Y. Y. Ü. Fen Bilimleri Enstitüsü Dergisi
Yıl : 2009, Cilt : 14, Sayı : 1, Sayfa : 28-32

Using Simulations to Approximate the Power Function of the One-tailed T-test

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Abstract: The power function of the one-tailed *T*-test is characterized by a rapid rise from close to zero for $\mu < \mu_0$ to one as μ becomes larger than μ_0. The larger the number of elements in a simulated sample, the faster the power rises to one, apparently until the mean value theorem takes effect and the sample averages begin to take on a normal distribution themselves which gives a limiting power function where all of the assumptions of the test are satisfied by default.
Key words: power function, simulation, T-test.

Simülasyonlarla Tek-Yönlü *T*-testi'nin Güç Fonksiyonunun Elde Edilmesi

Özet: Tek yönlü $\mu < \mu_0$ *T*-testinin güç fonksiyonu sıfıra yakınken μ değeri μ_0 değerini aştıkça bire hızla yaklaşan bir fonksiyon olarak karakterize edilebilir. Simülasyondaki birim sayısı arttıkça, güç bire daha hızlı yaklaşır. Bu durum ortalama değer teoremi etkisini gösterinceye ve örnek ortalamaları normal dağılım şeklini alıncaya kadar devam eder ki bunun ötesinde zaten test için gerekli varsayımlar otomotik olarak sağlanmıs olur.
Anahtar kelimeler: güç fonksiyonu, simülasyon, *T*-testi.

Introduction

In this study, the power of the one sample *T*-test is analyzed using simulations for a generating distribution where the assumptions of the test are satisfied, for distributions where the assumption about the mean are relaxed, and for where the assumptions about the underlying distribution are relaxed. The statistical software package Minitab is used to generate the simulations of the data for the underlying distributions used in this study (Ryan, Joiner and Cryer, 2005). Different sample sizes are used for analyses of the power of the *T*-test.

The highlights of this study are the usefulness of simulations and some of their properties, as well as defining what the power of a statistical test.

Theoretical Background

The one-sample *T*-test is a statistical test of the assumption that a sample mean is equal to the mean of the assumed normal distribution underlying the data (Wackerly *et al.*, 2002). Based on the presumption that the mean of the sample data taken is distributed according to the *t*- distribution, comparison values for chosen type I error probabilities, α, can be calculated numerically or taken from a table for certain values of α. The T-statistic:

$$T = \frac{(\bar{x} - \mu)}{\left(s/\sqrt{n}\right)}$$

has a *t*-distribution with (*n* – *1*) degrees of freedom.

For the one-tailed *T*-test, the null hypothesis ($H_0 : \mu = \mu_0$) is rejected if the sample T-statistic is larger than the table value for a given α. This would indicate that there is a probability of $1-\alpha$ that the two means are not the same as assumed in the null hypothesis. The assumptions of the *T*-test are that the sampled data are independently and identically distributed with a normal distribution having a mean of μ and standard deviation σ. The null hypothesis then assumes that the unknown population mean is equal to the hypothesized value of μ. This study focuses on the power of the one-tailed *T*-test where the assumptions are satisfied and where the assumption that the means are equal is relaxed and where the underlying distribution is not normal, separately.

The power of a statistical test is simply the probability that the test will reject the null hypothesis when it is false (Navidi, 2006). To find an approximate power function for the *T*-test for a normal underlying distribution with varying

μ, and for an exponential distribution with varying θ, simulations have to be used to test at different sample sizes. The number of rejected simulations divided by the total number of repetitions gives an approximate value for the power of the test for each sample size and underlying distributions of the data.

Study Design and Procedures

In this study, the approximate power function of the one-tailed *T*-test for varying μ with an underlying normal distribution with $\sigma = 1$, and for varying θ with an exponential underlying distribution are considered. Minitab macros for both underlying distributions are created (see Table 1). The T-statistic varies with the average value of the sample, the sample standard deviation, and the number of elements in the sample. The computer macro for approximating the power of the *T*-test has to account for this. The assigned value of θ for the *T*-test is taken as 0.05. The samples are tested against the null hypothesis that μ is qual to μ_0 (i.e. $H_0 : \mu = \mu_0$), and the alternative hypothesis that μ is greater than μ_0 (i.e. $H_a : \mu > \mu_0$). The null hypothesis is then rejected at the assigned α value where the T-statistic is greater than the tabulated T-value for given α and $n-1$ degrees of freedom (the parameter of the t-distribution).

