Traveling Wave Solutions For Two Physical Models via Extended Modified Kudryashov Method

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ABSTRACT: In this paper, we propose the extended modified Kudryashov method (EMKM) for solving the Biswas-Milovic equation and Gerdjikov-Ivanov equation which are commonly special cases of Schrödinger equation in mathematical physics. We received many new extended traveling wave solutions when the special values of the parameters are taken for these equations which are pointed out by rational function, exponential function and hyperbolic function forms. The results show that EMKM is advantageous mathematical technique for solving nonlinear partial differential equations.

Keywords: Biswas-Milovic equation, Gerdjikov-Ivanov equation, extended modified Kudryashov method.

Genişletilerek Düzenlenmiş Kudryashov Yöntemi ile İki Fiziksel Modelinin Hareketli Dalga Çözümleri

ÖZET: Bu makalede, matematiksel fizikte yer alan Schrödinger denkleminin özel durumları olan Biswas-Milovic denklemi ve Gerdjikov-Ivanov denklemini çözmek için genişletilerek düzenlenmiş Kudryashov yöntemini (EMKM) öneriyoruz. Bu denklemler için parametrelerin özel değerleri alındığında rasyonel fonksiyon, üstel fonksiyon ve hiperbolik fonksiyon formları ile gösterilen birçok yeni genişletilmiş dalga çözümü elde edildi. Sonuçlar, EMKM'nin doğrusal olmayan kısmi diferansiyel denklemleri çözmek için etkili bir yöntem olduğunu göstermektedir.

Anahtar Kelimeler: Biswas-Milovic denklemi, Gerdjikov-Ivanov denklemi, genişletilerek düzenlenmiş Kudryashov yöntemi.

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INTRODUCTION

It is well known that many natural phenomena in science and engineering such as physics, chemistry, biology, image processing, signal propagation, fluid dynamics, quantum theory etc. are related to nonlinear partial differential equations (NPDEs). Different analytical and numerical approaches are used in literature for exploring the precise solutions of NPDEs. Some of the commonly used techniques are $\exp(-\phi(\xi))$ function method (Mirzazadeh et al., 2017; Arshed., 2018; Raza et al., 2018), exp-function method (Kadkhda and Jafari, 2017; Hosseini et al., 2018), first integral method (Taghizadeh et al., 2011), trial equation method (Biswas et al., 2018, Biswas et al., 2018), hyperbolic function method (Hosseini and Zabihi et al. 2018), G'/G- expansion method (Mirzazadeh et al., 2015; Mirzazadeh et al., 2017) and so on (Ege and Misirli, 2012; Hosseini and Samadani et al., 2018).

Nonlinear Schrödinger equation (NLSE) with any angular momentum provides significant applications in many areas of physics that work up models enhanced to labor quantum mechanical systems (Taghizadeh et al., 2011; Triki et al., 2011; Eslami and Neirameh, 2018; Hosseini et al., 2018; Hosseini et. al. 2018). Recently, Biswas and Milovic have proposed to NLSE a general model that explains some of the defects in fiber during long distance transmission of these pulses. These involve superficial changes in the temporal development of the pulse or fiber diameter errors. This model is usually discoursed as the Biswas-Milovic equation (BME). More recently, Biswas and Milovic (Mirzazadeh et al., 2015; Najafi and Arabi, 2016; Sayed et al., 2016; Zhou et al., 2016; Zayed and Al-Nowehy, 2017; Raza et al., 2018) discussed special cases of the Kerr law with constant coefficients and the law of nonlinear power in generalized NLSE.

Many natural phenomena such as weak nonlinear distribution wave fields, quantum field theory and nonlinear optics in science can be modeled and defined by the Gerdjikov-Ivanov equation (GIE), which is called derivative of Schrödinger's equation. Due to the various applications in science, more and more studies are emerging in literature (Triki et al., 2017; Arshed, 2018; Biswas et al., 2018).

In this work, in special, we will find the traveling wave solutions of the Biswas-Milovic and the Gerdjikov-Ivanov equations by extended Kudryashov method. While wave parameters obtained by many existing methods have the same values, parameter values vary in this method. So, the aim of this study is to achieve larger and faster wave solutions by increasing the parameters.

