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Divergent Thinking and Convergent Thinking: Are They Promoted in Mathematics Textbooks?

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Abstract

This study explores whether mathematics tasks in primary school mathematics textbooks provide opportunities for divergent and convergent thinking. Four mathematics textbooks (one from each of first to fourth grades) are examined for this purpose. A task is divided into three segments for the analysis and the segments are named as the beginning, the intermediary, and the end. These segments are analysed in terms of the numbers of entry points, solution methods, and correct outcomes respectively. The modes of the segments enable us to identify six different tasks. Tasks that definitively have an open-ending (multiple correct outcomes) are considered to have divergent thinking features and those which have only one correct outcome are deemed to have convergent thinking characteristics. The study reveals that the textbooks provide opportunities for both divergent and convergent thinking, yet more chances are particularly given for convergent thinking. The findings are discussed in relation to divergent and convergent thinking alongside creativity and some implications are provided for textbooks studies.

Key words: Divergent thinking, Convergent thinking, Mathematics textbook analysis, Creativity, Open-ended tasks

Introduction

Divergent thinking (DT) and convergent thinking (CT), which are often associated with creativity, are desirable skills for student learning in all disciplines including mathematics education. These two terms (together, separately or only DT) sometimes are used to describe creativity as creativity is also linked with "original thought and divergence from the norm" (Bennevall, 2016, p. 1; Cropley, 2006). DT is generally associated with multiple and alternatives outcomes while convergent one is linked with a single correct answer in responding to a posed question (Cropley, 2006). Alongside their descriptions and features, ideas about how to cultivate and foster them in students have also drawn attention. One aspect that has drawn particular attention is the tasks and their features. Open-ended tasks are one such type that has received more attention.

Given that cultivating and fostering DT and CT in students are closely related to tasks to which students are exposed, it is then important to examine school textbooks in terms of types of the tasks that they present. It is of great importance to examine textbooks in this respect as the research shows that textbooks, their contents and their structures have influences on both student learning and teacher teaching (Valverde et al., 2002; Haggarty and Pepin, 2002; Eisenmann and Even, 2011; Olsher and Even, 2014). It appears, however, that an examination of textbook tasks in terms of DT and CT has not been the focus of much research in mathematics education. With this gap in mind, this study sets out to examine the extent to which primary mathematics textbooks present tasks with DT and CT features.

After this brief introduction, this paper is structured as follows. First, literature review on DT and CT is provided in connection with creativity. Then, DT and CT are examined in close relation with the task issue and a particular attention is paid to open-ended tasks. Afterwards, the theoretical framework of the study is presented. Methodology, results, discussion, conclusions and educational implication sections follow this section respectively.

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Literature Review

This section presents features and definitions of DT and CT first and then provides a concise review on how these two have been studied. They are particularly examined in connection with the research on the tasks studies.

DT and CT are considered to be the two key important cognitive processes for creative thinking and Joy Paul Guilford (1967) is often given credit for making distinction between these two processes (Japardi, Bookheimer, Knudsen, Ghahremani & Bilder, 2018). Based on the relevant literature, Japardi et al. (2018, p. 59) describe DT as "the ability to disengage from prevailing modes of thought and expression to generate novel ideas and solutions" while CT as "the recruitment and interaction of different cognitive processes to find a common solution to a given problem." Similar to the description of Japardi et al. (2018), Cropley (2006, p.391) defines DT as one "involves producing multiple or alternative answers from available information". By the contrast, he defines CT as being "oriented toward deriving the single best (or correct) answer to a clearly defined question". Similar descriptions of DT and CT are put forward by many different researchers (e.g., Brophy, 2011; Kwon, Park and Park, 2006; Balka, 1974) and common to all is that DT is associated with *variability* and CT is linked with *orthodoxy* (Cropley, 2006).

As far as research on DT and CT is concerned, it seems that they have mainly been researched in relation to the issues of creativity, personality traits, cognitive processes, mathematics learning performances, and tasks types and features. First and most noticeably, creativity is the main factor that DT and CT have received a close attention in research. Even though there are different views, it is mainly DT that is being associated with creativity. For instance, based on a meta-analysis, Kim (2008) found that a significantly higher relationship do exist between creative achievement and DT test scores. Cropley (2006) approaches creative thinking in terms of novelty and associates DT with generation of the novelty while CT with the evaluation of the novelty. He emphasizes the important role of CT and notes that without CT, DT may not be fruitful for the production of effective novelty.

In terms of personality traits, the question of "Why do some people like to come up with multiple possibilities, whereas others stick to the first solution that comes to their mind?" is raised by Wronska, Bujacz, Gocłowska, Rietzschel & Nijstad (2019) and these contrasting preferences are examined in the light of theoretical lens of 'need for cognitive closure (NFC)'. DT and CT tasks are used to see how participants in high or low in NFC perform and the findings show that participant high in NFC demonstrated more negative emotion and felt less capable while solving divergent task. Shen, Hommel, Yuan, Chang & Zhang (2018) examined the relationship between risk-taking and CT and their findings revealed that low risk takers demonstrated better achievement in CT and also there was no significant connection between risk-taking and DT. Brophy (2001) examined performance, creative problem solving activities and attributes of participants inclined to CT or DT and tested several hypotheses about the personality and cognitive traits in relation with DT and CT. Divergent performance was, for instance, found to be significantly higher than convergent performance for participants with high ambiguity tolerance and similarly divergent performance was found to be significantly higher than convergent performance for participants with an extraversion inclination.

