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Investigation of Linear Vibration Behavior of Middle Supported Nanobeam

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Abstract: In this study, linear vibration of middle supported nanobeam, which is commonly used in nano electro-mechanical systems, is analyzed. Eringen's nonlocal elasticity theory is used to capture nanoscale effect. Equation of motion of nanobeam is derived with the Hamilton principle. Multiple scale methods, which is one of the perturbation techniques, is performed for solving the equation of motion. Support position and nonlocal effect are focused on the research. The results are presented with graphs and table. In conclusion, when the nonlocal parameter is getting a raise, more nanoscale structure is obtained. Highest rigidity and linear natural frequency are received with mid-position of the support.

Keywords: Nanobeam, linear vibration, nonlocal elasticity, perturbation methods

Ortadan Destekli Nano Kirişin Doğrusal Titreşim Davranışının İncelenmesi

Öz : Bu çalışmada, nano elektromekanik sistemlerde yaygın olarak kullanılan ortadan destekli nano kirişin doğrusal titreşimi analiz edilmiştir. Nano ölçeği yakalayabilmek için Eringen'in yerel olmayan elastisite teorisi kullanılmıştır. Nano kirişin hareket denklemi Hamilton prensibi ile elde edilmiştir. Hareket denklemini çözmek için pertürbasyon tekniklerinden biri olan çok ölçekli metot uygulanmıştır. Orta mesnet pozisyonu ve yerel olmayan etki araştırmada odak nokta olmuştur. Sonuçlar grafikler ve tablo ile sunulmuştur. Sonuç olarak, yerel olmayan parametre artmasıyla daha fazla nano ölçekli yapı elde edilmektedir. Orta mesnet pozisyonu tam orta konumda iken en yüksek rijitlik ve doğrusal doğal frekans değeri elde edilmiştir.

Anahtar Kelimeler: Nanokiriş, doğrusal titreşim, yerel olmayan elastisite, perturbasyon metodu

1. Introduction

When recent studies on technology are examined, it is seen that the technological trend is moving towards smaller and faster. The ultimate point of scientific research and technology is that not only is product performance faster, but its dimensional parameters are also required to be at the lowest as possible level as. For this reason, micro/nano electro-mechanical systems known as MEMS and NEMS are frequently seen in scientific research and technological studies in recent years.

Nowadays, due to their importance in technological research, nanoscale structures, nanoparticles which are one-billionth of a meter, like beams, bars, plates, are found their places in many application areas such as optics [1,2], electricity [3], energy [4], chemistry [5], health [6], thermal [7] biomechanics [8], biology, medicine [9].

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Researchers have developed several numerical methods in order to continue their work on nano-sized due to the difficulty in doing experimental studies in different conditions for nano-sized structures and requiring high economic value workstations for simulation studies. These developed numerical methods are divided into two parts, basically like continuum mechanics and atomistic approaches [10]. The atomistic approach is divided into two different branches like molecular dynamics [11], density functional theory [12]. These methods often consume much time and could not capture enough small size effect.

Therefore, continuum mechanics theory is used more widely. Different continuum theories are derived from catching the nearest effect for nano size. They have modified couple stress theory [13], micropolar elasticity theory [14], the strain gradient theory [15], surface elasticity theory [16] and Eringen' s nonlocal elasticity theory [17]. Nonlocal elasticity theory, which is applied in this work, is commonly used in the nanostructure. Peddieson et al. [18] have developed a nonlocal Euler Bernoulli beam model using the nonlocal elasticity theory. They focused on bending problems for the cantilever-free and simple-free supported beams model. Reddy [19] investigated Euler-Bernoulli, Timoshenko, Reddy and Levinson beam theories by using Eringen' s nonlocal differential constituent relations and Hamilton method. The bending, buckling and vibration behaviour of the nonlocal parameter is performed for a simply supported beam. It is observed that when the nonlocal effect increases, the bending displacement grow up whereas the buckling load and natural frequencies decrease. Nix and Gao [20] show that the indentation size effect for crystalline materials can be accurately modelled using the concept of geometrically necessary dislocations. Reddy and Pang [21] evaluated the static bending, vibration, and buckling responses of Euler-Bernoulli and Timoshenko beams using Eringen's nonlocal elasticity theory. Simple-simple, clamped-clamped, clamped-simple, propped clamped-simple boundary conditions were examined. Aydoğdu [22] studied bending, buckling and free vibrations of nanobeams such as Euler-Bernoulli, Timoshenko, Reddy, Levinson beam theories by using nonlocal elasticity theory. It is observed when the nonlocal parameter increases, the rigidity of the beam decreases. In addition, the bending and buckling rate is increased in a high nonlocal parameter. Bağdatli [23] investigated the non-linear vibration behaviour of the Euler-Bernoulli nanobeam model for different boundary conditions based on nonlocal elasticity theory. It is stated that with the increase of the nonlocal parameter, the linear and non-linear frequency parameters decrease. It is clarified that the highest frequency response was obtained for clamped–simple support boundary conditions.

