



## THE INFLUENCE OF PRESSURE STIFFNESS ON THE TORSIONAL BUCKLING OF LAMINATED COMPOSITE CYLINDERS UNDER EXTERNAL HYDROSTATIC PRESSURE

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**ABSTRACT:** In this study, the influence of pressure stiffness on the torsional buckling of thin and moderately thick laminated composite perfect cylinders under external hydrostatic pressure is investigated. A degenerated-curved shell element is used to obtain the numerical results. First, a verification problem is solved and the element is validated for stability analysis including pressure stiffness. Then, cross-ply and angle-ply laminated composite cylinders are examined for selected length-radius ratios and stacking sequences. The results obtained show that, pressure stiffness may have a stabilizing effect against torsional buckling especially for thick orthotropic cylinders.

**Keywords:** Pressure stiffness, torsional buckling, composite cylinder, hydrostatic pressure

### Basınç Rijitliğinin Hidrostatik Basınç Etkisi Altındaki Katmanlı Kompozit Silindirlerin Burulmalı Burkulmalarına Etkisi

**ÖZ:** Bu çalışmada hidrostatik basınç altındaki ince ve orta kalınlıkta katmanlı kompozit kusursuz silindirlerin burkulmasında basınç rijitliğinin etkisi araştırılmıştır. Sayısal neticeler dejenere-eğri bir kabuk elemanı kullanılarak elde edilmiştir. Öncelikle bir doğrulama problem çözülmüş ve eleman basınç rijitliğini içeren stabilite tahkiki için doğrulanmıştır. Daha sonra, dik-açılı ve eğik-açılı katmanlı kompozit silindirler seçilen uzunluk/çap değerleri ve katman düzenleri için incelenmiştir. Elde edilen neticeler basınç rijitliğinin özellikle kalın ortotropik silindirlerde burulmalı burkulmayı engellediğini göstermektedir.

**Anahtar kelimeler:** Basınç rijitliği, burulmalı burkulma, kompozit silindir, hidrostatik basınç

## INTRODUCTION

In this study, the influence of pressure stiffness ( $PS$ ) on the torsional buckling of thin and moderately thick laminated composite perfect cylinders under external hydrostatic pressure is investigated.

One of the most comprehensive studies on buckling of composite shells was performed by Seide *et al.*(1968) who has prepared the NASA-SP8007 monograph. Nemeth and Starnes (1998) has improved this monograph by considering imperfections. However, the literature on the influence of  $PS$  on shell stability is not extensive.  $PS$  is known to have a detrimental effect on the stability of shells under compressive loads and the related functional was derived by Koiter (1967). Schokker *et al.* (1996), Sridharan and Kasagi (1997) and Kardomateas (1996, 2000) have also considered the influence of  $PS$  on buckling of composite cylinders. In more recent studies, Cagdas and Adali (2011) have shown that due to pressure stiffness, a reduction of up to 30% in buckling pressure is possible for cross-ply laminated cylinders under hydrostatic pressure. The influence of  $PS$  has recently been considered for filament wound cones by Cagdas (2017), for cylindrical panels by Smitha and Nandakumar (2018), for thick

laminated composite shells by Khayat *et al.* (2016), and for functionally graded shells under external pressure by Khayat *et al.* (2017). In a more recent study, Basaglia *et al.* (2019) have considered *PS* for pipes under hydrostatic pressure and have stated that the buckling pressure will be overestimated if *PS* is not taken into account.

As shell thickness increases, the flexural stiffness terms increase much more rapidly than the shear stiffness terms. Thus, for thicker cylinders, torsional buckling may become more critical than flexural buckling. However, it will be shown in this study that this may not be the case taken into account.

A degenerated composite shell element based on the shell element developed by Kant (1992) is used to conduct the related structural analyses. This same element was used by Cagdas and Adali (2012a, 2012b) for linearized stability analysis and design optimization of composite shell structures. This element was recently modified by Cagdas (2017) for stability analysis considering *PS*. Similar shell finite elements were used by Ram and Babu (2002), and Barbosa and Ferreira (2009), Bakshi and Chakravorty (2014). ANSYS shell element "Shell 99" is a similar shell element, which can be used for stability analysis of thin-walled composite structures; see, for example Öner *et al.* (2007) and Shen *et al.* (2017).