An initial indexing value has to be assigned outside of the macro in Minitab. The number of repetitions for the simulation (R) and the number of elements (n) in each sample also has to be assigned outside of the macro for efficient computing. A column of values for μ and for θ has to be defined for each underlying distribution to find an approximate value of the power at each value of μ or θ for that distribution. The rest of the computation, including simulating data for each underlying distribution, running a *T*-test on each sample, and finding an approximate value for the power of the T-test for each value of the mean for both the normal and the exponential distributions is done within the macros.

The power value for each given value of the mean of the underlying distribution is stored in a column in Minitab so that it can be graphed against its corresponding underlying mean. The error in these approximate values can be approximated using the calculated power values as the approximate p-values of a binomial distribution. The null hypothesis is rejected for each data sample from a given underlying distribution with an approximate probability equal to the value of the power. The standard deviation for the approximate values for power is then;

$$s = \sqrt{p(1-p)/n} \quad \text{and} \quad error = \pm 1.96 * s.$$

Table 1. Minitab macro used to generate the data for the various sample sizes for the normal underlying distribution:

```
SET C99                                  Data in C99 is values of μ to be tested
LET K1 = index number (1 to start)
LET K2 = number of elements, n
LET K3 = number of repetitions, R
EXECUTE 'power' m                        Where 'm' was the number of μ values tested

'Power.mtb'
LET K8 = K2-1
LET K9 = K2+1
LET K10 = K2+2
LET K11 = K2+3
LET K12 = K2+4
INVCDF 0.95 K6;
t K8.
LET K5 = C99(K1)
RANDOM K3 C1-CK2;
NORMAL K5, 1.
RMEAN C1-CK2 CK9
RSTD C1-CK2 CK10
LET CK11 = SQRT(K2)*CK9/CK10
LET CK12 = CK11>K6
MEAN CK12 = K4
LET C100(K1) = K4
LET K1 = K1+1
END
```

The output values for the power at given μ are the corresponding elements of C100. To simulate the data for the exponential distribution, to do the *T*-test on each sample, and to calculate the approximate values of the power at given values of θ, a new column C99 is created with respect to the fact that θ is greater than zero for the exponential distribution. The only line in the macro given in Table 1 that needs to be changed than is that:

NORMAL K5, 1. was changed to EXPONENTIAL K5.

These macros are run for sample sizes of 5, 10, 15, 25, 35 and 50 for each of the distributions. The plots of the simulation results are presented in Figure 1 for normal distribution and in Figure 2 for exponential distribution case. The comparison of the two cases where the null and the alternative hypotheses, respectively, are $H_0 = \mu = \mu_0 = 1$ and $H_a : \mu > \mu_0$ with the sample size of $n = 50$ is shown in Figure 3.