Ghayesh and Farajpour [24] studied vibration behaviour under force by nonlocal strain gradient model for nanoscale tubes. The equations of motion of the clamped nanotubes were obtained by using the Hamilton principle. The effects of nonlocal parameter, strain gradient parameter, dimension parameter on characteristic amplitude and frequency were examined. It has been found when the nonlocal parameter increases, the resonance frequency decreases but the transverse motion amplitude increases. It is emphasized that the values of the high strain gradient parameter significantly increase the resonant frequency. It is stated that the size parameter has little effect on vibration amplitudes. Romano et al. [25] investigated elastic nanobeams using nonlocal integral models. The boundary effects for nanobeam were explained by theoretical and numerical analysis. Arani et al. [26] studied the non-linear vibration response of viscous fluid conveying viscoelastic carbon nanotubes based on the nonlocal elasticity theory and modified couple stress theory. Nanotubes were modelled using Timoshenko beam theory and placed in a 2-dimensional magnetic field. The equations of motion were solved based on the energy method and the Hamilton principle. The results are compared with the Galerkin method results. It is realized that the magnetic field has a vital role in stability. It is stated that increasing the flow velocity reduces the fundamental natural frequencies, and increasing the nonlocal parameter at constant flow speed increases the fundamental frequencies.

In this paper, three supported nanobeams, which does not coincide in literature, are focused. Eringen’s nonlocal theory and Hamilton principle were used to obtain the equation of motion. Equations were transformed into non-dimensional form and solved by the help of perturbation technics. Thanks to solution the linear vibration behaviour of three supported nanobeam linear vibration, which is the main target of this work, is examined. Linear natural frequencies, mode shapes were obtained, and the results were compared and interpreted.

2. Materials and Methods

Schematically the three supported nanobeam is shown in Figure 1. The help of the Hamilton principle obtained the equations of motion with boundary conditions of the nanobeam. Local continuum theory fails to capture the dimension effect for nanostructures. Therefore, Eringen’ s nonlocal continuity theory has been used to give nano properties to the structure. The obtained equations of motion and boundary conditions were transformed non-dimensional equations to clean the material properties and geometric parameters. Then the non-dimensional equation of motion was subjected to perturbation analysis, and the dimensionless linear equations of motion were obtained. The resulting equations were solved with the help of boundary conditions. The fundamental natural frequencies and mode shapes are presented in table and figures. The determined natural frequencies values were verified by evaluated with mode shapes.