MATERIALS AND METHODS

Extended Modified Kudryashov Method

A given nonlinear partial differential equation (NPDE) is written in several independent variables as:

$$P\left(\theta, \theta_t, \theta_x, \theta_y, \theta_z, \theta_{xy}, \theta_{yz}, \theta_{xz}, \dots\right) = 0 \tag{1}$$

where P is some function, $\theta = \theta(x, y, z, , \dots, t)$ is a dependent variable or unknown function to be determined and the subscript indicates partial derivative.

First, we investigate the traveling wave solutions of Equation 1. of the form:

$$\theta(x, y, z, , \dots, t) = \theta(\sigma), \ \sigma = \kappa(x + \vartheta t) \ \text{or} \ \sigma = x - \vartheta t \tag{2}$$

where κ and ϑ are arbitrary constants. Then Equation 1. degrades to a nonlinear ordinary differential equation in the form:

$$O(\theta, \theta_\sigma, \theta_{\sigma\sigma}, \theta_{\sigma\sigma\sigma}, \dots) = 0. \tag{3}$$

Then, we assume that the analytic solutions of Equation 3. can be formed as in the form:

$$\theta(\sigma) = \sum_{k=1}^{M} b_k \psi^k(\sigma) \tag{4}$$

where $\psi = \dfrac{1}{\sqrt{1 \mp e^{2\sigma}}}$ and the function ψ is the solution of

$$\psi_\sigma = \psi^3 - \psi. \tag{5}$$

In compliance with the method, we suppose that the solution of Equation 3. can be stated as in the form:

$$\theta(\sigma) = b_M \psi^M + \cdots. \tag{6}$$

For the purpose of determining the value of the pole order for general solution of Equation 3., we balance the highest order nonlinear terms $\theta^m(\sigma)\theta^n(\sigma)$ and $(\theta^m(\sigma))^p$ in Equation 3. then we have

$$M = \frac{2(n-pm)}{p-m-1}. \tag{7}$$

Lastly, substituting Equation 4. into Equation 3. and equating the coefficients of ψ^k to zero, we obtain a system of algebraic equations. By solving this algebraic system, we obtain the analytic solutions of Equation 3.

RESULTS AND DISCUSSION

Biswas-Milovic Equation

We first apply the method to Biswas Milovic equation in the form:

$$i(q^m)_t + \alpha(q^m)_{xx} + \beta F(|q|^2)q^m = 0, \tag{8}$$

where α and β are constants and q is the function of (x,t). For the solutions, we can choose the wave transformation as follows:

$$q(x,t) = \theta(\sigma)e^{i(-\kappa x + \omega t + \varphi)}, \quad \sigma = x - \vartheta t \tag{9}$$

where $\kappa, \omega, \vartheta \neq 0$ are Equation 9. into Equation 8. and seperating it into imaginary and real parts yields $\vartheta = -2m\alpha\kappa$ and

$$\alpha(\theta^m)'' - (m\vartheta + \alpha m^2 \kappa^2)\theta^m + \beta F(\theta^2)\theta^m = 0. \tag{10}$$

We will consider the following two forms of nonlinearty.

Kerr law nonlinearity

$$F(\theta) = \theta$$

so that the Equation 8. collapses to

$$i(q^m)_t + \alpha(q^m)_{xx} + \beta(|q|^2)q^m = 0. \tag{11}$$

In this case, Equation 10. simplifies to

$$\alpha m(m-1)(\theta')^2 + \alpha m\theta\theta'' - (m\omega + \alpha m^2 + \kappa^2)\theta^2 + \beta\theta^4 = 0. \tag{12}$$

We employ the balance principle by,

$$\theta(\sigma) = \sum_{k=1}^{M} b_k \psi^k(\sigma)$$

where $\psi = \dfrac{1}{\sqrt{1 \mp e^{2\sigma}}}$ and the function ψ is the solution of

$\psi_\sigma = \psi^3 - \psi$. Then we find that $M = 2$.

