

Research Article

Mathematical understanding and reasoning of vocational school students in agriculture-based mathematical tasks

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Abstract

Mathematical understanding and reasoning are important in solving agriculture problems. This research aims to describe students' mathematical understanding and reasoning in agriculture-based mathematical tasks. This mathematical task corresponds to the competency of students' expertise to minimize the gap between mathematics learned at school and the workplace. This research used a descriptive case study method. Participants are students of the vocational crops and horticulture agribusiness of 11th graders students. Data collected through tasks, observations, and interviews. Data analysis through constant comparative techniques to find out students' understanding (coherence, correspondence, and connection) and reasoning (algorithmic or creative). The results show students' mathematical understanding and reasoning was influenced by the design of tasks and students' experiences. Both algorithmic and creative reasoning, should by the plausibility of the reality of workplace practice in agriculture to affects the ability of coherence and correspondence of students' mathematical representations. Mathematical knowledge and experience affect the whole process of solving the tasks. The results of this research have implications for the design of mathematical tasks in vocational schools in agriculture to support problem-solving in the workplace. The development of mathematical tasks can continue to be done given the many problems in agriculture involving mathematics in solving them.



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Introduction

Vocational mathematics is mathematics used at work (Bakker, Groenveld, Wijers, Akkerman, & Koeno, 2014). Mathematics in the workplace is routine and invisible (Bakker, Hoyles, Kent, & Noss, 2006; FitzSimons & Björklund Boistrup, 2017), such as black boxes (Williams & Wake, 2007). Mathematics for the workplace includes levels of mathematical thinking and problem-solving (National Council of Teachers of Mathematics, 2000). In vocational education, mathematics learning aims to prepare students for the workplace (Bakker, 2014; FitzSimons, 2014). Therefore, learning mathematics in vocational schools should be adjusted to the skills chosen by students so that it is useful in a variety of problems in the workplace.

There are gaps between mathematics learned in school and mathematics used in the workplace. Several studies have shown it. Differences in the use of mathematical concepts occur when the practice of bending the power lines between learning in school and being used in the workplace (references at school are formal trigonometry, while at work is a rule of practice) (Roth, 2014). Observations of pipe trade training students show that it is often difficult for students to connect their mathematical understanding of the problem of pipe production (LaCroix, 2014). This happens because school mathematics is not applied in everyday life or at work (Nunes, Schliemann & Carraher, 1993). The connection between mathematical concepts learned in school and at work is very important to overcome the

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gaps that occur.

The development of mathematics learning following the workplace has been done by researchers. The following research show this effort. The application of inquiry-based learning and connection to the world of work is carried out to support 21st-century skills (Maass & Engeln, 2019). The design of computer equipment for calculating the concentration of chemical dilution is used to develop students' proportional reasoning abilities in vocational high school laboratory technicians (Bakker, et al. 2014). Integrating mathematics, statistics with knowledge in the workplace through a boundary-crossing approach (Bakker & Akkerman, 2014). Research into mathematical calculations in the field of nursing (Coben & Weeks, 2014). Research on the meaning of mathematics is based on the perspective of telecommunications regulatory technicians (Triantafillou & Despina, 2010). Research on students' mathematical understanding of technical calculations in glass factories (Magajna & Monaghan, 2003). Analysis of nursing activities on drug administration and monitoring of fluid balance to be used as ideas in the design of didactic strategies in teaching vocational mathematics (Pozzi, Noss & Hoyles, 2003). This is very promising for researchers in the field of vocational mathematics education because of the many fields of expertise offered at vocational schools.

The field of agriculture is the focus of this research because it uses a lot of mathematics in solving problems. When working, farmers use practical and mathematical skills (Muhrman, 2015). Rasor stated that agricultural students must have mathematical knowledge and understanding, namely knowledge of simple calculations and understanding of statistical methods and probabilities to support the interpretation of experimental results (C, 1923). Agricultural students need mathematics in solving various problems in agriculture both from the aspect of mathematical concepts and mathematical thinking processes.

The teacher can implement mathematical concepts in workplace situations through mathematical tasks. A mathematical task is a form of social practice, carried out by teachers and students collectively (Johnson, Corel, & Clarke, 2017). The mathematical task is an artifact that refers to the theory of socio-didactical tetrahedron which is carried by Rezat & Strässer (2012). This theory pays attention to the importance of the relevance of mathematics in society. Mathematical tasks should be able to communicate and inform mathematics that is more coherent and implemented in the future (Thanheiser, 2017). Besides, mathematical task design pays attention to learning objectives, task types, and task variables (Yeo, 2017). Therefore, it is important to design mathematical tasks that have to learn goals that support implementation in the workplace.

