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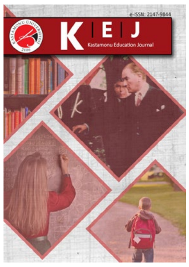

|Araştırma Makalesi / Research Article|

Fourth Grade Primary School Students' Solutions to the Questions on Fractions and the Models They Use in Solving

İlkokul 4. Sınıf Öğrencilerinin Kesirler Konusundaki Sorulara Yönelik Çözümleri ve Çözümde Kullandıkları Modeller

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Abstract

Purpose: This study aims to define the solutions towards fractions and solving models case of fourth grade primary school students.

Design/Methodology/Approach: Case study of qualitative research was used in this study. Selected by means of convenient case sampling, 113 fourth grade primary school students were included in this research. Fractions test developed by the researchers was applied to the students and the data obtained from the students were analyzed with descriptive analysis.

*Findings: P*rimary school 4th grade students were able to rank equal fractions of units and denominators, and add and subtract operation in fractions and solve problems that require these operations. It was seen that students who could model fraction types (simple, improper, whole number of fraction) could not determine the fraction of a multiplicity, but they could show the fraction of the multiplicity on at model. Students modelling types of fractions (proper, improper, mixed) are not able to define a fraction of multiplicity whereas they seem to determine the fraction on a model.

Highlights: It has been determined that students who can model ordering fractions with equal denominators cannot model ordering unit fractions, addition and subtraction, and problems that require these operations.

Öz

Çalışmanın amacı: Araştırmada ilkokul 4. sınıf öğrencilerinin kesirler konusuna yönelik çözümleri ve çözümleri modelleme durumlarını belirlemek amaçlanmıştır.

Materyal ve Yöntem: Nitel araştırma yaklaşımlarından özel durum yöntemi ile yürütülen araştırmaya 113 ilkokul 4. sınıf öğrencisi dahil edilmiştir. Öğrencilere araştırmacılar tarafından geliştirilen kesirler testi uygulanmış, öğrencilerden elde edilen veriler betimsel analiz ile analiz edilmiştir.

Bulgular: ilkokul 4. sınıf öğrencilerinin birim ve paydaları eşit kesirleri sıralayabildikleri, kesirlerde toplama ve çıkarma işlemleri ile bu işlemleri gerektiren problemleri çözebildikleri tespit edilmiştir. Kesir türlerini (basit, bileşik, türemiş) modelleyebilen öğrencilerin, bir çokluğun kesir kadarını belirleyemedikleri görülse de çokluğun kesir kadarını model üzerinde gösterebildikleri görülmüştür.

Önemli Vurgular: Paydası eşit kesirleri sıralamayı modelleyebilen öğrencilerin; birim kesirleri sıralamayı, toplama ile çıkarma işlemini ve bu işlemleri gerektiren problemleri modelleyemedikleri tespit edilmiştir.

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INTRODUCTION

Mathematics is a concept for general ideas and abstract expressions that include commonality of objects or events in the field (Ubuz, 1999). Primary and secondary school periods are especially important in the awareness of concepts by students or teaching by teachers (Duran, 2013; Zaslavsky and Shir, 2005). The abstractness of the concepts or the inability to be fully understood by the students causes the misconceptions of the concepts in the students (Batdal-Karaduman ve Elgün-Ceviz, 2018). In our country, there are misconceptions about mathematics such as geometry (Yenilmez and Yaşa, 2008), probability (Dawn, 2016), problem solving (Invincible and Yilmaz, 2008), environment, space, volume (Daghli, 2010), algebra (Kocakaya-Baysal, 2010) fractions (Gürel and Okur, 2016).

Fractions are among the most difficult mathematics concepts faced by primary school students (Hansen, 2014). Although fractions tell a quantity like integers, they differ from whole numbers in terms of how much the parts are ratherthan the whole and the reason why students make mistakes is that they do not understand the part-whole relationship well (Altun, 2014; Kocaoğlu and Yenilmez, 2010). Therefore, teachers need to be more careful when teaching the subject of fractions (Önal and Yormaz, 2017). Firstly, teachers should be interested in the subject and develop a positive attitude towards the course when teaching fractions (Altun and Çelik, 2018). Then they should conduct their courses with student-centered activities, enabling students to gain experience (Biber, Tuna and Aktaş, 2013; Ersoy and Ardahan, 2003; Kocaoğlu and Yenilmez, 2010). For this reason, they should use concrete and digital materials that have an important place in fraction teaching in their courses (Ergöl and Sezgin Memnun, 2020), and to structure the subject of fractions using different models for students with learning difficulties (Biber, Tuna and Aktaş, 2013; Kocaoğlu and Yenilmez, 2010).