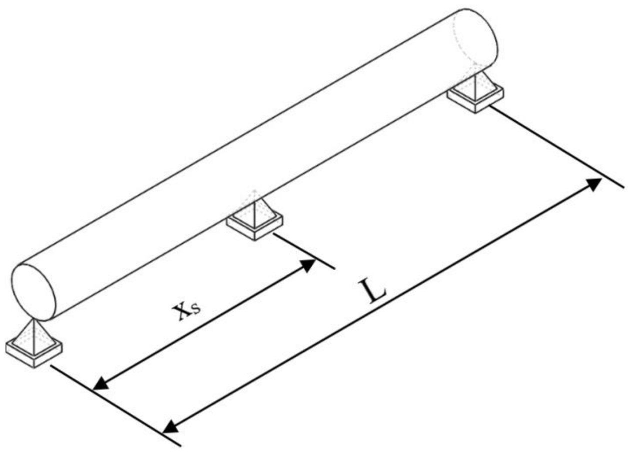

Figure 1. Three supported nanobeam

2.1. Obtaining Equations of Motion and Boundary Conditions of Three Supported Nanobeam

The equations of motion of the nanobeam were obtained using the Hamilton principle. In order to apply the Hamilton principle, we need to find the Lagrange of the nanobeam. The system’s Lagrange is obtained by subtracting potential energy from its kinetic energy. The potential and kinetic energies of the system are as follows:

$$T=\frac{1}{2}\int_{0}^{x_s}\rho A\left(\frac{\partial w_1^*}{\partial t^*}\right)^2 dx^*+\frac{1}{2}\int_{x_s}^{L}\rho A\left(\frac{\partial w_2^*}{\partial t^*}\right)^2 dx^* \tag{1}$$

$$\begin{aligned}V=&\frac{1}{2}\int_{0}^{x_s}\left(EI\frac{\partial^2 w_1^*}{\partial x^{*2}}+(e_0a)^2 N\frac{\partial^2 w_1^*}{\partial x^{*2}}-(e_0a)^2\rho A\frac{\partial^2 w_1^*}{\partial t^{*2}}\right)\frac{\partial^2 w_1^*}{\partial x^{*2}}dx^*+\frac{1}{2}\int_{0}^{x_s}N\left(\frac{\partial w_1^*}{\partial x^*}\right)^2 dx^*\\&+\frac{1}{2}\int_{0}^{x_s}\left(EI\frac{\partial^2 w_2^*}{\partial x^{*2}}+(e_0a)^2 N\frac{\partial^2 w_2^*}{\partial x^{*2}}-(e_0a)^2\rho A\frac{\partial^2 w_2^*}{\partial t^{*2}}\right)\frac{\partial^2 w_2^*}{\partial x^{*2}}dx^*+\frac{1}{2}\int_{0}^{x_s}N\left(\frac{\partial w_2^*}{\partial x^*}\right)^2 dx^*\end{aligned} \tag{2}$$

The expressions in the equations of kinetic and potential energy, respectively: x_s position of the middle support of the nanobeam, ρ density, A cross-sectional area, w_1 transverse displacement in the first region of the beam, w_2 transverse displacement in the second region of the beam, E elasticity module, I the area moment of inertia relative to the neutral axis of the beam, $e_0 a$ nonlocal parameter defined by Eringen, N the axial force is acting on the nanobeam. The mathematical expression of the Hamilton principle:

$$\delta\int_{t_1}^{t_2} L \,\mathrm{dt}^* = 0 \tag{3}$$

where L is Lagrange of the system. When Eq. (1) and Eq. (2) is substituted in the Hamilton Principle, the equation of motion of nanobeam is obtained like in Eq. (4) for two-region. Boundary conditions of the nanobeam are given in Eq. (6-8)

$$EI\frac{\partial^4 w_1^*}{\partial x^{*4}} + \rho A\frac{\partial^2 w_1^*}{\partial t^{*2}}\left(1-\frac{\partial^2}{\partial x^{*2}}(e_0 a)^2\right) = \frac{EA}{2L}\left(\int_0^{x_s}\left(\frac{\partial w_1^*}{\partial x^*}\right)^2 dx^* + \int_{x_s}^{L}\left(\frac{\partial w_2^*}{\partial x^*}\right)^2 dx^*\right)\left(\frac{\partial^2 w_1^*}{\partial x^{*2}} - (e_0 a)^2\frac{\partial^4 w_1^*}{\partial x^{*4}}\right) \tag{4}$$