Lithner (2008) stated three mathematical abilities that are important in solving mathematical tasks, namely problem solving, conceptual understanding, and reasoning. On the other hand, problems in the workplace require mathematical understanding (FitzSimons, 2014; Swanson, 2014) and reasoning (FitzSimons & Björklund Boistrup, 2017; Bakker, et al. 2014; Bakker & Akkerman, 2014). These abilities can be active with the support of contextual, conceptual, and procedural knowledge (Sáenz, 2009). Mathematical tasks that present problems at the workplace can be designed to facilitate the development of mathematical understanding and reasoning.

Many opinions about mathematical understanding from various perspectives. For example, Greeno (1978) states the criteria for mathematical understanding are coherence, correspondence, and connection. Mathematical understanding as an instrumental or relational understanding (Skemp, 1976; Piere & Schwarzenberger, 1988). Mathematical understanding emphasizes the concept development process which is characterized by rediscovering concepts, presenting concepts, and using examples and comparisons for analog reasoning (Cai & Ding, 2017). More specifically, we can see the growth of students' mathematical understanding using dynamical theory (Pirie & Kieren, 1989). Besides, we can see the level of student understanding that consists of primitive knowing, image-making, image-having, property noticing, formalizing, observing, structuring, and inventorying (Pirie & Martin, 2001). These opinions indicate that mathematical understanding applies following the context of the problem.

Mathematical reasoning has also been variously identified by researchers. The reasoning is the mathematical foundation in logical thinking (Ross, 1998). The function of reasoning is verification, explanation, systematization, discovery, communication, theoretical construction, and exploration (Yackel & Hanna, 2003). The reasoning transfers can occur between students (Hershkowitz, Tabach & Dreyfus, 2016). The reasoning is a thought process that is influenced by students' competencies and environment which consists of two main types namely creative and imitative reasoning (Lithner, 2008). The reasoning can focus on a particular concept, for example, reversible reasoning in inverse function problems (Ikram et al. 2020). Mathematical reasoning can also be seen when students solve mathematical tasks. For example, the results of research on students when solving mathematical tasks show that their reasoning abilities are still shallow, most choices of strategy and implementation without considering intrinsic mathematical

properties (Lithner, 2000, 2003). Therefore, the development of mathematical tasks continues to be done to improve student reasoning (for example, Johnson, McClintock, & Hornbein, 2017). Likewise, with vocational students, they need to develop mathematical reasoning abilities that they will use later in the workplace.

Problem of Research

The mathematical task is one of the tools to overcome the gap between mathematics learned at school and used at work. Besides, the complexity of problems at work requires vocational school students to get used to solving problems in workplace situations. Agriculture is one of the fields that need mathematics in solving problems. Based on the literature (for example, FitzSimons & Björklund Boistrup, 2017; Bakker, et al. 2014), mathematical understanding and reasoning are two important aspects of solving problems in the workplace. The problem in this study is divided into two parts as follows:

- Sub-problem 1 : How are students' mathematical understanding viewed from aspects of coherence, correspondence, and connection?
- Sub-problem 2 : How are students' mathematical reasoning viewed from aspects of imitative or creative?

This research focuses on the ability of mathematical understanding and reasoning when vocational students solve agricultural-based mathematical tasks. The exploration of these abilities is carried out in-depth, both to the process and the results of thinking.

Method

This study used a qualitative approach with a case study design (Hancock & Algozzine, 2006). The stages of the study are described in the following Table 1.