The model, emerges at the end of the modelling process, which acts as a bridge between mathematics and real life (Ortiz and Dos Santos, 2011; Sriraman, 2006). Modelling; it provides the opportunity to produce more flexible and analytical solutions to events by concretizing mathematical knowledge and looking at real-life problems from a mathematical perspective (Berry and Houston, 1995; Mousoulides, Christou and Sriraman, 2006). Therefore, in the modelling process, it is important for students to interpret problem situations, organize information, predict solutions and try possible solutions, and create models for an in-depth understanding of events (Biembengut and Hein, 2010; Lesh and Doerr, 2003). For this purpose, it is necessary to understand the complex systems of the modelling process, to develop shareable tools using different disciplines and to work as a team (Chan, 2013). In this way, students will be able to easily make mathematical definitions, read data from tables and charts, justify their explanations, improve their discussions, learn meaningfully and improve their mathematical attitudes (Işık and Es, 2019; Lesh and Doerr, 2003; Tural Sönmez, 2019; Watters, English and Mahoney, 2004).

In mathematics education, mathematical modelling is important because traditional methods are inadequate in gaining the ability to use mathematics in daily life (Lingefjard, 2012; Peter, 2018). For this reason, it is necessary to use appropriate and different models to expand and deepen the fraction understanding of both students and teachers. Because models are used to help the student develop new concepts and relationships, establish the relationship between concepts and symbols, and to measure the student's level of understanding (Olkun and Toluk Uçar, 2012). These models are region/area, number line, and set models (van de Walle, 2012). In the region/area model from these models, the fraction number is embodied as a specific part of a region, while number line models compare lengths and measurements instead of fields. The number line model used in the length models, on the other hand, qualifies the fraction number as a real number. In the set model some of the objects in a set are represented. In other words, a set of objects creates a fraction of a group of objects that are a subset of the whole and the whole (Olkun and Toluk Uçar, 2012; van de Walle, 2012). In order for students to discover these concepts and models in fractions, teachers need to know pedagogical practices, be able to use appropriate models and give opportunities to students in a classroom environment (Aydoğdu-İskenderoğlu, 2017; Utley and Reeder, 2012).

When looking at the literature; studies aimed at identifying conceptual misconceptions regarding fractions (Adıgüzel, et al., 2018; Altıparmak, et al., 2017; Ayyıldız and Altun, 2013; Flores, Hinton and Taylor, 2018; Pesen, 2007; Pesen, 2008; Trivena, Ningsih and Jupri, 2017; Türkdoğan, et al., 2015), teacher opinions on the teaching of fractions (Doğan and Temur, 2011; Gökkurt, Soylu and Demir, 2015; Kar and Işık, 2015; Toptaş, Han and Akın, 2017), the situations of elemantary school students are examined, opinions and metaphors about fractions are taken (Altun and Çalik, 2018; Ergöl and Sezgin Memnun, 2020; Stafylidou and Vosniadou, 2004). It is seen that studies are carried out to determine the mistakes that primary school students make in sorting fractions, in additing and subtracting fractions, and to examine the effect of realistic mathematics education in making sense of decimal fractions (Önal and Yorulmaz, 2017; Phu Loc, Huu Tong and Thai Chau, 2017; Uça and Saracaloğlu, 2017). However, it is seen that there are no studies that reveal the status of primary school students in all achievements of the subject of fractions in the Primary Mathematics Teaching Program such as modelling fractions, sorting fractions, collecting and removing and solving problems requiring these operations. Therefore, it is necessary to determine the status of the 4th grade students in the final semester of primary school in all achievements of the fractions in the Primary Mathematics Curriculum.

It is seen that various studies have been carried out with primary, elemantary school students, preservice teachers and teachers related to modelling skills (Çiltaş and Işık, 2013; Delice and Kertil, 2015; Deniz and Akgün, 2018; Eraslan and Kant, 2015; Erdoğan, 2019; Hıdıroğlu and Bukova Güzel, 2015; Kal, 2013; Kertil, 2008; Olkun, Şahin, Akkurt, Dikkartın and Gülbağcı, 2009;

Şahin and Eraslan, 2016; Tekin Dede and Bukova Güzel, 2013; Tural Sönmez, 2019; Ulu, 2017). Primary and elementary school teachers' opinions on teaching fractions by model (Gökkurt, Soylu and Demir, 2015; Toptaş, Han and Akın, 2017), a study examining the relationship between fractions modelling and mathematics attitudes of secondary school students (Işık and Es, 2019). In this sense, no previous studies have found any studies for all gains. In addition, you can use the Primary Mathematics Curriculum in all achievements regarding 4th grade of primary school, examining the modelling skills of students (modelling fractions, types of fractions, problems requiring addition and subtraction in fractions, problems requiring addition and subtraction in fractions) will fill the lack of literature. Therefore, the difficulties in modelling fractions and fractions, which are not used much in daily life according to whole numbers, are at the beginning of abstract and difficult subjects of mathematics and are one of the subjects that primary school students are most mistaken for, are worth investigating especially at the primaryy school level (Albayrak, 2000; Kocaoğlu and Yenilmez, 2010; Okur and Çakmak-Güzel, 2016). Identifying the misconceptions about fractions in the 4th grade and the modelling status of the students will help teachers plan their next lessons and consider course processing with different methods and techniques and materials in order to prevent the misconceptions that students will experience in the coming years.

Accordingly, in this research, primary school 4th grade students' solutions to the subject of fractions and their modelling situations will be determined. Depending on this purpose, the following sub-problems will be searched for answers.

