formed the same as you construct?*
> *P1: Different.*
> *R : Why?*
> *P1: Because quadrilateral relations are formed by definition. So, if the definition changes then the relations between quadrilaterals also changes, so I reconstruct the quadrilateral relations like this (Figure 4)*
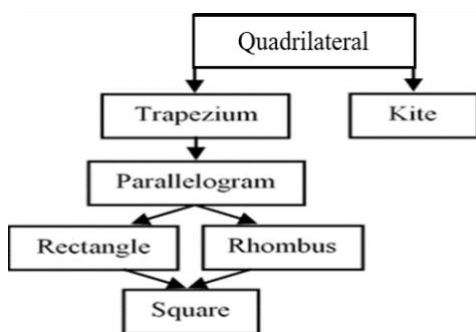

Figure 4.
The Quadrilateral Relations Reconstruction Results by P1

> *R : How can you reconstruct the quadrilateral relations like a Figure 4?*
> *P1: I noticed the definition of the trapezium given and the definition of the other quadrilateral I know. Then I make a quadrilateral relations based on the definitions that I know of.*
> *R : What is the difference between the quadrilateral relations in Figure 3 and Figure 4?*
> *P1: In Figure 3 I did not connect the trapezium and the parallelogram whereas in Figure 4 I had connected based on the existing definition. The other quadrilateral relations do not change, because only the definition of a trapezium has changed.*

The interview showed that in the process of reconstructing the concept of quadrilateral relations is carried out by taking P1 into account again the definition of the quadrilateral. The incompatibility of the definition obtained with the definition is known to cause P1 to change the schema of the definition of trapezium through accommodation. The known definition of trapezium P1 changed into a trapezium is a quadrilateral which a pair of opposite sides parallel. This change in the definition of trapezium causes the relationship between quadrilaterals also to change. The new definition of (accommodation results) especially the definition of a trapezium is used by P1 in reconstructing the concept of quadrilateral relations. The aspect to consider is the immediate family. Trapezium and kite have an immediate family the same, namely quadrilateral so that the trapezium and kite are connected with the quadrilateral. Based on the new definition, the relationship between the parallelogram and trapezium is formed through the aspect of special distinction by adding the condition to have two pairs of opposite sides are parallel in the special distinction of trapezium so parallelogram is formed. This cannot be done when the trapezium is defined as a quadrilateral which exactly a pair of opposite sides parallel. So that there are differences between the quadrilateral relations, especially trapezium and parallelogram based on a known definition of P1 and based on a new definition. In addition to accommodation, P1 also assimilates in reconstructing quadrilateral relations. This is done because not all relationships between quadrilaterals change due to changes in the definition of a trapezium. There are still schemes in accordance with the situation at hand, so some schemes owned by P1 can still be used to respond to the conditions they face. According to Blake & Pope (2008) assimilation is the process by which a person uses or changes the environment to fit an existing structure.

Through assimilation, P1 connects the parallelogram and rectangle because the rectangle can be formed by adding a condition that having four right angles at a parallelogram. The parallelogram is connected with the rhombus because the rhombus can be formed by adding the condition that has four congruent sides at the parallelogram. A rectangle is connected by a square because a square can be formed by adding the condition that has four congruent sides at the rectangle. A rhombus is connected to a square because a square can be formed by adding the condition of having a right angle at the rhombus. The concept of quadrilateral relations that have been constructed and the concepts of quadrilateral relations formed after the reconstruction process carried out by P1 are presented in Table 2 below.

Table 2.
Quadrilateral Relations Results of Construction and Reconstruction by P1

The quadrilateral relations results of construction	The quadrilateral relations results of reconstruction