Table 1.
Stages of Case Study Research

Research Stages	Description of Activities
Setting the stage	Established the mathematical understanding and reasoning when vocational students complete agricultural-based mathematical tasks as observed phenomena.
Determining what we know	<ol style="list-style-type: none"> 1. Designed agriculture-based mathematical tasks based on various references and information from agribusiness and mathematics teachers. This unusual mathematical task design is given to students in a mathematics class. 2. Test of the validity and reliability of mathematical tasks. 3. Chosen students of vocational of Food Crop and Horticulture Agribusiness class XI as participants. This expertise is a part of agriculture. Students in this class have learned and practiced food crops and horticulture.
Selecting a design	Chosen the type of case study, which is a descriptive design. The researcher wants to present a complete description of students' understanding and mathematical reasoning in a specific context, which is when students complete mathematical tasks in agricultural situations.
Observation	Conducted observations while students complete tasks.
Interview	Conducted interviews with students with typical task completion cases.
Documentation	<ol style="list-style-type: none"> 1. Analyzed student understanding and reasoning from student answer sheets. 2. Playback video and audio students during the mathematical task completion process.
Summarizing and interpreting the information	<ol style="list-style-type: none"> 1. Summarized the result of the observations. 2. Interpreted students' understanding and reasoning from the results of observations, interviews, and student answer sheets refers to Greeno's mathematical understanding (1978) and Lithner's mathematical reasoning (2008) theory. 3. Grouped of students' responses based on the specificity of task completion (called a case).
Reporting finding	Conducted synthesis of findings following the objectives of the study.
Confirming case study finding	<ol style="list-style-type: none"> 1. Compare research findings with other relevant studies. 2. Discussed the results of research with mathematics and agribusiness teachers.

Participants

Participants in this research were selected based on the purposeful sampling technique. Selection of participants with several considerations to obtain information that is useful in investigating the ability of mathematical understanding and reasoning in solving agriculture-based mathematical tasks. Participants are students of the vocational crops and horticulture agribusiness of 11th graders students' (fourth semester) in Ciamis, Indonesia. Participants were chosen one class (23 people) to be observed in the process of solving tasks given to students after the three-dimensional sub-topic was given. Participants have learned the topics of geometry and plant cultivation. This research was conducted in 2019.

Researchers got information about the mathematical abilities of participants from mathematics teachers and information about the ability of students' theory and practice of agribusiness from agribusiness teachers. These abilities are seen from the assessment of the process and student learning outcomes for three semesters. The characteristics of the participants are presented in Table 2 below.

Table 2.
Characteristics of the Students Participating in the Study

Variable	Characteristics	Frequency	%
Gender	Male	16	70
	Female	7	30
Family's background	Agriculture sector	18	78
	Non-agricultural	5	22
	Excellent	5	22
Mathematical ability of students (mathematics teacher perspective)	Good	7	30
	Medium	5	22
	Poor	6	26
Student knowledge on food crop and horticulture (agribusiness teacher perspective)	Excellent	6	26
	Good	11	48
	Medium	5	22
	Poor	1	4

Data Collection

Data collected through three tasks, observation, and interviews. The three tasks are designed by taking the agricultural context. The design involves agribusiness teachers. Researchers conducted a documentation study and preliminary interviews of agribusiness teachers to get an agricultural context that was following the topic of the three dimensions. Based on this information, the researcher then makes mathematical tasks.

The observation was carried out as long as students solved agricultural-based mathematical tasks. The researcher notes typical task solve cases to be followed up in the interview process. Observation is aimed at how students carry out the process of coherence, correspondence, and connection representation of the given task contexts that will affect the type of student reasoning.

The interview is the final process of data retrieval. The researcher's interpretation of the results of student answers and the results of observations were confirmed through interviews. Interviews were conducted using one-on-one interview techniques. The focus of the interview is to explore students' thought processes. The main questions given to students are about how to do coherence, correspondence, and connections from contexts to the task and how does the environment influences the idea of solving the task.

The mathematical tasks designed are real-world or word-problems to identify mathematical understanding and reasoning abilities in agricultural situations. This situation has never been experienced by students in mathematical tasks before, both in class and in mathematics textbooks. Task-1 and Task-3 design to encourage students to do creative reasoning, while Task-2 encourages imitative reasoning. Table 3 below presents the tasks that have been given to students.