1. What is the modelling status of fraction types of primary school 4th grade students?
2. What is the status of primary school 4th grade students ordering fractions with equal units and denominators correctly and modeling the order?
3. What are the situations of primary school 4th grade students to determine the fraction of a multiplicity and show it on the model?
4. What is the situation of primary school 4th grade students in addition and subtraction in fractions, solving problems that require these operations, and modelling their solutions?

METHOD

Research Model

In order to define the solvings towards fractions and solving models case of fourth grade primary school students, qualitative research deigns and case studies were used. The case study focuses on an up-to-date event, case, situation, group, and individuals, allowing for in-depth research (Bassey, 1999; Stake, 1995; Yin, 1994: Akt. Ekiz, 2009). Based on this study, case study was referred because of deep research on all achievements of fractions and modelling of the 4th grade primary school students.

Study Group

Selected via convenient case sampling, 113 (58 female, 55 male) 4th grade primary school student who study in a district, Trabzon, Turkey have participated in research. Convenient case sampling is associated with making it easier or easier for people or groups to participate in the research process (Ekiz, 2009). Convenient case sampling has been applied in order to speed up the research and make it easy for students to reach.

Data Collection Tools

In this study, open-ended fraction test have been applied which are developed by researchers. Before the fractions test was developed, the primary school 4th grade mathematics curriculum was examined. In the program, it is seen that there are these gains related to fractions: "1- *Recognizes simple, improper and mixed fractions and shows them with models, 2- Compares and sorts unit fractions, 3- Specifies a specified simple fraction of a multiplicity, 4- Compares up to three fractions with equal denominators, 5- Collects and subtract denominators with equal fractions, 6- Solves problems that require fractional addition and subtraction.*" (MEB, 2018). Questions have been prepared about the gains. The questions prepared are presented to the opinion of 6 classroom teachers who are teachers of the students in the working group ranging from 10-20 years of service. Teachers evaluated the prepared questions in terms of students' level of conformity and language. Afterwards, the questions were presented to two experts in mathematics education. Finally, the test was applied as a pilot study to 10 students in the 4th grade of primary school. Some of the questions of the test are symbolic and some are prepared as verbal problems. In the first question of the test, composed of 6 items in total, students are expected to model fractions types; in the second, order unit fractions by modelling them; right in the third, recognize a fraction of multiplicity and show it with model; in the fourth, order equivalent fractions by modelling them; in the fifth, solve addition and subtraction problems of fractions by modelling them; and in the sixth question, which consists of two different problems that require addition and subtraction, students are expected to solve the problems by modelling.

***Gain 1**: Show the fractions given below with the model.*

$\frac{2}{5}$ $\frac{6}{5}$ $3\frac{1}{5}$

***Gain 2**: Model the following unit fractions and sort them from large to small.*

$\frac{1}{8}$ $\frac{1}{3}$ $\frac{1}{6}$

***Gain 3:** 3/7 of the 56 kilograms of apples in a case are rottenAccording to this, how many kilograms of apples are left? (Solve by drawing a model.)*

***Gain 4:** Line the following fractions from small to large by modelling them.*

$\frac{9}{12}$ $\frac{5}{12}$ $\frac{7}{12}$

***Gain 5:** Model the following operations.*

a) $3\frac{1}{9}+\frac{4}{9}=$

b) $\frac{7}{11}-\frac{5}{11}=$

Gain 6:

***a)** Ahmet's gone 3/10 and then 5/10 of a road. What time did Ahmed go on the road? (make the solution by modelling it.)*

***b)** 11/12 of a tank is filled with water. Since 5/12 of the water in this tank and the fruit trees are iringged, how many of the tanks are left with water? (make the solution by modelling it.)*

Figure 1. Fractions open-ended success test

Data was collected in 2nd term of 2018-2019 educational year after the achievements related to fractions subject were completed. Suggesting the fact that 40 minutes was sufficient for students to complete the tests, 6 classroom teachers adjusted one course hour in a day on which all the students were available in order that they did not affect one another; and then all teachers executed tests at the same time.

Data Analysis

In the descriptive analysis, the data is interpreted and summarized according to the previously determined themes (Yıldırım ve Şimşek, 2013: s. 256). Questions that students are to model fractions requisite answers classified as True (T) and False (F). Unit and equivalent fractions have been divided into categories: both ordering and modelling are true (OTMT), ordering is true but modelling is false (OTMF), ordering is false but modelling is true (OFMT), all answers are false (AAF) and questions are unanswered (U). Defining a fraction of multiplicity, operating addition and subtraction, solving related problems and modelling have also been separated into categories: both operation and modelling are true (OTMT), operation is true but modelling is false (OTMF), operation is false but modelling is true (OFMT), all answers are false (AAF) and questions are unanswered (U). The answers of the students were evaluated separately by 2 researchers. Similarity between the two assessments; calculated according to Miles and Huberman's (2004) formula. According to this formula; common results are divided by the sum of common results and non-common results and multiplied by face. The similarity from the researchers' codes was found to be 95%. The inconsistent results were discussed by two researchers and concluded in common. Related questions are quoted from the students' answer sheets, mistake subjects and shown in the tables. The answers of the students are evaluated individually, while the mistakes in the modelling encountered in each question; MM1, MM2,... codes are given. A total of 15 modelling mistakes were found in the students' answers. The mistakes made by the students regarding the concepts were evaluated by the researchers according to the following criteria. (see Table 1).