With regard to cognitive processes, DT and CT also have received attention in a discipline such as neuropsychology as a part of studies on "cognitive and physiological processes underlying creativity" (Japardi, Bookheimer, Knudsen, Ghahremani & Bilder, 2018, p.59). In such studies, functional magnetic resonance imaging (fMRI) is used to evaluate brain function in the course of DT and CT while participants undertake tasks such as the Alternate Uses Task (AUT) and Remote Associates Task (RAT). For instance, the meta-analysis of neuroimaging investigations and fMRI studies by Wu et. al. (2015, p.1) showed that "distributed brain regions were more active under DT tasks (DTTs) than those under control tasks".

DT and CT have been the focus of attention in terms of mathematics learning and teaching as well. For instance, Kwon et al. (2006) developed a program based on open-ended problems for cultivating DT, applied it to 7th grade middle school students and measured its effect through pre- and post-testing. The findings showed that those students who received treatment on DT performed better than those who did not get the treatment on fluency, flexibility, and originality components of DT. In another study, Imai (2000) examined DT in relation with overcoming fixation through mathematical open-ended problems. The researcher found that overcoming fixation was related to DT in terms of its flexibility and originality components and showed that diverse and novel ideas came from those students who could overcome fixation.

Lee (2017) examined secondary mathematics teacher candidates' modifications of textbook tasks for DT and CT during a university course on learning to teach mathematics, which also had a focus on task features for creativity education. Only 23.9% of modified tasks were considered to be appropriate for creativity and these tasks were mainly categorized as following convergent-divergent model (CDM) and divergent-convergent model (DCM), which are proposed by Foster (2015). In the CDM, the beginning of the task has a closed entrance and the later phase is more open to the other possibilities. In the DCM, it is the other way around. In line with many other researchers (e.g., Cropley, 2006), Lee (2007) associated DT with "creating variability" and CT with "exploring variability" and noted that CT was the most effective one in modifying textbook tasks as it was used in the beginning as an enticer and in the end as an elaborator.

The issue of task undoubtedly is the most important aspects of studies on DT and CT. Different task types and tasks with various features have been used to both assess and develop creative thinking. As this study is specifically concerned with tasks, this aspect of the literature review is separately presented below.

Tasks, Divergent Thinking and Convergent Thinking

The issue of the task has always been essential to any discussion on creativity, DT and CT. Task and its features have been especially considered to be important for assessing, characterising and fostering creativity, DT and CT. For instance, the ability to overcome fixation and the ability for divergent production are two constructs which are generally employed to both assess and recognise creative thinking (Haylock, 1985; Haylock, 1987; Haylock, 1997). Divergent production is linked to originality and flexibility in mathematical tasks and that this type of task provides opportunity for many possible acceptable responses. Problem-solving, problem-posing and redefinition tasks are considered to meet the criteria that divergent production tasks require (Haylock, 1997).

In the study of Leikin and Lev (2007), multiple solution tasks are connected with creativity in mathematics and used to assess the creativity of the responses. Drawing from related studies in the literature, fluency, flexibility and novelty are taken as components of creativity; the numbers of solutions are linked with fluency, the pace of procedure solving and switches between alternative solutions are related to fluency and the conventionality of solutions are associated with novelty (ibid.). Silver (1997) also emphasizes the use of problem-solving and problem-posing tasks and argues that the use of these tasks can contribute to fluency, flexibility and novelty dimensions of creativity. The utilisation of ill-structured and open-ended problems is especially emphasised in this regard. Balka (1974) also refers to the role of divergent and open-ended items in identifying mathematical creativity in classrooms.

Based on a literature review on tasks for creativity, Bennevall (2016) provides some examples of creative open-ended tasks which are classified as insight, problem-solving, problem-posing, redefinition, open-classical analogy and generative tasks. Kwon et al. (2006, p.53) also refer to the importance role that open-ended problems can play in the development of DT and note that "in the course of searching for diverse solutions and various approaches, students can put forward many ideas freely (fluency), and make other attempts to devise new strategies to tackle the problem where others fail (flexibility), and think up very clever and unexpected ideas (originality)." They further note that open-ended problem was very effective in fostering mathematical creativity and DT, and employed the following types of open-ended problems to foster DT in students through a program adopted an open-ended approach: (1) overcoming fixations, (2) multiple answers, (3) multiple strategies (4) strategy investigations, (5) problem posing, (6) active inquiry tasks, and (7) logical thinking. These problems show not only how open-ended problems are classified but also reveal what types of tasks are considered to develop DT.