Table 3.
Task to Identify Students' Mathematical Understanding and Reasoning Abilities

Tasks variable	Tasks situation
The task with open methods and closed answers	<p>Task-1</p> <p>A farmer will make aquaponics in preparation for fish and vegetable cultivation. Aquaponics consists of one pond and five pipes. The walls of the pond are made of glass with dimensions of length, width, and height of 1.2 meters, 1.2 meters and 0.8 meters respectively. The pipe is placed above the pool which is parallel to the pond.</p> <ol style="list-style-type: none"> How long are all the pipes needed for aquaponics? How high is the water in the pond if the farmer only fills 4/5 of the height of the pond? <p>Note: Aquaponics is an agricultural system that combines aquaculture and hydroponics. The aquaponics system is mutually beneficial because it gets two farming commodities, namely vegetables and fish at the same time.</p>
The task with open methods and answers	<p>Task-2</p> <p>A farmer will do cucumber cultivation. He prepares 'bedengan' and completes them in the 'lanjaran' of a triangle model. Lanjaran has a length of 2 meters. The width of the bedengan is 1 meter. Determine the angle formed by the two uppers of lanjaran (tied by a rope).</p> <p>Note: Bedengan is a place to grow plants that are cultivated. Lanjaran or ajir is a stick to support or propagate plants.</p>
The task with open methods and answers	<p>Task-3</p> <p>A farmer owns a 5 bata paddy field. The farmer will do the "minapadi" business. Around the land is made a trench with a depth of 80 centimeters and installed mulch. The trench will be used for fish farming. Mulching is used to anticipate water leaks and reduce weed growth. Determine the mulch area needed by the farmer.</p> <p>Note: Minapadi (mina = 'fish' and padi = 'rice') is a form of combined farming that utilizes a pool of paddy water planted with rice as a pond for fish farming. Bata is a traditional unit that shows a large scale.</p>

Mathematical tasks are designed by utilizing the context of crop cultivation. Mathematical tasks that are designed are discussed with agribusiness teachers (3 people) to see the suitability of the context or situation of the task with the reality of plant cultivation theory and practice. Mathematical tasks are first tested on students outside the participants. The results of the tests of the validity and reliability of mathematical tasks empirically are high.

Mathematical tasks are given to students in three different meetings according to the three-dimensional sub-topic given by the teacher. While students work on tasks, researchers conduct observations and recordings through video. After students finish solving the task, the researcher interviews students based on the findings of typical task completion.

Data Analysis

This research triangulated through three types of data, namely student answer sheets, field notes, and interview transcripts. All three are used to corroborate findings throughout the study. These data are analyzed by constant comparative techniques. Direct observations produce field notes that are verified by playing back recordings. Observation results are preliminary data on the findings that will be compared with the results of student answers and the results of the interview.

Data analysis of observation, student answers, and interview results was conducted by comparing them with indicators of mathematical understanding (Greeno, 1978) and reasoning (Lithner, 2008), based on Table 4. Then, the data were classified according to the cases that emerged. After that, the findings that were produced were discussed with the mathematics teachers and the agribusiness teachers of food crops and horticulture. The teachers understand the characteristics of all participants because they are involved in the class of students every day.

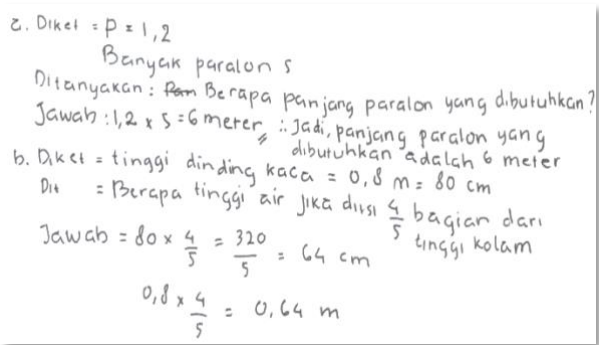
Table 4.
Indicators of Mathematical Understanding and Reasoning

Mathematical Ability	Indicator
Mathematical Understanding	<ol style="list-style-type: none"> The internal coherence of representation by connecting information with other things that are known. Correspondence with true/actual meaning. Connection of specific problems or situations with concepts; connection procedure with the concept; connection procedure with the concept; concept connection with concept.
Algorithmic reasoning	<ol style="list-style-type: none"> The choice of strategy is to remember the solution algorithm (no need to create a new solution). If students are not thorough, reasoning can be implemented wrongly.
Creative reasoning	<ol style="list-style-type: none"> Creativity: students create a sequence of reasoning that was not experienced before, or re-create. Plausibility: states predictive arguments and verification of strategy choices; explain why the application of strategies and conclusions is true or reasonable. Anchoring: arguments based on the intrinsic mathematical nature of the components of reasoning.

Results

This section will describe students' mathematical understanding and reasoning abilities for each task. The description is a special case that occurs in the solution of a task. Each task has two cases of solve. The following are the findings.

