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On the recursive sequence $x_{n+1}=\dfrac{x_{n-14}}{1+x_{n-2}x_{n-5}x_{n-8}x_{n-11}}$

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ABSTRACT

In this paper, given solutions fort he following difference equation

$$x_{n+1}=\frac{x_{n-14}}{1+x_{n-2}x_{n-5}x_{n-8}x_{n-11}},\quad n\in\mathbb{N}$$

where the initial conditions are positive real numbers. The initial conditions of the equation are arbitrary positive real numbers. We investigate periodic behavior of this equation. Also some numerical examples and graphs of solutions are given.

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1. Introduction

The study and solution of nonlinear rational difference equation of high order is quite challenging and rewarding. Lately there's a lot of interest in studying the global attractivity, boundedness character, periodicity and the solution form of nonlinear difference equations. For some works in this field, for example: [1-45].

Elabbasy et al. [10-11] investigated the global stability, periodicity character and gave the solution of some special cases of the following difference equations

$$x_{n+1}=ax_n-\frac{bx_n}{cx_n-dx_{n-1}},\quad x_{n+1}=\frac{\alpha x_{n-k}}{\beta+\gamma\prod_{i=0}^{k}x_{n-i}}.$$

In [15] Elsayed dealed with the dynamics and found the solution of the following rational recursive sequences

$$x_{n+1}=\frac{x_{n-5}}{\pm1\pm x_{n-1}x_{n-3}x_{n-5}}.$$

Simsek et. al. [31,32,33,34], studied the following problems with positive initial values

$$x_{n+1}=\frac{x_{n-3}}{1+x_{n-1}},\quad x_{n+1}=\frac{x_{n-5}}{1+x_{n-2}},\quad x_{n+1}=\frac{x_{n-5}}{1+x_{n-1}x_{n-3}},\quad x_{n+1}=\frac{x_{n-3}}{1+x_nx_{n-1}x_{n-2}}$$

respectively.
In this work the following non linear difference equation was studied

$$x_{n+1} = \frac{x_{n-14}}{1 + x_{n-2}x_{n-5}x_{n-8}x_{n-11}} \tag{1}$$

where $x_{-14}, x_{-13}, ..., x_{-1}, x_0 \in (0, \infty)$ is investigated.

2. Main results

Let $\overline{x}$ be the unique positive equilibrium of the equation (1), then clearly

$$\overline{x} = \frac{\overline{x}}{1 + \overline{xxxx}} \Rightarrow \overline{x} + \overline{x}^5 = \overline{x} \Rightarrow \overline{x}^5 = 0 \Rightarrow \overline{x} = 0$$

so, $\overline{x} = 0$ can be obtained.

For any $k \geq 0$ and $m > k$ notation $i = \overline{k, m}$ means $i = k, k+1, ..., m$.

Theorem 1: Consider the difference equation (1). Then the following statements are true.

a) The sequences

$$(x_{15n-14}), (x_{15n-13}), \ldots, (x_{15n-1}), (x_{15n})$$

are being decreasing and

$$a_1, a_2, ..., a_{14}, a_{15} \geq 0$$

are existed and such that

$$\lim_{n\to\infty} x_{15n-14+k} = a_{1+k} \text{ for } k = \overline{0,14}.$$

b) $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, ...)$ is a solution of (1) having period 15.

c) $\prod_{k=0}^{4} \lim_{n\to\infty} x_{15n-3k-2} = 0,\ \prod_{k=0}^{4} \lim_{n\to\infty} x_{15n-3k-1} = 0,\ \prod_{k=0}^{4} \lim_{n\to\infty} x_{15n-3k} = 0,$ or $\prod_{k=0}^{4} a_{3k+2} = 0,\ \prod_{k=0}^{4} a_{3k+1} = 0,\ \prod_{k=1}^{5} a_{3k} = 0.$

d) If there exist $n_0 \in ¥$ such that $x_{n+1} \leq x_{n-11}$ for all $n \geq n_0$, then