While the aforementioned researchers focus on open-ended problems and their roles in fostering creativity, DT and CTD, Foster (2015) directly introduces the terms of convergent and divergent task and provides definitions for them. Foster (2015, p.14) defines convergent task as "the one which has a single, correct answer, which may be arrived at by a range of different methods" and defines divergent task as "is more open-ended and provokes a more diverse range of outcomes." He also introduces a two-phase Convergent-Divergent Model (CDM). The idea behind the CDM is that the task starts with a safer beginning and then provides the opportunity for a divergent phase. Foster (2015) notes that alternative models are also possible and that a task can start with a divergent phase and then directs the learners for more convergent lines of activities.

All these studies suggest that tasks and their features are crucial for both assessing and fostering creativity and the associated cognitive process such as DT and CT. With particular regard to the focus of this study, it is hence important to examine whether curriculum materials provide opportunity for such tasks or not. An overall examination of textbook research studies show that textbooks have been subjected to the analysis in terms of problem types or task types and the results show that the textbooks of many countries mainly provide closed-ended problems or tasks (Zhu and Fan, 2006; Han, Rosli, Capraro and Capraro, 2011; Yang, Tseng and Wang, 2017; Glasnovic Gracin, 2018; Bingolbali, 2019). The literature also shows that textbook tasks have not been the focus of particular attention in terms of DT and CT and this study is aimed to fill the gap in this respect. The theoretical framework of the study presented below is helpful to show how this aim is intended to be realised.

Theoretical Framework

This section presents the theoretical framework of the study and explains how tasks with DT and CT features are conceptualized in this study.

As outlined above, tasks with DT features are often associated with open-ended tasks or problems while tasks with CT features are often linked with close-ended tasks or problems. In alignment with the extant literature, in this study, we also take tasks with CT features as "the one which has a single, correct answer, which may be arrived at by a range of different methods" and consider tasks with DT features as "more open-ended and provokes a more diverse range of outcomes" (Foster, 2015, p.14). This description of divergent and convergent tasks by Foster mainly focuses on the outcomes, yet the process of reaching to the outcomes is not particularly emphasized. More clarifications are hence needed to conceptualise tasks with DT and CT features.

Based on the work of Reitman (1965) with regard to well-structured and ill-structured problems, Leung (1997) also focuses on 'starting situation' and 'goal situation' to describe an open-ended task. Pehkonen (1997, p.8) considers a problem closed when "… its starting situation and goal situation are closed, i.e. exactly explained" and regards a problem as an open one when "… the starting situation and/or the goal situation are open". Pehkonen (1997) focuses on the beginning and the end of the task and decides if a task is open or closed accordingly. Bennevall (2016, p. 8) construes Pehkonen's description of an open task and states that when a task is open it is "open either in the beginning or in the end or at the both".

Bennevall (2016, p.8), citing Nohda (2000), also states that some other researchers see "the possibility of an open *intermediary* between the beginning and the end". The *intermediary* refers to different or alternative strategies employed to reach the outcome in a problem or task. Sometimes a task will have a closed beginning and a closed end, but it might require multiple solution methods and hence have an open intermediary. Some other possibilities are also feasible and a task can have an open beginning, open intermediary and open end as well.

In this study, in order to decide if a task has DT or CT features, we divide a task into three segments and examine each segment separately: i.) the beginning, ii.) the intermediary, and iii.) the end. The beginning of a task can have one or multiple entry points. The intermediary refers to solution strategies/methods or ways that a task explicitly addresses for carrying out the solution. The end refers to the outcomes of the task and it can either have one correct outcome or multiple ones. We employ this framework to analyse textbook tasks to show what sorts of the beginning, the intermediary and the end that they have and decide if the related task have DT or CT features. Tasks which ultimately have an open-ending are considered to have DT features while tasks which enable only one correct outcome are considered to have CT features. The framework is used as a data analysis tool as well and more exemplary descriptions are provided in the methodology section.

Methodology

As a qualitative research method, document analysis is employed in this study (Bowen, 2009). Patton (2015, p. 222) draws attention to the importance of document analysis method and notes that "while observation, interviews, and fieldwork dominate qualitative methods, analysis of documents and text is taking on increased importance in an information and communications age when texting has emerged as a verb". Closely related to content and thematic analysis, document analysis involves an iterative process of scrutinising, reviewing and interpreting the content, recognising patterns within data, forming emergent themes and constructing categories for the data analysis (Bowen, 2009). Document analysis requires more than just a sheer description of the content. More importantly, production of empirical knowledge and development of understanding are expected to come from the process of evaluating the document (ibid.).

In this study, mathematics textbooks are used as documents for the analysis. The textbooks' contents are examined and reviewed, and this process enables us to construct categories to analyse the data in terms of DT and CT. We think that this process leads us to develop categories, as provided in the data analysis framework below, that contributes to the development of understanding of mathematics textbooks' tasks analysis in terms of DT and CT.