Table 5.
Students' Response to Task-1 Case-1

Students' Response	Translation
 <p> a. Diket = p = 1,2 Banyak paralon 5 Ditanyakan: Berapa panjang paralon yang dibutuhkan? Jawab: $1,2 \times 5 = 6$ meter. Jadi, panjang paralon yang dibutuhkan adalah 6 meter. </p> <p> b. Diket = tinggi dinding kaca = 0,8 m = 80 cm Dit = Berapa tinggi air jika diisi $\frac{4}{5}$ bagian dari tinggi kolam Jawab = $80 \times \frac{4}{5} = \frac{320}{5} = 64$ cm $0,8 \times \frac{4}{5} = 0,64$ m </p>	<ol style="list-style-type: none"> Given: p (= length) = 1.2 many pipes 5 Asked: How long is the pipe needed? Solution: $1,2 \times 5 = 6$ meters So, the required pipe length is 6 meters. Given: height of glass wall = 0.8 m = 80 cm Asked: how much water if filled $\frac{4}{5}$ part of the height of the pond Solution: $80 \times \frac{4}{5} = \frac{320}{5} = 64$ cm $0,8 \times \frac{4}{5} = 0,64$ m

The students' response in Table 5 represents the first case in task-1. Indications students who have the ability to coherence can connect all information on the task with their knowledge (size and length of the pipe, pool dimensions). Besides, they know the purpose of the task. The indication of correspondence ability is to interpret the length of the pipe, to interpret the height of the water to the height of the pond (the concept of comparison). An indication of connection capability is the connection of context to the concept (making relations of numbers), the connection of procedures (performing unit and computational conversions). The indications of coherence, correspondence, and connection ability indicate that students have mathematical understanding abilities in solving Task-1.

Students can create their line of thought in solving tasks. That can be seen from the process of coherence, correspondence, and connection. During the interview, students stated that he had gotten this task for the first time. Students also already know the aquaponics system, but have never carried out the practice of aquaculture through aquaponics. In this case, students experience creative reasoning.

Table 6
Students' Response to Task-1 Case-2

Students' Response	Translation
	<p>Solution:</p> <p>a. Pipe length needed. Given the length of 1 pipe is 1.2 m and there are 5 pipes then $1,2 \times 5 = 6$ meters.</p> <p>b. Given: height of pond = 0,8 m Volume = 4/5 Asked: high water in a pond? Solution: $\frac{\text{high}}{\text{volume}} = \frac{0,8}{4/5} = 0,64$ meters</p>

The students' response in Table 6 represents the second case in task-1. They answered correctly in Task-1 (a). They can do coherence, correspondence, and connections. Conversely, in Task-1 (b), students fail at the connection process, namely making connections (operations) of numbers. Based on the interview, they have not correctly interpreted the nature of the comparison. They see the word 'part' as 'division operation'. Besides, they also have not computed correctly.

Based on interviews, students utilize line and field position knowledge at the coherence and correspondence process. Next, they make a mathematical connection at the connection process. Even though the connection process (Task-1, b) is wrong, students have tried to create creative lines of thought.

Table 7.
Students' Response to Task-2 Case-1

Students' Response	Translation
	<p>Given: length of lanjaran (AC dan BC) = 2 m. length of bedengan = 1 Asked: angle between two lanjaran or $\angle ACB$ Solution:</p> $c^2 = a^2 + b^2 - 2ab \cos \angle C = 2ab \cos \angle C$ $\cos \angle C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{2^2 + 2^2 - 1^2}{2 \cdot 2 \cdot 2} = \frac{7}{8}$ <p>So, the angle between two lanjaran is 29°</p>