$$\lim_{n\to\infty} x_n = 0.$$

e) The following formulas can be generated:

$$x_{15n+1+k} = x_{-14+k}\left(1 - \frac{x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{1 + x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}} \sum_{j=0}^{n} \prod_{i=1}^{5j} \frac{1}{1 + x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right),\ k = \overline{0,2}$$

$$x_{15n+4+k} = x_{-11+k}\left(1 - \frac{x_{-2+k}x_{-5+k}x_{-8+k}x_{-14+k}}{1 + x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}} \sum_{j=0}^{n} \prod_{i=1}^{5j+1} \frac{1}{1 + x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right),\ k = \overline{0,2}$$

$$x_{15n+7+k} = x_{-8+k}\left(1 - \frac{x_{-2+k}x_{-5+k}x_{-11+k}x_{-14+k}}{1 + x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}} \sum_{j=0}^{n} \prod_{i=1}^{5j+2} \frac{1}{1 + x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right),\ k = \overline{0,2}$$

$$x_{15n+10+k} = x_{-5+k}\left(1 - \frac{x_{-2+k}x_{-8+k}x_{-11+k}x_{-14+k}}{1 + x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}} \sum_{j=0}^{n} \prod_{i=1}^{5j+3} \frac{1}{1 + x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right),\ k = \overline{0,2}$$

$$x_{15n+13+k} = x_{-2+k}\left(1-\frac{x_{-5+k}x_{-8+k}x_{-11+k}x_{-14+k}}{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}\sum_{j=0}^{n}\prod_{i=1}^{5j+4}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right),\ k=\overline{0,2}\ .$$

f) If $x_{15n+1+k} \to a_{1+k} \neq 0$, $x_{15n+4+k} \to a_{4+k} \neq 0$, $x_{15n+7+k} \to a_{7+k} \neq 0$, $x_{15n+10+k} \to a_{10+k} \neq 0$, then $x_{15n+13+k} \to a_{13+k} = 0$ as $n \to \infty$. $k = \overline{0,2}$.

Proof

a) Firstly, from the (1)

$$x_{n+1}\left(1+x_{n-2}x_{n-5}x_{n-8}x_{n-11}\right) = x_{n-14}$$

is obtained. If $x_{n-2}x_{n-5}x_{n-8}x_{n-11} \in (0,\infty)$, then $1+x_{n-2}x_{n-5}x_{n-8}x_{n-11} \in (1,\infty)$. Since

$$x_{n+1} < x_{n-14},$$

$n \in ¥$,

$$\lim_{n\to\infty} x_{15n-14+k} = a_{1+k},\ \text{for}\ k=\overline{0,14}$$

existed formulas are obtained.

b) $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, ...)$ is a solution of (1) having period 15 .

c) In wiew the (1),

$$n = 15n \Rightarrow x_{15n+1} = \frac{x_{15n-14}}{1+x_{15n-2}x_{15n-5}x_{15n-8}x_{15n-11}}$$

is obtained. If the limits are put on both sides of the above equality,

$$\lim_{n\to\infty} x_{15n+1} = \lim_{n\to\infty}\frac{x_{15n-14}}{1+x_{15n-2}x_{15n-5}x_{15n-8}x_{15n-11}}\ ,$$

is obtained. Then

$$\prod_{k=0}^{4}\lim_{n\to\infty} x_{15n-3k-2} = 0\ ,\ k=\overline{0,2}\ .$$

d) If there exist $n_0 \in$ □ such that $x_{n+1} \le x_{n-11}$ for all $n \ge n_0$, then, $a_1 \le a_4 \le a_7 \le a_{10} \le a_{13} \le a_1$, $a_2 \le a_5 \le a_8 \le a_{11} \le a_{14} \le a_2$, $a_3 \le a_6 \le a_9 \le a_{12} \le a_{15} \le a_3$.