The data source: textbooks and their selection criteria

Primary schooling lasts four years in Turkey (from 6 or 7 years to 10 or 11 years). This study focuses on four primary school mathematics textbooks (Altay, Gümüş, Yaman, Özer & Akar, 2018; Atlı, Doğangüzel, Güneş & Şahin, 2018; Genç, Güleç, Şahin & Taşcı, 2018; Özçelik, 2018) for the data analysis. The textbooks are electronically provided on the webpage linked to Ministry of National Education. All selected textbooks are approved by the National Educational Board for their compatibility with the official mathematics curriculum and the related regulations of the Ministry. The textbooks are prepared by both the Ministry and some private educational companies. For each grade level only one textbook is chosen for the study. For some grades, more than one textbook was provided (at most three textbooks were available for a grade) and for some others only one textbook was available on the webpage. When one textbook was available, it was directly taken for the data analysis. If two textbooks were available, we chose the one officially prepared by the Ministry. When the numbers of available textbooks were more than two and two of them were prepared by the Ministry, one of them was randomly selected for the data analysis.

The textbooks employed different titles to present their contents. Such titles as 'task', 'task basket', 'the reminder', ' let's recall', 'let's study', 'let's learn', 'did we learn?', 'let's enjoy', 'fun time', 'your turn', 'exercise', 'example', 'fun time' and 'unit evaluation' were used to present the contents in all four textbooks. As we specifically focused on tasks in this study, 'task' and 'task basket' were the ones which we examined for the data analysis. Even though some textbooks prefer the term 'task basket' instead of 'task', it should be noted that it is the same as 'task' and we hence consider 'task basket' as 'task' for our analysis. Teachers are expected to use these tasks as a classroom activity to mainly either introduce a concept or to consolidate what students have just learnt. Some task examples from different textbooks are provided in the following section.

The data analysis frameworks

In the light of theoretical framework of the study, two complementary data analysis frameworks are developed. The first framework is concerned with determining if a task has one or more entry point(s) or solution method(s) or final outcome(s). For this purpose, a task is divided into three segments: the beginning, the intermediary and the end. The beginning segment is defined in terms of entry point(s), the intermediary is defined in terms of solution method(s) and the end is defined in terms of outcome(s) (see Table 1).

Table 1. Task segments in terms of entry points, solution methods and outcomes

	Task segments	Entry/method/outcome mode
Task	Beginning (B)	B1.One entry point B2. Multiple entry points
	Intermediary (I)	I1. One solution method I2. Multiple solution methods
	End (E)	E1. One outcome E2. Multiple outcomes

Table 1 presents both task segments and their entry/method/outcome mode. We explain how we define and differentiate task segments first and then state how we determine their entry, method and outcome modes. The beginning of the task refers to the numbers of choices that the readers are entitled in starting the task. When they have a single choice, the beginning is considered to permit one entry point (closed entry) and when they are given a choice to start the task, the beginning is considered to allow multiple entry points. Task-1 below, for instance, is concerned with forming a 4-digit number and naming the digit with the largest place value. This is a 4^{th} grade task and students are expected to know about 3 digit numbers and their place values from the previous year. The first two steps is a preparation for the 3^{rd} and 4^{th} steps and we take the first two steps as the beginning segment of the task (the 3^{rd} and 4^{th} steps are related and they are taken as the end segment). As Task-1 allows students to choose any three-digit number to start the task in the first two steps, the beginning of the task is considered to enable multiple entries.

Task-1

1. Let's write down a three-digit natural number.
2. Let's write down hundreds, tens and ones of this natural number and identify place value of each digit in the number.
3. If the natural number had one more digit, how many digits would the new number have?
4. How would the digit with the largest place value in the new number be named in terms of the value that it holds? Explain why? (A 4th grade task from Özçelik , 2018, p. 17).

The intermediary segment refers to solution method(s) that the task explicitly addresses for reaching the outcome. In this study, when tasks "contain an explicit requirement for solving the problem in multiple ways" (Leikin & Levav-Waynberg (2008, p.234), we define such tasks as having an open-intermediary and hence requiring multiple solution methods. Normally every problem, question or task can be solved with different strategies or solution methods. However, if there is no explicit reference to the solution methods or strategies in the task instructions, we regard such a task as having only one solution method. Task-1 above does not state an explicit instruction regarding a solution method. We, therefore, take Task-1 as having one solution method for the data analysis.

Task-2: How many pencils?

Materials: 5 plastic glasses, pencils, notebook

Let's do it step by step

- Put four pencils into each of three glasses.
- Find the total numbers of pencils in three glasses through addition and skip counting.
- Continue the task by changing the numbers of glasses and the pencils.
- For example, 5 glasses and 3 pencils inside each of them, 4 glasses and 5 pencil inside each of them, 2 glasses and 5 pencils inside each of them. (A 2rd grade task from Atlı, Doğangüzel, Güneş & Şahin, 2018, p. 164).

Task-2, however, explicitly requires finding the numbers of pencils through two ways: addition and skip counting. We thus take Task-2 as having multiple solution methods. Nevertheless, Task-3 below is considered to have one solution method, as there is no clear reference in the task instruction for using different methods to make a pattern.