Table 1. Categorization of the mistakes made by students

Abbreviations	Mistakes
MM1	Not all are drawn the same size
MM2	Re-scanning of the scanned part in the first fraction in the second fraction
MM3	The part removed in the whole is ultimately shown as existing
MM4	Show subtraction with the addition model
MM5	Split the whole into more than denominators
MM6	Inability to divide the whole into equal parts
MM7	Unable to show the removed model on the model even though it has drawn
MM8	Inability to draw the result of the model
MM9	Divide the model by the share and paint the denominator
MM10	Draw only the result of a process
MM11	Show only subtrahend of the model
MM12	Irrelevant modelling
MM13	Showing each fraction in the ranking with different wholes
MM14	Showing the numenator and denominator by relocating
MM15	Fractional parts as much as fractions in showing the multi-multi-length

FINDINGS

In this part of the study, students' ability to model fractions, model and sort equal fractions of unit fractions and denominators, determine by modelling the fraction of a multiplicity, model and solve the process of collecting and removing fractions and solving fraction problems using model will be examined.

Findings for Research Question 1

In this part of the study, students' modelling status of fraction types was examined and the data obtained were presented in Table 2.

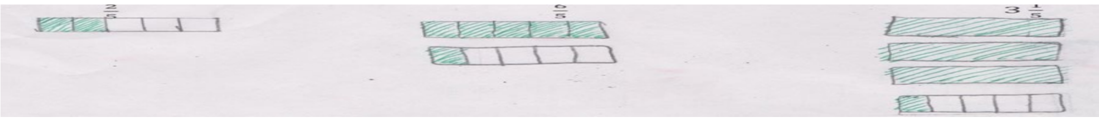
Table 2. Students' Ability to Model Fraction Types

Answers	Simple fraction	İmproper fraction	Whole number of fraction
True	112	93	82
False	1	20	30
Unanswered	0	0	1

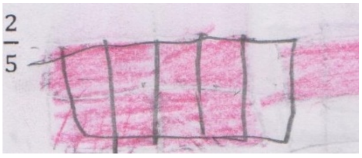
Almost all of the students are able to model simple fractions, the vast majority can model improper fractions, and nearly three-quarters can model whole number of fractions. When it comes to modelling fraction types; it is seen that there is a decrease in the number of students from the simple fraction to the whole number of fractions.

The correct answers that students give when it comes to modelling fraction types and the answers to the most mistakes of all types are shown in Figure 2**.**

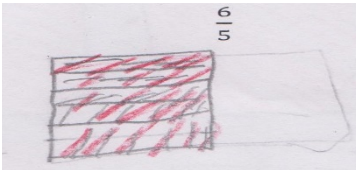
a. Accurate modelling of fraction types of students with code Ö1



b. Wrong modelling of student MM12 in simple fraction



c. MM12 coded incorrect modelling of student Ö66 in improper fraction



d. MM1 coded incorrect modelling of student Ö28 in whole number of fraction

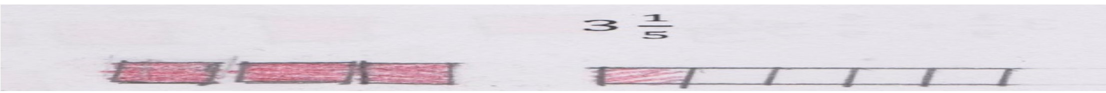


Figure 2. Modelling fraction types

It is seen that only 1 of the students models the simple fraction in a way that is irrelevant to the share and denominator (MM12). For example, the student appears to have scanned more than as much as desired despite dividing the fraction into 5 equal parts. In the improper fraction; 16 of the students made the mistake code MM12, 2 MM9, 1 student MM1 and MM6. The sample answer shows that the student can show the whole fraction (5/5) and not 5/6 (see Figure 1.c).

In the whole numbered fraction, 21 students made the mistake code MM1, 9 students MM12, 1 student MM13 and MM14. In the sample answer (see figure 1.d.), it is seen that the fractions are not the same size, although the whole numbered fraction shows correctly what is desired. In this context, it is understood that the students do not understand that the fractions in the whole numbered fractions should be the same size.

Findings for Research Question 2

In this part of the study, students' ability to sort and model units and denominators equal fractions was examined and the data obtained were shown in Table 3.

Table 3. Students' are sorted and modeled unit fractions, denominator equal fractions

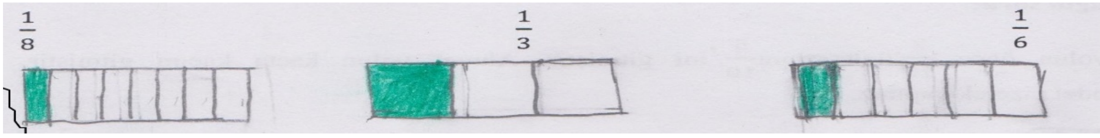
Students' ability to sort and model unit fractions and denominators equal fractions					
Fractions	**OTMT**	**OTMF**	**OFMT**	**AAF**	**U**
Unit fractions	39	41	4	29	0
Denominator equal fractions	57	4	38	12	2

80 of the students did the unit fraction ranking correctly (OTMT, OTMF), 33 students got it wrong (OFMT, AAF); 43 students (OTMT, OFMT) do modelling correctly, while 70 students (SDMH, TH) appear to have mismodeled modelling. In this sense, it can be stated that students can sort unit fractions but cannot model them.