e)
Subracting x_{n-14} from the left and right-hand sides (1) we obtain:

$$x_{n+1} - x_{n-14} = \frac{1}{1+x_{n-2}x_{n-5}x_{n-8}x_{n-11}}\left(x_{n-2} - x_{n-17}\right). \qquad (2)$$

From (2), for $n \ge 3$ following formula is produced.

$$
\begin{aligned}
x_{3n-8}-x_{3n-23}&=\left(x_1-x_{-14}\right)\prod_{i=1}^{n-3}\frac{1}{1+x_{3i-2}x_{3i-5}x_{3i-8}x_{3i-11}}\\
x_{3n-7}-x_{3n-22}&=\left(x_2-x_{-13}\right)\prod_{i=1}^{n-3}\frac{1}{1+x_{3i-2}x_{3i-5}x_{3i-8}x_{3i-11}}\\
x_{3n-6}-x_{3n-21}&=\left(x_3-x_{-12}\right)\prod_{i=1}^{n-3}\frac{1}{1+x_{3i-2}x_{3i-5}x_{3i-8}x_{3i-11}}
\end{aligned}
\qquad (3)
$$

Replacing n by $5j$ in (3) and summing from $j=0$ to $j=n$, we obtain:

$$x_{15n+1+k}-x_{-14+k}=\left(x_{1+k}-x_{-14+k}\right)\sum_{j=0}^{n}\prod_{i=1}^{5j}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}},\ k=\overline{0,2}$$

Also, $5j+1$ inserted in (3) by replacing n, $j=0$ to $j=n$ is obtained by summing

$$x_{15n+4+k}-x_{-11+k}=\left(x_{4+k}-x_{-11+k}\right)\sum_{j=0}^{n}\prod_{i=1}^{5j+1}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}},\ k=\overline{0,2}$$

Also, $5j+2$ inserted in (3) by replacing n, $j=0$ to $j=n$ is obtained by summing

$$x_{15n+7+k}-x_{-8+k}=\left(x_{7+k}-x_{-8+k}\right)\sum_{j=0}^{n}\prod_{i=1}^{5j+2}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}},\ k=\overline{0,2}$$

Also, $5j+3$ inserted in (3) by replacing n, $j=0$ to $j=n$ is obtained by summing

$$x_{15n+10+k}-x_{-5+k}=\left(x_{10+k}-x_{-5+k}\right)\sum_{j=0}^{n}\prod_{i=1}^{5j+3}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}},\ k=\overline{0,2}$$

Also, $5j+4$ inserted in (3) by replacing n, $j=0$ to $j=n$ is obtained by summing

$$x_{15n+13+k}-x_{-2+k}=\left(x_{13+k}-x_{-2+k}\right)\sum_{j=0}^{n}\prod_{i=1}^{5j+4}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}},\ k=\overline{0,2}$$

Now we obtained of the above formulas:

$$
\begin{aligned}
x_{15n+1+k}&=x_{-14+k}\left(1-\frac{x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}\sum_{j=0}^{n}\prod_{i=1}^{5j}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right),\ k=\overline{0,2}\\
x_{15n+4+k}&=x_{-11+k}\left(1-\frac{x_{-2+k}x_{-5+k}x_{-8+k}x_{-14+k}}{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}\sum_{j=0}^{n}\prod_{i=1}^{5j+1}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right),\ k=\overline{0,2}\\
x_{15n+7+k}&=x_{-8+k}\left(1-\frac{x_{-2+k}x_{-5+k}x_{-11+k}x_{-14+k}}{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}\sum_{j=0}^{n}\prod_{i=1}^{5j+2}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right),\ k=\overline{0,2}\\
x_{15n+10+k}&=x_{-5+k}\left(1-\frac{x_{-2+k}x_{-8+k}x_{-11+k}x_{-14+k}}{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}\sum_{j=0}^{n}\prod_{i=1}^{5j+3}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right),\ k=\overline{0,2}\\
x_{15n+13+k}&=x_{-2+k}\left(1-\frac{x_{-5+k}x_{-8+k}x_{-11+k}x_{-14+k}}{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}\sum_{j=0}^{n}\prod_{i=1}^{5j+4}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right),\ k=\overline{0,2}\,.
\end{aligned}
$$