Task-3: Let's make a pattern

Materials: A4 paper, coloured cardboard, glue, scissors

Let's do it step by step

1. Draw a big square on an A4 paper.
2. Split this big square into 16 small ones using horizontal and vertical lines.
3. Use your cardboard to draw geometric shapes with different colors and then cut them out.
4. Stick these shapes onto small squares in order to make a pattern.
5. Did you notice that your work is a good example of mathematical aesthetics? (A 2rd grade task from Atlı, Doğangüzel, Güneş & Şahin, 2018, p. 157).

NB: Extension such as 5th step in Task-3 or ones put at the end of the task for further similar work is not taken into account in our analysis.

The end refers to final outcomes, which are expected to emerge as an answer after carrying out the task. When more than one correct outcome is requested or ostensibly available, the task is considered to have multiple outcomes. If one correct outcome is an answer then the task is categorized as having on correct outcome. For example, while Task-3 has multiple correct outcomes as the readers can make different patterns, Task-4 below allows only one correct outcome to be stated.

Task-4: This is very lightweight, this is very heavyweight

Materials: Two school bags, notebooks, books, pencil, pencil box etc.

1. Let's be a pair with our desk mate.
2. Let's unpack our school bags and take our all belongings out.
3. Let's put notebooks and books into one bag.
4. Let's put the rest of our belongings into the other bag.
5. Let's carry the bags one by one.
6. Which bag was more difficult to carry? Why? (A 1st grade task from Altay, Gümüş, Yaman, Özer & Akar, 2018, p.27)

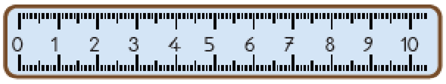
Alongside tasks presented so far, some tasks are presented through only a few instructional steps and it is important to explain how we differentiate task segments for such tasks as well. Task-5 below, for instance, has three steps and the first two steps are mainly introductory. The task is, in fact, consisted of the third step. The student is asked to explain what they can measure with their ruler. This step, in other words the task, has a multiple entries and multiple correct outcomes. However, since no explicit reference is given to solution methods, the task is considered to have a closed intermediary.

Task-5: Let's get to know the ruler

Materials: A4 paper, ruler and scissor

Let's practice it step by step

- Examine the image of the ruler.



- Make your own ruler with A4 paper.
- Explain what you can measure with your ruler. (A 2rd grade task from Atlı, Doğangüzel, Güneş & Şahin, 2018, p. 263).

In addition to the above framework, the following framework in Table 2 is developed by the researchers to decide about whether the textbooks present tasks with CT or DT features. Based on the preliminary analysis of the tasks in the analyzed textbooks, six different categories emerged to make decision about task types (see Table 2). The first three were considered to have CT features while the last three were considered to have DT ones.

Table 2. Framework for task analysis in terms of DT and CT features

Categories	**Task segments**		
CT Type-1	Beginning One entry point	Intermediary One solution method	End One outcome
CT Type-2	Beginning Multiple entry points	Intermediary One solution method	End One outcome
CT Type-3	Beginning One entry point	Intermediary Multiple solution methods	End One outcome
DT Type-1	Beginning Multiple entry points	Intermediary One solution method	End Multiple outcomes
DT Type-2	Beginning One entry point	Intermediary One solution method	End Multiple outcomes
DT Type-3	Beginning Multiple entry points	Intermediary Multiple solution methods	End Multiple outcomes

As Table 2 shows, the end segment is the main indicator in that it determines whether a task has CT or DT feature. Tasks with CT features have only one correct outcome. Both CT Type-1 (see Task-4) and CT Type-2 (see Task-1) have one correct outcome even though they have different beginnings. Although CT Type-3 requires multiple solution methods, it still has the characteristic of CT (see Task-2). Tasks with DT features also have three types. DT Type-1 (see Task-3) is considered to have an open entry (the beginning has multiple entries), a closed intermediary (multiple solution methods are not explicitly required) and an open end (multiple correct outcomes are possible). DT Type-2 has one entry point, one solution method and an open end. DT Type-3 has multiple entries, asks for multiple solution methods openly and enables multiple correct outcomes.

Data analysis

For the data analysis, the first framework in Table 1 is used to decide whether the beginning, the intermediary and the end segments of the task have an open or closed entry, intermediary and outcome. Each task in each textbook is examined in this regard and they are categorized accordingly. Upon the completion of the first stage of data analysis, the second framework is applied to each task again in order to identify which types of tasks that the textbooks present (e.g., CT Type-1 or DT Type-1). This analysis enables us to eventually determine whether the tasks have DT or CT characteristic. Afterwards the frequency tables are constructed and the data is quantified for each grade level. Some tasks from the textbooks are also provided to exemplify different types of tasks and hence substantiate the overall quantified data analysis.

The initial data analysis was conducted on the tasks by both authors together to identify whether the tasks have an open or closed entry, intermediary and end and hence have DT or CT features. This process leads us to form and develop both frameworks. After the frameworks were formed and construed, both authors analyzed the tasks independently. It was found that both authors had a major agreement over the allocation of the tasks into the related categories. Agreement was reached for all tasks where there was a disagreement and hence all tasks were allocated to the related categories based on the mutual agreement.