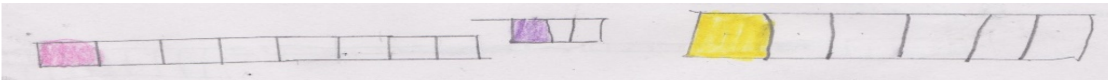
More than half of students were found to be correct in sorting equal fractions of denominators (61 students; OTMT, OTMF), the vast majority (95 students; OTMT, OFMT) also appears to be doing the modelling correctly. In this context, it can be concluded that students have no problems with both sorting and modelling of equal fractions of denominators.

The following illustrations of students' modelling status for sorting fractions equal to units and denominators are shown below (see Figure 3).

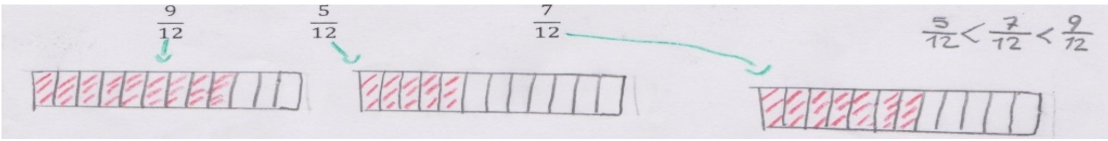
a- Correct modelling of unit fractions of student with code Ö8;



b- MM1 coded incorrect modelling of the student with the code Ö56 for sorting unit fractions



c- Accurate modelling of the student with the code Ö19 on sorting the denominators equal fractions



d- MM12 coded erroneous modelling of the student with code Ö70 on sorting equal fractions of denominators

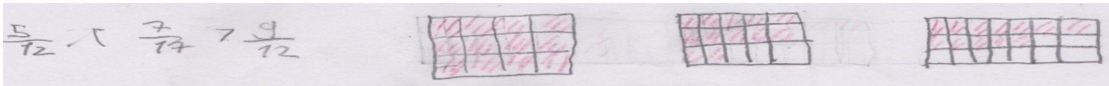


Figure 3. Errors in modelling fractions sorting

It was observed that 43 of the students correctly showed the unit fractions ranking on the model, 53 students made the mistake coded MM1, 9 students made the error coded MM12 and 8 students made the mistake coded MM13. Students were expected to draw models of the same size on all three simple fractions, while they were expected to cut as many of them into equal parts (see Figure 3.a.), but nearly half of the students drew the desired model for each fraction, but the three fraction models were not the same size (MM1) (see Figure 3.b.).

Students were expected to cut all boots of the same size and each whole into equal parts for three fractions with equal denominators (see Figure 3.c). It appears that most students modeled the correct question irrelevantly by 14 students (HM12) (see Figure 3.d.).

Findings for Research Question 3

At this stage of the research, the students' ability to model the fraction of a multiplicity was examined and the data obtained were shown in Table 4.

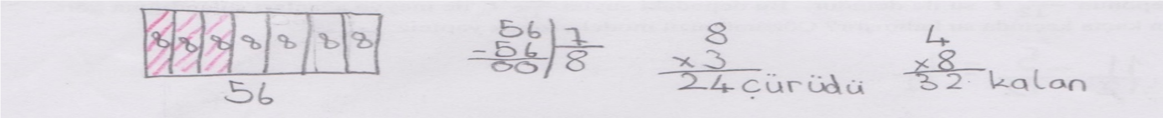
Table 4. Students' determination status by modelling the fraction of a multiplicity

Students' ability to identify and model the fraction of a multiplicity				
OTMT	OTMF	OFMT	AAF	U
36	13	34	28	2

Nearly half of the students appear to have correctly answered the fraction of the multiplicity (49 students; IDMD, IDMH). When looking at the ability of a multiplicity to show fractions on the model; it is seen that 70 of the students (IDMD, IHMD) can show in the correct model.

The answers of the students to show the fraction of a multiplicity on the model are shown in Figure 4 below.

a- Accurate modelling of the student with the code Ö75



b- Incorrect modelling of student with code 31 code HM12 on the subject

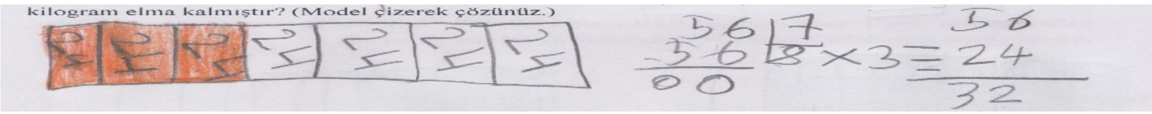


Figure 4. Modelling the fraction of a multiplicity

It was found that 70 of the students were able to accurately model the fraction of a multiplicity, while 34 students made the error coded MM12, 6 students MM11, 1 student MM6 and MM7 when modelling.