First, the shell element used is validated by comparing the results obtained here with the analytical results presented by Kardomateas (1996) and with the numerical results presented by Cagdas and Adali (2012) for an orthotropic composite cylinder problem. Then, numerical results are presented for cross-ply and angle-ply laminated composite cylinders. The results obtained in the present study show that *PS* considerably interferes with the buckling behavior of relatively thick laminated composite cylinders.

## RESEARCH SIGNIFICANCE

The results presented in this study will provide further insight into the issue of laminated composite cylinder buckling under hydrostatic pressure. It is aimed to show that the inclusion of pressure stiffness eliminates the torsional buckling mode and increases the buckling parameter for cross-ply laminated/orthotropic composite perfect cylinders under hydrostatic pressure. This finding is of practical importance as construction of orthotropic cylinders may be advantageous or more economical.

## PROBLEM DEFINITION

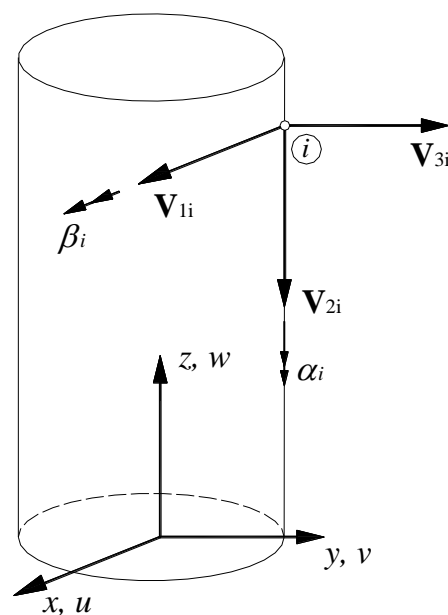


Figure 1. The global, and nodal coordinate systems

The shell element is based on the following displacement field;

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \sum_{i=1}^{nn} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{mid} S_i(\xi, \eta) + \frac{1}{2} \sum_{i=1}^{nn} S_i \zeta \mathbf{V}_{3i} \tag{1}$$

where,  $nn, S_i(\xi, \eta), \mathbf{V}_{3i}$  denote the total number of nodes, the value of the  $i^{\text{th}}$  shape function at  $(\xi, \eta)$ , and the unit surface normal vector at node  $i$ . respectively. The range of  $\xi$  and  $\eta$  are -1 to 1 and the range of  $\zeta$  is  $-t/2$  to  $t/2$ , where  $t$  denotes thickness.  $\alpha_i$ , and  $\beta_i$  denote the rotations about  $\mathbf{V}_{2i}$ , and  $\mathbf{V}_{1i}$  as shown in Fig. 1. For the problem under consideration,  $\mathbf{V}_{3i}$  is selected as the outward unit normal and the unit vector  $\mathbf{V}_{1i}$  is calculated by multiplying  $\mathbf{V}_{3i}$  with the unit vector in  $z$  direction,  $\mathbf{e}_3$ ; i.e.  $\mathbf{V}_{1i} = \mathbf{V}_{3i} \times \mathbf{e}_3$  as shown in Fig. 1. The details of formulation will not be given here as related information can be found elsewhere; for example see Reddy (2003). Note that, the lamination angle is taken as the angle between the fiber direction and  $\mathbf{V}_{1i}$ .

**NUMERICAL RESULTS**

First a verification problem is solved and then numerical results are presented for cross-ply and angle-ply laminated cylinders.

**Verification Problem**

Circumferentially reinforced graphite/epoxy cylinders of length  $L$ , external radius 1 m., constant thickness  $H$ , and length to external radius ratio  $L/R_{ext} = 10$  under external pressure are analyzed here using the 2D shell element modified for the  $PS$  effect. At the ends of the shells, only radial and circumferential displacements are set equal to zero. Comparisons are made with the analytical results for orthotropic cylinders under external pressure, obtained by Cagdas and Adali (2011) and Kardomateas (1996) who have considered the effect of non-conservative loading.