Findings

In this section, we provide the findings from all four primary mathematics textbooks to show the extent to which they include tasks with features of CT or DT.

Table 3 below shows that 71% of tasks in the first grade textbook (G1) have CT features (CT Type-1 and CT Type-2) while only 29% of the tasks have DT features (DT Type-1). The findings hence reveal the majority of the tasks (64%) in the first grade textbook have one entry point, do not explicitly ask for multiple solution methods and enable only one correct outcome. Only one task classified as having CT features has multiple entry points, yet it does not allow multiple outcomes. It is also revealed that 29% of the tasks have DT features in the first grade textbook and these tasks provide the opportunity for multiple entries and multiple outcomes. No task which explicitly requires multiple solution methods is provided in the first grade textbook.

Table 3. Findings from all four textbook analyses

Categories	Task Segment	Entry point/Solution method/ Outcome	G1-Book (n=14)	G2-Book (n=49)	G3-Book (n=30)	G4-Book (n=20)	All books (n=113)
CT Type-1	B I E	One entry point One solution method One outcome	9 (64%)	22 (45%)	11 (37%)	8 (40%)	50 (44%)
CT Type-2	B I E	Multiple entry points One solution method One outcome	1 (7%)	3 (6%)	5 (17%)	5 (25%)	14 (12%)
CT Type-3	B I E	One entry point Multiple solution methods One outcome	-	1 (2%)	-	-	1 (1%)
DT Type-1	B I E	Multiple entry points One solution method Multiple outcomes	4 (29%)	23 (47%)	13 (43%)	7 (35%)	47 (42%)
DT Type-2	B I E	One entry point One solution method Multiple outcomes	-	-	-	-	-
DT Type-3	B I E	Multiple entry points Multiple solution methods Multiple outcomes	-	-	1 (3%)	-	1 (1%)

For the second grade textbook (G2), Table 3 shows that 53% of the tasks have CT features and 47% of them have DT characteristics. More specifically, it is found that 45% of tasks with CT features have one entry point and one correct outcome (CT Type-1) and even though 6% of the tasks with CT features have multiple entries, they still have one correct outcome (CT Type-2). There is only one task (see Task-2 above) with multiple solution methods in the second grade textbook and this task is categorized as being CT Type-3. With regard to DT, the findings reveal that 47% of the tasks have multiple entries and multiple correct outcomes. Grade-2 textbook is the one that presents the most tasks with DT features.

With regard to third textbook (G3), 54% of the tasks have the features of CT and 46% of the tasks have DT features. Table 3 shows that 37% of the tasks have one entry point and one correct outcome. The percentage of the tasks enabling multiple entry points is 17 and these tasks have CT features (CT Type-2). Table 3 also demonstrates that 43% of the tasks, which have DT features, enable multiple entries and multiple correct outcomes. One task enables multiple entries, asks for multiple solution methods explicitly and allows different outcomes. With regard to fourth grade textbook, Table 3 shows that 65% of the tasks have CT features (CT Type-1 and CT Type-2) and 35% of the tasks have the features of DT (DT Type-1). It is also revealed that none of the tasks in the fourth grade textbook explicitly requires multiple solution methods.

With regard to overall findings, Table 3 shows that 57% of the tasks have CT features and 43% of them have DT features. The findings also reveal that 44% of the tasks allow only one entry point for starting the task and one correct outcome as a result in CT Type-1 category. Even though 12% of the tasks in Type-2 of CT category allow multiple entries, they still have the features of CT. The findings also show that 42% of the tasks enable multiple entries and multiple outcomes and hence have the features of DT (DT Type-1). There are only two explicit references (2%) to multiple solution methods in the task instructions in the four textbooks (CT Type-3 and DT Type-3). It is also found that there is only one task with multiple entries, multiple solution methods and multiple correct outcomes in all four textbooks (DT Type-3).

Table 3 shows that the tasks in all textbooks have five distinctive types. Alongside tasks presented earlier in the methodology section, we qualitatively present here the three most common types of task examples. A common type of task (CT Type-1) is the one that enables one entry point, does not ask for multiple solution methods explicitly and provides the opportunity for only one correct outcome. Task-6 below, for instance, determines a closed entry, does not ask for multiple solution methods explicitly and enables one correct outcome to be stated at the end. As presented in Table 3 above, although there are some variations, 44% of tasks in the four textbooks are presented in such or similar manner.

Task-6

Materials: Place value blocks

- Let’s place ‘9 hundreds, 9 tens and 9 ones’ place value blocks on our desk.
- Which number did you model it with these place value blocks?
- Let’s put 1 more ‘ones block’ on our desk.
- Let’s change this new ‘10 ones blocks’ with ‘1 tens block’.
- Let’s change this new ’10 tens blocks’ with ‘1 hundred block’.
- Explain which number does this new ’10 hundreds blocks’ represent? (A 4th grade task from Özçelik , 2018, p. 10)

The textbooks also include tasks that enable multiple entries and yet allow one correct answer as a result. Task-7 below, for instance, allows the readers to write down any multiplication at the beginning. The task requires the reader to state the commutative rule of multiplication and hence directs him/her to come with one correct outcome. This task is a good example of divergent starting and convergent ending.