Thus we have

$$\theta(\sigma) = b_0 + b_1\psi(\sigma) + b_2\psi^2(\sigma) \tag{13}$$

and substituting derivatives of $\theta(\sigma)$ with respect to σ in Equation 13. we obtain

$$\theta'(\sigma) = 2b_2\theta^4(\sigma) + b_1\theta^3(\sigma) - 2b_2\theta^2(\sigma) + b_1\theta(\sigma),$$

$$\theta''(\sigma) = 8b_2\theta^6(\sigma) + 3b_1\theta^5(\sigma) - 12b_2\theta^4(\sigma) - 4b_1\theta^3(\sigma) + 4b_2\theta^2(\sigma) + b_1. \tag{14}$$

Substituting Equation 13. and Equations 14. into Equation 12. we have a system of algebraic equations. By solving this algebraic system, we find the following solutions of Equation 11. as follows:

Case 1: When $b_0 = 0$, $b_1 = 0$, $b_2 = -\dfrac{2\sqrt{m\omega}}{\sqrt{4\beta - \beta\kappa^2}}$, $\alpha = -\dfrac{4m\omega}{\kappa^2 - 4}$,
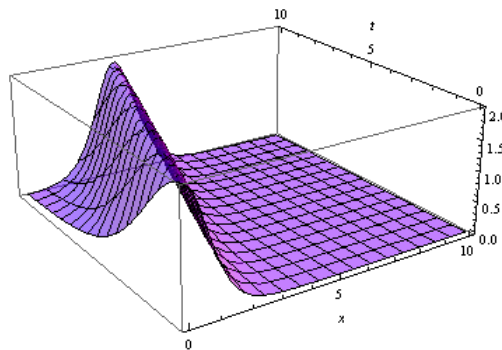
then

$$q(x,t) = -\sqrt{\frac{\kappa^2 - 4\alpha}{4\beta - \beta\kappa^2}} \frac{e^{i\left(-\kappa x + \frac{(\kappa^2 - 4)\alpha}{4m}t + \varphi\right)}}{\sinh(2x - 2\vartheta t) + 2\cosh^2(x - \vartheta t)}.$$

Case 2: When $b_0 = 0$, $b_1 = 0$, $b_2 = \dfrac{2\sqrt{m\omega}}{\sqrt{4\beta - \beta\kappa^2}}$, $\alpha = -\dfrac{4m\omega}{\kappa^2 - 4}$,

then

$$q(x,t) = \sqrt{\frac{\kappa^2 - 4\alpha}{4\beta - \beta\kappa^2}} \frac{e^{i\left(-\kappa x + \frac{(\kappa^2 - 4)\alpha}{4m}t + \varphi\right)}}{\sinh(2x - 2\vartheta t) + 2\cosh^2(x - \vartheta t)}.$$

Figure 1. The solution $q(x,t)$ for $m = -\dfrac{1}{2}$, $\omega = 1$, $\beta = -\dfrac{1}{3}$, $\kappa = 1$, $\alpha = -\dfrac{2}{3}$

Case 3: When $b_0 = -\frac{2\sqrt{-(2+\kappa^2)m\omega}}{\beta(\kappa^4-2\kappa^2-8)}$, $b_1 = 0$, $b_2 = 2\sqrt{\frac{-(2+\kappa^2)m\omega}{\beta(\kappa^4-2\kappa^2-8)}}$, $\alpha = -\frac{4m\omega}{\kappa^2-4}$,

then

$$q(x,t) = -\frac{1}{2}\sqrt{\frac{\alpha}{\beta}}(tanh(x-\vartheta t)+1)e^{i(-\kappa x+\frac{(\kappa^2-4)\alpha}{4m}t+\varphi)}.$$