The material properties are given below;

$$E_1 = 140 \text{ GPa}, E_2 = 9.1, E_3 = 9.9 \text{ GPa}, G_{12} = 4.3 \text{ GPa}, G_{13} = 4.7 \text{ GPa}, G_{23} = 5.9 \text{ GPa}, \nu_{12} = 0.3, \nu_{13} = 0.02, \nu_{23} = 0.49.$$

The results obtained for  $L/R_{ext} = 10$  and for  $R_{ext}/R_{int}$  ratios between 1.05 and 1.25 are presented in Table 1 in comparison with the results obtained by Kardomateas (1996) and Cagdas and Adali (2011), where  $n=2$  for all cases. In Table 1,  $\bar{q}_{cr}, R_{int}$  and  $R_{ext}$  denote external pressure, internal radius, and external radius of the cylinder, respectively. The pressure acts at the outer surface, so the axial compressive force acting on top of the cylinder is calculated as  $f = q\pi(R_{int} + H)^2$ . The non-dimensional buckling parameter is defined as;

$$\bar{q}_{cr} = \frac{q_{cr} R_{ext}^3}{E_1 H^3} \tag{2}$$

The percentage change in buckling pressure  $\eta(\%)$  is defined as;

$$\eta(\%) = \frac{\lambda_{cr,PS} - \lambda_{cr}}{\lambda_{cr}} \times 100 \tag{3}$$

where  $\lambda_{cr}$  denotes buckling parameter and subscript  $PS$  indicates that  $PS$  is taken into account.

**Table 1.**  $\bar{q}_{cr}$  for orthotropic cylinders with  $L/R_{ext} = 10$ .

$\frac{R_{ext}}{R_{int}}$	Elasticity <sup>(1)</sup>	FEM <sup>(2)</sup> (without <i>PS</i> )	FEM <sup>(2)</sup> (with <i>PS</i> )	Present	Present	$\eta(\%)$
				2D Shell 40×8 mesh (without <i>PS</i> )	2D Shell 40×8 mesh (with <i>PS</i> )	
1.05	0.2576	0.3447	0.2687	0.3444	0.2686	-22.0
1.10	0.2513	0.3211	0.2581	0.3192	0.2573	-19.4
1.15	0.2347	0.2878	0.2467	0.1484 *	0.2447	64.9
1.20	0.2166	0.2535	0.2308	0.0873 *	0.2276	160.7
1.25	0.1978 (?)	0.2227	0.2128	0.0583 *	0.2007	244.3

<sup>(1)</sup> Kardomateas (1996), <sup>(2)</sup> Cagdas and Adali (2011), \* torsional buckling mode

As can be observed from Table 1, the results obtained using the 2D shell model are in good agreement with the elasticity solution presented by Kardomateas (1996) and in excellent agreement with the results obtained using the axisymmetric shell model with and without the inclusion of the *PS* effect for  $R_{ext}/R_{int} \leq 1.10$ . However, when *PS* was not taken into account, a torsional buckling mode was observed for the case  $R_{ext}/R_{int} \geq 1.15$ . This torsional buckling mode shape, captured by the 2D shell model only, is shown in Fig. 2(a). However, with the inclusion of *PS*, the buckling mode shape changes as shown in Fig. 2(b) and the buckling parameter increases and gets closer to the one calculated using the axisymmetric model. Similarly, it can be seen from Figures 2(c) and 2(d) that, for  $R_{ext}/R_{int}$ , torsional buckling does not occur when *PS* is taken into account. Also, as can be seen from Table 1, when *PS* is taken into account, the torsional buckling mode shape vanishes and this implies that *PS*, which is expected to be detrimental, unexpectedly stabilizes the cylinders with  $R_{ext}/R_{int} \geq 1.15$  against torsional buckling. In Table 1, the variable  $\eta$  values are also presented as 64.9%, 160.7%, and 244.3% for  $R_{ext}/R_{int} = 1.15, 1.20, \text{ and } 1.25$  respectively.