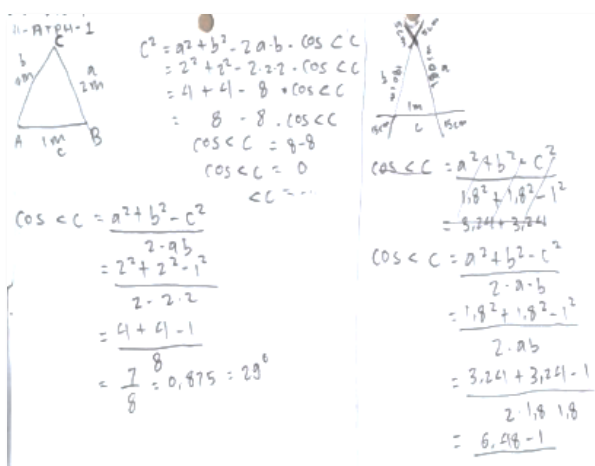


Figure 1.
Students' Response to Task-2 Case-2

The students' response in Table 7 represents the first case and Figure 1 represents the second case in task-2. Students in the process of coherence and correspondence can connect information from the task and correct it on the triangular sketch. The students in the first case are only fixated on the situation and the numbers in the task. In contrast to students in the second case, students realize and try to interpret the task based on the perspective in the field (the workplace). They are aware of the plausibility of the coherence and correspondence process. The next process is the connection, students determine the formula, do the substitution and computation. Based on interviews, students make connections by taking formulas they have learned in class. That is, at the connection process, student reasoning is algorithmic.

Table 8.
Students' Response to Task-3 Case-1

Students' Response	Translation
	<p>Given: 5 bata 1 bata = 14 m² So, 14x5=70 m². Depth=80 cm. Mulch width of 80 centimeters is stretched to = 1 m. Mulch is needed 4.5 m if pulled to 5 m. The required mulch length is 13.5 m because it is stretched to 14 m.</p> $p + l = 13,5 + 4,5 = 18$ $18 m + 18 m = 36$ $p \times l = 36 \times 0,8 = 28,8 m^2$ <p>So, the area needed is 28.8 m².</p>

The students' response in Table 8 represents the first case in task-3. In the coherence and correspondence process, they can identify information and relate it to practical knowledge (unit of 'bata', landform, mulching elasticity, mulch position). In the connection process, they can connect the task context with the circumference formula and the area of the rectangle. Next, they do the computing. From the aspect of reasoning, students can determine their reasoning sequence, can provide reasonable completion arguments, and can provide arguments supported by mathematical properties. The reasoning is categorized as creative reasoning.

Table 9.
Students' Response to Task-3 Case-2

Students' Response	Translation
	<p>Given: 1 bata=14,28 m² 5 bata=71,4 m² Depth of the trench =80 cm Mulch expands, so it requires only 90%.</p> $L = p \times l$ $L = 14,28 m^2 \times 5 m^2 = 71,4 m^2$ <p>Because mulch can expand when exposed to heat, the required mulch = 71,4 m² × 5 m² × 90% = 51,408 m² So, the mulch needed is 51,408 m²</p>

The students' response in Table 9 represents the second case in task-3. From the aspect of understanding, students can identify information, relate it to the practical knowledge, and correctly interpret unit conversions. The misunderstanding occurs at the position of mulching. Although they have not been able to complete this task, the reasoning they do is categorized in creative reasoning.

Discussion and Conclusion

The findings show that students who solve tasks are students who have the ability to coherence, correspondence, and connections. The coherence process in this study is demonstrated by the ability of students to associate all information on the task with prior knowledge. Students can represent information in agricultural situations in mathematical forms (geometric, algebraic, or number operations). In this situation, students can connect information with other things they know (Greeno, 1978). Also, students can assimilate situations to be following the individual's knowledge exiting (Skemp, 1976).

Correspondence is more specific than coherence because it must interpret each sentence and know what concepts are involved in that context. Coherence can appear earlier then continue the process of correspondence or vice versa. The results of the interview show that students who do coherence usually determine the keywords to find out the purpose of the task question. On the other hand, students who did earlier correspondence will make variables of the elements that are known in advance. The process of coherence and correspondence is the process of transition between contexts to mathematical concepts. Transitions occur between abstract mathematics and concrete mathematics at work (Bakker, 2014). This is a contextualized mathematical re-contextualization process with workplace activities (FitzSimons & Björklund Boistrup, 2017).

Based on the findings, there are three types of connections, namely context connections with concepts, connections between concepts, and connection procedures. Connection context-concept is a process of determining the appropriate mathematical tools (formulas, rules, or procedures) to complete the task. In this process, students must be able to adopt appropriate procedures from given task situations (Piere & Schwarzenberger, 1988). Students must know the relationship of concepts to the task situation, reality, or experience (Greeno, 1978). Students must determine general rules that are appropriate to the context (Skemp, 1976). In connections between concepts, students must be aware of the relationship between concepts (Piere & Schwarzenberger, 1988). The procedure connection includes understanding operations and mathematical relationships (Kilpatrick, Swafford, & Findell, 2001) either realized (knowing why the rules are used) or not realized by students (Skemp, 1976) because it is an ordinary experience (Greeno, 1978).