Description process of construction and reconstruction quadrilateral relations by Participants 2 (P2)
Quadrilateral relations had constructed by Participants 2 (P2) based on forms of the quadrilateral. Even though P2 has a scheme about the definition of a quadrilateral, P2 does not pay attention to the definition like P1, but P2 pays attention to quadrilateral forms. Schemes about quadrilateral forms are used to construct quadrilateral relations. Schemas are cognitive structures that individuals use when interacting with their environment (Heo, Han, Koch & Ayidin, 2011). According to (Tasheva, 2018), a schema is mentally controlling a person's behavior when adapting to the environment. Schemes are mental patterns that guide behavior (Slavin, 2017). By using the existing scheme, the process of constructing quadrilateral relations is carried out through an assimilation process. Assimilation is the

process of inserting a stimulus into an established scheme (Zhiqing, 2015). P2 represents a parallelogram that has a shape similar to the rectangle, so the parallelogram is connected with the rectangle. Furthermore, a rectangle is connected by square because have similar shapes. The shape aspect is also used in constructing the relationship between a kite and a rhombus. P2 represents the kite as having a similar shape with a rhombus so that the kite connected to a rhombus. A trapezium is represented as a quadrilateral that has a different shape from the other quadrilaterals so that the trapezium is not connected with the others. In constructing the relationship between quadrilaterals, P2 not only pays attention to form but also indirectly pays attention to the routine attributes especially the parallel sides of the quadrilaterals. Routine attributes are attributes that are often used when learning quadrilaterals. The parallel side attributes that are considered by P2 are routine attributes (Budiarto, Rahaju, & Hartono, 2017). The schematic of quadrilateral relations that had been constructed by P1 is presented in Figure 5 below.
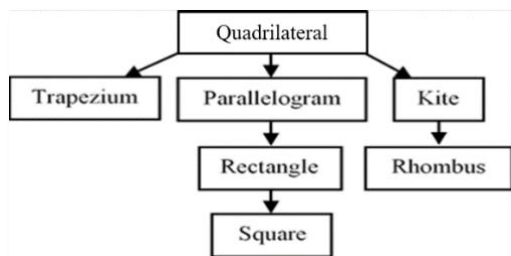

Figure 5.

The Schematic of Quadrilateral Relations Construction Results by P2

Besides that, based on the construction process carried out by P2, it was revealed that P2 had a scheme about the definition of a trapezium. A trapezium is a quadrilateral which exactly a pair of opposite sides parallel. P2 also had a scheme about the definition of parallelogram, kite, rectangle, rhombus, and square. A parallelogram is a quadrilateral with two pairs of sides that are parallel and congruent. A kite is a quadrilateral with two distinct pairs of congruent adjacent sides. A rectangle is a quadrilateral with two pairs of opposite sides is congruent, parallel, and has four right angles. A rhombus is a quadrilateral with four opposite sides that are congruent and opposite angles are congruent. A square is a quadrilateral with four opposite sides are congruent and four right angles.

When given a different definition of the trapezium from the definition he knows, the concept of inter-quadrilaterals relations owned by P2 becomes "disturbed". So, P2 must reconstruct the concepts they have. The process of reconstructing the concept of inter-quadrilaterals relations by P2 was revealed in the interview between the Researcher (R) and Participants 2 (P2) below.

> R : *If the definition of trapezium is changed to "a trapezium is a quadrilateral with a pair of opposite sides is parallel", is the quadrilateral relations formed the same as you construct?*
> P2: *Different*
> R : *Why?*
> P2: *Because the shape of the trapezium will change, so the quadrilateral relations also changes, so I reconstruct the quadrilateral relations like this (Figure 6)*
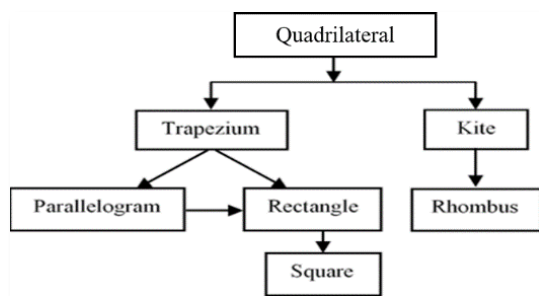

Figure 6.