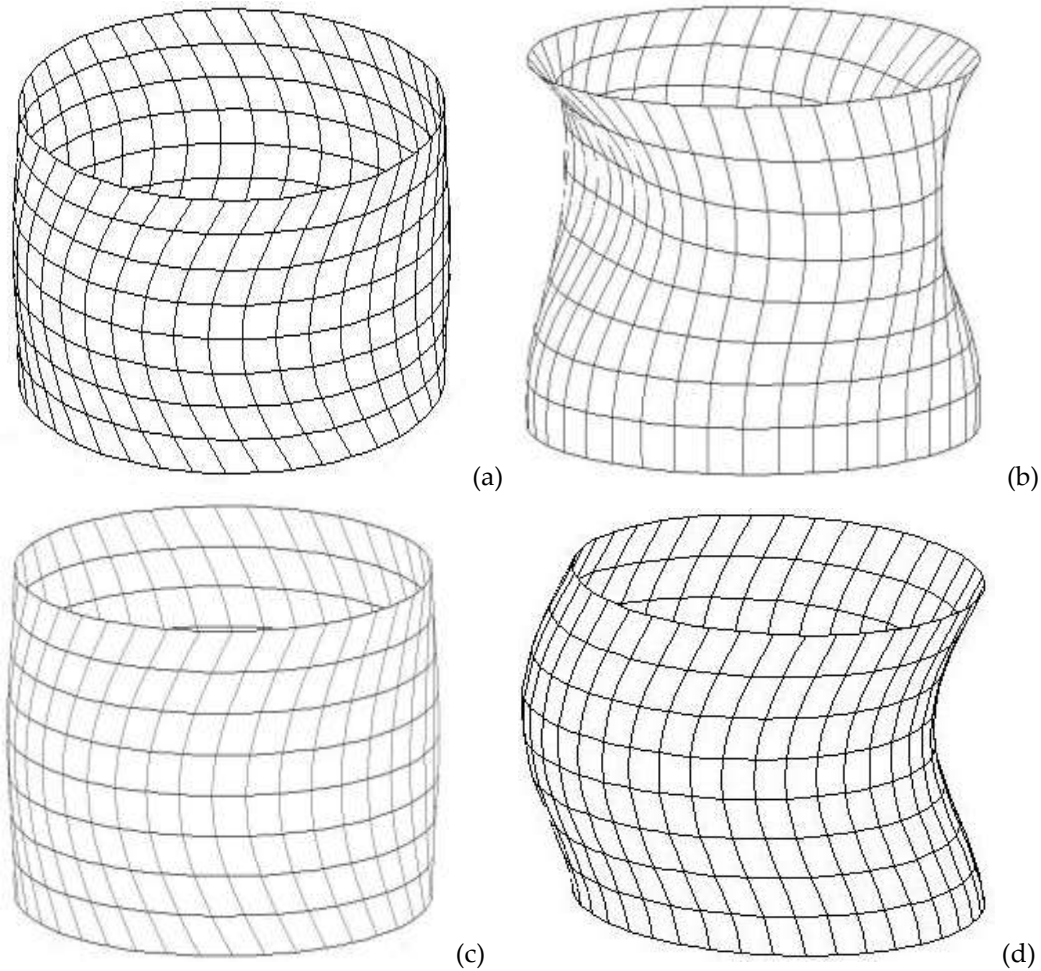
Next, the problem is solved again for the fully clamped boundary conditions, except for the axial displacement at one of the ends, but the buckling parameters did not significantly change and therefore the results obtained are not presented here. It therefore may be stated that the influence of *PS* is not related with the rotational boundary conditions.

It should also be stated that, even though the formulation of the 2D degenerated shell element employed here is not based on a specific shell theory and is not as sophisticated as the shell theory based axisymmetric shell element of Cagdas and Adali (2011), where the thickness/radius ratio is also taken into account, the results obtained using the 2D shell element, presented in Table 1, are found out to be more accurate for some cases as complicated buckling mode shapes can be better captured by the 2D shell model comparing with the axisymmetric model.