In the modelling of the fraction of a multiplicity, the students were first expected to divide the entire fraction into 7 equal parts, show the decaying apple and the remaining apple part, then write down the amount per piece and determine the desired amount (see Figure 4.a.).

In the question answered correctly by almost three-quarters of students (70), it is seen that the wrong modelers are not able to determine (irrelevant modelling) in each unit in the model (see Figure 4.b.).

Findings for Research Question 4

In this part of the research, students' ability to solve and model problems requiring the process of addition and subtraction fractions was examined.

Table 5. Students' situations of solving and modelling problems that require the addition, subtraction process and these operations in fractions

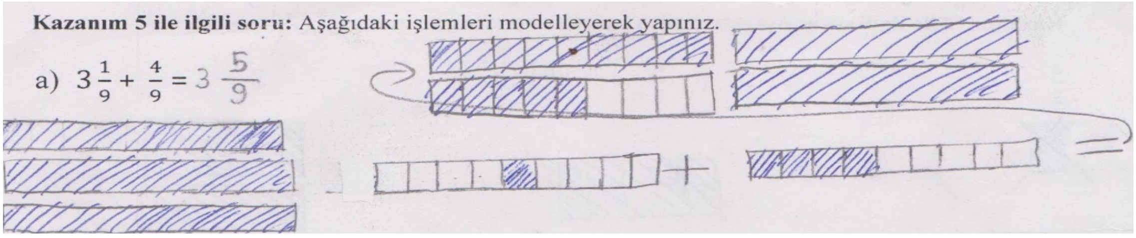
Operation-problem	Students' addition and subtraction and modelling skills in fractions				
	OTMT	OTMF	OFMT	AAF	U
Addition	5	85	0	20	3
Subtrction	12	81	0	19	1
Addition problem	42	49	3	10	9
Subtraction problem	18	63	1	15	16

Although the vast majority of students (90 students; OTMT, OTMF) did the addition correctly, almost all of them (108 students; OTMF, AAF, U) were unable to model the collection process. Although the vast majority of students (93 students; OTMT, OTMF) also did the subtraction correctly, it appears that they did the modelling incorrectly (101 students; OTMF, AAF, U). In this sense, it can be concluded that the students did the addition and subtraction process correctly in the fractions but could not show the procedures on the model.

It is seen that 91 of the students (OTMT, OTMF) solved the fraction problem that required the addition process, while 68 students (OTMF, AAF, U) could not show it on the model. Although 71 students (OTMT, OTMF) solved problems requiring subtraction, 94 students (OTMF, AAF, U) could not show on the model. In this context, it can be concluded that students solve problems that require addition and subtraction in fractions but cannot show them on the model.

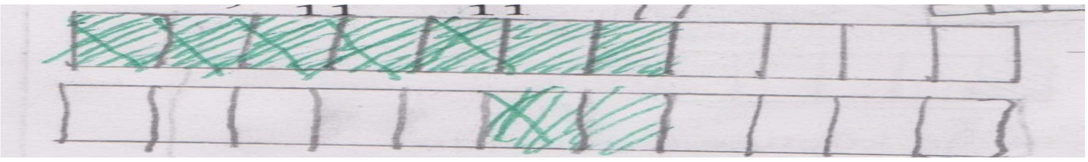
The status of students to show the addition, subtraction and problems that require these operations on the model is presented in figure 5.

a- Accurate modelling of the addition process of the student with the code Ö101

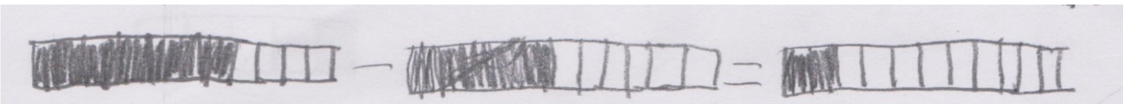


b- MM2 coded incorrect modelling of student with code Ö58 on addition

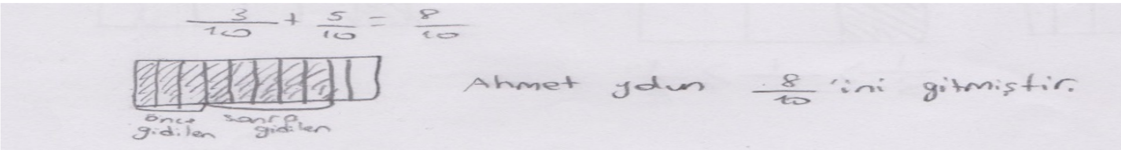
c- Accurate modelling of the subtraction process of the student coded Ö47



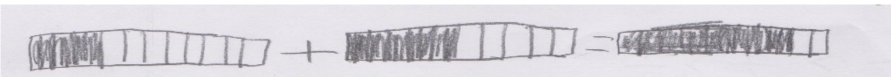
d- Incorrect modelling of the subtraction process of the student coded Ö33