Case 4: When $b_0 = \sqrt{\frac{2(-4+\kappa^2)m\omega}{\beta(\kappa^4-2\kappa^2-8)}}$, $b_1 = 0$, $b_2 = -\sqrt{\frac{-(2+\kappa^2)m\omega}{\beta(\kappa^4-2\kappa^2-8)}}$, $\alpha = -\frac{4m\omega}{\kappa^2-4}$,

then

$$q(x,t) = \left(\frac{(\kappa-2)\sqrt{(\kappa+2)\alpha}}{\sqrt{2\beta(\kappa-4)}} - \frac{i\sqrt{\alpha}}{2\sqrt{\beta}(sinh(2x-2\vartheta t)+2cosh^2(x-\vartheta t))}\right)e^{i(-\kappa x+\frac{(\kappa^2-4)\alpha}{4m}t+\varphi)}.$$

Case 5: When $b_0 = -\sqrt{\frac{2(-4+\kappa^2)m\omega}{\beta(\kappa^4-2\kappa^2-8)}}$, $b_1 = 0$, $b_2 = \sqrt{\frac{-(2+\kappa^2)m\omega}{\beta(\kappa^4-2\kappa^2-8)}}$, $\alpha = -\frac{4m\omega}{\kappa^2-4}$,

then

$$q(x,t) = \left(-\frac{(\kappa-2)\sqrt{(\kappa+2)\alpha}}{\sqrt{2\beta(\kappa-4)}} + \frac{i\sqrt{\alpha}}{2\sqrt{\beta}}\frac{1}{sinh(2x-2\vartheta t)+2cosh^2(x-\vartheta t)}\right)e^{i(-\kappa x+\frac{(\kappa^2-4)\alpha}{4m}t+\varphi)}.$$

Case 6: When $b_0 = \sqrt{\frac{2(-4+\kappa^2)m\omega}{\beta(\kappa^4-2\kappa^2-8)}}$, $b_1 = 0$, $b_2 = -\sqrt{\frac{8(-4+\kappa^2)m\omega}{\beta(\kappa^4-2\kappa^2-8)}}$, $\alpha = -\frac{m\omega}{\kappa^2+2}$,

then

$$q(x,t) = i\sqrt{\frac{2\alpha}{\beta}}(tanh(x-\vartheta t)+1)e^{i(-\kappa x+\frac{(\kappa^2+2)\alpha}{4m}t+\varphi)}.$$

Case 7: When $b_0 = -\sqrt{\frac{2(-4+\kappa^2)m\omega}{\beta(\kappa^4-2\kappa^2-8)}}$, $b_1 = 0$, $b_2 = \sqrt{\frac{8(-4+\kappa^2)m\omega}{\beta(\kappa^4-2\kappa^2-8)}}$, $\alpha = -\frac{m\omega}{\kappa^2+2}$

then

$$q(x,t) = i\sqrt{\frac{2\alpha}{\beta}}(tanh(x-\vartheta t)+1)e^{i(-\kappa x+\frac{(\kappa^2+2)\alpha}{4m}t+\varphi)}.$$

Power law nonlinearity

For power law nonlinearity

$$F(\theta) = \theta^n$$

so that Equation 8. subsides to

$$i(q^m)_t + \alpha(q^m)_{xx} + \beta(|q|^{2n})q^m = 0. \tag{15}$$

The parameter n enunciates law nonlinearity of the power in Equation 15. In this case, Equation 11. reduces to

$$\alpha m(m-n)(\tau')^2 + \alpha mn\tau\tau'' - (m\omega + \alpha m^2\kappa^2)n^2\tau^2 + \beta n^2\tau^4 = 0 \tag{16}$$

where $\theta = \tau^{\frac{1}{n}}$. Then, we employ the balance principle and find that $M = 2$ then we can write the solution of Equation 16. in the form:

$$\tau(\sigma) = b_0 + b_1\psi(\sigma) + b_2\psi^2(\sigma) \tag{17}$$

By differentiating $\tau(\sigma)$ in Equation 17. two times with respect to σ, we obtain $\tau'(\sigma)$ and $\tau''(\sigma)$. Then putting the terms τ, τ' and τ'' into Equation 16. we have an algebraic equation system and solving this system, we find the following results:

Case1: When

$$b_0 = \sqrt{\frac{2(8+\kappa^2m(3+2m)^2)m\omega}{\beta(2+\kappa^2m^3)(8+\kappa^2m(3+2m)^2)}}, \quad b_1 = 0, \quad b_2 = -\sqrt{\frac{8(8+\kappa^2m(3+2m)^2)m\omega}{\beta(2+\kappa^2m^3)(8+\kappa^2m(3+2m)^2)}}, \quad \alpha = -\frac{m^2\omega}{\kappa^2m^3+2}, \quad n = m,$$

then

$$q(x,t) = e^{i\left(-\kappa x - \frac{(2+\kappa^2m^3)\alpha}{m^2}t+\varphi\right)}\left(\sqrt{\frac{2(8+\kappa^2m(3+2m)^2)m\omega}{\beta(2+\kappa^2m^3)(8+\kappa^2m(3+2m)^2)}}\tanh(x-\vartheta t)\right)^n.$$

Case2: When

$$b_0 = -\sqrt{\frac{2(8+\kappa^2m(3+2m)^2)m\omega}{\beta(2+\kappa^2m^3)(8+\kappa^2m(3+2m)^2)}}, \quad b_1 = 0, \quad b_2 = \sqrt{\frac{8(8+\kappa^2m(3+2m)^2)m\omega}{\beta(2+\kappa^2m^3)(8+\kappa^2m(3+2m)^2)}}, \quad \alpha = -\frac{m^2\omega}{\kappa^2m^3+2}, \quad n = m,$$

then

$$q(x,t) = e^{i\left(-\kappa x - \frac{(2+\kappa^2m^3)\alpha}{m^2}t+\varphi\right)}\left(-\sqrt{\frac{2(8+\kappa^2m(3+2m)^2)m\omega}{\beta(2+\kappa^2m^3)(8+\kappa^2m(3+2m)^2)}}\tanh(x-\vartheta t)\right)^n.$$

Case 3: When

$$b_0 = 0, \quad b_1 = 0, \quad b_2 = -\sqrt{\frac{8m\omega}{\beta(8+\kappa^2m(3+2m)^2)}}, \quad \alpha = -\frac{(3+2m)^2m\omega}{m(8+\kappa^2m(3+2m)^2)}, \quad n = \frac{1}{2}(3+2m),$$

then

$$q(x,t) = -e^{i\left(-\kappa x - \left(\frac{8\alpha}{(3+2m)^2}+\kappa^2m\alpha\right)t+\varphi\right)}\sqrt{\frac{8m\omega}{\beta(8+\kappa^2m(3+2m)^2)\sinh(2x-2\vartheta)+\cosh(2x-2\vartheta)+1}}.$$

Case 4: When

$$b_0 = 0, \quad b_1 = 0, \quad b_2 = \sqrt{\frac{8m\omega}{\beta(8+\kappa^2 m(3+2m)^2)}}, \alpha = -\frac{(3+2m)^2 m\omega}{m(8+\kappa^2 m(3+2m)^2)}, n = \frac{1}{2}(3+2m),$$

then

$$q(x,t) = -e^{i\left(-\kappa x - \left(\frac{8\alpha}{(3+2m)^2}+\kappa^2 m\alpha\right)t+\varphi\right)}\sqrt{\frac{8m\omega}{\beta(8+\kappa^2 m(3+2m)^2)\sinh(2x-2\vartheta)+\cosh(2x-2\vartheta)+1}}.$$

Case 5: When

$$b_0 = \frac{2\sqrt{m\omega(4+2\kappa^2 m^3)}}{16\beta+\kappa^2\beta m\left(18+m(3+2m)\left(8+\kappa^2 m^2(3+2m)\right)\right)}, \quad b_1 = 0, \quad b_2 - \frac{2\sqrt{m\omega(4+2\kappa^2 m^3)}}{16\beta+\kappa^2\beta m\left(18+m(3+2m)\left(8+\kappa^2 m^2(3+2m)\right)\right)},$$

$$\alpha = -\frac{m^2\omega}{\kappa^2 m^3+2}, \quad n = \frac{1}{2}(3+2m),$$

then

$$q(x,t) = e^{i\left(-\kappa x - \left(\frac{8\alpha}{(3+2m)^2}+\kappa^2 m\alpha\right)t+\varphi\right)}\frac{2\sqrt{m\omega(4+2\kappa^2 m^3)}}{\left(16\beta+\kappa^2\beta m\left(18+m(3+2m)\left(8+\kappa^2 m^2(3+2m)\right)\right)\right)\cosh(x-\vartheta t)}.$$