$$EI\frac{\partial^4 w_2^*}{\partial x^{*4}} + \rho A\frac{\partial^2 w_2^*}{\partial t^{*2}}\left(1-\frac{\partial^2}{\partial x^{*2}}(e_0 a)^2\right) = \frac{EA}{2L}\left(\int_0^{x_s}\left(\frac{\partial w_1^*}{\partial x^*}\right)^2 dx^* + \int_{x_s}^{L}\left(\frac{\partial w_2^*}{\partial x^*}\right)^2 dx^*\right)\left(\frac{\partial^2 w_2^*}{\partial x^{*2}} - (e_0 a)^2\frac{\partial^4 w_2^*}{\partial x^{*4}}\right) \tag{5}$$

$$w_1^*(0,t) = 0, w_1^{*\prime\prime}(0,t) = 0 \tag{6}$$

$$w_1^*(x_s) = 0, w_2^*(x_s) = 0, w_1^{*\prime}(x_s) = w_2^{*\prime}(x_s), w_1^{*\prime\prime}(x_s) = w_2^{*\prime\prime}(x_s) \tag{7}$$

$$w_2^*(L,t) = 0, w_2^{*\prime\prime}(L,t) = 0 \tag{8}$$

2.2. Non-Dimensioning of Equations of Motion and Boundary Conditions

The ( ) * marked expressions are given in the equations represent dimensional parameters. First, the non-dimensioning process starts from variable parameters. The non-dimensional parameters aim to study the equations under general form are obtained $x = \frac{x^*}{L}, w_1 = \frac{w_1^*}{r}, w_2 = \frac{w_2^*}{r}, \zeta = \frac{x_s}{L}, t = \frac{1}{L}\sqrt{\frac{EI}{\rho A}}t^*, \gamma = \frac{e_0 a}{L}$. Final expression of the equation of motions can be written non-dimensional forms are the following,

$$w_1^{iv} + \ddot{w}_1 - \gamma^2 \ddot{w}_1'' = \frac{1}{2}\left[\int_0^{\eta} w_1'^2 dx + \int_{\eta}^{1} w_2'^2 dx\right]\left[w_1'' - \gamma^2 w_1^{iv}\right] \tag{9}$$

$$w_2^{iv} + \ddot{w}_2 - \gamma^2 \ddot{w}_2'' = \frac{1}{2}\left[\int_0^{\eta} w_1'^2 dx + \int_{\eta}^{1} w_2'^2 dx\right]\left[w_2'' - \gamma^2 w_2^{iv}\right] \tag{10}$$

where, $(\dot{\ })$ refers to the derivative according to time, $(\ )'$ refers to the derivative according to location and γ represent another form of nonlocal effect. Non-dimensional boundary conditions of nanobeam are present in follows:

$$w_1(0,t) = 0\,,\; w_1''(0,t) = 0 \tag{11}$$

$$w_1(\eta) = 0\,,\; w_2(\eta) = 0\,,\; w_1'(\eta) = w_2'(\eta)\,,\; w_1''(\eta) = w_2''(\eta) \tag{12}$$

$$w_2(1,t) = 0\,,\; w_2''(1,t) = 0 \tag{13}$$

2.3. Perturbation Analysis

Perturbation theory is a numerical method applied at the point where the analytic (implicit) solution of the interested equation cannot be solved. In this study, the method of multiple scales, one of the perturbation methods Nayfeh [27], will be used. In the case of variable amplitude problems in the equation concerned, the multi-scale method should be used. In this method, fast (T_0=t) and slow (T_1=εt) time scales should be defined. The new time derivatives transform given below:

$$\frac{\partial}{\partial t} = D_0 + \varepsilon D_1 + ... \tag{14}$$

$$\frac{\partial^2}{\partial t^2} = D_0^{\,2} + 2\varepsilon D_0 D_1 + ... \tag{15}$$

where $D_n = \dfrac{\partial}{\partial T_n}$.

$$w_1^{iv} + \ddot{w}_1 - \gamma^2 \ddot{w}_1'' = \frac{1}{2}\left[\int_0^\eta w_1'^2\,dx + \int_\eta^1 w_2'^2\,dx\right]\left[w_1'' - \gamma^2 w_1^{iv}\right] + \bar{F}\cos(\Omega t) - 2\bar{\mu}\dot{w}_1 \tag{16}$$

$$w_2^{iv} + \ddot{w}_2 - \gamma^2 \ddot{w}_2'' = \frac{1}{2}\left[\int_0^\eta w_1'^2\,dx + \int_\eta^1 w_2'^2\,dx\right]\left[w_2'' - \gamma^2 w_2 i^{v}\right] + \bar{F}\cos(\Omega t) - 2\bar{\mu}\dot{w}_2 \tag{17}$$