### Cross-ply Laminated Cylinders

Numerical results obtained for graphite-epoxy cylinders with S3 B.C., stacking sequence  $[90^\circ/0^\circ]_s$ , and varying  $R_{ext}/R_{int}$  ratios are presented for  $L/R_{ext} = 5$ , and 10 in Tables 2 and 3, respectively. It can be observed from Table 2 that, the stabilizing influence of *PS* is minimal for  $L/R_{ext} = 5$  as flexural buckling occurs for the  $R_{ext}/R_{int}$  ratios considered. However, as can be observed from Table 3, the influence of *PS* is much higher for  $L/R_{ext} = 10$  due to the fact that a greater surface area is under hydrostatic pressure comparing with the case  $L/R_{ext} = 5$ . This behavior was also observed by Cagdas and Adali (2011). It can be observed from Table 3, that the percentage increase is nearly 250% for  $R_{ext}/R_{int} = 1.25$ , which also was

the case for the verification problem solved. The buckling mode shapes for  $R_{ext}/R_{int} = 1.25$  with and without *PS* effect are given in Figures 3(a) and 3(b), which clearly show that the inclusion of *PS* in stability analysis yields a flexural buckling mode shape instead of a torsional one.



**Figure 2.** Buckling mode shapes for cross-ply laminated cylinders under hydrostatic pressure with  $L/R_{ext} = 10$  (a)  $R_{ext}/R_{int} = 1.15$ , without *PS* (b)  $R_{ext}/R_{int} = 1.15$ , with *PS* (c)  $R_{ext}/R_{int} = 1.3$ , without *PS* (d)  $R_{ext}/R_{int} = 1.3$ , with *PS*

**Table 2.**  $\bar{q}_{cr}$  for  $[90^\circ/0^\circ]_s$  cross-ply laminated cylinders with  $L/R_{ext} = 5$ .

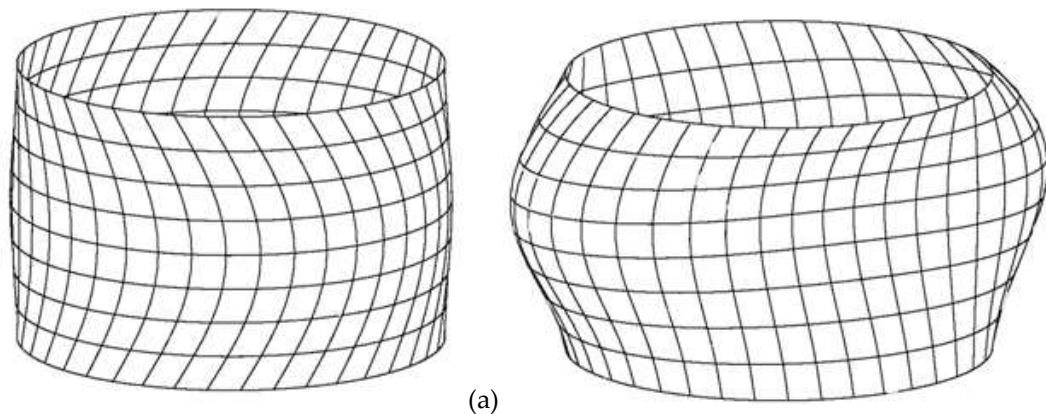
$\frac{R_{ext}}{R_{int}}$	Present 2D Shell 40×8 mesh (without <i>PS</i> )	Present 2D Shell 40×8 mesh (with <i>PS</i> )	$\eta(\%)$
1.05	0.6503	0.51410	-20.9
1.10	0.3711	0.29810	-19.7
1.15	0.3067	0.25120	-18.1
1.20	0.2714	0.22704	-16.3
1.25	0.20677	0.20868*	0.9

\* torsional buckling mode

**Table 3.**  $\bar{q}_{cr}$  for  $[90^\circ/0^\circ]_s$  cross-ply laminated cylinders with  $L/R_{ext} = 10$ .

$\frac{R_{ext}}{R_{int}}$	Present	Present	$\eta(\%)$
	2D Shell 40×8 mesh (without PS)	2D Shell 40×8 mesh (with PS)	
1.05	0.3601	0.2774	-23,0
1.10	0.3075	0.2712	-11,8
1.15	0.1483	0.2286	54,1
1.20	0.0872	0.21650	148,3
1.25	0.0583	0.2037	249,4

\* torsional buckling mode



**Figure 3.** Buckling mode shapes for cross-ply laminated cylinders under hydrostatic pressure with  $L/R_{ext} = 10$ , and  $R_{ext}/R_{int} = 1.25$  (a), without PS (b) with PS