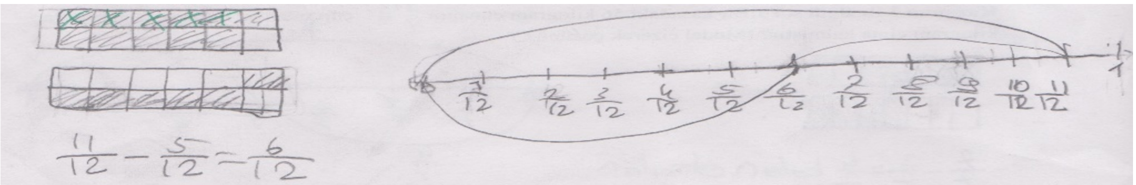
e- Accurate modelling of the problem that requires the addition process of the student coded Ö29



f- MM2 coded incorrect modelling of student Ö3 on the subject



g- Accurate modelling of the problem requiring subtraction of student with the code Ö112



h- Incorrect MM3 coded modelling of student with code Ö56 on the subject

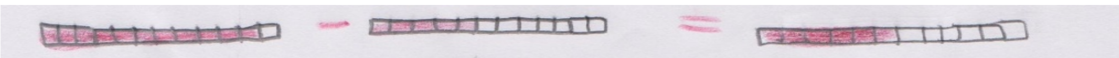


Figure 5. Modelling problems that require these operations with addition, subtraction

While 5 of the students were seen to correctly model the addition process in the fractions; 38 students were found to have made the error coded MM2, 28 students MM12, 21 MM10, 15 MM1, 2 students MM8 and 1 student MM6. When modelling the addition process in fractions, students are expected to first mark the first fraction on the same whole, then show the second fraction in parts that were not previously marked in the fraction, and ultimately all the marked parts are expected to be marked (see Figure 5.a.). It has been observed that most of the students made the mistake of marking the part they marked in the first fraction again in the second fraction (HM2) (see Figure 5.b.).

It turned out that 12 of the students correctly modelled the subtraction process in fractions, 37 students made MM3, 30 students made MM12, 21 students made MM10, 6 students made MM8 and 4 students made the MM7 error. In the subtraction process, it is expected to determine the desired number of the marked parts in the second fraction after the desired part is shown in the first fraction, and to show the unmarked parts of the first fraction in the resulting section (see Figure 5.c.). It is seen that most of the students determined the desired number of the parts marked in the first fraction in the second fraction, but showed the part they removed again in the resulting section (HM3) (see Figure 5.d.).

While it is seen that 45 of the students can correctly model solving the problem that requires addition; Thirty-three were found to have made the mistake, MM2, 12 students MM10, 8 students MM12, 4 students MM8 and 2 students MM4. When modelling the problem that requires addition in fractions, students were expected to first mark the first fraction on the same whole, then to show the second fraction in parts that had not been previously marked in the fraction, and as a result, all the marked parts were expected to be marked (see Figure 5.e.), but it was observed that most of the students had the mistake of marking the part they marked in the first fraction again in the second fraction (see Figure 5.f.f).

While 19 of the students were found to have correctly modeled the solution of the problem that required subtraction; It was determined that 29 of the students made the mistake coded MM3, 18 students MM12, 12 students MM8, 10 students MM10, 3 students MM4 and MM7, 2 students MM11 and 1 student MM5.

In the problem requiring subtraction, it is expected to determine the desired number of the marked parts in the second fraction after the desired part is shown in the first fraction, and to show the unmarked parts of the first fraction in the resulting section (see Figure 5.g.). Most of the students also determine the desired amount of the parts marked in the first fraction, as in the question that requires removal in the fractions, but show the part they extracted again in the resulting section (see Figure 5.h.).

CONCLUSION, DISCUSSION AND RECOMMENDATIONS

At the and of study; primary school 4th grade students were able to rank equal fractions of units and denominators, and add and subtract operation in fractions and solve problems that require these operations. It was seen that students who could model fraction types (simple, improper, mixed) could not determine the fraction of a multiplicity, but they could show the fraction of the multiplicity on at model. Students modelling types of fractions (proper, improper, mixed) are not able to define a fraction of multiplicity whereas they seem to determine the fraction on a model.

The study found that students were able to sort both denominator equal fractions and unit fractions. Önal and Yorulmaz, (2017) found that the students sorted the fractions like natural numbers. In fact, many studies have shown that students are mistaken about this (Biber, Tuna and Aktaş, 2013; Soylu and Soylu, 2005; Stafylidou and Vosniadou, 2004; Tuna and Aktaş, 2013). hese misconceptions; it is seen by the students that only the denominator of the fraction is observed in the form of making the large incision look larger or thinking that the value of the fraction increases in cases where the value of the denominator or share is decreased (Bingölbali and Özmantar, 2012; Demiri, 2013; Okur and Çakmak-Güzel, 2016; Stafylidou and Vosniadou, 2004). The reason why the students in the study were successful in sorting the fractions may be because teachers theoretically know the subject of fractions well (Chick, Pham and Baker, 2006). In addition, there are some rules that can be used to sort both denominator equal and unit fractions, which may have been taught to students by teachers. However, it may be that the students in the study did not have a good understanding of the part-whole relationship required to learn the fractions underneath their difficulty in modelling when sorting the unit fractions (Charalambous and Pitta-Pantazi, 2005). The fact that students do not understand the whole-part relationship may be because teachers have not spent enough time modelling in lessons. The reason these students make no mistake in modelling the ordering of equal fractions of denominators is that they can easily model the same fractions, all of which consist of simple fractions, and after determining the desired parts (share), they can concretely realize which is more and which is less.