Transverse displacement, which is non-linear term, is transformed of $w_{1,2} = \sqrt{\varepsilon}\, y_{1,2}$. A straightforward asymptotic expansion for displacement in two regions of the nanobeam as follows:

$$y_1(x,t,\varepsilon) = \varepsilon^0 y_{11}(x,T_0;T_1) + \varepsilon^1 y_{12}(x,T_0;T_1) \tag{18}$$

$$y_2(x,t,\varepsilon) = \varepsilon^0 y_{21}(x,T_0;T_1) + \varepsilon^1 y_{22}(x,T_0;T_1) \tag{19}$$

Obtained linear equations of motion for two-part of nanobeam in Eqs. (20)-(21).

<u>First region:</u>

$$O(1):\; y^{iv}{}_{11} + D_0^{\,2} y_{11} - \gamma^2 D_0^{\,2} y_{11}'' = 0 \tag{20}$$

<u>Second region:</u>

$$O(1):\; y^{iv}{}_{21} + D_0^{\,2} y_{21} - \gamma^2 D_0^{\,2} y_{21}'' = 0 \tag{21}$$

2.4. Solving the Linear Problem

In this section, solutions of first-order equations are performed to obtain fundamental linear frequencies and mode shapes. The first and second region equations are identical. Therefore, only the first region equations solution is clarified. Second region equation is given directly. A solution assumption is given as follows:

$$y_{11}\left(x,T_0;T_1\right)=\left[A\left(T_1\right)e^{i\omega T_0}+cc\right]Y_1(x) \tag{22}$$

$$y_{21}\left(x,T_0;T_1\right)=\left[A\left(T_1\right)e^{i\omega T_0}+cc\right]Y_2(x) \tag{23}$$

where *cc*, ω, $A\left(T_1\right)$ represent to the complex conjugate, natural frequency, complex amplitude respectively. Eq. (22-23) is substituted in Eq. (20-21), the following form is obtained:

$$Y_1^{iv}(x)-\omega^2 Y_1(x)+\gamma^2\omega^2 Y_1(x)''=0 \tag{24}$$

$$Y_2^{iv}(x)-\omega^2 Y_2(x)+\gamma^2\omega^2 Y_2(x)''=0 \tag{25}$$

After required mathematical operations are performed, for the first and region solution equation is obtained in Eq. (26-27).

$$Y_1(x)=c_1e^{ir_1x}+c_2e^{ir_2x}+c_3e^{ir_3x}+c_4e^{ir_4x} \tag{26}$$

$$Y_2(x)=c_5e^{ir_1x}+c_6e^{ir_2x}+c_7e^{ir_3x}+c_8e^{ir_4x} \tag{27}$$

Using the boundary conditions, natural frequencies is found for the three supporting nanobeams.

3. Results and Discussion

The main target of this study is that examination the non-linear vibration of the three supported nanobeam. The position of the middle support was monitored by changing its position on the beam. In addition, the nonlocal coefficient on the beams was examined. The effects of variable parameters on the nanobeam were studied by finding natural frequencies using boundary conditions. Mode shapes of the detected natural frequencies were drawn for verification.

The first five-mode shapes of the nanobeam for γ= 0.1 and η=0.1 are shown in Fig. 2. In order to examine the effect of the nonlocal coefficient, three supported nanobeam mode shapes with different γ values at η=0.5 are shown in Fig. 3 obtained natural frequency results for different γ values are given in Table 1.

When examining the natural frequencies of the three supported nanobeam at different locations and γ values, it is observed that the natural frequency decrease with the increase of the nonlocal coefficient. In addition, amplitudes decrease with the increase of nonlocal coefficient.

When the position of the supports is examined, it is seen that the natural frequencies increase when the middle support moves away from the other bearing. As shown in Table 1, the highest linear natural frequency is obtained when the middle support is located at η=0.5.