Case 6: When

$$b_0 = -\frac{2\sqrt{m\omega(4+2\kappa^2 m^3)}}{16\beta+\kappa^2\beta m\left(18+m(3+2m)\left(8+\kappa^2 m^2(3+2m)\right)\right)}, \quad b_1 = 0, \quad b_2 = \frac{2\sqrt{m\omega(4+2\kappa^2 m^3)}}{16\beta+\kappa^2\beta m\left(18+m(3+2m)\left(8+\kappa^2 m^2(3+2m)\right)\right)},$$

$$\alpha = -\frac{m^2\omega}{\kappa^2 m^3+2}, \quad n = \frac{1}{2}(3+2m),$$

then

$$q(x,t) = -\frac{2\sqrt{m\omega(4+2\kappa^2 m^3)}e^{i\left(-\kappa x - \left(\frac{8\alpha}{(3+2m)^2}+\kappa^2 m\alpha\right)t+\varphi\right)}}{\left(16\beta+\kappa^2\beta m\left(18+m(3+2m)\left(8+\kappa^2 m^2(3+2m)\right)\right)\right)\cosh(x-\vartheta t)}.$$

Gerdjikov-Ivanov Equation

$$iq_t + \alpha q_{xx} + \gamma q_{xt} + \beta|q|^4 = i(\varsigma q^2 q_x + \alpha q_x + \lambda(|q|^2 q)_x) + \Theta(|q|^2)_x q. \tag{18}$$

The dependent variable $q(x,t)$ which is a complex function, indicates wave profile. Indicates the distance along the fiber x, which is the independent variable and the independent variable t indicates the time dimensionlessly. In Equation 18. the first term indicates linear temporal evolution and the second term refers the scattering of velocity group, on the left hand side of the equation the third term exemplifies spatio-temporal scattering and and finally the fourth term is accountable for qunitic-nonlinearity. On the right side, α is the scattering between the modal, the coefficient λ is the self-correcting term and Θ indicates nonlinear scattering. The equilibrium between nonlinearity and speed distribution causes a soliton.

By considering the traveling wave transformation:

$$q(x,t) = \theta(\sigma)e^{i(-\kappa x + \omega t + \varphi)}, \sigma = x - \vartheta t$$

where ϑ is the solution velocity, κ is the solution frequency, ω is the soliton wave number and φ is the phase constant.

Appliying traveling wave transformation in Equation 16. and seperating it into real and imaginary parts, we obtain

$$\vartheta(1 - \gamma\kappa) + \alpha + 2\alpha\kappa - \gamma\omega + 2\Theta\theta^3 + (\varsigma + 3\lambda)\theta^2 = 0. \tag{19}$$

Equating the coefficients of the linearly independent functions to zero gives

$$\vartheta = -\frac{\alpha + 2\alpha\kappa - \gamma\omega}{(1 - \gamma\kappa)}$$

whereever $\gamma\kappa \neq 1$ along with the constrait conditions $\Theta = 0$ and $\varsigma + 3\lambda = 0$.

The real part gives

$$(\alpha - \gamma\vartheta)\theta'' - (\omega + (\alpha + \gamma)\kappa^2 + \alpha\kappa)\theta + (\varsigma - \gamma)\kappa\theta^3 + \beta\theta^5 = 0. \tag{20}$$

Balancing the highest order derivative and the nonlinear term in Equation 20. gives $M = 1$. So,

$$\theta(\sigma) = b_0 + b_1\psi(\sigma) \tag{21}$$

Substituting the derivatives into Equation 20. and accumulating the coefficient of each power of ψ^k and setting each of coefficient to zero, then solving the resulting algebraic equation system we obtain the following solutions:

Case 1: $b_0 = 0$, $\quad b_1 = -\left(\frac{3\gamma\vartheta - \alpha}{\beta}\right)^{\frac{1}{4}}$, $\quad \omega = \alpha - \gamma\vartheta - \alpha\kappa - \alpha\kappa^2 - \gamma\kappa^2$, $\quad \varsigma = \frac{3\kappa - 4\sqrt{3\beta(\gamma\vartheta - \alpha)}\lambda}{3\kappa}$.