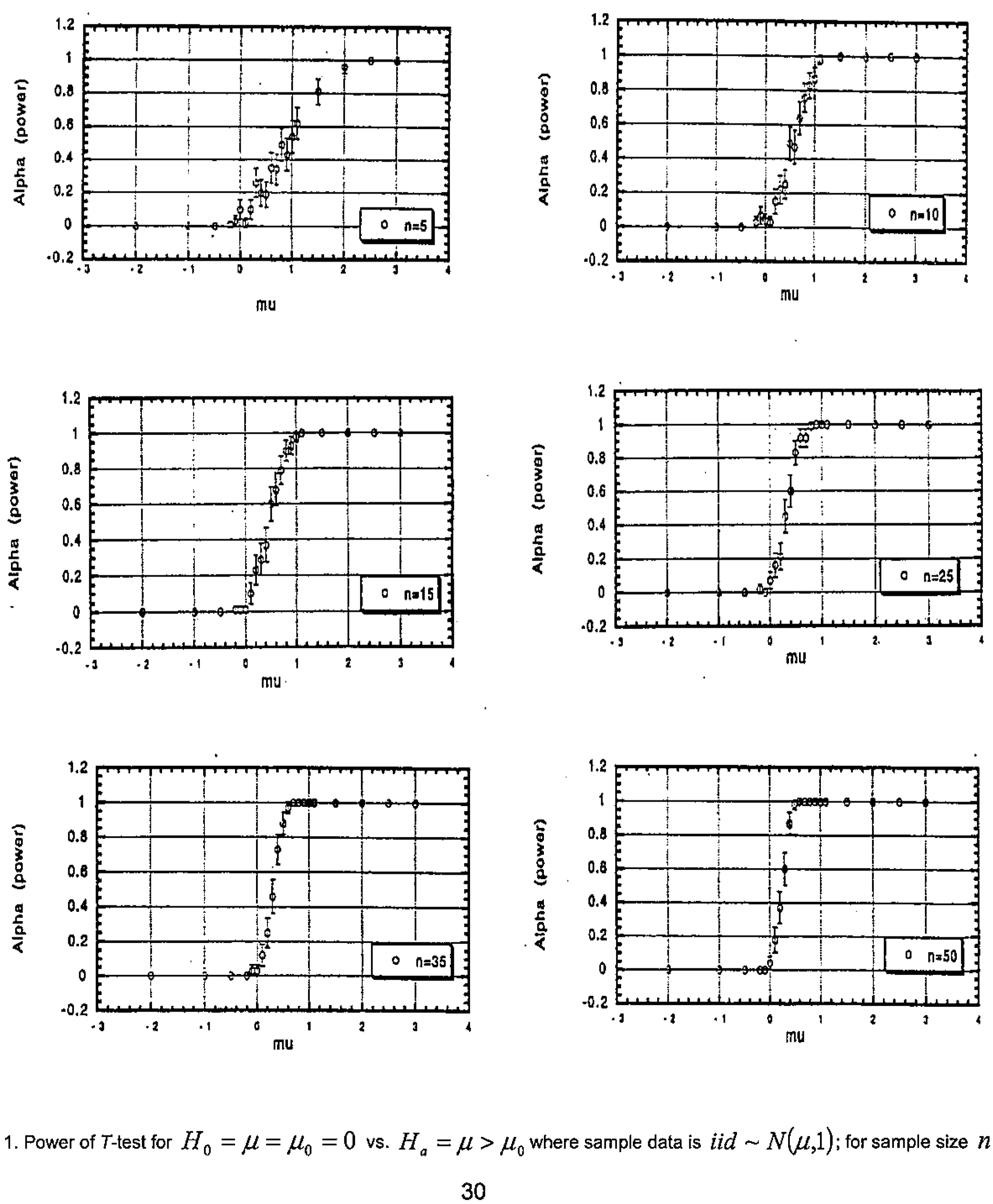

Figure 1. Power of *T*-test for $H_0 = \mu = \mu_0 = 0$ vs. $H_a = \mu > \mu_0$ where sample data is $iid \sim N(\mu,1)$; for sample size n.