f) Suppose that $a_{1+k}=a_{4+k}=a_{7+k}=a_{10+k}=a_{13+k}=0$ for $k=\overline{0,2}$. By e) the following formulas are produced below

$$\lim_{n\to\infty} x_{15n+1+k}=\lim_{n\to\infty} x_{-14+k}\left(1-\frac{x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}\sum_{j=0}^{n}\prod_{i=1}^{5j}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right)$$

$$a_{1+k}=x_{-14+k}\left(1-\frac{x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}\sum_{j=0}^{\infty}\prod_{i=1}^{5j}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\right)$$

$$a_{1+k}=0\Rightarrow\frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}=\sum_{j=0}^{\infty}\prod_{i=1}^{5j}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}} \tag{4}$$

Similarly;

$$a_{4+k}=0\Rightarrow\frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-2+k}x_{-5+k}x_{-8+k}x_{-14+k}}=\sum_{j=0}^{\infty}\prod_{i=1}^{5j+1}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}} \tag{5}$$

Similarly;

$$a_{7+k}=0\Rightarrow\frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-2+k}x_{-5+k}x_{-11+k}x_{-14+k}}=\sum_{j=0}^{\infty}\prod_{i=1}^{5j+2}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}} \tag{6}$$

Similarly;

$$a_{10+k}=0\Rightarrow\frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-2+k}x_{-8+k}x_{-11+k}x_{-14+k}}=\sum_{j=0}^{\infty}\prod_{i=1}^{5j+3}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}} \tag{7}$$

Similarly;

$$a_{13+k}=0\Rightarrow\frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-5+k}x_{-8+k}x_{-11+k}x_{-14+k}}=\sum_{j=0}^{\infty}\prod_{i=1}^{5j+4}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}} \tag{8}$$

From (4) and (5),

$$\begin{gathered}\frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}=\sum_{j=0}^{\infty}\prod_{i=1}^{5j}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}>\\ \frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-2+k}x_{-5+k}x_{-8+k}x_{-14+k}}=\sum_{j=0}^{\infty}\prod_{i=1}^{5j+1}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\end{gathered} \tag{9}$$

thus, $x_{-14+k}>x_{-11+k}$. From the (5) and (6),

$$\begin{gathered}\frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-2+k}x_{-5+k}x_{-8+k}x_{-14+k}}=\sum_{j=0}^{\infty}\prod_{i=1}^{5j+1}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}>\\ \frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-2+k}x_{-5+k}x_{-11+k}x_{-14+k}}=\sum_{j=0}^{\infty}\prod_{i=1}^{5j+2}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\end{gathered} \tag{10}$$

thus, $x_{-11+k}>x_{-8+k}$. From the (6) and (7),

$$\begin{gathered}\frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-2+k}x_{-5+k}x_{-11+k}x_{-14+k}}=\sum_{j=0}^{\infty}\prod_{i=1}^{5j+2}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}>\\ \frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-2+k}x_{-8+k}x_{-11+k}x_{-14+k}}=\sum_{j=0}^{\infty}\prod_{i=1}^{5j+3}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}}\end{gathered} \tag{11}$$

thus, $x_{-8+k}>x_{-5+k}$.From the (7) and (8),

$$\frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-2+k}x_{-8+k}x_{-11+k}x_{-14+k}} = \sum_{j=0}^{\infty}\prod_{i=1}^{5j+3}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}} > \frac{1+x_{-2+k}x_{-5+k}x_{-8+k}x_{-11+k}}{x_{-5+k}x_{-8+k}x_{-11+k}x_{-14+k}} = \sum_{j=0}^{\infty}\prod_{i=1}^{5j+4}\frac{1}{1+x_{3i-2+k}x_{3i-5+k}x_{3i-8+k}x_{3i-11+k}} \tag{12}$$

thus, $x_{-5+k} > x_{-2+k}$.