It is seen in Figure 3, the mode shapes at the high natural frequencies have higher amplitudes than the mode shapes at the low natural frequencies. When Fig.3 is considered, it is noticed that the amplitudes decrease with the increase of nonlocal coefficient.

Table 1. Natural frequencies of three supported beams for different γ and η values.

		η				
		0.1	**0.2**	**0.3**	**0.4**	**0.5**
$\gamma = 0.1$	ω_1	16.766	19.748	23.870	29.316	33.427
	ω_2	47.429	54.996	63.410	87.906	98.329
	ω_3	83.849	94.907	92.753	132.507	166.514
	ω_4	121.492	132.506	163.628	197.437	233.521
	ω_5	158.821	161.844	226.075	260.620	299.359
		η				
		0.1	**0.2**	**0.3**	**0.4**	**0.5**
$\gamma = 0.2$	ω_1	14.383	16.517	19.256	22.502	24.582
	ω_2	33.496	37.386	411.679	39.708	58.380
	ω_3	52.281	57.119	56.725	74.935	91.097
	ω_4	70.420	74.839	90.227	106.455	123.248
	ω_5	88.0875	89.825	106.977	122.522	155.127
		η				
		0.1	**0.2**	**0.3**	**0.4**	**0.5**
$\gamma = 0.3$	ω_1	11.976	13.4752	15.305	17.326	18.501
	ω_2	24.771	27.188	29.376	28.666	40.487
	ω_3	36.870	39.791	39.682	51.219	71.381
	ω_4	48.588	51.219	61.430	72.105	83.048
	ω_5	60.077	61.2280	72.358	82.705	104.135
		η				
		0.1	**0.2**	**0.3**	**0.4**	**0.5**
$\gamma = 0.4$	ω_1	10.012	11.117	12.430	13.825	14.595
	ω_2	19.398	21.121	22.618	22.180	21.139
	ω_3	28.272	30.343	30.306	38.781	46.714
	ω_4	36.925	38.781	46.430	54.394	62.523
	ω_5	45.445	46.296	54.551	62.310	78.292
		η				
		0.1	**0.2**	**0.3**	**0.4**	**0.5**
$\gamma = 0.5$	ω_1	8.499	9.360	10.366	11.413	11.974
	ω_2	15.861	17.195	18.324	18.015	24.820
	ω_3	22.867	24.468	24.456	31.164	37.488
	ω_4	29.731	31.164	37.281	43.635	50.107
	ω_5	36.507	37.181	43.747	49.954	62.704

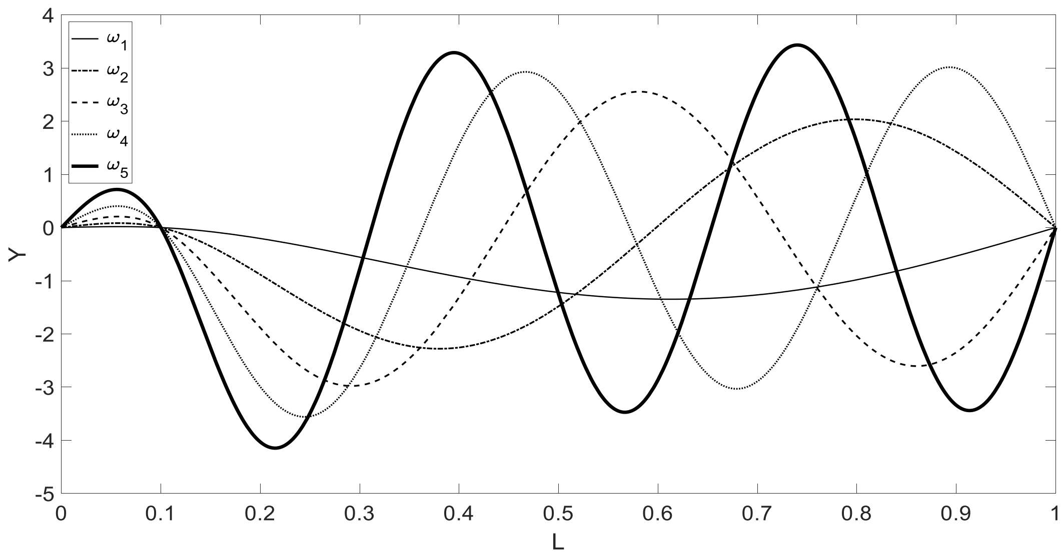

Figure 2. The first five mode shapes of the three supported nanobeam for $\gamma = 0.1$ and η=0.1