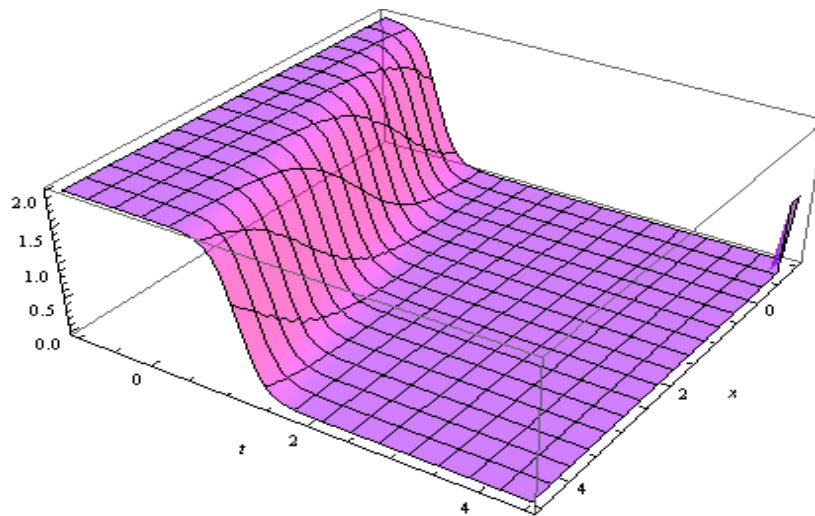Inserting the above coefficients into Equation 21., we obtain the following solution of Equation 18.:

$$q(x,t) = -\left(\frac{3\gamma\vartheta - \alpha}{\beta}\right)^{1/4}\frac{e^{i(\kappa x + (\alpha - \gamma\vartheta - \alpha\kappa - \alpha\kappa^2 - \gamma\kappa^2)t + \varphi)}}{\sinh(2x - 2\vartheta) + \cosh(2x - 2\vartheta)} .$$

Case 2: $b_0 = 0$, $\quad b_1 = -\left(\frac{3\gamma\vartheta - \alpha}{\beta}\right)^{\frac{1}{4}}$, $\quad \omega = \alpha - \gamma\vartheta - \alpha\kappa - \alpha\kappa^2 - \gamma\kappa^2$, $\quad \varsigma = \frac{3\kappa - 4\sqrt{3\beta(\gamma\vartheta - \alpha)}\lambda}{3\kappa}$.

Inserting the above coefficients into Equation 21., we obtain the following solution of Equation 18.:

$$q(x,t) = \left(\frac{3\gamma\vartheta - \alpha}{\beta}\right)^{1/4}\frac{e^{i(\kappa x + (\alpha - \gamma\vartheta - \alpha\kappa - \alpha\kappa^2 - \gamma\kappa^2)t + \varphi)}}{\sinh(2x - 2\vartheta) + \cosh(2x - 2\vartheta)} .$$

Figure 2. The solution $q(x, t)$ for $\alpha = 1$

Remark

    Although the solitary wave solutions obtained by extended modified Kudryashov method, the increase values of the parameters may affect the wavelength and velocity of the wave. Increasing values of parameters may affect wave length and wave velocity.

CONCLUSION

    In this study, the extended modified Kudryashov method has been proposed to construct analytic solutions of evolutionary equations with constant coefficients. Using the proposed method, we have achieved our goal that obtaining the analytical solutions of the Biswas-Milovic equation and Gerdjikov-Ivanov equation. The Kudryashov method yieleded more cases of traveling wave solutions. Moreover, changes in parameters affect both the wavelength and the velocity of the wave. The resulting solutions may be important for certain specific physical events. It can be concluded that this method is standard and effective, allows us to solve complex algebraic calculations.

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