Task-7

- Write down a multiplication which has two factors (*a multiplicand and a multiplier*).
- Find the product of the multiplication and note it down.
- Exchange factors in the multiplication (*which you first wrote above*).
- Find the product of this new multiplication and note it down.
- Determine the relationship between the numbers (*products*).
- Write down a general rule for the products of both multiplications based on the relationship you determined. (A 4th grade task from Özçelik , 2018, p. 77)

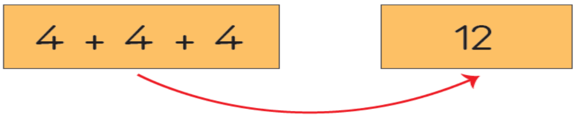
Another common task type is the one which enables both multiple entries and multiple outcomes (DT Type-1). Task-8, for instance, allows the students to write down any repeated addition that they want and hence every student can have a different outcome at the end. The findings reveal that 42% of the tasks in the four textbooks have DT Type-1. Furthermore, there is no clear reference to multiple solution methods and that is why Task-8 is categorized as having one solution method.

Task-8: Let's find it

Materials: cardboard, pencils

Let's do it step by step

- Put a hundreds chart on a large size cardboard and stick it to blackboard.
- For every each of you, write down a repeated addition on a small size cardboard.
- Find the product of your repeated addition on the hundreds chart. Go to the blackboard and stick your answer onto it. (A 2rd grade task from Atlı, Doğangüzel, Güneş & Şahin, 2018, p. 168).



The findings also reveal that there was no DT Type-2 task in the four textbooks. There was only one task for each category of CT Type-3 (one entry point, multiple solution methods and one outcome) and DT Type-3 (Multiple entries, multiple solution methods and multiple correct outcomes). In what follows, we discuss all these findings in term of DT and CT alongside the issue of creativity.

Discussion

The findings reveal that there are more tasks with CT features (57%) than those with DT features (43%) in the four primary mathematics textbooks which we analyzed for this study (see Table 3). More clearly, the findings show that 44% of tasks are classified as CT Type-1 in which tasks allow one entry point, have one correct outcome and do not explicitly require multiple solution methods. Unlike tasks in CT Type-1, 12% of the tasks in CT Type-2 allow multiple entry points, yet they still have CT features. It is also found that 43% of tasks have DT features in all four textbooks and all these tasks have multiple entry points and multiple correct outcomes. Only one task with DT features also explicitly requires multiple solution methods. There are only two tasks (out of 113) which openly ask for multiple solution methods in the four textbooks (CT Type-3 and DT Type-3). Further to all these findings, it is also found that tasks with the most DT features are encountered in Grade-2 textbook (47%) while tasks with the most CT features are provided in Grade-1 textbooks (71%). DT Type-2 task is not found in any textbook at all.

These findings show that the analyzed textbooks provide more opportunity for tasks with one correct outcome and hence less opportunity for tasks with multiple outcomes. Furthermore, it is also found that multiple solution methods are not explicitly expressed in the task instructions and there are indeed only two open references to multiple solution methods in the four textbooks. With this in mind, we think that these findings raise some key concerns about the types, natures and qualities of the tasks that deserve a profound discussion. We discuss these findings with regard to DT, CT and creativity respectively as the focus of the study requires.

Kwon et al. (2006, p.53) refer to different types of tasks for the development of DT by noting such tasks as (1) overcoming fixations, (2) multiple answers, (3) multiple strategies (4) strategy investigations, (5) problem posing, (6) active inquiry tasks, and (7) logical thinking. 'Multiple answers' task type of Kwon et al. (2006) is similar to DT task types in our study and our findings show that 43% of the tasks are considered to have 'multiple answers' and hence are open-ended. Although our tasks are not examined in relation with the task types that Kwon et al. (2006) outline for the development of DT and there are not diverse opportunities for different task types that they note, our findings point to the fact that there is an opportunity for the development of DT in the analysed textbooks as a significant percentage (43%) of open-ended tasks (those with DT features) are presented.

With regard to open-ended questions (ones with multiple correct outcomes), it is often reported that students have difficulties in answering them. The main reported reason is that students are not given sufficient opportunities in their learning environment for such tasks (Stein & Smith, 1998; Cai, 2000; Zhu & Fan, 2006; Yang, Tseng & Wang, 2017). Students' learning environment is affected by the opportunity provided in the textbooks and it has been, in fact, reported that textbooks have influences on classroom instruction (Valverde et al., 2002; Haggarty and Pepin, 2002; Eisenmann and Even, 2011; Olsher and Even, 2014). Some earlier studies have shown that a little opportunity is provided for open-ended questions or problems in the textbooks (Zhu ve Fan, 2006; Han, Rosli, Capraro ve Capraro, 2011; Yang,Tseng, Wang, 2017; Glasnovic Gracin, 2018). Unlike to the findings reported by these studies, our findings show that a good opportunity for open-ended tasks is provided in the textbooks that we analyse for this study. When effectively employed by the teachers, the use of such tasks can contribute to the development of DT and creativity in students.