The study found that students made no mistake in solving the process of addition and subtraction fractions. However, in some studies, it has been revealed that students write the same fractional denominator and share in the fractions as the natural number, and in the extraction they think of the denominator and the share separately and subtract the small number from the large number, and they act just like in natural numbers (Kar and Işık, 2015; Önal and Yorulmaz, 2017). Some studies have also found that students are wrong to addition and subtraction fractions (Biber, Tuna and Aktaş, 2013; Trivena, Ningsih and Jupri, 2017). It is generally seen that the errors made in the addition process are in the form of separate considerations of the shares and denominators and the collection among themselves (Biber, Tuna and Aktaş, 2013; Soylu and Soylu, 2005). The reason the subtraction cannot be performed may be because students think it is difficult to remove in fractions (Trivena, Ningsih and Jupri, 2017). The fact that the students in the study also had a good understanding of the relationship between the share and denominator in the fractions may have led them to make mistakes in addition and subtraction fractions. However, students made mistakes in showing the solution of the addition and subtraction process on the model, and the reason for this is that; It may be because teachers point out that only shares should be collected when performing the procedures in the solution of the addition and subtraction of only equal fractions in the primary school mathematics curriculum. Because it is seen that teachers in primary schools do not have enough knowledge about modelling fractions (Aydoğdu-İskenderoğlu, 2017). Teachers who do not have this competence can focus only on theoretical knowledge when teaching fractions to students, but they may not dwell on modelling the processes as much or at all as they do on modelling the types of fractions to embody them. For these reasons, students are likely to make mistakes, as in this research, both in modelling the collection and extraction processes and in modelling the solution of the problems that require these processes.

The study found that students did not make mistakes in solving problems that require the process of additing and subtracting fractions, but they made mistakes in determining the fraction of a multiplicity asked as a problem. The study of Biber, Tuna and Aktaş (2013) is similar to the results of the research, but the results of this research are at odds with the research of Başgün and Ersoy (2000). Although the reason why students were wrong in solving fraction problems depends on the correct modelling (Biber, Tuna and Aktaş, 2013), the students in the study had problems with modelling even though they solved the problems correctly. The reason why students have trouble solving the problems of finding the fraction of a multiplicity; Fractional problems at the primary school level, which only involve the addition and subtraction process, may be due to the fact that more than four students who have previously acquired four processing skills have learned what words such as increase, addition, addition, deficiency, separation, subtraction mean in the problem. Because students who learn these words correctly can decide which action to take to solve the problem correctly. Students who decide which action to take may have concluded the correct way to solve the problems by warnings of teachers that only the shares should be processed when additing and subtracing the denominator in equal fractions.

Although the students in the study were able to model the types of fractions and the fraction of a multiplicity; it was determined that they made mistakes such as drawing all of them in equal fractions of different sizes, addition in fractions and showing the scanned part in the first fraction as scanning, subtraction and the problem requiring removal as the result. Other studies have found that students were wrong to understand the whole of the piece, that is, to co-cut fractions, which are the nature of fractions, or to write fractions that are broken into equal parts (Haser and Ubuz, 2002; Lamon, 2011; Okur and Çakmak-Güzel, 2016; Önal and Yorulmaz, 2017; Pesen, 2007, Phu Loc, Huu Tong and Thai Chau, 2017). However, in the study of Uça and Saracaloglu, (2017) it was seen that the students were able to form a whole relationship in parts. This may be because of classroom teachers do not know enough about modelling and cannot use it adequately (Toptaş, Han and Akın, 2017). In one study, teachers were asked to evaluate the answers of their students and teachers were found to be inadequate in explaining the misconceptions of their students (Karaağaç and Köse, 2015). It has been observed that most secondary school teachers start with activities suitable for fraction teaching, but have incomplete information about the models and subjects they use in fraction teaching (Gökkurt, Soylu and Demir, 2015), In this context, it can be considered that teachers have the misconceptions about fractions. Failure of teachers with misconceptions to teach modelling of different achievements may be the reason why students cannot model what is desired.

Recommendations

1. Problem solving, addition, subtraction, determining the specified amount of a multiplicity, modelling fractions equal to the unit and denominator, students who have problems with the problem can be processed with concrete materials (orange, cake, fraction sets, fraction cards...) to eliminate their problems.
2. Teachers, who are the priority in solving the problem that students often experience in modelling fractions, can be asked to plan enriched course activities focused on errors by explaining the mistakes that students make about fractions.
3. Qualitative studies can be carried out to determine why students have difficulty modelling fractions.
4. Longitude studies can be carried out to see if students continue to make mistakes after being taught their mistakes in modelling.
5. The achievements of students at each class level in modelling the subjects that contain all the gains in fractions can be examined and compared.

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Statements of publication ethics

We hereby declare that the study has no unethical issues and that research and publication ethics have been observed carefully.

Researchers’ contribution rate

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