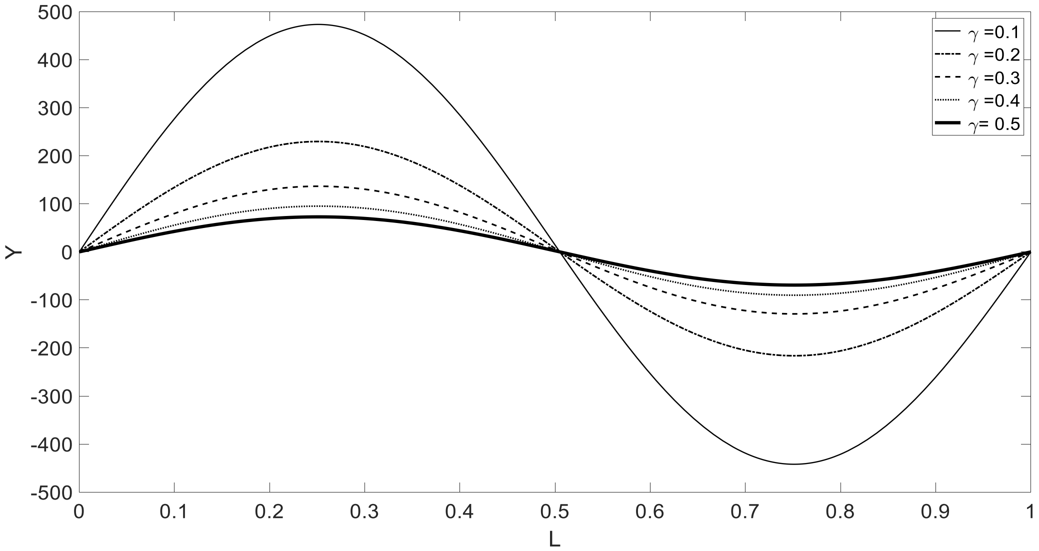

Figure 3. First linear natural frequency mode shapes for five different γ values

4. Conclusions

The present study, the linear vibration behaviour of the three supported nanobeam is investigated. The results are presented in tables and graphs. As expected, the reduction of the natural frequency and amplitudes of mode shapes is observed with the increase of the nonlocal parameter. It is seen that the support position contributes significantly to the natural frequency. It has been shown that determining the desired frequency range or distancing at specific frequencies can be carried out easily by changing the positions of the supports. It is not found multi supported previous nanobeam work. Therefore, it is evaluated that this study will be a new light for this area.

When the results of the three supported nanobeam are examined, the linear natural frequencies tend to increase in general by shifting the support to the middle position. However, this trend is not valid for all-natural frequencies of the structure. In some cases, it is observed that the natural frequencies values do not change or decrease slightly due to the fact that the structure cannot be resonance in that mode. Increasing the nonlocal coefficient, reduce to the natural frequencies. The suitability of the nanostructure increases with the increase of this coefficient. Nanoscale suitability of structure increases with the increase of this coefficient. When the nonlocal parameter for the three supported nanobeam is increased from 0.1 to 0.5, a reduction of 50.70% is observed for the first natural frequency and a reduction of 64.18% for the fifth natural frequency.

The performed work for the linear behaviour of three supported nanobeams is summarized as follows:

- Increasing nonlocal parameter reduces linear natural frequency and amplitudes.
- In case the supports are placed in the reinforcement zones of the beams linear natural frequencies are increasing, the middle support close to the head and end position of the beams, the natural frequency values decrease.
- The amplitudes of the mode shapes are getting increase at the higher modes of the structure.

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