The students' abilities of coherence, correspondence, and connection can determine the sequence of student reasoning. Based on findings, the knowledge and experience of practice in the workplace affect the coherence and correspondence abilities of students' mathematical representations. Students' coherence and correspondence abilities help in the process of connection between agricultural contexts and mathematical concepts. The process of connection between concepts and procedures involves mathematical conceptual knowledge. This is consistent with the opinion of Lithner (2008) that the sequence of reasoning is influenced by the thought process, mathematical competence, and student environment.

Identification of thinking processes from the way students choose strategies, pour them on the answer sheets and or express them during interviews. In Task-2, students choose strategies to remember the rules and analogize the results of coherence and correspondence with procedures that they have learned. In Task-1 and Task 3, students make their settlement procedures following the context of the task and the reality at the workplace. The selection of these strategies shows that the use of mathematics is subjective (Roth, 2014).

Contextual, conceptual, and procedural knowledge are important aspects of activating mathematical competence (Sáenz, 2009). Contextual knowledge is useful for connecting physical or virtual knowledge objects with mathematics (FitzSimons, 2014). The context, concepts, and procedures for solving tasks must be connected logically to believe that the construction and implementation of task completion are correct (Lithner, 2017). These conditions are indicated by plausibility and anchoring in reasoning.

Plausibility is an important finding in this study. Lithner (2008) states that plausibility and anchoring are indicators of mathematical creative reasoning. The research findings show that plausibility and anchoring are needed in algorithmic reasoning type. The reason is that the concept, formula, or procedure chosen should be following the task

context and mathematical properties (conceptual and procedural). This kind of reasoning is included in plausible reasoning criteria (Lithner, 2000, 2003).

Student learning experiences can be obtained from the environment. Agriculture-based mathematical tasks in this study are designed as an environment to support student reasoning. Design of tasks attention to task types and variables. The findings show that task design influences the sequence of reasoning. This is consistent with the opinion (Lithner, 2017). Finally, we can conclude about mathematical understanding and reasoning in solving agriculture-based mathematical tasks. This conclusion is related to indicators that can be used to identify students' mathematical understanding and reasoning abilities.

Coherence, correspondence, and connection are three indicators that can identify students' mathematical understanding in solving agricultural-based mathematical tasks. First, coherence is the ability of individuals to associate all information on a task with prior knowledge. Coherence has criteria: determining the scope of material (mathematical concepts) that correspond to all information; represent the results of understanding in a model (sketch, picture, graphic, equation, inequality, or function). Second, correspondence is the ability to correctly interpret the context (sentence or picture) representation with the concept it understands. Correspondence has the following criteria: determining the elements of the sketch; determine the variables and values (constants) in an function. Third, connections consist of context connections with concepts, connections between concepts, and procedure connections. Context with the concept is the ability to determine the appropriate mathematical tools (formulas, rules, or procedures) to solve the task. The connection between concepts is the ability to connect several relevant concepts to support task completion. The connection procedure is the ability to perform various procedures (number operations, conversions, or algorithms) to support task completion.

There are two types of reasoning in the case of solving agriculture-based mathematical tasks, namely algorithmic and creative reasoning. Algorithmic reasoning is indicated by choosing a strategy that suits the context of the task (plausibility) and mathematical properties (anchoring) based on previous learning experiences and implementing them correctly. Creative reasoning is indicated by making their strategies following the context of the task (plausibility) and mathematical properties (anchoring) and implementing them correctly.

Recommendations

Agriculture involves a lot of mathematics in solving problems. This opens up opportunities for researchers interested in researching the area of vocational mathematics education in agriculture. Future studies can identify other mathematical abilities in solving problems in agriculture. Research can also be carried out to explore the more specific mathematical needs of a socio-cultural. The results of his research will be very useful for mathematics teachers in vocational schools to design mathematical tasks for students who are following their competency expertise.

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Limitations of Study

This research only describes the students' mathematical understanding and reasoning abilities when solving mathematical tasks to a group of class 11 students in an agricultural school. Mathematical tasks that are designed in the context of agribusiness are very limited to a socio-cultural area.

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