From here we obtain $x_{-14+k} > x_{-11+k} > x_{-8+k} > x_{-5+k} > x_{-2+k}$. We arrive at a contradiction which completes the proof of theorem.

3. Example

Example 3.1: If the initial conditions are selected in accordance with Lemma 1 and Theorem 1;

x[-14] = 0.999999999999999; x[-13] = 0.99999999999999; x[-12] = 0.9999999999999; x[-11] = 9.999999999999; x[-10] = 9.99999999999; x[-9] = 9.9999999999; x[-8] = 8.999999999; x[-7] = 7.99999999; x[-6] = 6.9999999; , x[-5] = 5.999999; x[-4] = 4.9999999; x[-3] = 3.9999; x[-2] = 2.999; x[-1] = 1.99; x[0] = 0.9;

$$x_n = \left\{\begin{array}{l} 0.50025,0.502515,0.526341,0.666778,0.667786,0.678583,0.750063,0.75063,0.756763,0.800039, \\ 0.800394,0.804256,0.832389,0.823893,0.739291,0.375266,0.377674,0.403224,0.561523,0.562608, \\ 0.574373,0.65776,0.658379,0.665167,0.717284,0.717676,0.722025,0.757118,0.748722,0.665291, \\ 0.31257,0.314982,0.340698,0.505124,0.506201,0.517986,0.605816,0.606423,0.613186,0.668847, \\ 0.669228,0.673535,0.711593,0.703219,0.620095,0.272833,0.275222,0.300796,0.468286,0.469337, \\ 0.480937,0.571088,0.571671,0.578248,0.635834,0.636192,0.64032,0.680044,0.671665,0.588569, \\ 0.244554,0.246913,0.27226,0.441617,0.442639,0.454,0.545609,0.546164,0.55251,0.611337, \\ 0.611668,0.615577,0.656398,0.648005,0.564824,0.222993,0.225322,0.25042,0.421059,0.422052, \\ 0.433162,0.525799,0.526325,0.53243,0.592148,0.592452,0.59613,0.637754,0.629342,0.546025, \\ 0.205788,0.208088,0.232934,0.404529,0.405493,0.416354,0.509772,0.510272,0.516136,0.576544,... \end{array}\right\}$$

solutions are obtained and the graphs of the solutions are shown below.

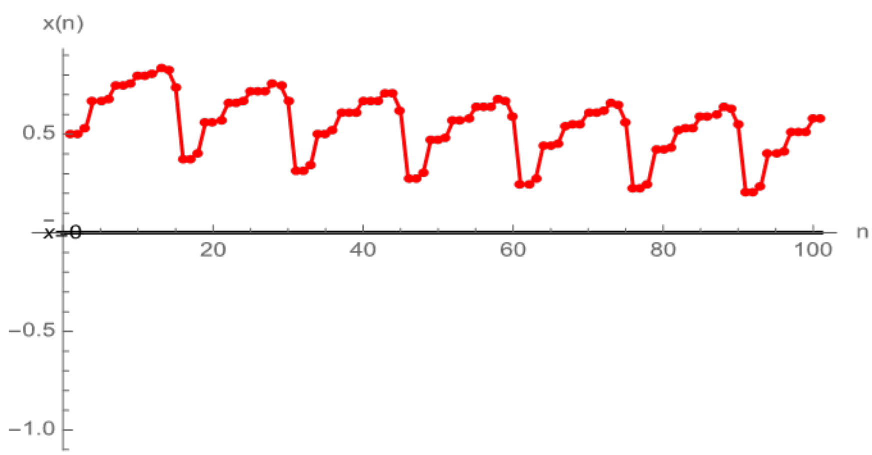

Figure 3.1 x_n *solutions graph*

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