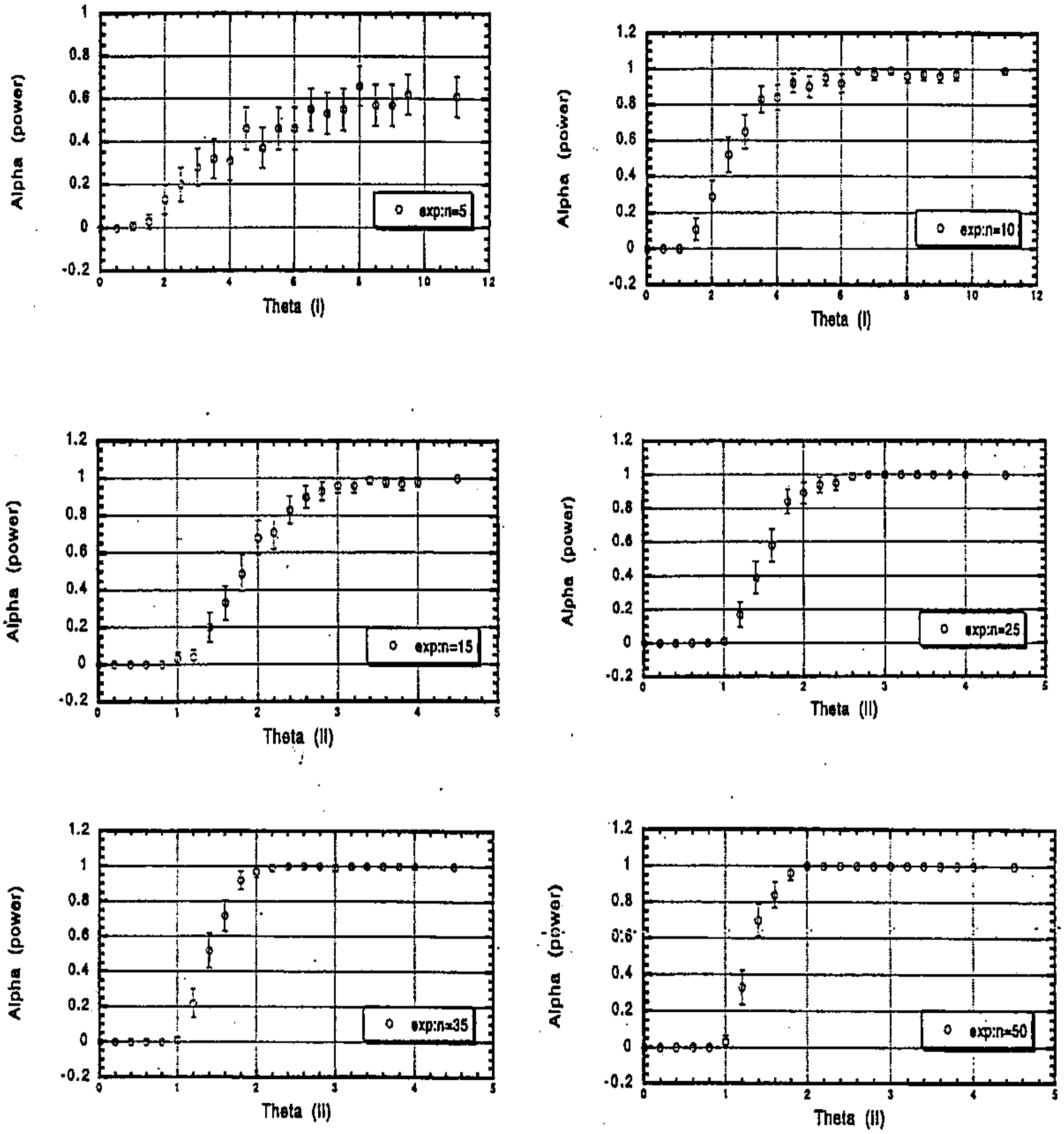

Figure 2. Power of *T*-test for $H_0 = \mu = \mu_0 = 1$ vs. $H_a = \mu > \mu_0$ where sample data is $iid \sim \exp(\theta)$; for sample size n.

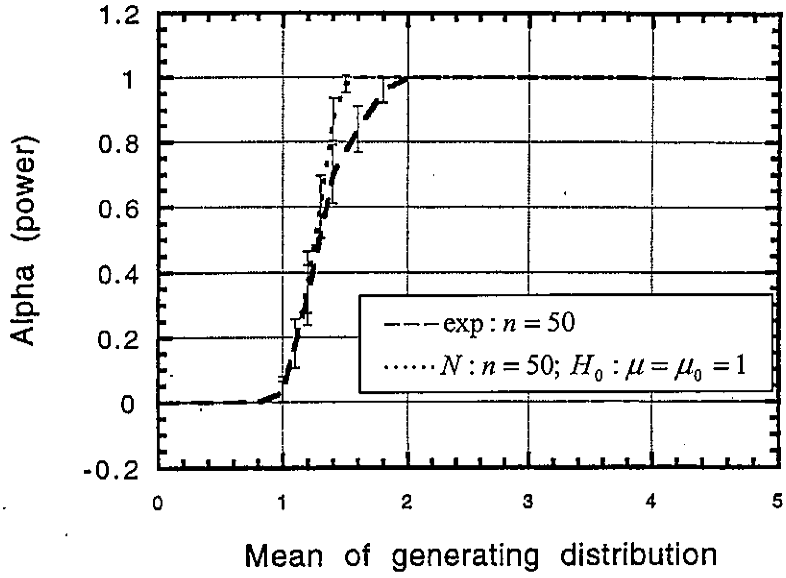

Figure 3. Power function for $n = 50$ for $X \sim iid\ N(\mu,1)$ and for $X \sim iid\ Exp(\theta)$ of the one-tailed T-test for $H_0 = \mu = \mu_0 = 1$ vs. $H_a = \mu > \mu_0$.

Discussion and Conclusion

As the sample size increases the power functions for the one-tailed *T*-test for the exponentially distributed data samples and the normally distributed samples seem to converge. Further study of this would be of interest to see if this could be due to the mean value theorem taking effect for sample averages of large samples with equal means. The results of the study are interesting in themselves in that they show the strong dependence of the power function on the sample size. The power function seems more dependent on sample size than on whether or not the assumption that the sample data being tested is normally distributed, at least when compared to exponentially distributed sample data.

References

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