### Angle-ply Laminated Cylinders

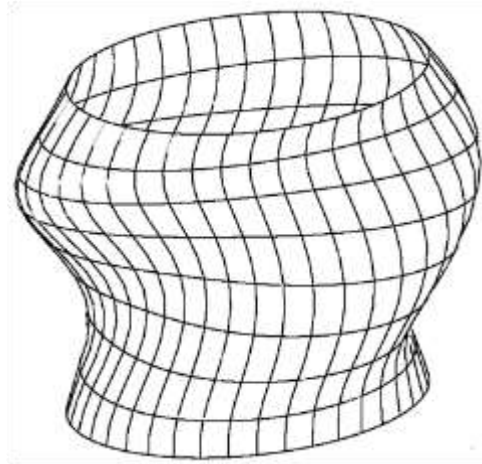
Finally, numerical results are presented in Table 4 for angle-ply graphite-epoxy cylinders with S3 B.C., stacking sequence  $[55^\circ/-55^\circ]_s$ ,  $L/R_{ext} = 10$ , and varying  $R_{ext}/R_{int}$  ratios. The lamination angle is selected as equal to  $55^\circ$  as this angle maximizes the static strength; see Messenger *et al.* (2002).

**Table 4.**  $\bar{q}_{cr}$  for  $[55^\circ/-55^\circ]_s$  angle-ply laminated cylinders under hydrostatic pressure,  $L/R_{ext} = 10$ .

$\frac{R_{ext}}{R_{int}}$	Present	Present	$\eta(\%)$
	2D Shell 40×8 mesh (without PS)	2D Shell 40×8 mesh (with PS)	
1.05	0.1888	0.1459	-22.7
1.10	0.1617	0.1276	-21.1
1.15	0.1418	0.1142	-19.5
1.20	0.1301	0.1065	-18.1
1.25	0.1216	0.1011	-16.9

It can be observed from Table 4, that PS has no positive influence for all of the  $R_{ext}/R_{int}$  values considered and the maximum decrease percentage is equal to 22.7% for  $R_{ext}/R_{int} = 1.05$ . The buckling

mode shape for  $R_{ext}/R_{int} = 1.25$  shown in Figure 4 is almost the same for both cases, which indicates that for the angle-ply cylinders considered, the buckling mode did not change with the inclusion of  $PS$ .



**Figure 4.** Buckling mode shape for angle-ply laminated cylinders under hydrostatic pressure with  $L/R_{ext} = 10$ , and  $R_{ext}/R_{int} = 1.25$ , with and without  $PS$

## CONCLUSIONS

In this study, the influence of pressure stiffness ( $PS$ ) on torsional buckling of thin and moderately thick laminated composite perfect cylinders under external hydrostatic pressure is investigated using a degenerated shell finite element.

First, verification problems are solved and the super parametric shell finite element used in this study is validated for stability analysis including pressure stiffness. The results obtained for the verification problems show that the torsional buckling mode shape vanishes for orthotropic cylinders with  $R_{ext}/R_{int} \geq 1.15$  when  $PS$  is taken into account. In order to more thoroughly investigate the problem, buckling parameters are presented for cross-ply and angle-ply laminated composite cylinders under external hydrostatic pressure and the following observations are made;

*i.* For cross-ply laminated cylinders, the influence of  $PS$  decreases with decreasing  $L/R_{ext}$  ratio and increases with increasing thickness. The positive influence of  $PS$  is higher for  $L/R_{ext} = 10$ , and for the cross-ply cylinders considered, a nearly 250% increase in buckling pressure is observed for  $R_{ext}/R_{int} = 1.25$  and  $L/R_{ext} = 10$ . However, for  $R_{ext}/R_{int} \leq 1.1$ , the influence of  $PS$  is detrimental as expected.

*ii.* For angle ply cylinders, an increase in buckling pressure due to  $PS$  is not observed as the in-plane shear stiffness values of angle-ply cylinders are higher than the ones for cross-ply cylinders. It is also shown for angle-ply cylinders that, the buckling mode shape does not change with the inclusion of  $PS$ .

In conclusion, it may be stated that for composite cylinders under external hydrostatic pressure  $PS$  should be included in the finite element model, as failure to do so may lead the designer to reach wrong estimates about both the buckling pressure, and the mode shape.

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