The Quadrilateral Relations Results of Reconstruction by P2

> R : *How can you reconstruct the quadrilateral relations like Figure 6?*
> P2: *By forming a trapezium based on the given definition then compare with the quadrilaterals forms that I know of. I connect quadrilateral that have similar shapes while the rectangle that has a different shape I do not connect.*
> R : *What is the difference between the quadrilateral relations in Figure 5 and Figure 6?*

> *P2: In Figure 5, I did not connect the trapezium with the other quadrilateral because the shape is different. Whereas in Figure 6 I connected the trapezium with parallelogram and rectangle. Because the trapezium has two parallel sides, whereas the parallelogram and rectangle have two pairs of parallel sides, so that the trapezium is related to the parallelogram and rectangle. Then, because only the shape of the trapezium has changed so that the others quadrilateral relations remain the same.*

The interview showed that when reconstructing the concept of quadrilateral relations, P2 not only paid attention to the shape of the quadrilateral, but P2 also paid attention to the definitions it knew. The process of reconstructing the concept of quadrilateral relations carried out by P2 not only pays attention to aspects of quadrilateral shapes but also pays attention to the definition of the quadrilateral. Because the definition of the trapezium obtained does not match the scheme it has, so P2 changes its scheme to the definition of trapezium through accommodation. The known definition of trapezium P2 is trapezium is a quadrilateral which exactly a pair of opposite sides parallel transformed into a trapezium is a quadrilateral which a pair of opposite sides parallel. This change in definition has caused the concept of the quadrilateral relations by P2 also changed. Trapezium form based on the given definition is represented as a form similar to a parallelogram and rectangle so that the trapezium is connected with a parallelogram and rectangle. Besides the aspect of form, the relationship of trapezium, parallelogram, and rectangle are strengthened by a scheme of definitions known to P2. The aspect of definition that is considered by P2 is the special distinction at the parallelogram and rectangular which is to have two pairs of opposite sides is parallel and congruent. Changes in the definition of the trapezium do not cause changes in the shape of the parallelogram, rectangle, kite, rhombus, and square so that the quadrilateral relations are fixed. This causes the scheme which is owned by P2 can still be used. Because there is an appropriate scheme, so the P2 performs assimilation using the shapes of a parallelogram, rectangle, kite, rhombus, and square in reconstructing quadrilateral relations. The concept of quadrilateral relations constructed and the concepts of quadrilateral relations formed after the reconstruction process carried out by P2 are presented in Table 3 below.

Table 3.

Quadrilateral Relations Results of Construction and Reconstruction by P2

The quadrilateral relations results of construction	The quadrilateral relations results of reconstruction

Conclusion

Cognitive adaptation occurs through assimilation and accommodation. The existence of concepts related to the definition and forms of the quadrilateral that has been owned and in accordance with the reality faced makes the assimilation process can be done. Assimilation is done by integrating the information obtained into the schema of the corresponding quadrilateral definitions and forms so that the scheme can be used in constructing and reconstructing the concept of quadrilateral relations. This is in accordance with the opinion which states that assimilation is the process of integrating information obtained into an appropriate scheme (Zhiqing, 2015). However, because the information obtained is not in accordance with the schemes that have been owned causing the concept of quadrilateral relations to be disrupted, so that the concepts possessed a need to be reconstructed through accommodation. Accommodation is done by changing the scheme that is owned and adjusted to the reality faced or information obtained, then integrating the information obtained into the new scheme (accommodation results). Accommodation involves changing mindset to adapt to reality built with newly acquired information (Castillo et al. 2017). Then, a new scheme is used in reconstructing the concept of quadrilateral relations.

The results of this study contribute to help pre-service mathematics teachers in solving geometry problems, especially quadrilateral. In addition to ensuring that students have schemes about quadrilateral relations, students must also be trained so that they can use their schemes in the future or in different situations. This will greatly help students in solving problems.

Limitations and Recommendation

The researcher realizes that there are limitations in this study including (1) the participants are relatively few and only taken from one location; (2) the test is given to reveal the cognitive adaptation that occurred consisted of only one item so that it was less varied and comprehensive.

Based on the results of this study, the researcher recommends that in the problem solving process, pre-service mathematics teachers be trained to find the relationship between the problems faced and the knowledge (schemes) that they already have has actively and independently. In addition to ensuring that pre-service mathematics teachers have a scheme of the relation between quadrilaterals, it must also be ensured that pre-service mathematics teachers can use their scheme. This will help pre-service mathematics teachers to do the cognitive adaptation.

Acknowledgements

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