The issue of multiple solution methods or ways has also been closely associated with DT and creativity (e.g., Kwon et al. 2006; Levav-Waynberg & Leikin, 2012). Leikin & Levav-Waynberg (2008, p.234) define "multiple-solution connecting tasks (CT) as tasks that contain an explicit requirement for solving the problem in multiple ways". In this study, when multiple ways for solving the tasks is explicitly required we view such tasks as having multiple solution methods. Our findings, however, reveal that there were only two explicit references to the employment of multiple solution methods in carrying out tasks in the analysed textbooks. It is generally known that when multiple solution methods are not explicitly requested, students tend to solve tasks with one accustomed solution method. Adding a line like 'solve the task or problem with as many ways or methods as possible' to the task instruction can make a difference and direct students to look for different methods. Such a request can be made even if the task has both a closed beginning and the end and this, in turn, can enrich the quality of task and contribute to the development of students' DT skills. The study of Lee (2017), in fact, reveals that such modifications are feasible and routine tasks can be turned into creative ones.

As detailed in the literature review, both DT and creativity have been defined through (or with) fluency, flexibility and originality components as well (Haylock, 1997; Imai, 2000; Kwon et al., 2006; Leikin and Lev, 2007). Creative and DT tasks are often considered to enable students to show fluency in terms of generating ideas, demonstrate flexibility in terms of variability in their responses and reveal creativity in terms of producing exceptional responses. Our findings show that tasks with DT features have the potential to provide opportunities for students to generate different ideas at the beginning as such tasks allow multiple entries. Flexibility is concerned with qualitatively different sets of ideas and our observation is that requirements for different sets of ideas are not included in the task instructions. For instance, Task-8 enables students to generate different numbers for repeated addition (fluency), yet the task instructions do not have any requirement regarding different sets of responses (e.g., odd numbers, even numbers, numbers having a factor 7 etc.) that can help the students to produce ideas from distinctive categories. Similar remarks can also be made regarding the originality component and that there is no particular reference or explicit encouragement for finding original answers in the task instructions.

Even though CT is often associated with *orthodoxy* and receives less value than DT, Cropley (2006) draws attention to the important role of CT for creative thinking as well. He, in fact, notes that DT is not fruitful if CT is not employed in tasks requiring creativity and novelty. Lee (2007) also appears to value CT and notes that while DT is important for creating variability, CT is required for exploring variability. All these suggest that CT does not have less importance; on the contrary, it is complementary to DT in mathematics learning. Based on the work of Cropley (2006) and Lee (2007), we also think that CT is fruitful when it is used in tasks which require some features of DT as well. For instance, Task-7 asks for finding commutative property of multiplication and CT is required in this process. Even though 57% of tasks are categorised as having CT features and 12% of them are considered to have CT Type-3 features in the four textbooks, tasks similar to Task-7 are really provided. This particular discussion suggests that CT requires more than just reaching an outcome and it does not mean that every task with CT features in the textbooks assure the development of CT in a fruitful way.

Conclusion and Educational Implications

In 1974, Balka (p. 634) states that "with the increasing concern for creativity, it would seem appropriate for elementary teachers to consider divergent, open-ended items in their classes in an effort to identify and encourage mathematical creativity." Some studies have shown that such tasks are still rarely encountered in the actual classroom settings (Kasar, 2013). After nearly a half century, however, it is important to note that the analysed textbooks provide and include such tasks. Given that a textbook is regarded as 'a surrogate curriculum' (Robitaille et al. 1993, p. 50) and has impact on instruction (Valverde et al., 2002), it is a natural outcome that some of the tasks that Balka (1974) describes and those which require DT features will have some chances to be implemented in the real classroom settings by teachers who follow the analysed textbooks in our study. In spite of this chance, our findings also reveal that very little opportunity for the employment of multiple solution methods are provided in the textbooks and opportunities for developing CT skills through such tasks as Task-7 are also rarely realized. Our findings also point to the fact that the analysed tasks, as their instructions reveal, provide little opportunity for the development of flexibility and originality components of creativity.

In terms of educational implications, we contend that our framework provides ideas that can be used for developing tasks which can enable creativity, DT and CT skills. Bennevall's (2016) list of open-ended tasks for creativity and Kwon et al.'s (2006) list for DT are helpful in this regard. Our framework also delineates six different task types of CT and DT. We think that these task types are useful for the development of tasks in textbook writing and they can act as a guide in this regard. Further to this, the use of such open-ended tasks has also been found beneficial for not only teachers' professional development but also improving students' DT skills (Zaslavsky, 1995; Kwon et al., 2006). We believe that task types that our conceptual framework introduces can be used in the professional development of both pre-service and in-service teachers as well. Exposure to such an experience can enable them to have a close understanding of designing tasks with DT and CT features and that will eventually have influences on their students' learning too.

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